

Economic Growth in a Cooperative Economy

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Abstract

We develop and formalize an equilibrium concept for a dynamic economy in which production takes place in worker cooperatives. The concept rules out allocations of workers to cooperatives in which a worker in one cooperative could move to a different cooperative and make both herself and the existing workers in the receiving cooperative better off. It also rules out allocations in which workers in a cooperative would be made better off by some of the other workers leaving. We also provide a minimum-information equilibrium-selection criterion which refines our equilibrium concept. We illustrate the application of our concept and refinement in the context of an overlapping-generation economy with specific preferences and technology. The cooperative economy follows a dynamic path qualitatively similar to the path followed by a capitalist economy, featuring gradual convergence to a steady state with constant output. However the cooperative economy features a static inefficiency, in that, for a given aggregate capital stock, firm size is smaller than what a social planner would choose. On the other hand, the cooperative economy cannot be dynamically inefficient, and could accumulate capital at a rate that is higher or lower than the capitalist economy. As a result, steady-state income per worker could be higher or lower in the cooperative economy. We also present an illustrative calibration which quantitatively compares steady-state incomes and welfare in a cooperative and in a capitalist economy.

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1 Introduction

For the first time in many decades the capitalist organization of production is under discussion in several Western societies. In the United States, avowed socialists are among the most popular politicians in the country - and one of them has been a leading candidate to be nominated by a major party in the last two Presidential elections. Meanwhile, historically unprecedented percentages of opinion-poll respondents express positive views of socialism. Perhaps more significantly for future developments, socialism is viewed more favorably than capitalism among the youngest cohorts.¹ In the United Kingdom, leaders with a Marxist background, and with a recent history of advocating worker ownership of the means of production, have recently led the major opposition party, and might have succeeded in winning power had they not chosen an unpopular stance on Brexit. Disaffection with capitalism is also affecting political dynamics in several other countries.

A similar, vigorous debate is taking place among academics and public intellectuals. New books about the failures of capitalism appear on a monthly basis, and columns on the same topic are featured daily on the major newspapers. Major research programs, involving management scientists, sociologists, political scientists, and economists repudiate Friedmanite shareholder value and attempt to redefine the role and purpose of corporations.² Campaigns to redistribute power from shareholders to workers attract support from thousands of academics in social science disciplines.³

Macroeconomists have yet to make significant contributions to this important debate, and yet the institutional changes under discussion cry out for rigorous analysis of their general equilibrium and dynamic implications. What do they imply for aggregate productivity? How do they affect economic growth? This paper attempts to take a first step towards filling this gap.⁴

¹E.g. Pew Research centre, 2019

²E.g. the British Academy's *Future of the Corporation* programme, lead by Colin Mayer.

³E.g. the *Democratizing Work* campaign of the Summer of 2020.

⁴Microeconomists have been quicker to the mark, and have produced important normative insights in a partial equilibrium context (e.g. Magill, Quinzii, and Rochet (2015) and Hart and Zingales (2017)). But these contributions cannot substitute for positive assessments of the dynamic and general equilibrium consequences of alternative arrangements.

The alternative to shareholder capitalism that we study in this paper is the worker cooperative. This is a natural starting point for several reasons. First, cooperatives are frequently cited as possible remedies to the perceived crisis of capitalism, making an assessment of their growth implications directly relevant for the ongoing debate. Recent influential books which contain expressions of support for producer cooperatives as part of the needed revamp of the economic system include Block (2018), Cass (2018), and Collier (2018), which all appeared within a few months of each other. The earlier blockbuster on the consequences of inequality by Wilkinson and Pickett (2010) devotes its *entire* “normative” section to producer cooperatives, to the exclusion of all other remedies for the problems the book highlights. As we show in Appendix A.1, the phrase “worker cooperatives” has appeared more and more frequently among newly published (digitized) books since the mid-2000s. Positive media coverage of producer cooperatives seems also to have become more frequent, with stories centering on their ability to support the income and employment of their members during recent crisis periods; or on owner-managers transferring ownership to the workers as their individual contribution towards the transition to a post-capitalist model [for examples in prominent media, see Financial Times (2019) and New York Times (2021)].

Second, worker cooperatives have existed for nearly 200 years, and continue to exist virtually everywhere in the world.⁵ This provides a real-world basis to build the model on, and some confidence that the alternative to capitalism being studied has a chance to survive impact with reality. Third, as we discuss shortly, there is a pre-existing (if largely forgotten) tradition of economic modelling of worker cooperatives which we can relate our work to. Fourth, and perhaps most importantly, worker cooperatives can be thought of as a limiting case of many of the more nuanced ideas advanced by would-be reformers, which typically include less complete reallocations of control and ownership rights away from shareholders (or, essentially equivalently, reallocations of the weights of different stakeholders in corporation decision making). We submit that studying this limiting case is a useful first step towards a framework suitable for the study of more “interior”

⁵The worker cooperative movement has its origins in the industrial revolution. Then as now it emerged as a response to the perceived shortcomings of subordinate-labour capitalism.

forms of organization.⁶

We study a production economy inhabited by two-period lived overlapping generations, where only the young work, while both old and young consume. The capitalist version of this economy, characterized by individual property of capital and profit-maximizing firms, is entirely standard and its dynamic properties are well known. Consistent with real-world arrangements, we conceptualize cooperatives as *labor-managed* entities which allow *no individual ownership* of their assets. In our model this implies that cooperatives, and not any individuals, own their own capital stock, and that young workers come together to produce and collectively choose investment plans. Given that the firm is managed by young workers, its objective is to maximize the present value of their (common) lifetime utility. As in the capitalist economy, these cooperatives supply their output on a perfectly competitive product market.

