Agency, Firm Growth, and Managerial Turnover

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Motivation

- Firm growth sometimes requires major transformations.
  - e.g., launching a new product, venturing into new markets, adopting a new technology, making organizational change.
- If the incumbent is unable/unwilling to implement such changes, managerial turnover is needed to realize firm's growth potential.
- Question: How does the risk of 'growth-induced' turnover affect the cost of incentivizing managers during their tenure?

This Paper

- We introduce ‘growth-induced’ turnover in a dynamic model of managerial moral hazard.
- Exogenous growth opportunities arrive stochastically over time and taking them requires a change of management.
- Focus of our analysis:
  - Tradeoff between the benefits of always having the right guy at the top and the cost of incentive provision.
  - What is the optimal resolution of this tradeoff?
- Solve for optimal incentive contract in this novel environment.

Main Results

- A manager who is put at risk of being fired for the sake of growth needs to be "compensated" in the form of higher rewards upon good performance early on in his tenure.

- Exposing a manager to the risk of growth-induced turnover effectively makes him more impatient.
  - This makes it more costly to provide incentives and creates a tendency to front-load pay.

- It may be optimal for the firm to commit to forego growth opportunities after periods of good performance.
  - i.e., the firm may grant partial job protection to its managers.
  - More likely if poor growth prospects or severe moral hazard.

- Evidence for the U.S. is supportive of the model's predictions.
  - Firms with better growth prospects experience higher CEO turnover and use more front-loaded compensation.

Related Literature

- Broad theme: managers affect firm performance
  - Strategy: Bertrand-Schoar (2003), Roberts (2004).
  - Firm-CEO matching dynamics: Eisfeldt-Kuhnen (2012).
- Dynamic contracting literature
  - Without growth: DeMarzo-Sannikov (2006), DeMarzo-Fishman (2007), Biais et al. (2007), Sannikov (2008).
  - Investment: Clementi-Hopenhayn (2006), DeMarzo-Fishman (2007), Biais et al. (2010), DeMarzo et al. (2011).
  - Turnover: Spear-Wang (2005), Garrett-Pavan (2012).
- Empirical literature on CEOs
  - Turnover: Jenter-Lewellen (2014), Jenter-Kannaan (2015).
  - Compensation: Murphy (1999), Kaplan-Minton (2012), Clementi-Cooley (2010), Gopalan et al. (2014).

Outline

1. Two-period framework
2. Continuous-time stationary setup
3. Optimal dynamic contract
4. Empirical evidence

A Two-Period Model

- A firm hires a manager at $t = 0$ for at most two periods.
  - At $t = 1$, the manager can be fired and replaced at cost $\kappa > 0$.
- First-period cashflow Y_1 is y with probability p and 0 otherwise.
- A growth opportunity arrives at $t = 1$ with probability q.
  - Arrival of a growth opportunity is publicly observable.
  - Taking a growth opportunity requires a change of management.
  - Upon growth, second-period cashflow Y_2 is $(1 + \gamma)y$ with proba p and 0 otherwise. Else, Y_2 has same distribution as Y_1.
- Firm and manager(s) are risk-neutral, discount rates r and $\varrho > r$.
- Managers are protected by limited liability. Zero reservation value.

First Best

- Assume growth-cum-replacement is efficient:

$$\kappa < \frac{p\gamma y}{1+r}.$$

- Optimal dismissal policy under first best:
  - Replace and grow when a growth opportunity arises.
  - Otherwise, keep the manager ($\kappa > 0 \rightarrow$ termination is inefficient).

Moral Hazard

- Cashflows are privately observed. The manager can under-report.
- True cashflow Y_t vs. reported cashflow $\hat{Y}_t$.
- Manager gets benefit $\lambda \in]0,1]$ per unit of diverted cashflow.

One-Period Incentive Contract

- A manager hired at $t = 1$ to run the firm for one period receives contingent compensation $C(\hat{Y}_2) = \lambda \hat{Y}_2$.
- The firm's problem is to choose $C : \{0, y\} \to \mathbb{R}^+$ to maximize

$$p(y - C(y)) - (1 - p)C(0)$$

subject to the IC constraint

$$C(y) \geq \lambda y + C(0).$$

- This yields the solution $C(0) = 0$ and $C(y) = \lambda y$.
- Remark: Inducing truthful reporting is optimal.

Long-Term Incentive Contract

Contract chosen at $t = 0$ for initial manager (full commitment).

Dismissal policy

- Probability of dismissal conditional on a growth opportunity being available, $G(\hat{Y}_1)$. Also determines the growth policy.
- Proba of dismissal in the absence of a growth opportunity, $F(\hat{Y}_1)$.

