Agency, Firm Growth, and Managerial Turnover

Ronald W. Anderson†, Cecilia Bustamante‡ and Stéphane Guibaud§

November 18, 2011

Abstract

This paper analyzes the relation between firm growth and managerial incentive provision in a dynamic moral hazard environment. We characterize the optimal growth, firing and compensation policies specified in the long-term contracts signed between an infinitely-lived firm and the managers who sequentially run the firm. We show that the realized growth of firms depends both on the exogenous arrival of growth opportunities and the severity of moral hazard. Managerial turnover arises upon poor managerial performance to provide incentives, but can also be induced by growth when firms find it more profitable to grow with a new manager. Contracts rely on deferred compensation as a means of incentivizing agents; however, firms with better investment opportunities rely on more front-loaded compensation. We also identify a new component of agency costs which relates exclusively to managerial turnover, and relates to the spillover effect of the length of an existing managerial contract onto the present value of all future contracts signed by the firm.

1 Introduction

Firms extract value not only from operating their existing assets, but also from the expected future profits of their growth opportunities. The latter source of value creation typically involves implementing major changes of strategy, exploring new markets, developing new products, adopting innovative production techniques or changing the organization of labor within the firm. However incumbent managers, for a variety of reasons, may lack the vision or the skills that are necessary to lead the firm through a new growth phase. Firms often find that major management changes are needed to pursue their growth opportunities successfully.

This paper explores how growth-induced management turnover interacts with the provision of managerial incentives in a dynamic moral hazard model. We consider a

---

*Preliminary draft. Comments welcome. We are grateful to Mike Fishman and seminar participants at LSE for their comments. All responsibility for errors and views expressed is our own.

†London School of Economics, r.w.anderson@lse.ac.uk
‡London School of Economics, m.bustamante@lse.ac.uk
§London School of Economics, s.guibaud@lse.ac.uk
firm with assets in place and growth opportunities, which is run by a sequence of managers throughout its life-cycle. As in previous studies on optimal long term contracts with limited liability, firms can use the threat of early termination to discipline their incumbent managers, i.e., firms often fire their managers after periods of poor performance. But in contrast with previous studies, our paper stresses that firms may also fire their managers despite good performance if a change of management is the best or only option to seize valuable growth opportunities.

In our model, a risk-neutral manager is hired by a risk neutral, long-lived firm to run its existing assets. Cash flows are only observable by the manager, who can then underreport and divert them for his own private benefit. The firm can fire its incumbent manager at any point in time, and replace him at a cost. Growth opportunities are stochastic and may arrive in any period. We assume that growth is efficient under first-best and investment is contractible. In our baseline model, the firm needs to replace its current manager in order to pursue a growth opportunity. We later show that growth-driven turnover arises endogenously when it is more profitable for a firm to grow with a new manager. Upon taking up a growth opportunity, the firm pays the costs of investing and replacing the manager, and the scale of its business increases.

We solve for the optimal long-term contract signed between the firm and each of its successive managers at the time they are hired. As in other papers in the literature on dynamic moral hazard, a manager’s expected discounted payoff under the optimal contract, or continuation value, evolves over time and its sensitivity to cashflows is related to the severity of the agency problem. A key feature in our analysis is that the continuation value of the firm upon replacing an incumbent manager is endogenous (equal to the value of the firm under the newly hired manager), and contingent on the current availability of a growth opportunity. This contrasts with most of the existing dynamic contracting models where, upon firing the manager, the firm obtains an exogenously given liquidation value.

Our results in the baseline model are as follows. First, the realized growth of firms depends both on the technological features of the growth process and on the severity of moral hazard. That is, a firm’s corporate governance can be a key determinant of corporate growth. In our model, two firms with similar growth opportunities may end up having very different realized growth profiles just because they differ in the severity of the agency problem they face. A firm plagued with more severe agency problem may forego a growth opportunity and decide instead to retain its incumbent management, when the growth opportunity arises after a period of good performance. Throughout the paper, we therefore distinguish between two (endogenous) types of firms: low growth firms that may or may not undertake growth opportunities depending upon the past performance of the incumbent manager, and high growth firms that undertake all growth opportunities when they arise. In the former type of firms, underinvestment adds to the usual inefficiency that, for the sake of ex ante incentive provision, managers can be fired upon poor past performance in the absence of a growth opportunity.

Second, the probability of replacing an incumbent manager in our model depends not only on past and current performance, as summarized by the manager’s continuation value, but also on the availability of a growth opportunity. In all firms, the conditional probability of managerial replacement is higher in states of the world where
a growth opportunity is available. In low growth firms, the performance threshold being used to determine replacement decision is set at a higher level in these states, making replacement more likely. In high growth firm, the incumbent management is systematically replaced when a growth opportunity arises.

Third, we find that the optimal compensation scheme is readily implementable by a system of deferred compensation credit, bonuses and severance pay. Deferred compensation is used, along with the threat of inefficient replacement, in order to provide incentives in the best possible way. We show that the degree to which firms rely on back-loading of compensation is affected by their growth prospects. Namely, the extent of back-loading decreases with the quality of firms’ growth opportunities.

Lastly, we identify a new component of agency costs that arises in our framework, which is due to a form of contractual externality. When a firm offers a contract to a newly hired manager, it fails to take into account the spillover effect upon the expected amount of time before hiring future managers and thus the present value of compensation received by all future managers. The agency cost induced by this externality is naturally larger for low growth firms, where the arrival of a growth opportunity does not always result in managerial turnover. This externality of the current binding contracts of the firm on its future binding contracts does not arise in earlier papers in the literature, in which firms are liquidated at an exogenous value upon termination of the incumbent, and only, manager of the firm.

In an extension of the baseline model, we allow firms to grow with their incumbent managers, possibly at a different cost than when they grow with a new manager. Whenever it is sufficiently more costly to grow with the incumbent manager, e.g., because realizing a growth opportunity would require paying an army of external consultants to help the firm reinvent itself, all the results of the baseline model survive. However, under symmetric costs, an alternative set of predictions emerges. When the costs of growth are reasonably low, firms always undertake their growth opportunities, sometimes with their incumbent managers. Managers are only fired upon poor performance, but it is still the case that the performance threshold that determines the probability of replacement is higher when a growth opportunity is available.

Our paper relates to several strands in the finance and economics literature on dynamic contracting and the theory of the firm. The works by Quadrini (2004), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007a), Biais et al. (2010), DeMarzo et al. (2011), and Philippon and Sannikov (2011) explore, as we do, the link between dynamic moral hazard and contractible investment opportunities. Our framework differs from these papers in several dimensions. The key difference is that in our framework growth may entail replacing the current manager; whereas, all of the papers mentioned assume that a firm retains the same manager over its entire life-cycle. Furthermore, in contrast with Quadrini (2004), Clementi and Hopenhayn (2006), and DeMarzo and Fishman (2007a), we endogenize the liquidation value of the firm, and focus on managerial turnover rather than firm survival. Finally, in contrast with DeMarzo and Fishman (2007a) and DeMarzo et al. (2011), we consider growth opportunities which arrive stochastically.

---

1He (2008) considers an environment where growth is affected by non-observable effort.
Our paper relates to Spear and Wang (2005), who develop a dynamic contracting model where a firm can fire the incumbent manager, and hire a new one from an external labor market. Spear and Wang (2005) do not account for growth opportunities in their setup. Consequently, the economic determinants of managerial turnover in their model differ from the ones emphasized in our paper.

Our notion that the growth of a firm may require replacing the incumbent manager is found in many early contributions to the managerial theory of the firm. Penrose (1959) discusses why firms may operate successfully under competent managers but may still fail to take full advantage of their opportunities of expansion. Williamson (1966) elaborates on how management constraints affect the realized growth of firms. More recently, Roberts (2004) echoes Penrose by emphasizing the need for different organizational capabilities in the exploration and exploitation of firms’ investment projects. He discusses a number of business cases where this effect if prominent. In their empirical study of U.S. firms, Murphy and Zimmerman (1993) study a variety of measures of firm performance in the years preceding and following CEO turnover. They report a decline in capital expenditures in the year of CEO replacement followed by a sharp increase subsequently. In their theoretical analysis using a repeated moral hazard framework but without optimal contracting, Anderson and Nyborg (2011) show the link between managerial replacement and firm growth are affected the firm’s choice of debt or equity financing.

Finally, our paper relates to the empirical literature that highlights how managerial turnover and incentives relate to realized growth. In the context of venture capital, Kaplan, Sensoy and Stromberg (2009) find that the management team of firms in their early stages of growth firms undergo high turnover before the IPO. This is consistent with the prediction in our model that firms with high realized growth have high managerial turnover. The testable implications of our model on managerial turnover and growth also relate to the recent study by Jenter and Lewellen (2011) on CEO turnover and acquisitions. As in this paper, acquisitions are major investments in which target CEOs are either fired or forced to retire early; Jenter and Lewellen (2011) then show that all else equal takeovers are more likely when incumbent CEOs reach their retirement age and hence it is cheaper to replace them.

The paper proceeds as follows. Section 2 describes the model. Section 3 derives the optimal long-term contract, and provides an informal discussion of its main features. Section 4 provides an illustration in the stationary limit of the model. Section 5 employs numerical simulations to further analyze the implications of our model and quantify the effects. Section 6 extends our results to an alternative environment where the firm can grow with its current manager. All proofs are relegated in a separate Appendix.