Real-world cooperatives differ in the claims former workers have in the distribution of income, with traditional cooperatives tending to sever all links upon a worker's retirement or withdrawal from the membership, and other, often more successful cooperatives where former workers continue to receive payments. Coops in the celebrated Mondragon system, for example, which employs nearly 100,000 people in the Basque region of Spain, belong to the latter category.⁷ Our modelling choices mimic this model: old workers continue to participate in the distribution of income of the cooperative to which they were attached when young. As was noted in the early economics literature on labour-managed firms, in traditional cooperatives members' horizon when voting over investment is limited to their expected remaining time with the coop, and this tends to depress cooperative investment. By lengthening the planning horizon of young workers our institutional setup encourages greater investment by the coop, and potentially explains the apparent greater success of those coops which continue to confer distribution rights to former workers. Importantly, in an appendix we endogenize this arrangement and show that it can emerge as a feature of the equilibrium in the dynamic inter-generational game among subsequent cohorts of

⁶Early statements of the view that worker cooperatives are limiting cases of models of codetermination and/or collective bargaining include Law (1977), Aoki (1980), Svejnar (1982), and Miyazaki (1984).

⁷Classic economic analyses of the (still thriving) Mondragon experience include Bradley and Gelb (1983) and Whyte and Whyte (1988).

workers in the coop.

One of our main goals is to identify an appropriate equilibrium concept for a dynamic cooperative economy. This is challenging because it is not *a priori* obvious how young workers will sort themselves into the cooperatives that exist when they join the labor market, and also under what conditions they will decide to form new cooperatives rather than joining an existing one. Furthermore, any worker allocation mechanism has repercussions for investment, as a cooperative's current workers' incentive to invest depends on the expected employment of the cooperative in the future. We solve these challenges by developing a “minimum rationality” constraint on the admissible allocations. Part of this criterion is that workers in one cooperative cannot improve their lifetime utility by attracting a *willing* worker from another cooperative. After establishing our equilibrium concept, we provide an equilibrium-selection criterion which minimizes informational requirements. We explain later how our equilibrium concept borrows from and extends existing ideas in cooperative game theory, as well as how it relates to the literature on matching.

After developing the framework and the equilibrium concepts for a cooperative economy, we study a couple of examples. In these examples, we characterize the growth path of the cooperative economy and compare it to the growth path of the same economy when production takes place in the “standard” capitalist firms which feature in neoclassical growth theory. Our analysis is based on choices of technology and preferences for which we are able to develop qualitative, or at least quantitative results.

In our examples, the cooperative economy converges to a steady state level of income per efficiency unit of labor - just as the capitalist economy is well known to do. In general, steady state income, consumption, and welfare can be higher or lower in the cooperative economy, depending on parameter values. Still, there are some systematic differences. We uncover a form of *static inefficiency* in the cooperative economy: for a given aggregate capital stock, worker cooperatives are inefficiently small (or, equivalently, there are too many firms in the cooperative economy). On the other hand, in our one fully-solved example the capitalist economy features potential over-accumulation of capital, while the cooperative economy is always dynamically efficient. We

provide an exact decomposition of steady-state income differences between the cooperative and the capitalist economy into a *static efficiency* component and a *capital accumulation* component. The static efficiency component always favours the capitalist economy but, if the cooperative economy saves considerably more than the capitalist one, this can more than compensate for its lesser static efficiency, resulting in higher steady-state output and welfare.

We calibrate our model’s preference and technology parameters by matching the capitalist version of the model to relevant US data moments. In our baseline calibration the steady state output of the cooperative economy is 73% of what it is in the capitalist economy, resulting in a 28% welfare loss. All of this output gap is due to the static inefficiency of cooperatives: the aggregate saving rate is in fact slightly higher in the cooperative economy.

Needless to say these results are illustrative and more in the nature of a “proof of concept” for the modelling framework. Many of the failures that have led to dissatisfaction with capitalism, such as market power and concentration, environmental and other types of externalities, asymmetries in bargaining power vis-a-vis workers and labour-market monopsony, and others are omitted from the model, and so the deck is stacked against the cooperative economy. As we discuss in the Conclusions, our long-term goal is indeed to develop a quantitative framework which we can enrich with some of these elements, for a more objective comparison of the welfare properties of the two systems.⁸

The Golden Era of the theoretical economic analysis of worker cooperatives was the period between the late 1950s and the late 1970s, when some of the stars of the profession took an interest in the topic. Ward (1958), Domar (1966), and Sen (1966) set up static, partial equilibrium models focused on the determination of cooperative labor input (on the extensive and/or intensive mar-

⁸In an additional set of results, we study the *coexistence* of cooperative and capitalist firms. In particular, we (i) extend the theoretical framework to analyze dynamic equilibrium in a mixed capitalist and cooperative economy; and (ii) use the quantitative economy of Section 5 to study numerically whether there can be steady-state equilibria with only capitalist firms, only cooperatives, or coexistence of the two forms (hence, one limitation is that we do not have transitional dynamics – in particular, we have no convergence results to these various kinds of steady states). The result of this latter exercise is to discover that, depending on parameter values, all three configurations are possible: under some parameter values the existence of capitalist firms makes coops unviable; but the reverse is also true: in other regions of the parameter space only coops survive competition with capitalist firms. Finally, in yet another region the two types of firms coexist. These results are available in the Unpublished Appendix of this paper.

gin). Vanek (1970), Drèze (1976, 1989), Ichiishi (1977), Greenberg (1979), Drèze and Greenberg (1980), and Laffont and M. Moreaux (1983) provided general equilibrium analyses, and established conditions for the existence and Pareto optimality of equilibria in economies constituted by worker cooperatives.⁹ However, their analyses were still static. Furubotn and Peyovich (1973) and Furubotn (1976) argued that this gave them a blind spot for the anti-investment bias arising from the limited planning horizons of traditional cooperative members, who lose property rights in the cooperative's assets when they leave the firm.¹⁰

Conceptually, our paper can be understood as a step towards marrying Vanek and Dreze's general equilibrium analysis with Furubotn and Peyovich's dynamic (but partial equilibrium) one - while at the same time proposing a solution to the Furubotn and Peyovich critique (in the form of giving former workers a claim on current distributions).¹¹ However the modelling framework is completely different and much more in line with recognizable modern macroeconomic practice.