Compensation policy

- First-period pay $C_1(\hat{Y}_1) \geq 0$.
- Severance pay upon growth-induced turnover $C_{\mathrm{g}}(\hat{Y}_1) \geq 0$.
- Second-period pay (if incumbent is still running): $C_2(\hat{Y}_1, \hat{Y}_2) \geq 0$.

Incentive Constraints

- Second-period IC constraint

$$C_2(Y_1, y) \geq \lambda y + C_2(Y_1, 0).$$

- Define the conditional 'survival' probability

$$S(Y_1) = 1 - [qG(Y_1) + (1-q)F(Y_1)].$$

The **first-period IC constraint** requires

$$C_1(y) + qG(y)C_{\mathrm{g}}(y) + S(y)\frac{pC_2(y,y) + (1-p)C_2(y,0)}{1+\varrho}$$
$$\geq \lambda y + C_1(0) + qG(0)C_{\mathrm{g}}(0) + S(0)\frac{pC_2(0,y) + (1-p)C_2(0,0)}{1+\varrho}.$$

Optimal Compensation

- Second-period compensation

$$C_2(0,0) = C_2(y,0) = 0,$$
$$C_2(0,y) = C_2(y,y) = \lambda y.$$

- First-period compensation

$$C_1(0) = C_{\mathrm{g}}(0) = 0,$$

and

$$C_1(y) + qG(y)C_{\mathrm{g}}(y) = \lambda y - \Big(S(y) - S(0)\Big)\frac{p\lambda y}{1+\varrho}.$$

Impact of Dismissal Policy on Compensation

- First-period pay upon good reported performance is

$$\lambda y - \Big(S(y) - S(0)\Big)\frac{p\lambda y}{1+\varrho}.$$

- The **wedge** in survival probabilities is

$$S(y) - S(0) = [qG(0) + (1-q)F(0)] - [qG(y) + (1-q)F(y)].$$

- Any change in the dismissal policy that results in a smaller wedge is costly because it needs to be compensated by an increase in first-period pay upon good reported performance.

Optimal Dismissal/Growth Policies

i. Setting $F(y) = 0$ is always optimal.

ii. Setting $F(0) = 1$ is optimal if and only if

$$\kappa \leq \frac{p}{1-p}\frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{F(0)}.$$

iii. Setting $G(0) = 1$ is optimal if and only if

$$\kappa \leq \frac{p(1-\lambda)\gamma y}{1+r} + \frac{p}{1-p}\frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{G(0)}.$$

iv. Setting $G(y) = 1$ is optimal if and only if

$$\kappa \leq \frac{p(1-\lambda)\gamma y}{1+r} - \frac{p\lambda y}{1+\varrho} =: \hat{\kappa}_{G(y)}.$$

High-Growth vs Low-Growth Firms

- Assume for simplicity that

$$\frac{p\gamma y}{1+r} \leq \hat{\kappa}_{G(0)} \quad \Leftrightarrow \quad \frac{p}{1-p} \geq \frac{1+\varrho}{1+r}\gamma.$$

  This restriction implies that the optimal contract sets $G(0) = 1$.

- A firm's growth regime depends on how it optimally sets $G(y)$.
  - **High-growth firms** are such that $G(y) = 1$.
  - **Low-growth firms** are such that $G(y) = 0$.

- For some parameter values, the firm is in the low-growth regime.
  - Foregoes *efficient* growth opportunities after good performance.
  - Provides partial job protection to the manager.

- An increase in λ, r or κ, or a drop in γ, ϱ or p can induce a switch from the high-growth to the low-growth regime.

Empirical Predictions: Managerial Turnover

i. The likelihood of turnover is decreasing in performance

$$qG(y) + (1-q)F(y) \leq qG(0) + (1-q)F(0).$$

ii. The likelihood of turnover, $qG(Y_1) + (1-q)F(Y_1)$, is increasing in the quality of growth opportunities, γ, and in their arrival probability, q.

iii. The probability of turnover is higher when a growth opportunity arises, namely,

$$G(Y_1) \geq F(Y_1).$$

Moreover, $G(Y_1) - F(Y_1)$ is increasing in γ.

Empirical Predictions: Managerial Compensation

i. Compensation is increasing in performance, namely,

$$C_1(y) + qG(y)C_{\mathrm{g}}(y) > C_1(0) + qG(0)C_{\mathrm{g}}(0)$$
$$\text{and} \quad C_2(Y_1, y) > C_2(Y_1, 0).$$

ii. The average compensation profile is increasing over tenure, namely,

$$\bar{C}_1 = p[C_1(y) + qG(y)C_{\mathrm{g}}(y)] \ \leq \ \bar{C}_2 = p\lambda y.$$

iii. The average **level** of first-period compensation $\bar{C}_1$ is increasing in the quality of growth opportunities, γ.