2 The model

2.1 Setup

Time is discrete. We consider a project/firm that generates a stream of risky cashflows \{Y_1, Y_2, ..., Y_T\} over \(T\) periods (we later consider the stationary limit as \(T\) goes to
infinity). The project is run by an agent (the manager) who can underreport cashflows and divert them for his own private benefit. We assume that the agent gets $\lambda \leq 1$ for each unit of diverted cash, so that $\lambda$ captures the severity of moral hazard. In any period, an incumbent agent can be fired (at some cost) and replaced by a new agent. For simplicity, we assume that the value of an agent’s best outside option upon being fired is zero.

The firm cashflow in period $t$ is $Y_t = \Phi_t y_t$, where $\Phi_t$ is the size of the firm at the beginning of period $t$ and $y_t$ is iid with support $\mathcal{Y}$, $\min \mathcal{Y} = 0$, and $E(y_t) = \mu$ for all $t$. The firm size $\Phi$ evolves over time according to the stochastic arrival of growth opportunities. In each period, with probability $q$ and independently from current cashflow realization, the firm gets a chance to grow at rate $\gamma$. The state variable $\theta$ captures the availability of a growth opportunity in a given period, where $\theta = G$ with probability $q$ and $\theta = N$ with probability $1 - q$. We assume that taking up a growth opportunity involves paying some investment cost and hiring a new manager (we relax the latter assumption in Section 6). If a growth opportunity materializes in period $t$, given an initial size $\Phi_t$, the firm can grow to a size $(1 + \gamma)\Phi_t$ in period $t + 1$ at a cost of $(\chi + \kappa)\Phi_t$, where $\chi$ and $\kappa$ denote the proportional costs of scaling-up and replacing the manager, respectively. Otherwise, if there is no growth opportunity or if an available growth opportunity is not taken up, the size of the firm remains constant. In case no growth opportunity is available it still is possible to replace the manager at cost $\kappa \Phi_t$.

Figure 1 summarizes the timing within each period.

The assumption that growth necessarily entails replacing the incumbent manager captures the idea that the growth described by the model requires a new skill set and/or a change in corporate culture. The incumbent manager, whose human capital has to some degree become specific to the firm in its current form during his tenure, will have lost the flexibility to adapt his skills to new requirements. While we have in mind drastic changes of the firm, as a modeling convenience we capture this as a discrete change of scale of the firm, as indicated by the rate of cashflows. However, growth in our context may not involve an increase in physical capital. Instead growth could even be thought of as simply the result of finding better managers.

Agents and principal are risk-neutral with (continuously compounded) discount rates $\rho$ and $r$, respectively, with $\rho > r$. When considering the stationary limit of the model as $T \to \infty$, we impose that $q\gamma < e^r - 1$ (i.e., the average growth rate is lower than the principal’s discount rate) to ensure finite valuation.

We assume that the (proportional) replacement cost $\kappa$ is independent of whether a growth opportunity is available or not. Because of this cost, it would be inefficient to fire and replace an incumbent agent under first best, absent a growth opportunity. However, we assume that it is efficient to replace management to take up an available growth opportunity, which in the infinite horizon limit of the model amounts to the following parameter restriction

$$\frac{\gamma \mu}{e^r - 1} > \kappa + \chi.$$  \hspace{1cm} (1)

As a benchmark, it is useful to consider the value of the firm in the case of symmetric information about cashflows. Under condition (1), the optimal firm policy involves
taking up all available growth opportunities and firing managers only upon growth. Let $V_t(\Phi)$ denote the first-best value of the firm (ex-cashflow) in period $t$, before the growth opportunity realization. The sequence of first-best value functions is given recursively by

$$V_t(\Phi) = q \left[ - (\kappa + \chi) \Phi + e^{-r} \{(1 + \gamma) \Phi \mu + V_{t+1}([1 + \gamma] \Phi)\} \right] + (1 - q) e^{-r} \{\Phi \mu + V_{t+1}(\Phi)\},$$

where the recursion starts at $V_T(\Phi) = 0$, for all $\Phi$. The homogenous nature of the model allows us, in the infinite horizon limit, to write $V(\Phi) = v^* \Phi$, where

$$v^* = \frac{-q(\kappa + \chi) + e^{-r}(1 + q\gamma)\mu}{1 - e^{-r}(1 + q\gamma)}.$$  \hspace{1cm} (2)

### 2.2 Contracting

We now consider optimal second best contracting under asymmetric information. A contract is established between the investor and the manager at the outset of his tenure. When the latter is replaced, the contract is terminated and a new contract is established with a new manager. In the case of asymmetric information about cashflows, a contract specifies as a function of history (i.e., the sequence of payments received by the principal, as well as the history of growth opportunity realizations), circumstances under which an agent is fired (i.e., history-contingent firing probabilities), investment and growth, and non-negative cash compensation from principal to agents. For simplicity, we assume a contractual environment with full-commitment (no renegotiation) and we assume that the agent cannot save, and therefore consumes in each period his current compensation plus the diverted cashflow.\(^2\) The amount of diversion is the only decision over which the agent has control. In searching for an optimal contract, we restrict our attention to contracts that induce truthful reporting (indeed, since $\lambda \leq 1$ diversion is at least weakly inefficient). An optimal contract is one that satisfies the associated incentive-compatibility constraint and gives the maximum payoff to the principal subject to providing a certain payoff to the agent. For now, we assume that the contract is designed so as to give an expected discounted value of $\Phi w_0$ to a manager hired to run the firm at size $\Phi$.

### 3 The optimal contract

In this section, we characterize managerial compensation, managerial turnover, and realized firm growth under the optimal contract. Our derivation of the optimal contract follows the recursive approach pioneered by Green (1987) and Spear and Srivastava (1987). Let $w$ denote the size-adjusted continuation value of the current manager (i.e., for current size $\Phi$, the agent’s continuation value is $\Phi w$). In our context, history can

---

\(^2\) The assumption that the agent cannot engage in private saving is not crucial. What is required is that the rate of return available to the agent is less than or equal to $r$, i.e., private saving is weakly inefficient. Under this condition, even if allowed to do so, the agent would have no incentive to use private savings under the derived optimal contract. See DeMarzo and Fishman (2007b), Section 2.1 and Corollary 1.
be summarized by two state variables: the current size of the firm $\Phi$, and the agent’s size-adjusted continuation value $w$.

Given this simplified state space, the optimal contracting problem can be solved via dynamic programming. To this end, it is useful to introduce a number of value functions to keep track of the principal’s intertemporal payoff at different points within a period (as shown in Figure 1). We let $B_t^p(\Phi, w)$ denote the principal’s value under the optimal contract at the beginning of period $t$, before cashflow realization, given current size $\Phi$ and (size-adjusted) continuation value $w$ to be delivered to the agent; $B_t^g(\Phi, w)$ denotes the principal’s value in period $t$, after cashflow realization, but before the growth opportunity is realized; $B_{t,\theta}^\ell(\Phi, w)$ denotes the principal’s value conditional on a growth opportunity being available ($\theta = G$) or not ($\theta = N$) before replacement and growth decisions; $B_t^c(\Phi, w)$ denotes the principal’s value before compensation to the agent, conditional on the firm entering period $t + 1$ with size $\Phi$; and finally $B_{t+1}^e(\Phi, w)$ to be delivered to the manager as of the beginning of period $t+1$.

Our assumptions that firm cashflows and costs are all proportional to size guarantee that these value functions are all homogenous in current firm size.

**Lemma 1.** All value functions satisfy the following homogeneity property

$$B_t(\Phi, w) = \Phi B_t(1, w) \equiv \Phi b_t(w).$$

The analysis can therefore be simplified by applying dynamic programming techniques directly onto size-adjusted value functions. Given this considerable reduction in the state space, an optimal contract can be entirely characterized via a set of rules specifying the evolution of the state variable $w$, and a set of policy functions specifying the agent’s compensation and the optimal replacement/growth policy as a function of the current value of $w$ and of whether a growth opportunity is currently available or not ($\theta = G$ or $\theta = N$).

---

3See DeMarzo and Fishman (2007a), Biais et al. (2010).
3.1 Properties of the optimal contract

We solve for the size-adjusted value functions, the law of motion for the agent’s continuation value $w$, along with the optimal compensation, growth and replacement policies by backward induction, along the lines of DeMarzo and Fishman (2007b). The recursion starts in the final period with $b^\mu_{T,\theta}(w) = -w$ for $\theta = G, N$. Now suppose $b^\mu_{t+1,G}(w)$ and $b^\mu_{t+1,N}(w)$ are known. Then

$$b^g_{t+1}(w) = q b^\mu_{t+1,G}(w) + (1 - q) b^\mu_{t+1,N}(w),$$

which is simply an expectation over the realization of $\theta$ in period $t+1$. The beginning-of-period value function is obtained as

$$b^y_t(w) = \max \{ w, \mu + \mathbb{E}[b^\mu_{t+1}[w^\mu(y)]] \},$$

where the expectation is taken over the distribution of $y$, subject to promise-keeping condition $\mathbb{E}[w^\mu(y)] = w$, limited liability $w^\mu(y) \geq 0$, and incentive-compatibility constraint

$$w^\mu(y) \geq w^\mu(\tilde{y}) + \lambda(y - \tilde{y}), \quad \forall y \in \mathcal{Y}, \forall \tilde{y} \in [0, y].$$

The following lemma further characterizes the beginning-of-period value function, as well as the cashflow sensitivity of the agent’s updated continuation value.