Subsequent theoretical developments have returned to concerns, originally voiced by Alchian and Demsetz (1972), with cooperative members' incentives to provide effort (e.g. Holmström (1982), Kremer (1997)). Solutions to this problem have been identified in peer monitoring (e.g. Mirrlees (1976), Puterman (1982)),¹² and the repeated nature of the interaction among coop members, giving rise to the extension of the Folk Theorem to so-called "partnership games" (Radner (1986), Radner, Myerson, and Maskin (1986), Fudenberg, Levine, and Maskin (1994), and a conspicuous following). In order to keep the focus on the macroeconomic implications, in this paper we abstract from the intensive margin of effort. We do however note that, as pointed out by Bonin, Jones, and Puterman (1993), and confirmed in many successive surveys, shirking

⁹Or "labor-managed firms," as earlier writers prefer to call them, or the less politically-loaded "partnerships" which features most frequently in post-1990 writings. We use "worker cooperative," "producer cooperative," and "labor-managed firm" interchangeably.

¹⁰Atkinson (1972) and Sapir (1980) also attempted to inject dynamic considerations in the Ward (1958) model, but were not able to produce significant insights. The contributions cited here are only the landmarks of what became a huge literature full of extensions and generalizations of the results in the key papers. The *Journal of Comparative Economics*, in particular, was largely devoted to the study of labor-managed organizations well into the 1980s. A very comprehensive review of this literature (up to the mid-1980s) is in Bonin and Puterman (1987).

¹¹The implicit assumption being that, if our society turns to the cooperative mode of production, it will do so based on the best practice available.

¹²In particular, cooperative workers have much greater incentives to monitor each other's effort than subordinate employees on a fixed salary.

by workers or managers is virtually never reported as a concern in studies of real-world producer cooperatives.¹³

In the last two or three decades the focus of the research effort on worker cooperatives (and more generally of forms of worker participation in profit and/or management) has shifted from the development of theoretical models to the mobilization of empirical evidence. Excellent recent surveys of this large literature, which collectively covers a large variety of countries and industries, can be found in Pencavel (2013), and Jones (2018). Generally speaking, the evidence suggests that worker cooperatives tend to be (somewhat) more productive than conventional firms, to afford their workers greater income stability and job satisfaction, and to display comparable exit and investment rates. It must be acknowledged, however, that only rarely are these empirical results immune from concerns regarding selectivity.¹⁴

The paper is organized as follows. Section 2 describes the physical environment, including technology, demographics, and preferences. Section 3 describes the institutional setup with which we represent the “capitalist” system, and the maximization problems and equilibrium conditions that derive from it. These are familiar to all economic students. Section 4 sets out institutions, maximization problems, and equilibrium conditions for a cooperative economy. This is the main conceptual and methodological contribution of the paper. Section 5 solves the model, both under capitalist and under cooperative institutions, for the case in which individuals derive log utility from consumption and production is Cobb-Douglas. For this example we are able to

¹³Our deterministic environment also means that we abstract from differences in risk diversification between capitalist and cooperative economies. The theoretical literature has generally pointed to countervailing risk-diversification mechanisms operating in the two economies. Capitalist firms do a superior job with the diversification of capital income (e.g. Meade, 1972), but cooperatives are more likely to insulate workers from labor income volatility, particularly as arising from unemployment risk (Steinherr and Thisse, 1979, Miyazaki and Neary (1983), Bonin, 1983) and, thanks to their more equalitarian pay structure, provide better insurance against idiosyncratic productivity shocks (e.g. Lang and Gordon, 1995, Kremer, 1997). Hansmann (1996) reviews empirical evidence showing that cooperatives have more stable employment and that they are often found in highly capital-intensive and high-volatility industries, and concludes that, on balance, differences in risk diversification are probably not first order in comparing the two types of institutions. See also Drèze (1985) for equivalence results between stochastic capitalist and cooperative economies.

¹⁴We should mention a healthy parallel literature on other types of cooperatives. For example, Rey and Tirole (2007) study cooperative investment by groups of firms, and Hart and Moore (1996, 1998) study consumer cooperatives. We should also cite an important 1980s research program on profit sharing (e.g. Weitzman (1984, 1985), Meade (1986)), which had a particular focus on its potential role in dealing with stagflation.

develop closed from solutions and make a number of general statements about the comparative growth paths of the two economies. Section 6 presents a calibration of the model with slightly more realistic preferences and derive the main quantitative results. Section 7 evaluates the dependence of our numerical results on variations in the parameters and performs comparative statics exercises. Section 8 discusses some of the many directions in which we hope to take this project in future work, both to probe the robustness of our preliminary results and to investigate additional issues, such as inequality.