$\Rightarrow$ The **slope** of the profile, $\bar{C}_2 - \bar{C}_1$, is decreasing in γ.

Continuous-Time Stationary Model

- Infinitely-lived firm run by a sequence of managers.
- Firm's operations generate instantaneous cashflow

$$\Phi_t \, dY_t = \Phi_t(\mu \, dt + \sigma \, dZ_t), \qquad \mu, \sigma > 0,$$

  where Z denotes a standard Brownian motion and $\Phi_0 = 1$.
- Growth opportunities arrive stochastically, publicly observable.
  - Arrival time exponentially distributed with parameter q.
  - Taking an opportunity entails a change of management.
  - Upon growth, firm size Φ expands by a factor $(1 + \gamma)$.
- A manager hired to run the firm at size ϕ has reservation value $\bar{w}\phi$ and the cost of replacing him is $\kappa\phi$, where $\bar{w} \geq 0$, $\kappa > 0$.

First Best

- If the firm takes all opportunities

$$
\begin{aligned}
V^* &= -\bar{w} + \mathbb{E}\left[\int_0^{\tau} e^{-rt} dY_t + e^{-r\tau}\left[(1+\gamma)V^* - \kappa\right]\right] \\
&= \frac{\mu - q\kappa}{r - q\gamma} - \frac{r+q}{r - q\gamma}\bar{w}.
\end{aligned}
$$

- We restrict parameter values to be such that

$$
V^* > \max\left\{-\bar{w} + \frac{\mu}{r},\ 0\right\},
$$

i.e., taking all opportunities is efficient under first best.

Moral Hazard

- Cashflows are privately observed by the manager.
- Size-adjusted reported cashflows: $\hat{Y} = Y - A$.
  - Manager derives instantaneous benefit λdA from stealing.
- Let $\hat{\mathcal{F}}_t$ denote the information coming from the history of reported cashflows up to time t.

Incentive Contract

- The random dismissal time is

$$\tau = \min\{\tau_{\mathrm{d}}, \tau_{\mathrm{g}}\}.$$

  Two types of turnover: disciplinary and growth-induced.
  - τ_{d} is an $(\hat{\mathcal{F}}_t)$-stopping time.
  - Given the firm's growth policy G taking values in $\{0,1\}$, the intensity of growth-induced turnover is qG_t at time t.

$$\mathbb{P}\left(\tau_{\mathrm{g}} > t \,\middle|\, \hat{\mathcal{F}}_t\right) = \exp\left(-\int_0^t qG_s\, ds\right).$$

- Compensation:
  - Cumulative compensation process C, increasing, $C_{0-} = 0$.
  - Severance pay upon growth-induced turnover, $S_{\tau_{\mathrm{g}}}$.

- A contract is a mapping $\Gamma : A \mapsto \big(C(A), S(A), G(A), \tau_{\mathrm{d}}(A)\big)$.

Admissible Contracts

- Given a contract Γ and a stealing strategy A, the first manager's expected discounted payoff is

$$M(\Gamma, A) = \mathbb{E}\left[\int_0^\tau e^{-\varrho t}\left(dC_t + \lambda\, dA_t\right) + e^{-\varrho\tau} S_\tau \mathbf{1}_{\{\tau=\tau_{\mathrm{g}}\}}\right].$$

- A contract is *admissible* if it is such that

$$M(\Gamma, 0) = \sup_{A \in \mathcal{A}} M(\Gamma, A) \qquad \text{and} \qquad M(\Gamma, 0) \geq \bar{w}.$$

Dynamics of the Manager's Expected Payoff

- Define the manager's expected future payoff at time $t < \tau$ when he refrains from stealing ($A = 0$)

$$M_t = \mathbb{E}_t \left[\int_{]t,\tau]} e^{-\varrho(s-t)}\, dC_s + e^{-\varrho(\tau - t)}\, S_\tau \mathbf{1}_{\{\tau = \tau_{\mathrm{g}}\}} \right].$$

- Given any contract Γ, there exists a process β such that

$$dM_t = \left[\varrho M_t + q G_t (M_t - S_t) \right] dt - dC_t + \sigma \beta_t \, dZ_t, \quad \text{for } t < \tau.$$

β is the sensitivity of the manager's continuation value to cashflow shocks induced by the contract Γ.