**Lemma 2.** In any period $t$, $b^g_t$ is only defined for $w \geq \lambda \mu$. Moreover,

$$w^\mu(y, w) = w + \lambda(y - \mu), \quad w \geq \lambda \mu.$$  

The intuition behind Eq. (7) is that in order to induce the agent not to divert, his continuation value must have a sensitivity $\lambda$ to his payment to the principal. Hence the incentive-compatibility condition gives the slope of $w^\mu$ with respect to $y$, while the promise-keeping condition gives the level of the schedule. The fact that $b^g_t$ is only defined for $w \geq \lambda \mu$ comes from the limited liability constraint: indeed, $w$ needs to be high enough to guarantee that even for the lowest possible cashflow realization ($\min \mathcal{Y} = 0$), the continuation value $w^\mu(y)$ consistent with incentive-compatibility and promise-keeping constraints remains non-negative. Given $b^\mu_{t+1}$, the end-of-period value function in period $t$ is simply given by

$$b^e_t(w) = e^{-r} b^\mu_{t+1}(e^r w), \quad w \geq e^{-r} \lambda \mu,$$

where the domain of $b^e_t$ follows directly from that of $b^\mu_{t+1}$.

**Lemma 3.** For $t < T - 1$, $b^e_t$ is concave in $w$.

In a Modigliani-Miller world, increasing the agent’s value would merely amount to redistributing total firm value, and the principal’s value would simply be linearly decreasing in the agent’s value with a slope of $-1$. In the presence of moral hazard, costly replacement and firm growth, a change in $w$ also affects the principal’s value.

8
via its impact on the likelihood of inefficient firing. Under the contract the investor is committed to firing the agent following a string of bad cash flow realizations even though this may be costly (i.e., ex post inefficient) for the investor. When the manager’s current promise is low, this ex post bad outcome for the investor is relatively likely. Increasing the agent’s promise by some given amount hurts the investor by sacrificing some portion of future cash flows, but this is mitigated by the fact that it reduces the prospect of a costly turnover. When the manager’s current promise is relatively high, the prospect of turnover is slight and the benefit derived from reducing it is also slight.\footnote{Note that an increase in $w$ can also affect the growth prospects of the firm, as will become clear from Proposition 1.}

### 3.1.1 Cash compensation

The value function $b^c_t$ captures the principal’s value contingent on the incumbent manager being retained. The problem at this stage is to find the best possible way to compensate the agent over time, by employing the optimal mix of present versus future compensation. Formally, for $w \geq e^{-\rho \lambda \mu}$

$$b^c_t(w) = \max_{c,w} e^{-c} - c + b^c(w)$$

subject to the promise keeping condition $c + w^e = w$, the limited liability condition $c \geq 0$ and $w^e \geq e^{-\rho \lambda \mu}$.

**Lemma 4.** Let $\overline{w}_t$ such that $b^c_t'(\overline{w}_t) = -1$. The optimal compensation policy is

$$c_t(w) = \begin{cases} 0, & w \leq \overline{w}_t, \\ w - \overline{w}_t, & w > \overline{w}_t. \end{cases}$$

Therefore, $b^c_t(w) = b^c_t(w)$ for $w \leq \overline{w}_t$ and $b^c_t(w) = b^c_t(\overline{w}_t) - (w - \overline{w}_t)$ for $w > \overline{w}_t$.

Lemma 4 states that it is optimal to defer an agent’s compensation until his continuation value has reached the threshold $\overline{w}_t$. The optimal compensation threshold is determined by a basic tradeoff: delayed compensation is preferable because it keeps the agent’s promise from falling closer to the inefficient termination threshold, while early compensation is preferable because the agent is more impatient than the principal. Formally, the compensation threshold $\overline{w}_t$ is determined by comparing the marginal cost for the principal of present versus deferred compensation. By compensating the agent with $\Delta c$ in period $t$, the principal’s value is $-\Delta c + b^c_t(w - \Delta c)$. For a small $\Delta c$, this can be approximated by $b^c_t(w) - \Delta c(1 - b^c_t'(w))$, which shows that non-zero compensation is optimal if and only if $b^c_t'(w) < -1$.

### 3.1.2 Replacement and growth

We can now proceed with the construction of $b^{\ell}_t, \theta = G, N$, which will end the recursion. At this stage, given the realization of $\theta$ and the manager’s continuation value $w$, the contract specifies firing probability $p_{t,\theta}(w)$, severance pay $s_{t,\theta}(w)$, and the
updated continuation value $w^\tau_{t,\theta}(w)$ that the incumbent manager would get upon being retained. Note that if $\theta = N$, the principal’s continuation value (adjusted by current size) upon replacing the incumbent manager is:

$$\ell_{t,N} = e^{-r}b^y_{t+1}(w_0) - \kappa. \quad (11)$$

If instead a growth opportunity is available in period $t$, the principal’s continuation value upon firing the incumbent manager would be

$$\ell_{t,G} = \max \{ e^{-r}(1 + \gamma)b^y_{t+1}(w_0) - (\kappa + \chi); e^{-r}b^y_{t+1}(w_0) - \kappa \}, \quad (12)$$

where the second term corresponds to the case where the incumbent manager is fired but the growth opportunity is not undertaken. For economically interesting cases, it is reasonable to assume that replacing the manager is more valuable when there is a growth opportunity than when there is not. It follows that $\ell_{t,G} > \ell_{t,N}$. Then we can interpret $p_{t,G}(w)$ as the conditional probability of efficient replacement (i.e., conditional on $\theta = G$). Similarly, $p_{t,N}(w)$ captures the conditional probability of inefficient firing.

The optimal severance and replacement/growth policies are obtained by considering the following constrained maximization problem, separately for $\theta = G$ and $\theta = N$:

$$b^\ell_{t,\theta}(w) = \max_{p,s,w} p(\ell_{t,\theta} - s) + (1 - p)b^c_{t}(w^c) \quad (13)$$

subject to the promise keeping condition $ps + (1 - p)w^c = w$, the limited liability condition $s \geq 0$, $w^c \geq e^{-\rho}\lambda\mu$, and $p \in [0,1]$. To analyze this problem, it is useful to introduce for $\theta = G, N$

$$\delta_{t,\theta} = \sup \left\{ \frac{b^c_{t}(w) - \ell_{t,\theta}}{w} : w \geq e^{-\rho}\lambda\mu \right\}, \quad (14)$$

and

$$w_{t,\theta} = \begin{cases} \inf \{ w \geq e^{-\rho}\lambda\mu : b^c_{t}'(w) \leq \delta_{t,\theta} \}, & \text{if } \delta_{t,\theta} > -1, \\ \infty, & \text{otherwise}. \end{cases} \quad (15)$$

Graphically, $\delta_{t,\theta}$ and $w_{t,\theta}$ are determined by finding the line of maximum slope relating the termination point $(0, \ell_{t,\theta})$ to the curve representing $b^c_{t}(w)$. The slope of this line gives $\delta_{t,\theta}$, while $w_{t,\theta}$ is defined as the value of $w$ at the intersection/tangency point if $\delta_{t,\theta} > -1$ and $w_{t,\theta} = \infty$ otherwise. Note that (14) along with $\ell_{t,G} > \ell_{t,N}$ implies $\delta_{t,G} < \delta_{t,N}$.

**Proposition 1.** For any realization of $\theta \in \{G, N\}$, the optimal replacement policy can be described as follows:

(i) if $\delta_{t,\theta} > -1$, the probability of the incumbent agent being replaced is

$$p_{t,\theta}(w) = \begin{cases} 1 - w/w_{t,\theta}, & 0 \leq w < w_{t,\theta}, \\ 0, & w \geq w_{t,\theta}. \end{cases} \quad (16)$$
The agent receives no severance pay upon being fired, \( s_{t,\theta}(w) = 0 \), \( \forall w < \underline{w}_{t,\theta} \), and his continuation value upon being retained is

\[
w_{t,\theta}^c(w) = \begin{cases} 
  \underline{w}_{t,\theta}, & 0 < w < \underline{w}_{t,\theta}, \\
  w, & w \geq \underline{w}_{t,\theta},
\end{cases}
\]

Hence

\[
b_{t,\theta}^L(w) = \begin{cases} 
  \ell_{t,\theta} + \delta_{t,\theta} w, & 0 \leq w \leq \underline{w}_{t,\theta}, \\
  \ell_{t,\theta}^c, & w \geq \underline{w}_{t,\theta}.
\end{cases}
\]

(ii) if \( \delta_{t,\theta} \leq -1 \), the incumbent manager is replaced with probability one independently of the agent’s promised value, \( p_{t,\theta}(w) = 1 \) for all \( w \geq 0 \). Upon being replaced, the manager receives \( s_{t,\theta}(w) = w \), and

\[
b_{t,\theta}^L(w) = \ell_{t,\theta} - w, \quad \forall w \geq 0.
\]

### 3.2 Implementation of the optimal contract

Having formally derived the optimal contract in our setting, it is useful to summarize it informally and to discuss how it can be implemented in practice. The optimal contract between the firm and its manager sets out the conditions under which the manager will be compensated during his tenure at the firm and also those which will lead to his leaving the firm. These terms and conditions are chosen to maximize the value of payoffs to the firm’s owners subject to incentivizing the manager to truthfully report realized cashflows. Payments and retention/replacement decisions are made over time as a function of the value of promised deferred payments, \( w_t \), which evolves under the influence of the firm’s operating performance and changing growth opportunities. The contractual features in force at time \( t \) are summarized in three threshold values \( \underline{w}_t \), \( \underline{w}_{t,G} \), and \( \underline{w}_{t,N} \), and the manager receives qualitatively different treatment depending upon whether \( w_t \) is above or below these thresholds.