2 Physical Environment

As noted in the Introduction, a critical economic feature of producer cooperatives is the finite planning horizon of self-managing workers. These workers know that benefits accruing to the coop after they have left may escape them, potentially leading to severe under-investment (and failure to implement other choices with back-loaded returns). These considerations need to be taken into account when choosing the appropriate modelling of demographic. The simplest option is a two-period overlapping-generations framework, in which agents only work when young (for a profit-maximizing firm in the capitalist economy; as members of a cooperative in the cooperative economy), but consume when both young and old. This means that young cooperative workers make decisions which will affect cooperative outcomes after they have stepped down from their membership, as is the case in real-world cooperatives. The fact that all the workers making decisions within a cooperative are identical allows to identify an unambiguous objective for the coop, namely the maximisation of the utility of all its current workers.¹⁵

¹⁵In a model with a more general dynamic structure different workers in the same cooperative would have different horizons and different employment histories. Since workers accumulate claims on their previous cooperatives' revenues, these would create heterogeneity in preferences within the decision makers of a cooperative regarding investment decisions. For example workers closer to retirement may have different preferences vis-à-vis investment to workers further away from retirement. The problem of heterogeneous horizons might be solved within an infinite-horizon framework by using a dynamic model à la Barro or a perpetual-youth model à la Blanchard, but still additional "tricks" would be required to make former employment histories (namely the fact that workers will have accumulated claims of potentially differing expected value against previous employers) irrelevant for their preferences regarding the firm's current investment decisions. While clever devices along these lines could certainly be introduced, we do not think that pursuing them would move the model in the direction of greater

Formally, we endow agents with utility function

$$U(c^Y, c^O),$$

where c^Y (c^O) is consumption of the economy's final good when young (old). All agents in a generation are identical, and each young agent supplies one unit of labour inelastically. For simplicity, we assume that the population is constant and denote L the mass of each generation.¹⁶

There exists a technology that uses capital and labour as inputs to produce the final good according to the production function

$$F(k, l).$$

The capital used for production fully depreciates across periods, while investment of the final good generates new capital on a one-for-one basis.

3 Capitalist Economy

Our capitalist benchmark is a standard competitive equilibrium where profit-maximizing firms can enter and exit freely; young workers supply labour, consume and save in the form of capital; old workers rent out the capital they saved and use the proceeds to finance consumption; and all agents are price takers.

The prices of labor and capital at time t are denoted w_t and r_t , and they are in terms of the final good, which acts as numéraire. Conditional on entry, individual firms maximize profits taking current prices as given:

$$\pi(r_t, w_t) = \max_{k, l} \{F(k, l) - r_t k - w_t l\},$$

realism. Having said all this, it has to be acknowledged that conflict of interest among workers has been stressed as a key weakness of cooperatives by Hansmann (1996), though we don't know of compelling empirical evidence in support of this view. Other than the omission of an analysis of such potential conflicts, we struggle to conceive of insights about growth in a cooperative economy which would be fundamentally different in a more complex demographic framework, or an infinite-horizon one, from those we identify in our simpler OLG model.

¹⁶Extending the model to feature population growth is relatively easy, see Appendix A.11.

with factor demands denoted: $k(r_t, w_t)$ and $l(r_t, w_t)$. We assume that these factor demands are single-valued so that all active firms behave symmetrically, and we can omit firm subscripts.

Capital is owned by individuals. We assume that each period-0 old agent is endowed with some initial capital stock κ_0 .¹⁷ In each subsequent period, old workers can sell their savings in the form of capital stock κ_t at the market price r_t . At the same time, young workers become old capitalists by saving some of their labor income. In particular, the young solve the following program:

$$\begin{aligned} & \max_{c^Y, c^O, \kappa_{t+1}} U(c^Y, c^O) \\ \text{s.t. } & c^Y + \kappa_{t+1} = w_t \\ & c^O = r_{t+1}\kappa_{t+1}. \end{aligned}$$

The solution to this problem defines the optimal capital investment as a function of the prices w_t and r_{t+1} :

$$\kappa_{t+1} = \mathfrak{K}(w_t, r_{t+1}).$$

In equilibrium, markets for capital, labor and the final good clear, and free entry and exit drive firms' profits to zero. Denoting N_t the equilibrium measure of operating firms, the competitive equilibrium in each period is characterised by the following system:

$$\begin{aligned} \pi_t(r_t, w_t) &= 0 \\ N_t l(r_t, w_t) &= L \\ N_t k(r_t, w_t) &= L\kappa_t. \end{aligned}$$

A solution to this system defines the equilibrium prices and the number of firms as functions of the state variable κ_t : $r_t = r(\kappa_t)$, $w_t = w(\kappa_t)$, $N_t = N(\kappa_t)$. It follows that the dynamics in this

¹⁷Heterogeneous capital endowments among the initial old could trivially be allowed for, but all heterogeneity would immediately disappear with the first young generation.

economy are characterised by the following capital accumulation equation:

$$\kappa_{t+1} = \mathfrak{K}(w(\kappa_t), r(\kappa_{t+1})).$$

4 Cooperative Economy

This section works its way to the construction of a general-equilibrium concept for a dynamic cooperative economy. We begin by formalizing the concept of cooperative, and identifying the decisions which cooperative members make. Then we take up the more complex task of analysing how these decisions interact at the aggregate level and, in particular, we discuss the allocation of labour in the absence of a wage rate.

4.1 Concept of a Cooperative

Our conceptualization of cooperatives stresses two features which seem to most clearly distinguish this mode of organization from standard, externally-owned corporations: collective decision making by workers (labor management) and the non-tradability of productive assets. Self-management implies that decisions concerning the cooperative's size and investment are made collectively by the current workers of the cooperative. In our simplified context, where all workers are identical, this means that the objective function of the cooperative is the maximization of the present value of the lifetime utility of its current workers. Non-tradability means that capital is directly owned by the cooperative.