Dynamic Contract (C, S, G, β)

- State process (manager's "promise") evolves as

$$dW_t = \left[\varrho W_t + qG_t(W_t - S_t)\right] dt - dC_t + \beta_t \left(d\hat{Y}_t - \mu \, dt\right).$$

- Dismissal time $\tau = \min\{\tau_{\rm d}, \tau_{\rm g}\}$, where

$$\tau_{\rm d} = \inf\{t \geq 0 : \; W_t = 0\}.$$

- The manager's expected future payoff if he refrains from stealing is equal to W_t. Namely, on the event $\{t < \tau\}$,

$$W_t = \mathbb{E}_t \left[\int_{]t,\tau]} e^{-\varrho(s-t)} \, dC_s + e^{-\varrho(\tau-t)} S_\tau \mathbf{1}_{\{\tau=\tau_{\rm g}\}} \right].$$

- If $\beta \geq \lambda$, it is optimal for the manager not to steal.
  - Admissibility requires $\beta \geq \lambda$ and initial condition $W_{0-} = w_{\rm h} \geq \bar{w}$.

Augmented Drift

- The drift of the manager's promise is

$$\varrho W_t + qG_t(W_t - S_t).$$

- When a manager is exposed to the risk of growth-induced termination, he needs to be compensated for the potential loss.
- Compensation comes in the form of a higher continuation promise conditional on being retained.
- For $S = 0$, the drift rate of the promise is $\varrho + qG$.
  - Putting a manager at risk is akin to making him more impatient: his discount rate goes from ϱ to $\varrho + q$.

Homogeneity

- Stationarity and size homogeneity assumptions imply that the firm offers the "same" dynamic contract (C, S, G, β) and the "same" hiring promise w_{h} to all successive managers.

- Let $V(\phi, w)$ denote the firm's value function, which gives the firm's expected discounted profit given
  - current size ϕ,
  - current (size-adjusted) promise w to the incumbent manager.

- Size-homogeneity property:

$$V(\phi, w) = \phi V(1, w) =: \phi v(w).$$

The Firm's Stochastic Control Problem

The size-adjusted value function $v(w)$ is determined along with the optimal contract by the recursive equation

$$v(w) = \sup_{C,S,G,\beta} \mathbb{E}\Big[\int_0^{\tau} e^{-rt}(\mu dt - dC_t) - e^{-r\tau}\Big(\kappa + S_\tau \mathbf{1}_{\{\tau=\tau_{\mathrm{g}}\}}\Big) + e^{-r\tau}\Big(v(w_{\mathrm{h}})\mathbf{1}_{\{\tau=\tau_{\mathrm{d}}\}} + (1+\gamma)v(w_{\mathrm{h}})\mathbf{1}_{\{\tau=\tau_{\mathrm{g}}\}}\Big)\Big]$$

subject to $\beta \geq \lambda$, $W_{0-} = w$, where the *hiring promise* w_{h} is such that

$$w_{\mathrm{h}} = \max\{\bar{w}, w_\circ\}, \quad \text{with } w_\circ = \arg\max_{w>0} v(w).$$

Note: the constraint $w_{\mathrm{h}} \geq \bar{w}$ may not be binding as it may be optimal to give managers a rent in excess of their reservation value.

Hamilton-Jacobi-Bellman Equation

The firm's value function satisfies the HJB equation

$$\max\left\{\frac{\sigma^2\lambda^2}{2}v''(w) + \varrho w v'(w) - rv(w) + \mu \right.$$
$$\left. + q\left[(1+\gamma)v(w_\mathrm{h}) - \kappa + wv'(w) - v(w)\right]^+,\ -v'(w) - 1\right\} = 0$$

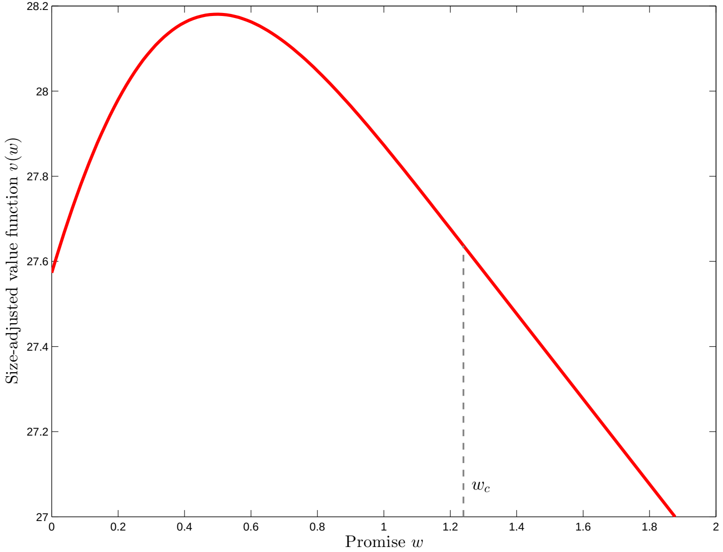
along with the boundary condition

$$v(0) = v(w_\mathrm{h}) - \kappa,$$

where $w_\mathrm{h} = \max\{\bar{w}, w_\circ\} =: \bar{w} \vee \arg\max_{w>0} v(w)$.