\( \underline{w}_t \) is the **bonus threshold**. In any period \( t \) following the realization and report of the firm’s cashflows, \( y_t \), the manager’s promise is adjusted linearly according to Equation (7). The sensitivity of this promise to cashflow changes reflects the severity of agency problems faced by the firm. The more severe the agency problem, the greater should be this sensitivity. If after adjustment the resulting promise lies above the bonus threshold, \( (w_t > \underline{w}_t) \), a bonus is awarded in that period equal to the excess, \( (w_t - \underline{w}_t) \), and the agent’s continuing promise is reduced to the threshold amount, \( \underline{w}_t \).

The other two threshold values may be thought of as **replacement thresholds**, and, as the replacement decision is made after the availability of a growth opportunity (or lack thereof) has been observed, these thresholds are conditioned on such opportunity being available or not. \( \underline{w}_{t,N} \) is the dismissal threshold when there is no growth opportunity available. If the manager’s current promise lies above this threshold, \( w_t > \underline{w}_{t,N} \), then he knows that he will be retained. In particular, if he has just been awarded a bonus, he is retained for sure when there is no growth opportunity (indeed, \( \underline{w}_{t,N} < \underline{w}_t \)). If rather the operating performance has been so poor that the manager’s promise is below the threshold, \( w_t < \underline{w}_{t,N} \), then he is at risk of being fired. In effect, he is given a lottery whereby with some probability he will be dismissed and will receive no further
payments from the firm. If he survives this, he stays with the firm and is awarded a continuing promise that is increased to the dismissal threshold amount, \( w_t = w_{t,N} \). The probability of dismissal is chosen so that this lottery is fair, i.e., its expected value equals the agent’s promise following the report of cashflow.

For one type of firm the logic of the dismissal decision when the growth opportunity is available is similar to the above; however, it is made by comparing the promise to the growth dismissal threshold \( w_{t,G} \) which is higher than that without growth (i.e., \( w_{t,G} > w_{t,N} \)). That is, risk of dismissal weakly increases if a growth opportunity arises. If the manager’s promise is above the threshold \( w_{t,G} \) he knows he is safe. If he is below this threshold he is given a fair lottery in which, if he is dismissed, he leaves the firm with no further compensation, and, if he survives, he is given a continuing promise which is increased to \( w_{t,G} \). However, this replacement rule does not apply in all firms. In a second type of firm, upon the arrival of the growth opportunity, the manager knows he will be dismissed for sure and upon leaving the firm will receive a severance pay equal to his current promise, \( s = w_t \).

In light of these comments on the replacement decision in the face of growth, it is useful to distinguish two categories of firms depending upon the attractiveness of their growth opportunities. A high growth firm is one that will undertake growth any time it has an opportunity, independently of the firm’s past operating performance. Other firms, which for simplicity we call low growth firms even though in practice they may grow quite fast, do not always take up an available growth opportunity. Instead, they will possibly retain the current manager and keep operating assets at the current scale, if past performance has been good enough and the manager has accumulated a high promised compensation. Proposition 1 shows that the distinction between high and low growth firms depends crucially on the quantities \( \delta_{t,G} \) and \( \delta_{t,N} \) defined by Eq. (14). Low growth firms are characterized by \( \delta_{t,N} > \delta_{t,G} > -1 \). High growth firms satisfy \( \delta_{t,N} > -1 \) and \( \delta_{t,G} \leq -1 \). Whenever \( \delta_{t,\theta} \geq -1 \), this quantity indicates the slope of the Pareto frontier describing the firm owners’ value as a function of the value promised to the current manager, as implied by the dismissal lotteries described above (i.e., for low enough promised values). The slope of the frontier itself can never fall below \(-1\), since the firm always has the option to pay cash immediately, in which case an increase in the manager’s promised value simply translates one-for-one into a reduced value for the firm owners.\(^5\)

High growth firms and low growth firms behave in dramatically different ways. While high growth firms always seize an opportunity to invest and grow, fully realizing their growth potential, low growth firms do not systematically take up available growth opportunities, thus wasting part of their growth potential. Hence, for the latter firms, an important source of agency cost is under-investment. For low growth firms, the probability of taking a growth opportunity, \( p_{t,G}(w) \), is decreasing in \( w \). That is, the better has been the operating performance recently, the less likely that the firm will take up a growth opportunity. These firms do not take up growth opportunities for high

\(^5\)Note that with a finite horizon \( T \), the same firm can possibly switch type over its life-cycle. In Section 4.3, we provide a mapping of high growth vs. low growth firms in the parameter space in the stationary limit of the model.
because the overall cost of taking up the growth opportunity is too high. This result contrasts with DeMarzo and Fishman (2007a) who find that investment is increasing in the agent’s promise because the return on investment is high then.

In the absence of a growth opportunity, both types of firms have similar policies. Eq. (28) describes \( p_{t,N}(w) \). The probability of replacement is positive and decreasing if and only if the promise is below the threshold, \( w < \bar{w}_{t,N} \). Taken together with the cashflow sensitivity of \( w^q \), this means that the probability of an agent being fired (conditional on \( \theta = N \)) increases with bad (past and current) performance. This replacement policy, combined with zero severance pay \( s_N(w) = 0 \), is used as a threat to provide the right incentives to the agent while respecting limited liability. Note that terminating the agent when \( \theta = N \) is inefficient and contributes to the agency costs induced by asymmetric information.

The optimal contract we have just described can be implemented fairly directly using standard employment contracts, and there is some evidence that features of our optimal contracts are used in practice. The bonus calculation in this contract is very much like the typical contract that was found by Murphy (2001) in his study of the bonus contracts of large U.S. firms in 1997. The key parameters he identifies are the performance target, the pay-performance-sensitivity (pps), and the bonus threshold. In our contracts, these are \( \mu, \lambda \), and \( \bar{w}_t \) respectively.

Our contract specifies an indefinite term with both the manager and the firm having the right to terminate at will.\(^6\) Actual employment contracts are often written in this way.\(^7\) In practice, it is not unheard of that following a period of poor performance when the manager was thought to be under threat of dismissal, the firm instead retains the manager and gives him an improved compensation package as a vote of confidence. This is analogous to the award of deferred compensation of \( w_\theta - w \) when the manager survives a dismissal threat.

The feature of our contracts that is perhaps most difficult to implement regards the payment of severance pay. Here empirical evidence is scanty and that which exists does not exactly support the idea that our optimal contracts are implemented in practice. In our framework, severance pay is paid only when the manager is dismissed as part of the pursuit of a growth opportunity. This feature relies crucially on the assumption that realized growth is verifiable. In some circumstances, this may be realistic – if the growth opportunity involves a significant investment (\( \chi > 0 \)) and would lead to a major increase in the rate of cash flow (as would be the case for high growth firms by our definition), it will be clear to all that new managers have been brought in to implement a change in direction for the firm. In other cases, e.g., when there is no capital expenditure and when new strategy will emerge only in the future, it is less clear that making severance pay contingent upon pursuit of a growth opportunity would be enforceable.

\(^6\)Our setup could easily be extended to incorporate a positive reservation value for the agent. With zero reservation value and limited liability, inducing the agent to remain in the contract is never an issue.

\(^7\)Of course, some employment laws may constrain this, e.g., by imposing a mandatory notice period which may vary with the tenure.
4 Optimal stationary contract

We now consider our model in the stationary limit where $T \to \infty$. This is a useful simplification because the key features of the optimal contract, adjusting for changes of scale as the firm grows, will be constant over the life of the firm. This allows us to better understand the relationship between these contract features and the deep underlying characteristics of the firm, in particular, the severity of managerial moral hazard and the frequency of growth opportunities.

To do this, we solve numerically for the value functions and associated replacement, growth, severance and compensation policies by iterating backward until convergence for a large value of $T$. When considering the stationary limit of the optimal contract, we drop all time subscripts. We assume size-adjusted cashflows are independently, identically and uniformly distributed on \{0, 1, 2, ..., 20\}, with mean $\mu = 10$. The moral hazard parameter is $\lambda = 0.9$. Discount rates for the principal and the agent are such that $e^{-r} - 1 = 6.5\%$ and $e^{-\rho} - 1 = 7\%$. The cost of firing and replacing a manager is equal to 2% of annual mean cashflow ($\kappa = 0.2$), while the investment cost required for the firm to scale up is set to 20% of annual mean cashflow ($\chi = 2$). We set the scale adjusted reservation compensation for a new manager at $w_0 = 14$. Other parameter values to be specified are $q$ and $\gamma$, capturing the likelihood and the magnitude of growth opportunities, respectively.