Any period t begins with a set of *incumbent* cooperatives, indexed by $i \in I_t$. An incumbent cooperative i is characterized by an inherited capital stock k_{it} and a set of former workers l_{it-1} . Each incumbent cooperative is allocated a set of workers l_{it} via a mechanism which we describe later (Section 4.3). These workers produce output $y_{it} = F(k_{it}, l_{it})$ (where we use l to denote both the set and the number of workers). A share τ of this output is immediately distributed to the former workers. Next, the current-period workers decide how much of the cash flow

(net of payments to the old) should be invested to put in place capital to be used in the next period, k_{it+1} . All non-retained earnings are distributed equally among current-period workers. These assumptions result in the following consumption levels for a representative young worker of incumbent cooperative i in period t :

$$c_{it}^Y = \frac{(1 - \tau)y_{it} - k_{it+1}}{l_{it}},$$

$$c_{it+1}^O = \frac{\tau y_{it+1}}{l_{it}}.$$

The sharing rule τ provides young workers with an incentive to agree on the retention of earnings for the purposes of investment. It should be clear from the equations above that if $\tau = 0$ young workers will wish to set k_{it+1} to 0 as well. This is the Furubotn-Peyovich critique of traditional cooperatives as it manifests itself in our model. In our view this critique largely explains why many traditional cooperatives tend to remain small over their life cycle, while those with post-retirement attachment (such as those in the Mondragon system) tend to flourish.

In the main text we treat τ as a constitutional principle of the cooperative, which entitles former workers to keep sharing in the coop's distributions. However in Appendix A.9 we generalize our model to nest an intergenerational game in which, in each period, the decisions whether to honor the payment τ is taken optimally by the young. The construct is in the spirit of a literature on endogenous pay-as-you-go social-security systems, conceived as time consistent equilibria in infinite-horizon intergenerational games (e.g. Kandori (1992), Cooley and Soares (1999)). In those models, as in ours, these transfers from the young to the old are supported by trigger strategies: young workers not making the transfer forego access to the transfer themselves when old. The difference from that literature is that our arrangement is essentially a *within firm* pay-as-you go system - rather than a society-wide one. We show that *all* of the analysis presented in the text of the paper is robust to the generalized version with endogenous τ . In particular, there is a range of values of τ which can be supported in dynamic equilibrium. We characterize this range. In the quantitative analysis, we make sure that the chosen τ falls within this range.¹⁸

¹⁸An implication of the analysis is that τ needs not be the same across coops. Imposing symmetry obviously

We focus on symmetric equilibria in which cooperatives adopt a perfectly egalitarian pay structure (within current workers and within former workers). We conjecture that standard arguments used in the context of capitalist economies could still be deployed to rule out equilibria with inequality within generations. In particular, workers receiving below-average pay in one cooperative could offer to undercut workers receiving above-average pay at another cooperative.

4.2 Continuation, Entry and Exit

The previous subsection describes the consumption of workers allocated to a *continuing* incumbent cooperative, which in our framework is an incumbent cooperative which is allocated some positive young membership l_{it} .

Our model also allows for entry and exit of cooperatives. Entering cooperatives have no capital stock, so they produce with labour only. They also have no former workers. Hence, young-worker consumption in an entering cooperative is $c_{0t}^Y = \frac{F(0, l_{0t}) - k_{0t+1}}{l_{0t}}$ - where we use the subscript 0 for workers belonging to, or inputs and outputs of, entering cooperatives.

An exiting cooperative at time t is an incumbent which is assigned no workers by the worker-allocation mechanism. Such a cooperative produces zero output and its capital stock is left idle. Because of full depreciation this cooperative does not continue to period $t + 1$. Note that the consumption of old workers attached to exiting cooperatives is 0.¹⁹

simplifies the analysis. Beyond that, which particular value of τ is selected from within the equilibrium range is not particularly important for the theoretical analysis. When we come to the quantitative work, we will assume that τ is chosen to maximize steady-state utility.

¹⁹We do not formally include the possibility of mergers. To formally include mergers, we should introduce a new decision stage. In particular, after workers have been allocated to incumbent coops and/or have created new coops, before production takes place workers from different coops could enter in agreements to pool their capital stock and work jointly in the enlarged firm. This would be a very challenging extension to work out because the allocation mechanism would be affected by expectations about how this merging game will play out. A justification for ruling out mergers could possibly be based on an irreversibility argument. When a new generation of workers enters the economy, the landscape is dotted by discrete lumps of capital, i.e. the incumbent coops. If this capital is very costly to reconfigure (e.g. literally move around in space) in the short term, mergers may be prohibitively costly. This same argument about irreversibility may also be used to justify abstracting from coop division, but, in any case, abstracting from division strikes us as much less restrictive. Because of the rationality requirement we have imposed on the worker allocation mechanism, a coop never has “too many workers.” And, given the number of workers “in possession” of an incumbent’s capital stock, it is very hard to think of reasons why they would want to split up in smaller units, at least under conventional technological assumptions.

4.3 Equilibrium Concept for the Cooperative Economy

We now discuss, jointly, how labor is allocated to cooperatives and how cooperatives make their investment decisions. Informally, we have in mind a decentralized mechanism in which workers are able to move freely into cooperatives, as long as these are willing to accept them. Therefore, workers sort into the cooperatives which generate highest utility levels until the market clears. This process takes into account the possibility that groups of workers might create a new cooperative without any initial capital. On the other hand, any remaining cooperative without any worker willing to join exits. Once workers have been allocated to cooperatives and production has taken place, workers collectively decide on the amount of earnings that should be retained to put in place as capital for the next period. In making this decision workers take into account the implications of the worker-allocation mechanism for the number of young workers joining the cooperative in that period.