There exists a point $w_\mathrm{c} < \infty$ such that v is strictly concave in $[0, w_\mathrm{c}]$ and $v'(w) = -1$ for all $w \geq w_\mathrm{c}$.

Illustration



Optimality Properties

1. The optimal contract has sensitivity to reported cashflows $\beta = \lambda$.
    - Minimizes prospect of inefficient turnover subject to IC constraint.
2. The optimal compensation policy is such that the manager receives transfers only if his current promise w is at least w_{c}.
    - When $v'(w) > -1$, deferring compensation is optimal.
3. The optimal compensation policy involves no severance payment, namely, $\Delta C_{\tau_{\mathrm{d}}} = 0$ and $S = 0$.
    - Giving cash to a departing manager is inefficient: better increase his promise conditional on him being retained.

Optimality Properties (cont'd)

4. Growth Optimality Condition: It is optimal for the firm to stand ready to take a growth opportunity if and only if the manager's current promise w is such that

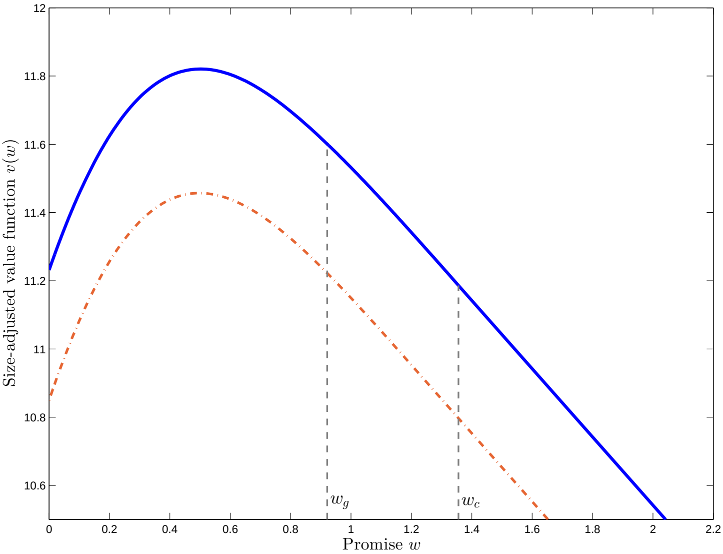
$$(1+\gamma)v(w_{\mathrm{h}}) - \kappa + wv'(w) \geq v(w).$$

   - When this condition fails, it is optimal to insulate the manager from the risk of growth-induced turnover.
   - Condition more likely to fail for high values of w (by concavity).

5. If partial job protection arises as part of the optimal contract, the firm foregoes growth opportunities if the manager's promise w is above the threshold w_{g}, where

$$w_{\mathrm{g}} = \sup\left\{w \geq 0 : \ (1+\gamma)v(w_{\mathrm{h}}) - \kappa + wv'(w) - v(w) \geq 0\right\} < w_{\mathrm{c}}.$$

Illustration (Low-Growth Firm)



Summary

- Cashflow sensitivity: $\beta = \lambda$.
- Severance: Zero.
  - Drift rate of the promise is $\varrho + qG$.
- Growth policy
  - High-growth firm: $G = 1$.
  - Low-growth firm: $G = \mathbf{1}_{[0,w_{\mathrm{g}}]}(W)$.
- Cumulative compensation process C: reflects W at w_{c}.
  - Transfers dC are akin to bonuses indexed on performance.
  - Signing bonus $\Delta C_0 > 0$ if $w_{\mathrm{c}} < \bar{w}$.

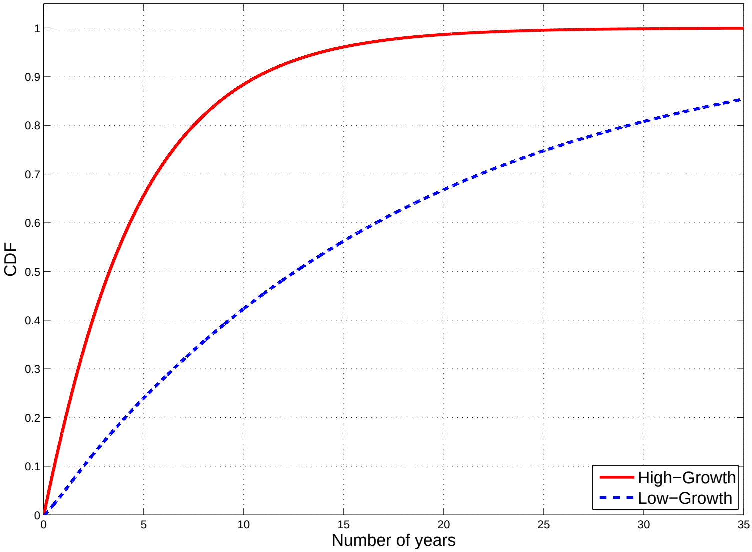
Determining Firm Type and Contractual Threshold(s)

- The firm's growth type as well as the thresholds w_{c}, w_{g} are endogenously determined and depend on parameter values.
- Analytical results are derived from the (extensive) analysis of two free-boundary problems. FBPs
- In particular, we show that w_{c} is decreasing in q.