4.1 Two baseline cases

Our analysis in Section 3.1 shows that the optimal stationary contract is entirely summarized by three threshold values $w_N$, $w_G$ and $\overline{w}$. Consider first the case where $q = 0.1$ and $\gamma = 0.25$. In this case, the optimal stationary thresholds become $w_N = 8.42$, $w_G = \infty$ and $\overline{w} = 32.96$. The fact that $w_G = \infty$ indicates that it is optimal to grow and replace the agent with probability 1 whenever a growth opportunity is available. That is, this is a high growth firm. Figure 2 represents the corresponding stationary value functions. Note that, $b_G'(w)$ decreases linearly with slope $-1$ and lies above $b_c'(w)$ for all $w$ indicates graphically that this is a case of high growth. The agent’s compensation threshold $\overline{w} = 32.96$ means that an agent who enters the job with an expected discounted payoff of $w_0 = 14$ must experience a sustained run of good cashflow realizations before receiving any cash compensation. Given that each year there is a one in ten chance of being replaced through growth, in this case most agents will only see compensation in the form of severance pay.

Suppose instead $\gamma = 0.1$, while all other parameters are kept the same. The optimal stationary thresholds become $w_N = 8.42$, $w_G = 19.5$ and $\overline{w} = 35.6$. Having reduced the rate at which the firm is allowed to grow upon arrival of a growth opportunity, we now have a firm which does not take up efficient growth opportunities systematically when available, but only if $w$ is below the threshold $w_G = 19.5$. This is a low growth firm. Figure 3 shows the stationary value functions in this case. Note that, in this case $b_G'(w)$ initially decreases linearly with slope greater than $-1$ and is tangent to $b_c'(w)$ at $w_G = 19.5$. In this firm, the agent is more likely to be replaced as a disciplinary measure in the absence of growth opportunities than through growth. This means that
there is a higher chance that the agent will receive cash compensation while still active despite the fact that the bonus threshold is higher now (35.6) than for the high $\gamma$ case. We will see that the net effect of this will mean that on average compensation will arrive much later for the agent in this lower growth case.

4.2 Sensitivity of contract terms

The realized earnings and growth performance of firms are the result of managers’ and owners’ responses to cashflow shocks and to the arrival of growth opportunities, and these reactions will be shaped by the terms of the contract as set out in the pay-performance sensitivity and in the thresholds, $w_N$, $w_G$, and $\overline{w}$. Thus understanding how these thresholds are affected by changes in the deep parameters of the model is an important step toward understanding how the earnings and growth experience of firms is determined.

Figure 4 depicts the three thresholds as functions of the severity of moral hazard, $\lambda$, and the arrival growth opportunity frequency, $q$, for a firm with a finite $w_G$, that is, for a low growth firm. The understanding of $w_N$, the dismissal threshold in the absence of growth opportunities, is quite straightforward because here we have an analytical formula: $w_N = e^{-\rho\lambda\mu}$. That is, the non-growth dismissal threshold is linearly increasing in $\lambda$ and independent of $q$. Intuitively, in the face of increased moral hazard, the principal will increase the dismissal threshold, thereby increasing the risk of disciplinary dismissal.

Next consider the impact of $\lambda$ on the bonus threshold, $\overline{w}$. It is increasing in $\lambda$ reflecting an increased benefit of deferred compensation. This is because the inefficient termination threshold is higher and the pay-performance sensitivity increases, implying that it takes a shorter run of poor performance for the no-growth dismissal threat to be active.

To understand the effect of increasing $\lambda$ on $w_G$, recall that an increase in this threshold means the agent’s promise is more likely to be below it, which in turn means that the probability that the firm will take a growth opportunity and fire the manager increases. That is, there is a positive relationship between $w_G$ and conditional probability of growth. In light of this, a higher $\lambda$ results in a higher $w_G$ because this has two benefits. There is a higher probability that the firm will undertake the attractive growth opportunity. And if no growth opportunity arrives, agent continues with a higher promise, $w = w_G$, which makes subsequent inefficient liquidation less likely.

We turn next to the impact of $q$ on $\overline{w}$ and $w_G$, again for low growth firm. A higher $q$ causes a fall in the bonus threshold, $\overline{w}$, implying that cash payouts will be made following a shorter run of good performance. This follows because, a higher $q$ implies higher unconditional probability of early termination, with no severance pay, as this is a low growth firm. Thus in order to deliver the reservation value, $w_0$, ex ante, the cash compensation needs to be paid earlier. Furthermore, for the same reason, in order to increase the probability of getting to the bonus threshold the growth dismissal threshold, $w_G$, decreases because this decreases the probability of dismissal, conditional on $\theta = G$.

Finally, for high-growth firms, by definition $w_G = \infty$. The sensitivities of $w_N$ and
\( \overline{w} \) are similar to those in the low-growth case and for similar reasons. Again, in our framework, \( w_N = e^{-\rho \lambda \mu} \). The bonus threshold \( \overline{w} \) is increasing in \( \lambda \) and decreasing in \( q \), as is the case for low-growth firms. \( \overline{w} \) falls with an increase in \( q \) because the marginal cost of earlier bonus payments decreases as \( q \) increases. This is because as \( q \) increases it is more likely that a growth opportunity will arrive soon, in which case it will be taken up for sure. Therefore the likelihood of inefficient replacement is reduced and the marginal benefit of deferred compensation is reduced.

### 4.3 What makes a firm grow fast?

Our baseline examples in Section 4.1 show that two firms that differ only in the size of the growth opportunity will have very different contracts for top management. These differences translate into very different policies toward growth opportunities with high-growth firms undertaking all opportunities that present themselves and low-growth firms undertaking opportunities only if incumbent management is not performing well.

It is also the case that differences only in agency costs may result in very different growth experiences. To see this, consider an example of two firms that have the same size of their growth opportunities (\( \gamma = 1.25 \)), the same probability of having a stochastic growth opportunity \( q = 0.10 \), and only differ in the degree of moral hazard \( \lambda \). All other parameters are as in our baseline cases. In this example, our model predicts that the firm with \( \lambda = 0.5 \) grows at an average rate of 1.25%. This is because it is a high-growth firm that undertakes all the growth opportunities that arise. Meanwhile, the firm with \( \lambda = 0.9 \) grows at an average rate of around 0.06.8 Stated otherwise, suppose the two firms start out life with identical scale of operations. Fifty years on, \( t = 50 \), the expectation is that the firm with low agency problems will have a scale (measured by the mean cashflow rate) that is 38% larger than the high agency cost firm.

This holds for other parameters as well. That is, we may have two firms that differ only slightly in their deep parameters, with one a high-growth firm and the other a low-growth firm. Figure 5 depicts regions of the parameter space corresponding to high-growth firms and low-growth firms. All parameters are set as in the second baseline case (low-growth firm) of Section 4 except for the two parameters depicted in the diagram.

To summarize, small differences in parameters can result in dramatically different growth and turnover behavior. Growing firms need a flow of good ideas for expanding markets and improving technology (high \( q \), high \( \gamma \)). They need to manage transitions well (low \( \kappa \), low \( \chi \)). And they need to keep agency problems under control, for example, through increased monitoring (low \( \lambda \)).

---

8The latter statement is based on simulations.
5 Management turnover, the timing of compensation and agency costs

5.1 Simulating the model

We now simulate the model to understand its implications for management turnover and the relative importance of deferred compensation. Simulations also allow us to assess the importance of the agency costs due to the contracting imperfections present in this framework.

Specifically we draw repeatedly a sequence of cashflows and growth opportunity realizations, keeping track of compensation, growth and termination decisions commanded by the optimal contract. We then characterize these histories using a variety of summary statistics. We focus on three statistics that we find particularly interesting. First we calculate the average longevity or ‘tenure’ of managers, which is inversely related to the replacement frequency. Second we calculate the unconditional probabilities of efficient termination (i.e. fire the agent to undertake growth) and inefficient termination (i.e. fire the agent without growing) as the corresponding realized sample frequencies. Third, to measure the extent to which the optimal contract relies on deferred compensation, we calculate the average duration of the agent’s compensation conditional on the agent receiving non-zero compensation during his tenure in the firm. This is calculated as the weighted average tenure years of the agent’s realized payments with weights calculated as the ratio of discounted cash flow to the sum of discounted cash flows.

For example, consider the results for the benchmark cases given in Section 4.1. For the high growth firm with $\gamma = 0.25$, average tenure of an agent is 7.7 years. The average probability of efficient termination is 10% per year, reflecting the fact that for a high growth firm any available growth opportunity is undertaken. The probability of inefficient termination is about 3% per year. And the average duration of compensation is 7.24 years. This reflects the fact that most agents receiving compensation do so in the event of growth and in the form of severance pay.

In contrast for the low growth firm with $\gamma = 0.1$, the average tenure is 195 years. The probability of inefficient termination is 0.33% which is higher than the probability of efficient termination (0.23%). That is, usually the firm passes up growth opportunities when they present themselves. The average duration of compensation is 22.5 years. Comparing results for the two cases, we see that even though most agents in high growth firms receive compensation in the form of severance pay, they receive this earlier than on average do agents in low growth firms.