Formally, in each period, the economy is characterised by a set of incumbent cooperatives I_t , and by a distribution of initial capital stocks: $\{k_{it}\}_{i \in I_t}$. For convenience, we assume the set of cooperatives is located on a continuum, and that in each period the set of incumbents I_t is a subset of the real line with finite Lebesgue measure. Denote $\bar{I} = \mathbb{R}$, with the interpretation that $\bar{I} \setminus I_t$ is the set of potential entrants. An arbitrary allocation of workers is a measurable function $l : \bar{I} \rightarrow \mathbb{R}_+$ with support of finite measure, such that:

$$\int_{\bar{I}} l_i di = L.$$

Note that entry and exit are captured by the fact that the support of l is not restricted to coincide with I_t . Note also that we impose full employment, assuming that any group of unemployed workers would optimally create a new cooperative.²⁰ The set of all such allocations is denoted \mathfrak{L} .

²⁰We do not mean to suggest that a cooperative economy would be less (or more) prone to some frictional unemployment than a private-ownership economy. Our omission of search frictions is purely to focus on long-run analysis, as is standard in growth theory. We regard the addition of search and matching frictions to a model of a cooperative-based economy as an interesting area of research to learn more about the business-cycle properties of these economies. Such an endeavor would be particularly fruitful since, as mentioned earlier, empirical evidence suggests that employment is less cyclical in worker cooperatives.

Our relevant equilibrium object is a worker allocation mechanism, that is a mapping:

$$(I_t, \{k_{i,t}\}_{i \in I_t}) \mapsto \mathbb{L}(I_t, \{k_{i,t}\}_{i \in I_t}) \in \mathfrak{L}.$$

Given such a mechanism and any current allocation of workers, the continuation of the economy is characterised by optimal investment decisions taking as given the behaviour of other cooperatives. Denote $\mathcal{U}_i(l)$ the continuation utility of the young workers assigned to cooperative $i \in \bar{I}$ by allocation l , with the convention that $\mathcal{U}_i(l) = -\infty$ if i is not allocated workers by l . That is, for an incumbent cooperative:

$$\mathcal{U}_i(l) = \max_{k_{i,t+1}} U \left(\frac{(1-\tau)F(k_{i,t}, l_i) - k_{i,t+1}}{l_i}, \frac{\tau F(k_{i,t+1}, \mathbb{L}_i(I_{t+1}, \{k_{j,t+1}\}_{j \in I_{t+1}}))}{l_i} \right),$$

while for an entering cooperative:

$$\mathcal{U}_i(l) = \max_{k_{i,t+1}} U \left(\frac{F(0, l_i) - k_{i,t+1}}{l_i}, \frac{\tau F(k_{i,t+1}, \mathbb{L}_i(I_{t+1}, \{k_{j,t+1}\}_{j \in I_{t+1}}))}{l_i} \right).$$

The last two expressions define the utility level workers can rationally expect by joining the various cooperatives in the economy. At the same time, they provide information to workers who have joined a particular cooperative about the consequences of allowing further workers to join in. Hence, we can use these objects to define an equilibrium as one in which there exist no reallocation in which the transfer of a worker to a different cooperative makes *both* this worker and the original members of this cooperative better off.

Formally, an *Equilibrium with Cooperatives* is a sequence of distributions of capital stocks $\{k_{it}\}_{i \in \bar{I}}$ and a mapping \mathbb{L} , such that for any state $(I_t, \{k_{\iota,t}\}_{\iota \in I_t})$, any allocation $l \in \mathfrak{L}$, and any two cooperatives $i, j \in \bar{I}$, if $l_i < \mathbb{L}_i(I_t, \{k_{\iota,t}\}_{\iota \in I_t})$ and $l_j > \mathbb{L}_j(I_t, \{k_{\iota,t}\}_{\iota \in I_t})$, then:

$$\mathcal{U}_i(\mathbb{L}(I_t, \{k_{\iota,t}\}_{\iota \in I_t})) \geq \mathcal{U}_i(l), \quad (1)$$

and

$$\text{either } \mathcal{U}_i\left(\mathbb{L}(I_t, \{k_{\iota,t}\}_{\iota \in I_t})\right) \geq \mathcal{U}_j(l), \quad (2)$$

$$\text{or } \mathcal{U}_j\left(\mathbb{L}(I_t, \{k_{\iota,t}\}_{\iota \in I_t})\right) > \mathcal{U}_j(l). \quad (3)$$

Note that our equilibrium concept is of the Markov variety, in that the worker allocation mechanism, and the continuation values on which it is based, are allowed to depend only on the current distribution of capital stocks.

In words, we are considering a feasible reallocation of workers across cooperatives, including a movement of workers from cooperative i to cooperative j .²¹ Condition (1) says that in an equilibrium this reallocation must not be beneficial to the remaining workers of cooperative i (or these workers would wish to reduce the membership). Furthermore, either the reallocation does not make the reallocated workers better off [condition (2)], or it makes the workers of the receiving cooperative worse off [condition (3)]. Note that the subscripts i and j can equally apply to continuing, entering, and exiting cooperatives.