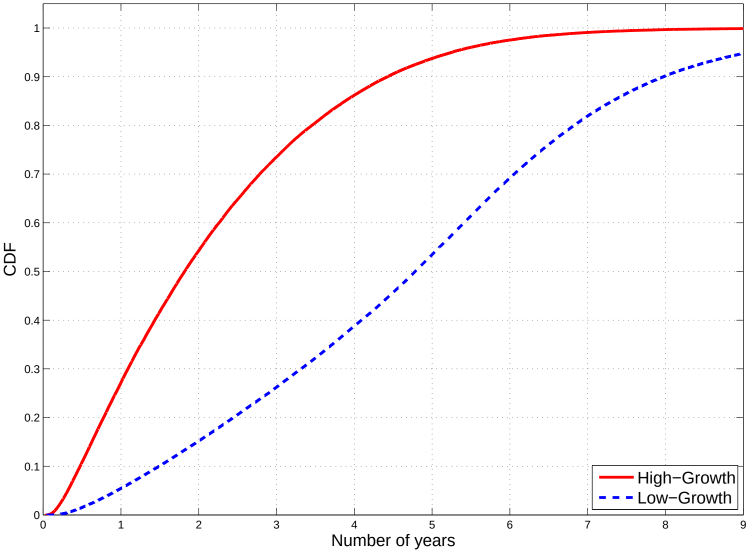
Discussion: Low-Growth Firms

- For some parameter values, the firm optimally foregoes efficient growth opportunities after good performance.
  - This makes it cheaper to incentivize its managers.
- More likely if λ, σ or κ are high, or if γ or q are low.
- Job protection, if at all, is granted after good performance.
  - Intuition: losses due to agency problems are diminished after good performance $\rightarrow$ increases the value of continuing with the incumbent manager net of the foregone benefit of growth.
- Negative relationship between growth and past performance.
  - In contrast with dynamic moral hazard models where the firm grows with the incumbent.
  - In such settings, return on investment is higher after good cashflows, due to a reduction in agency costs.

Numerical Example: Tenure Length



Numerical Example: Compensation Duration



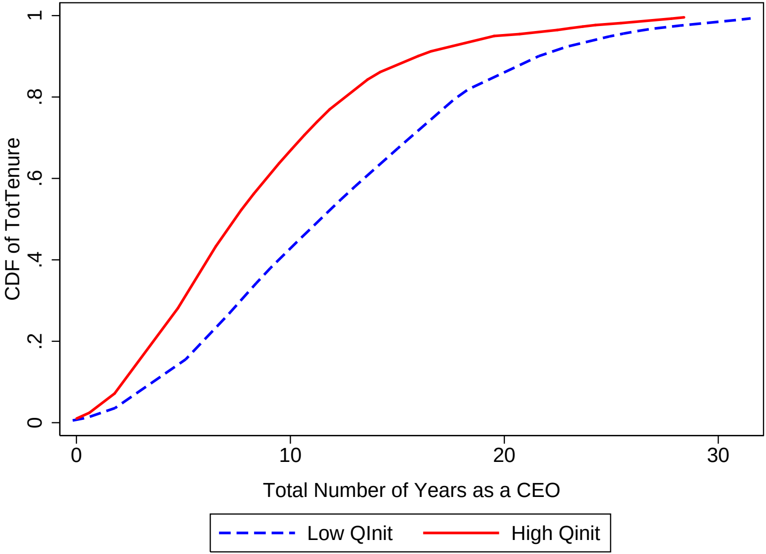
Data/Evidence

- Merge data from ExecuComp, CRSP, Compustat for U.S. public companies over 1992-2014.
- 4,514 CEO episodes, including 2,510 'complete' episodes.
  - 27,992 episode-year observations.
- $QInit_j$ captures *ex ante* growth prospects.
  - Equal to the firm's average Q in the year before CEO was hired.
- $RatioQ_{j,t}$ is a proxy for the availability of a growth opportunity.
  - Equal to the ratio of $Q_{j,t}$ over $QInit_j$.
- $CAR_{j,t}$ is a measure of past performance.
  - Computed based on recent cumulative abnormal returns.