5.2 Comparative statics

In this section, we further explore predictions from our model in terms of its comparative statics with respect to some key parameters. Specifically, we solve our model for alternative values of these parameters and then simulate the model assuming the same realizations for underlying cashflow shocks and growth opportunities. We record the
histories of management turnover, whether turnover takes place for growth or for disciplinary reasons, and the compensation histories for each of the firm’s managers. The parameters we vary are $q$, the probability of having a stochastic growth opportunity, and $\lambda$, the severity of agency problems. The default values of these parameters take on when the other parameter is varied are $q = 0.1$ and $\lambda = 0.9$. Other parameters are as in our baseline cases of Section 4.1.

5.2.1 Management turnover

In our model managers are replaced either to facilitate growth or because a history of poor operating results leads to dismissal. The exact conditions under which managers are replaced are sensitive to both the growth prospects of the firm and to the severity of agency problems faced by the firm.

Representing the quality of the growth prospects by the frequency of arrival of growth opportunities, $q$, we show the sensitivity to this parameter of average manager tenure. This is depicted in the left panel of Figure 6 for a high growth firm with $\gamma = 0.25$. From the figure we see that as the probability of growth opportunity in a year rises from 5% to 25% the average tenure of the agent declines from 14 years to something under 4 years. A similar negative sensitivity to increases in $q$ holds for low growth firms (e.g., with $\gamma < 0.1$), with the difference that, for a given $q$, the average tenure is much higher.

Thus tenure falls and turnover rises for firms with better growth prospects. To our knowledge this hypothesis has not been submitted to direct empirical testing. However, there is some indirect evidence which is supportive of the hypothesis. Specifically, Mikkelson and Partch (1997) compare top management turnover intensity in two successive five-year periods with very different mergers and acquisitions activity. They find that in the active take-over period of 1984-1988, 33% of firms in the sample underwent complete management changes (i.e., replaced all of the president, CEO and Chairman); whereas this intensity was only 17% in the subsequent period 1989-1993 when take-over activity was low. Interestingly their notion of complete management corresponds better to our model which associates turnover and major changes of direction than does most of the literature which has focused exclusively on CEO turnover. While they do not specifically make a link of management turnover and firm growth, the two periods they cover coincide with very different experiences of firm growth and investment. Specifically, in the 1984-88 period U.S. annual non-residential investment spending increased 28%; whereas, between 1989 and 1993 it increased only 12.5%.\footnote{Based on annual U.S. National Income Statistics.}

In the right panel of Figure 6 we see the consequences of increasing the severity of managerial moral hazard. As the rent extraction efficiency ($\lambda$) of the agent rises the average longevity declines. This is a reflection of the fact that the optimal contract relies more heavily on the threat of termination in the face of more severe moral hazard. Again, a similar pattern is found for low growth firms as well.
5.2.2 Efficient and inefficient replacement probabilities

As already noted, turnover may occur for growth or for discipline. These two kinds of managerial turnover are affected differently by changes in the firm’s underlying characteristics. To distinguish these effects, we calculate the average frequency of these two types of turnover in the simulated histories and plot these as functions of $q$ and $\lambda$ in Figure 7. The top row pertains to the high growth case, with $\gamma = 0.25$ as above. In high growth firms the unconditional probability of replacement for reasons of growth are higher than the probability of disciplinary replacement. Since all growth opportunities are taken up in these firms, this frequency increases linearly in $q$. The probability of disciplinary (or inefficient) turnover is slightly increasing in $q$ as well. To understand this effect, recall that $w_G = \infty$ in the high growth case and that $w_N$ is insensitive to changes in $q$. Therefore, the slight increase in the unconditional probability of disciplinary dismissals is due to the indirect effect of increased probability of growth dismissals which eliminates a disproportionate number of incumbent managers with little track record or who have recently been enjoying good operating performance. This leaves relatively more agents with poor recent performance who are under threat of disciplinary dismissal.

The effect of more severe agency problems on dismissal frequencies in high growth firms is given in the upper right panel of Figure 7. Since all growth opportunities are taken up, changes in $\lambda$ have no effect on the efficient dismissal probability. The probability of inefficient dismissal is slightly increasing in $\lambda$. This reflects an increased reliance on the termination threat when moral hazard is more severe.

The sensitivities of dismissal probabilities for low growth firms are given in the bottom row of Figure 7. As for high growth firms, efficient dismissal probability is increasing in $q$, but now the inefficient dismissal probability is slightly decreasing in $q$. Recalling that in low growth firms, growth opportunities are taken only when incumbent managers have been performing poorly, we see that more such managers are eliminated through growth when growth arrives more frequently (i.e., as $q$ increases). In the right panel, the probability of inefficient replacement increases with increasing $\lambda$ reflecting greater reliance on the dismissal threat (increased $w_N$). Thus more managers are replaced before any growth opportunity arrives, implying a decline in the unconditional efficient dismissal probability, as seen in the figure.

5.2.3 Compensation duration

To assess the consequence of changing parameters for the reliance on front loading of compensation, we have calculated the realized duration of compensation including in the calculation all agents who receive bonuses during their tenure and/or severance pay upon leaving the firm. These sensitivities are given in Figure 8. From the top row we see that for both high and low growth firms an increase in $q$ reduces the duration of compensation. That is, when growth opportunities arrive more frequently, firms optimally rely on more front-loading of compensation. In the case of high growth firms, the effect is very direct—the more frequent growth opportunities translate into more frequent dismissals with associated severance pay. For low growth firms, the effect works through the decreased bonus threshold, as depicted in Figure 4.
The second row of Figure 8 shows the effect of increasing $\lambda$. For the high growth firm
the average duration of compensation falls slightly as $\lambda$ rises. The reason for this is that
a higher $\lambda$ increases the probability of leaving the firm for poor performance without
ever receiving compensation. Therefore among managers receiving compensation, a
larger proportion receive this in the form of severance pay, which occurs relatively
early. In contrast, for low growth firms, increases in $\lambda$ result in increased compensation
duration. Recalling the fact that for low growth firms average tenure is relatively long
and most managers leave for disciplinary reasons rather than growth, we see the effect
is through the increased bonus threshold, $w$. Most low growth firm managers receiving
any compensation do so after a sustained run of good performance but they are made
to wait longer to receive that bonus.

Again, to our knowledge, there are no empirical studies that directly test whether
these effects on the timing of compensation hold. However, recently Kaplan and Minton
(2008) have studied the evolution of top CEO turnover since 1990, a period that saw
very rapid increases in the amount of top management compensation. They find ev-
didence of more rapid turnover, especially after 2000. They argue that the observed
increases in CEO pay are compensation for shorter tenure. This is consistent with our
theory in which high growth will be associated with shorter tenure and more front-
loading of compensation.

5.3 Agency costs

In this section we assess the loss of value caused by the non-contractibility of cashflows.
In our framework with repeated growth options, the first-best value of the firm is the
expected present discounted value of all cashflows net of dismissal and investment costs
when the firm undertakes all growth opportunities that present themselves but does
not dismiss any manager in the absence of growth. Under the optimal contract in
the face of non-contractible cashflow, the firm will fall short of this value for several
distinct reasons. First, as in previous studies of agency in a dynamic setting, under
the optimal contract the firm will dismiss managers for disciplinary reasons following
a series of poor cashflow realizations even though this is ex post inefficient. Second,
there is an inefficiency due to the reliance on deferred compensation when managers
are more impatient than investors, $\rho > r$. Third, under the optimal contract the firm
will sometimes retain an incumbent manager and pass-up growth opportunities even
though growth is ex post efficient. Finally, there is a more subtle form of agency costs
which we have not emphasized in our discussion until now. This is due to the fact that
at the time of agreeing a contract with an incoming manager the firm does not take
into account the spill-over effect on the timing of future managers’ hiring. As noted in
the Introduction, this effect is absent in the previous literature.

Specifically, the second best value of the firm is the expected present value of all
cashflows that accrue to the principal and to all managers who successively run the firm
under optimal contracts as set out in Proposition 1. Two subtleties should be noted
in calculating this second best value. First, cash flows to agents are discounted at the
agents’ discount rate, $\rho$; whereas, investor cash flows are discounted at rate $r$. Since
$\rho > r$, the promise to an agent is worth less to the agent than it costs the firm. Second,
the calculation of agent cash flows includes payments to all agents, both current and future. Thus in the stationary case we can write the size-adjusted, beginning-of-period second-best value of the firm as

\[ v(w) = b^*(w) + w + f(w), \]

(20)

where \( f(w) \) denotes the expected discounted value of payoffs to future agents as a function of the current agent’s promised value, \( w \).\(^{10}\) To assess the extent of agency costs, the total value of the firm under the optimal contract \( v(w) \) can be compared to the beginning-of-period, first-best value of the firm, \( \mu + v^* \).