Our equilibrium concept for a dynamic cooperative economy has elements in common with equilibrium concepts in cooperative game theory as well as in models of matching. Cooperative game theorists study coalition formation and typically seek stable coalition structures which, like in our model, are robust to defection from subsets of agents.²² However in our model the “coalition formation game” is re-played in every period by a new set of agents and, more importantly, the entire distribution of investment decisions taken by the coalitions that exist at time t operate as state variables for the time $t + 1$ game, and in turn this game’s outcome is payoff relevant for agents making decisions at time t . In this sense, the coalition-formation aspect of the model is much more complex than in typical cooperative games, and the definition of equilibrium had to be generalized accordingly. This is compensated to a considerable extent by the fact that we

²¹We emphasize that a reallocation is not restricted to only include the aforementioned movement of workers from i to j . It is possible that some other workers leaving i also join a different cooperative $j' \neq j$, or that workers joining j do not all come from i but possibly an alternative cooperative $i' \neq i$. Importantly, conditions (1) - (3) are required to hold *for any* pair i and j such that i would lose workers and j gain extra workers.

²²Indeed the tools of cooperative game theory have been deployed for the study of (static) cooperative economies. See, e.g., Ichiishi (1977), Greenberg (1979), Drèze and Greenberg (1980), and Farrell and Scotchmer (1988).

work with a homogeneous-agent model.

In the previous paragraph the first reference to a “coalition formation game” was hedged by quotation marks, because this terminology is arguably slightly misleading. In typical cooperative games coalitions are formed in a sort of vacuum, and the output of the coalition depends exclusively on its size and composition. In our model, however, workers form coops around and inside existing lumps of capital. They don’t so much form coalitions but they attach themselves to an existing coop – represented by the stock of capital inherited from the past (and its former workers). In this sense, our equilibrium concept is as much about forming coalition as it is about matching workers to incumbent coops – hence the link with the matching literature. Compared to the matching literature, however, we offer a somewhat axiomatic definition of equilibrium (based on stability from deviations) rather than the more standard description of a search environment.²³

4.4 Further Equilibrium Conditions

The concept of equilibrium in the previous section is inspired by minimal requirements of rationality and efficiency. Needless to say, these general principles are hardly sufficient as a basis for a study of economic growth in a cooperative economy. What is needed is a further refinement allowing us to focus on a subset of equilibria which are tractable for the modeller, and do not impose unrealistic information requirements on the agents in the model. In particular, the generic decentralization of the equilibrium definition in the previous section requires knowledge by each agent of the strategies of all agents in all future generations. This is in sharp contrast to the equilibrium in the capitalist economy where agents need only know current wages and interest rates.

The particular restriction we impose on our equilibria is as follows: the worker allocation mechanism assigns to each incumbent cooperative a number of workers which depends only on

²³See, however, Sasaki and Toda (1996), and a small following literature on matching with externalities. Our mechanism to assign workers to coops is similar to the concept of *Optimistic Stability* in the working paper version of their article. In particular, when evaluating the stability of an allocation, no alternative reallocation is a priori deemed unreasonable.

that cooperative's capital stock k_{it} . Formally, we only consider equilibria in which, for $t > 0$, there is a mapping $\mathcal{L}(k_{jt})$ such that $\mathbb{L}_j(I_t, \{k_{it}\}_{i \in I_t}) = \mathcal{L}(k_{jt})$ for $j \in I_t$. Note that the above is a statement about the allocation of workers only to incumbent cooperatives on path. We do not impose restrictions on the allocation of workers to entering cooperatives.

It can easily be seen that if $\mathcal{L}(k_{jt})$ is an allocation mechanism in an equilibrium as defined in the previous section, then each cooperative has an investment policy rule which also depends only on that cooperative's capital stock, $\mathcal{K}(k_{jt})$. Furthermore, in Appendix A.2 we prove that the following functional equation holds:

$$(\mathcal{L}(k_{jt}), \mathcal{K}(k_{jt})) \in \arg \max_{l, k} U\left(\frac{(1-\tau)F(k_{jt}, l) - k}{l}, \frac{\tau F(k, \mathcal{L}(k))}{l}\right). \quad (4)$$

In words, focusing only on equilibria in which an incumbent's allocation of workers depends only on that incumbent's initial capital stock is equivalent to focusing on equilibria in which each incumbent cooperative chooses current employment and investment so as to maximize the utility of current young workers, taking as given the fact that all future generations will follow the same strategy. Importantly, this maximization is unconstrained.

It is important to stress some implications and limitations of our refinement. Equation (4) implies that, for $t > 0$, incumbent cooperatives are never constrained in the number of members they can attract, i.e. we are implicitly ruling out growth paths along which cooperatives would like to attract more members, but are prevented from doing so because all workers are already "taken up" by other coops. Nevertheless, in Sections 5 and 6 we show by example that under standard growth-theoretic assumptions about preferences and technology equilibria fulfilling our operational concept emerge naturally.

Importantly, in our refinement the independence of the labour allocation from the full distribution of capital stocks to incumbents only applies for $t > 0$. Hence, we allow for the possibility that, at time 0, there are "too many coops for too few workers," in the sense that the unconstrained optimal membership of at least some coops exceeds the number of members the coop can attract. In the examples we work out later, we will see that this can result in a burst of

exit at time 0. The possibility of exit at time 0 due to insufficient access to workers is generally useful because it makes the existence of equilibria independent of the initial distribution and size of the capital stock, and thus makes it potentially possible to study “MIT-type” shocks, i.e. unanticipated permanent changes in endowments or technology.