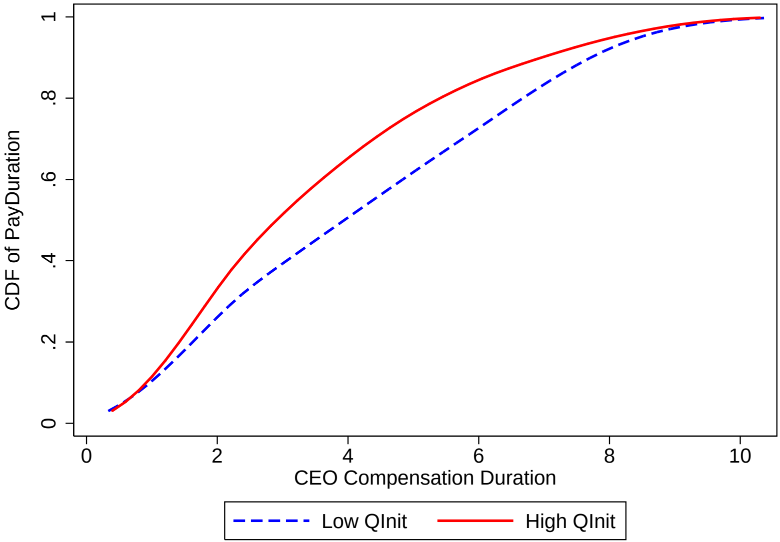
Summary Statistics

Variable	Mean	Sd	p25	p50	p75	N
TotTenure	6.492	5.001	3.000	5.000	9.000	2,510
Turnover	0.084	0.278	0.000	0.000	0.000	27,992
LnTotPay	7.915	1.053	7.172	7.927	8.663	27,958
QInit	1.788	1.212	1.099	1.371	1.981	27,992
RatioQ	1.069	0.420	0.876	1.000	1.164	27,992
CAR	-0.001	0.244	-0.147	-0.005	0.129	27,992
ROA	0.038	0.078	0.013	0.040	0.075	27,992
LnAssets	7.700	1.699	6.452	7.602	8.887	27,992

Tenure Length: High QInit vs Low QInit



Compensation Duration: High QInit vs Low QInit



Determinants of CEO Turnover (probit)

	(A) Coefficients b/se	(B) Marginal Effects b/se	(C) Coefficients of Variation (in percentage points)
QInit	0.047***	0.007***	0.848
	(0.009)	(0.001)	
RatioQ	0.089***	0.014***	0.588
	(0.030)	(0.005)	
CAR	-0.595***	-0.090***	-2.196
	(0.053)	(0.008)	
ROA	-0.806***	-0.122***	-0.952
	(0.131)	(0.020)	
LnAssets	0.040***	0.006***	1.019
	(0.006)	(0.001)	
N	27,992	27,992	27,992
Year fixed effects	Yes	Yes	Yes

Determinants of CEO Compensation

	(A) *LnTotPay*	(B) *LnTotPay*
TenureYear	0.017***	0.008*
	(0.004)	(0.003)
QInit	0.073***	0.054***
	(0.011)	(0.014)
QInit × *TenureYear*	-0.008***	-0.005**
	(0.002)	(0.002)
CAR	0.556***	0.435***
	(0.026)	(0.026)
LnAssets	0.430***	0.199***
	(0.007)	(0.019)
Firm Fixed Effects	No	Yes
Industry Fixed Effects	Yes	No
Year Fixed Effects	Yes	Yes
R-squared	0.532	0.663
N	27,615	27,615

Conclusion

- When does a firm engage in transformative change — i.e., major transformations that entail a change of management?
  - Would Steve Jobs have been called back at Apple Inc. in 1997 if the company had done well under previous CEO?
- This paper suggests a mechanism through which transformative change may only occur following poor performance.
- Incumbent CEOs are insulated from the risk of growth-induced turnover after good performance, thus hindering growth.
  - $=$ Efficient way to resolve the tension between the benefit of always having a manager who is the right man to implement value-enhancing changes and the cost of incentive provision.
- Evidence suggests that the risk of growth-induced turnover affects level and slope of CEOs' compensation profiles.