Figure 9 depicts values under the second best optimal contract for the high growth (\( \gamma = 0.25 \) in the top panel) and low growth firms (\( \gamma = 0.1 \) in the bottom panel) as set out in Section 4.1. The left panel gives the value for the principal and the incumbent agent, \( b(w) + w \). The middle panel gives the present value of compensation to future agents who are not party to the current contract but who are affected by the current contract and the current promise to the incumbent agent, \( f(w) \). The right panel gives the sum of all these components, that is, the second best value of the firm defined above, \( v(w) = b(w) + w + f(w) \). These can be compared to the corresponding first best values (\( \mu + v^* \)) of 260.39 and 189.37, respectively. The second-best value function \( v(w) \) shows only little sensitivity to the current agent’s promise \( w \). Agency costs amount to roughly 5% of first-best value for the high growth case and 15% in the low growth case. That is, agency costs represent about fifteen months of expected cashflows for the high-growth firm and about thirty-six months of expected cashflows for the low-growth firm.

The principal reason why agency costs are less for the high-growth firm is because it undertakes all investment opportunities, even under the second-best contract, whereas a low-growth firm suffers from under-investment.

In the left panels of Figure 9 we see that for both high and low growth firms the combined value to the principal and the incumbent manager is increasing in the promise to this manager. This reflects the relaxation of agency problems affecting the two parties to the current contract, and this is an effect already seen in previous dynamic agency models. Interestingly, the second-best firm value, taking into account the effect on future managers, is not increasing and concave in \( w \). This is seen in the right panel of Figure 9 where, for both high-growth and low-growth firms, \( v(w) \) becomes decreasing beyond a certain point.

Why? The answer is that the second best contract is designed so as to maximize investor value subject to the incentive compatibility condition (6) \( \text{vis à vis} \) the incumbent agent. This condition does not take into consideration the consequences for future agents. Thus incentivizing the current agent with a higher promise may come at the cost of reducing payoffs to future agents. Specifically, if the current agent will be succeeded by future agents at stochastic stopping times \( \tau_i, i = 1, 2, 3, \ldots \), the expected present values of the amounts they will receive, \( \mathbb{E}[e^{-\rho \tau_i} \Phi_{\tau_i} w_0] \), are both missing and affected by the current \( w \) since this affects the distribution of stopping times.

\(^{10}\) The last term, \( f(w) \), does not appear in earlier contributions to the literature on optimal long-term contracts where there is a single agent and the “liquidation” value of the firm is exogenous. For instance, the liquidation value of the firm is set equal to zero in Biais et al. (2007), and DeMarzo and Fishman (2007b) take it to be equal to an exogenous fraction of the first-best value in their discussion of agency costs.
As can be seen from the central panel of Figure 9, the present value of payoffs to future agents, \( f(w) \), is decreasing in the current promise. In the case of low growth firms there are two separate effects. A higher promise \( w \) tends to decrease the probability that the incumbent will be replaced for disciplinary reasons. It is also reduces the probability of replacing the agent in order to undertake growth. In the case of high growth firms, by definition, growth opportunities are undertaken whenever they appear, independently of \( w \). Thus only the first effect is present. This is the reason that the value \( f(w) \) is less sensitive to changes in \( w \) in the high growth case than in the low growth case. Note that as \( w \) increases from 10 to 30, \( f(w) \) declines by about 5 for the high-growth firm and by about 9 for the low-growth firm.

6 Extension

Our maintained assumption so far was that in order to pursue an opportunity to grow, the incumbent manager had to be replaced. We now consider a more general environment where upon the arrival of a growth opportunity, the firm can decide to grow either with a new manager or with the incumbent manager. Endogenizing the choice of managerial replacement upon growth makes the analysis of the model more complex. However, the economic forces we have highlighted so far remain at play, and the analysis will help clarify under which circumstances our conclusions from earlier sections still hold, and how they need to be modified in other cases.

We now let \( \chi^i \) denote the (size-adjusted) cost of taking the growth opportunity with the incumbent manager, and \( \chi^n \) the cost of growing with a new manager.\( ^{11} \) The derivation of the optimal contract follows the same logic as in Section 3.1, except for the construction of \( b_G^i(w) \).\(^{12} \) The continuation value upon replacement \( \ell_G \) involves \( \chi^n \) and is defined as

\[
\ell_G = \max \{ e^{-r}(1 + \gamma)b^i(w_0) - \kappa - \chi^n; e^{-r}b^i(w_0) - \kappa \}. \tag{21}
\]

In particular, whenever \( \chi^n \) is sufficiently small, \( \ell_G \) reflects the value obtained when the newly hired manager implements the available growth opportunity — captured by the first term of (21). The key novel feature of the optimal contract in the extended environment is that, when faced with a growth opportunity, the firm needs to decide whether if retained, the incumbent manager would keep running the firm at the same size or at an expanded size. Formally, we define

\[
\hat{b}_G^i(w) = \max_{p, s, w} p(\ell_G - s) + (1 - p)b^i(w^c) \tag{22}
\]

subject to the promise keeping condition \( ps + (1 - p)w^c = w \) the limited liability condition \( s \geq 0, w^c \geq e^{-\rho}\lambda \mu \), and \( p \in [0, 1] \). We also define

\[
\hat{b}_G^e(w) = \max_{p, s, w} p(\ell_G - s) + (1 - p)[(1 + \gamma)b^e(w^c) - \chi_i] \tag{23}
\]

\(^{11} \)We assume that \( \gamma\mu/(e^r - 1) > \min(\chi^i, \chi^n + \kappa) \), so that the first-best policy in steady state involves taking all growth opportunities. Under first best, the firm grows with new managers if and only if \( \chi^i > \chi^n + \kappa \).

\(^{12} \) For notational convenience, we drop all time subscripts in this section.
subject to the alternative promise keeping condition \( ps + (1 - p)(1 + \gamma)w^c = w \). The value function \( \hat{b}_G^c \) corresponds to the case where upon retaining its incumbent manager the firm does not take up the growth opportunity. The value function \( \hat{b}_G \) corresponds to the alternative case where, if not fired, the incumbent manager does implement the growth opportunity. The firm chooses optimally whether to grow or not upon retaining an incumbent manager, hence we can define for a given promised value \( w \)

\[
b^c_G(w) = \max\{\hat{b}_G^c(w), \hat{b}_G^c(w)\}. \tag{24}
\]

Note that whenever \( b^c_G(w) = \hat{b}_G^c(w) \), the probability of managerial replacement \( p_G(w) \), which appears as \( p \) in (23), does no longer also denote the probability of growing conditional on \( \theta = G \), since growth is implemented either with or without the incumbent manager. In order to analyze the construction of \( \hat{b}_G^c \) and the associated policy functions, we first focus separately on the construction of \( \hat{b}_G^c \) and \( \hat{b}_G^c \). Given \( b^c \), the construction of the former follows the logic of Proposition 1 for the case \( \theta = G \). To obtain the latter however, we need to introduce \( \hat{\delta}_G \) and \( \hat{w}_G \) accordingly as

\[
\hat{\delta}_G = \sup \left\{ \frac{b^c_G(w) - \ell_G}{w} : w \geq (1 + \gamma)e^{-\rho}\lambda\mu \right\}, \tag{26}
\]

and

\[
\hat{w}_G = \begin{cases} 
\inf \{w \geq (1 + \gamma)e^{-\rho}\lambda\mu : \hat{b}^c_G(w) \leq \hat{\delta}_G \}, & \text{if } \hat{\delta}_G > -1, \\
\infty, & \text{otherwise.} 
\end{cases} \tag{27}
\]

**Lemma 5.** The construction of \( \hat{b}_G^c \) given \( b^c \) proceeds as follows:

(i) if \( \hat{\delta}_G > -1 \), the replacement probability is

\[
p_G(w) = \begin{cases} 
1 - \frac{w}{\hat{w}_G}, & 0 \leq w < \hat{w}_G, \\
0, & w \geq \hat{w}_G. 
\end{cases} \tag{28}
\]

Severance pay is \( s_G(w) = 0 \), \( \forall w \), and continuation value upon being retained (adjusted by end-of-the-period size) is

\[
w^c(w) = \begin{cases} 
\frac{\hat{w}_G}{1 + \gamma}, & 0 < w < \hat{\delta}_G, \\
\hat{w}_G, & w \geq \hat{w}_G. 
\end{cases} \tag{29}
\]

Hence

\[
\hat{b}^c_G(w) = \begin{cases} 
\ell_G + \hat{\delta}_G w, & 0 \leq w \leq \hat{w}_G, \\
(1 + \gamma)b^c \left( \frac{w}{1 + \gamma} \right) - \chi^i, & w \geq \hat{w}_G. 
\end{cases} \tag{30}
\]

(ii) if \( \hat{\delta}_G \leq -1 \), then \( p_G(w) = 1 \) for all \( w \geq 0 \), severance is given by \( s_G(w) = w \), and

\[
\hat{b}^c_G(w) = \ell_G - w, \quad \forall w \geq 0. \tag{31}
\]
We are now in a position to characterize the optimal replacement, growth and severance policies. These depend both quantitatively and qualitatively on the deep parameters of the model. We outline these policies and describe their dependence on the cost of growing with the incumbent manager, \( \chi^i \), and the size of the growth opportunity, \( \gamma \). For now, we focus on firms for which \( \chi^n \) is small, so that newly hired managers always implement available growth opportunities.