Firm's Expected Profit (two-period model)

$$
\begin{aligned}
V = p\Bigg(& y - C_1(y) + qG(y)[V_{\mathrm{g}} - C_{\mathrm{g}}(y)] + (1-q)F(y)V_{\mathrm{d}} \\
& + \Big(1 - [qG(y) + (1-q)F(y)]\Big)\frac{p[y - C_2(y,y)] - (1-p)C_2(y,0)}{1+r}\Bigg) \\
+(1-p)\Bigg(& -C_1(0) + qG(0)[V_{\mathrm{g}} - C_{\mathrm{g}}(0)] + (1-q)F(0)V_{\mathrm{d}} \\
& + \Big(1 - [qG(0) + (1-q)F(0)]\Big)\frac{p[y - C_2(0,y)] - (1-p)C_2(0,0)}{1+r}\Bigg),
\end{aligned}
$$

in which expression

$$
V_{\mathrm{d}} = \frac{p(1-\lambda)y}{1+r} - \kappa \quad \text{and} \quad V_{\mathrm{g}} = \frac{p(1-\lambda)(1+\gamma)y}{1+r} - \kappa.
$$

Firm's Profit (continuous-time model)

- Given an admissible contract Γ, the firm's expected discounted profit at $t = 0$ is

$$F(\Gamma) = \mathbb{E}\bigg[\int_0^{\tau} e^{-rt} \, (\mu \, dt - dC_t) \\ + e^{-r\tau} \Big([V_{\mathrm{d}} - \kappa] \mathbf{1}_{\{\tau = \tau_{\mathrm{d}}\}} + [V_{\mathrm{g}} - S_{\tau} - \kappa] \mathbf{1}_{\{\tau = \tau_{\mathrm{g}}\}} \Big) \bigg].$$

- When the same contract Γ is offered to all managers, the firm's continuation values after dismissal of the first manager are

$$V_{\mathrm{d}} = F(\Gamma) \quad \text{and} \quad V_{\mathrm{g}} = (1 + \gamma) F(\Gamma).$$

HJB Equation

Let $u : \mathbb{R}_+ \to \mathbb{R}$ be a concave C^2 function that satisfies the Hamilton-Jacobi-Bellman equation

$$\max\left\{ \frac{\sigma^2\lambda^2}{2} u''(w) + \varrho w u'(w) - r u(w) + \mu \right. \\ \left. + q\left[(1+\gamma)u(w_{\mathrm{h}}) - \kappa + w u'(w) - u(w)\right]^+ ,\ -u'(w) - 1 \right\} = 0$$

with boundary condition

$$u(0) = u(w_{\mathrm{h}}) - \kappa,$$

where $w_{\mathrm{h}} = \bar{w} \vee \arg\max_{w>0} u(w)$, and is such that $u'(w) = -1$ for some $w < \infty$. Then

$$v(w) = u(w) \qquad \text{for all } w \geq 0.$$

Free Boundary Problem: High Growth

back

Find a free-boundary point $w_{\rm c}$ and a function u that satisfies the ODE

$$\frac{\sigma^2\lambda^2}{2}u''(w) + (\varrho + q)wu'(w) - (r+q)u(w) + \mu + qV_{\rm g} = 0$$

in the interval $(0, w_{\rm c})$, is given by

$$u(w) = u(w_{\rm c}) - (w - w_{\rm c}), \quad \text{if } w > w_{\rm c},$$

and satisfies the boundary conditions

$$u(0) = u(w_{\rm h}) - \kappa, \quad u'(w_{\rm c}) = -1 \quad \text{and} \quad u''(w_{\rm c}) = 0, \tag{1}$$

where

$$w_{\rm h} = \bar{w} \vee \arg\max_{w>0} u(w), \qquad V_{\rm g} = (1+\gamma)u(w_{\rm h}) - \kappa. \tag{2}$$

Free Boundary Problem: Low Growth

back

Find two free-boundary points w_c and $w_g < w_c$ and a function u that satisfies the ODE

$$\frac{\sigma^2\lambda^2}{2}u''(w) + (\varrho + q)wu'(w) - (r+q)u(w) + \mu + qV_g = 0$$

in the interval $(0, w_g)$, satisfies the ODE

$$\frac{\sigma^2\lambda^2}{2}u''(w) + \varrho wu'(w) - ru(w) + \mu = 0$$

in the interval (w_g, w_c), is given by

$$u(w) = u(w_c) - (w - w_c), \quad \text{if } w > w_c,$$

satisfies the boundary conditions given by (1), and satisfies the requirement that

$$u(w_g) - w_g u'(w_g) = (1+\gamma)u(w_h) - \kappa,$$

where w_h and V_g are defined as in (2).

Numerical Example (baseline parameter values)

- Parameter values: $r = 7\%$, $\varrho = 16\%$, $\mu = 1$, $\sigma = 1$, $q = 0.2$, $\lambda = 0.4$, $\kappa = 0.3$, $\bar{w} = 1$.
  - High growth: $\gamma = 0.25$, Low growth: $\gamma = 0.10$.
- Average turnover rate (annualized): 21.4% vs 5.5%.
- Median tenure length: 3.3 years vs 12.6 years.
  - data: 5 years.
- Average compensation duration: 2.2 years vs 4.8 years.
  - data: 3.7 years.