Consider first a situation where \( \chi^i \) is high. This is tantamount to assuming that growth entails change of management. Intuitively, implementing growth with the current manager is very costly, so that it is optimal for the firm never to do so. Hence Proposition 1 and our discussion in Section 3.2 apply. Figures 10 and 11 depict value functions for the low-growth and high-growth case, respectively. Note that in the case of a high growth rate \( \gamma \), growth is a very appealing option, therefore the optimal solution in (22) involves setting \( p = 1 \). And because growing with the manager is very costly, it is also optimal to set \( p = 1 \) in (23), therefore \( \bar{b}^G_{t,G}(w) = \hat{b}^G_{t,G}(w) = \ell_G - w \) for all \( w \geq 0 \). Note also that Figure 11 illustrates a case where non-zero severance pay arises as part of the optimal contract. It turns out that this combination of relatively large growth opportunity but high cost of growing with the incumbent is the only configuration of the model that gives rise to positive severance pay.

Now consider a situation where \( \chi^i \) is relatively low, so that \( \bar{b}^G_{t,G}(w) = \hat{b}^G_{t,G}(w) \). This is depicted in Figures 12 and 13 for low and high values of \( \gamma \), respectively. An alternative set of predictions emerges as set out in Part (i) of Lemma 5. Firms always undertake their growth opportunities, just as do high growth firms of the baseline model. However, they sometimes grow with their incumbent managers. The conditional probability of replacement is weakly decreasing in past performance. Replacement never occurs after a sustained period of good performance. Furthermore, the probability of firing is higher when a growth opportunity is available than when it is not since \( w_G > w_N \). Finally, firms pay no severance to their managers when they fire them.

Figure 14 depicts a more complex configuration that can arise for intermediate values for \( \chi^i \). The thresholds \( w_G \) and \( \hat{w}_G \) are both finite, and \( \bar{b}^G_{t,G} \) intersects \( \hat{b}^G_{t,G} \) from below at some point \( \tilde{w} \). Observe that \( w_G < \tilde{w} < \hat{w} \). Optimal replacement, growth and severance policies are as follows. First, there is a dismissal threshold \( w_G \). For \( w < w_G \), an incumbent manager is replaced with probability \( 1 - w/w_G \). Second, the growth policy depends on the current promised value \( w \) in a subtle way: when \( w \in (0, w_G) \), the firm grows upon firing the incumbent and does not not grow upon retaining him; when \( w \in (w_G, \tilde{w}) \), the firm does not grow (i.e., the incumbent manager keeps running the firm at its current size); when \( w \geq \tilde{w} \), the firm grows (with the incumbent manager).13 Finally, the firm pays no severance upon firing.

The developments of this section show that many of the qualitative properties of the optimal contract we obtained in the benchmark model where growth necessarily entailed the replacement of the incumbent manager extend to the more general specification where the firm can grow with either the incumbent or a new manager. Now, in this extended framework (and relaxing any constraint on \( \chi^n \)), we focus on the basic

---

13 The growth policy of the firm for \( w > w_G \) follows the same logic as in DeMarzo and Fishman (2007a) where growth only occurs for high enough values of \( w \).
question of which manager, incumbent or newly appointed, will in fact be given the task of growing the firm. We have the following result.

**Proposition 2.** Consider an infinitely lived firm with a sequence of potential growth opportunities that can either be taken by an incumbent manager at an investment cost $\chi^i$ or by a new manager at an investment cost $\chi^n$. Then

1. the firm ever undertakes growth with a new manager if and only if $\chi^n < e^{-r}\gamma b^0(w_0)$,
2. the firm ever undertakes growth with an incumbent manager if and only if there exists $w$ such that $b^i_G(w) > b^n_G(w)$.

In general, the firm may undertake growth with an incumbent at some points in its history when the conditions are right and with a new manager at other times. This will depend upon the deep parameters of the model and the history of cash flows. This is illustrated for different values of costs of investment for the incumbent ($\chi^i$) and the new manager ($\chi^n$) in Figure 15 for a firm with large growth opportunities ($\gamma = 0.25$) and Figure 16 for a firm with smaller growth opportunities ($\gamma = 0.1$). These are drawn over ranges of $\chi^i$ and $\chi^n$ such that the firm takes up all growth opportunities under the first-best policy.

Focusing on the high growth case, Figure 15, we see that when costs of undertaking the investments are relatively low for both incumbents and new managers, then the firm will undertake growth with an incumbent or a new manager. In these cases, good recent cash flow history will favor the incumbent—high cash flows will lead to an increase in his promised compensation and, given a higher promise, to a higher probability he will be retained to undertake growth. If recent cash flow experience is poor, the incumbent will be at risk of being replaced by a new manager who will then undertake growth. If we consider cases where the incumbents have a natural disadvantage relative to new managers in undertaking growth (high $\chi^i$ and low $\chi^n$), we see that the firm will undertake growth only with new managers. In this region the model gives the same qualitative results we have presented in Sections 3 to 5. There is also a region where the new manager is relatively poor at growing the firm (high $\chi^n$) and where growth is undertaken by the incumbent if it is undertaken at all. This occurs toward the northeast corner of the figure. In this region given the high investment costs with either new or incumbent managers, the value of the growth opportunity will be relatively low, but not so low as to never be worthwhile undertaking. In this region growth will be undertaken only if recent cash flow experience has been good and then only with the incumbent. Why? For these cases the difference in costs of undertaking the investment with the incumbent ($\chi^i$) will be similar to those incurred by firing the incumbent and undertaking growth with a new manager ($\chi^n + \kappa$). The main difference between the incumbent and the new manager is that the incumbent will start the post-growth phase of his career with a high promise, $w$, which means that agency costs will be relaxed relative to what they would be had the new manager been hired with promise $w_0$.

It is worth noting that when new managers’ costs ($\chi^n$) are high and incumbent’s costs ($\chi^i$) are low, we do not find that growth will be undertaken only by incumbents. The reason is that in our framework there will be times following a streak of bad cash
flows when the incumbent must surely be replaced with a new manager. If this occurs when a growth opportunity is present it will still be interesting to let the firm grow despite the high $\chi^a$ incurred in pursuing it. For, this will give a platform for further growth which in all likelihood will be undertaken by the incoming manager after he has become the incumbent and thus endowed with the low $\chi^i$. Finally, we see that in the extreme northeast of the Figure, cost of investment with either manager are sufficiently high to kill off growth under the second-best contract even though it would be undertaken under first-best.

To summarize, when the firm facing a growth opportunity can choose whether or not to grow and in either case whether or not to keep the incumbent manager, a variety of qualitative predictions emerge depending upon the deep parameters of the model. If the costs of growing with an incumbent manager are sufficiently high and those of a new manager sufficiently low we recover the results seen in our benchmark model where growth takes place only with new managers and incumbent managers are replaced both to facilitate growth and as a disciplinary measure. If in contrast, costs of growth with an incumbent are not very high, then growth may or may not take place with an incumbent depending upon his past performance. The better the past performance the more likely he will be retained. Conditional upon a given past performance, the firing probability is weakly higher in the face of a growth opportunity than in its absence.

7 Conclusion

In this paper we have explored the relationship between managerial compensation and growth in a dynamic agency framework. In contrast with previous studies, we consider a long-lived firm with growth prospects that can hire a sequence of managers over time. In this setting management replacement may occur not only to discipline management but also possibly to facilitate growth. This framework produces new insights on managerial compensation and turnover. We find that the firm’s growth trajectory depends on the severity of agency problems as well as the quality of its growth opportunities. We show how optimal contracts in firms with growth opportunities can be implemented with a system of deferred compensation credit and bonuses that are similar to that found in practice. We find that firms with very good growth prospects tend to rely less on back-loading of compensation than firms with poor growth prospects. We also identify a new component of agency costs which relates exclusively to managerial turnover. This new component of agency costs is due to the spillover effect of the length of an existing managerial contract onto the present value of all future contracts signed by the firm.

Our study suggests a number of open issues concerning the relation between growth and incentive provision. In our framework, the growth event is modeled very simply. Within a single time period, a growth opportunity appears, and the firm decides whether or not to take it up and whether or not to replace the incumbent with a new manager. In reality, many growth opportunities may require or at least benefit from a prolonged transition during which outgoing and incoming management need to coop-
erate. Extending our model in this direction might yield new predictions on optimal managerial contracts.

In a different vein, it would be interesting to explore the determinants of the growth opportunity arrival process which here we have treated as exogenous. In particular, current management may need to allocate their efforts between producing cash flows from assets in place and developing new opportunities for growth. There may be a trade-off between two activities in that they may both require top management time but also because they use different management skills.
References


Figure 2: Value functions for high growth firm ($\gamma = 0.25$)

Figure 3: Value functions for low growth firms ($\gamma = 0.1$)
Figure 4: Threshold sensitivities: low growth firm
Figure 5: High growth and low growth regions of parameters space
Figure 6: Average tenure in high-growth firms

Figure 7: Average dismissal rates
Figure 8: Compensation duration

Figure 9: Second Best Values for high growth firms (top) and low growth firms (bottom)
Figure 10: Extension: High $\chi^i$ and low $\gamma$

Figure 11: Extension: High $\chi^i$ and high $\gamma$
Figure 12: Extension: Low $\chi^i$ and low $\gamma$

Figure 13: Extension: Low $\chi^i$ and high $\gamma$
Figure 14: Extension: intermediate $\chi^j$
Figure 15: Who grows the firm? (big growth opportunity, $\gamma = 0.25$)
Figure 16: Who grows the firm? (small growth opportunity, $\gamma = 0.10$)