Another important feature of our refinement is that it is fully consistent with entry and imposes no restriction on the allocation of workers to cooperatives - other than the restriction imposed by the aggregate labour supply. In particular, it must be the case that:

$$\forall t, \quad \int_{I_t} \mathcal{L}(k_{it}) di \leq L.$$

Therefore, in any period, once incumbents have been allocated workers, new entering cooperatives are created and allocated workers. This allocation of workers to new cooperatives follows the restrictions imposed in section 4.3, and in particular takes into account the economy’s resource constraint in terms of labour supply. Importantly though, cooperatives may be constrained only upon entry, but expect to be allocated workers as incumbents in future periods according to the mapping \mathcal{L} .

In an equilibrium as defined in this section all behaviour is pinned down by an initial distribution of capital stocks and the mappings \mathcal{L} and \mathcal{K} . The capital accumulation dynamics within a cooperative are pinned down by the equilibrium mapping:

$$k_{jt+1} = \mathcal{K}(k_{jt}).$$

As cooperatives in the economy may differ only in their capital stock, we can then easily study aggregate dynamics as resulting from the sum of individual independent decisions using the same mapping \mathcal{K} . The precise algorithm we follow to solve for the equilibrium is detailed in Appendix A.3.

5 An Example with Closed Forms

In this section, we use specific functional forms for preferences and the production technology which allow us to characterise analytically employment as well as the capital accumulation dynamics both in the capitalist and cooperative economies. We use these results to compare the two economies in terms of output, efficiency, and welfare.

The production function for production units with positive inputs ($k, l > 0$) takes the form

$$F(k, l) = Ak^\alpha(l - \underline{l})^\beta, \quad (5)$$

where $A > 0$, $\underline{l} \in (0, L)$, $\alpha > 0$ and $\beta > 0$ are constant parameters.

Relative to the familiar neoclassical growth model, this production function features the slightly unusual property that there is a fixed cost, in the form of a minimum of \underline{l} units of labor which are required independently of the scale of operation. This assumption is a direct legacy of the older static literature on cooperatives, which showed that in the absence of a fixed cost of production there is no equilibrium with positive cooperative size.²⁴ The intuition will be apparent below. Needless to say the assumption that production involves fixed costs is entirely realistic.

Since fixed costs of production introduce a form of increasing returns to scale, in order for the model to have an equilibrium under the capitalist form of organization we need decreasing returns to scale in the variable inputs, i.e.

$$\alpha + \beta < 1.$$

The assumption of decreasing returns to variable inputs is also realistic, and it is usually motivated by span-of-control considerations.

²⁴More accurately the existence of cooperatives requires that at low levels of membership the marginal product of labour exceeds average income. In models of capitalist economies the omission of fixed costs of production is without loss of generality due to the replication argument. This is not the case in modelling cooperatives.

Since the cooperative model features potential entry, we will also need an assumption for production in production units with $k = 0$. However, it will turn out that we do not need a specific functional form. Hence, for now we simply assume that $F(0, l) > 0$. We will add some mild restrictions to this below.²⁵

As for preferences, in order to derive closed form results we assume for now that agents obtain log-utility from consumption, with a discount factor $\delta \in (0, 1]$:

$$U(c^Y, c^O) = \log c^Y + \delta \log c^O.$$

5.1 Capitalist Economy

Using these functional forms, we can solve for the capitalist equilibrium as outlined in section 3. The procedure to find the equilibrium is entirely standard and hence we relegate the details to Appendix A.4. Here we only discuss the main aspects of the equilibrium.

The only slightly unfamiliar feature of the capitalist equilibrium is that, because of the fixed production cost, it features an optimal firm size:

$$l_{cap} = \frac{1 - \alpha}{1 - \alpha - \beta} l, \quad (6)$$

where the subscript *cap* will be helpful later to distinguish firm size in a capitalist equilibrium from firm size in the cooperative economy. The optimal firm size would generally depend on state variables, such as the aggregate capital stock. This will be the case in the example in the next section. However, under the particular combination of functional forms in this section, the optimal firm size is constant over time both under capitalist and under cooperative arrangements. It is this constancy that allows us to solve the model in closed form, and hence it is a valuable simplification.

²⁵Assuming a different functional form for the production function in units with $k > 0$ and units with $k = 0$ may seem unpalatable. In the Unpublished Appendix to this paper we repeat the analysis with a more general production technology that are common to all firms. Equilibrium outcomes are identical and, in particular, all the formal and quantitative results from Section 5.3 onwards are identical.

Despite this slightly unfamiliar feature the dynamics of the economy are qualitatively the ones we have come to expect from standard growth models. In particular, individual capital holdings evolve according to

$$\kappa_{t+1} = \frac{\delta}{1+\delta} A(1-\alpha)^\alpha \beta^\beta \left(\frac{1-\alpha-\beta}{\underline{l}} \right)^{1-\alpha-\beta} \kappa_t^\alpha, \quad (7)$$

where, recall, κ_t is the savings decision by a member of the period t young. It follows from this functional form that κ_t converges to a steady-state value.

5.2 Cooperative Economy

We study the cooperative economy following the approach presented in section 4.4. We follow a “conjecture and verify” strategy. The conjecture is that in the equilibrium, if one exists, cooperative firm size is constant, or $\mathcal{L}(k) = l_{coop}$. If this is so, then the cooperative solves the problem:

$$\max_{l_t, k_{t+1}} \log \left(\frac{(1-\tau)Ak_t^\alpha(l_t - \underline{l})^\beta - k_{t+1}}{l_t} \right) + \delta \log \left(\frac{\tau Ak_{t+1}^\alpha(l_{coop} - \underline{l})^\beta}{l_t} \right). \quad (8)$$

The necessary and sufficient first-order conditions for this problem are:

$$-\frac{1}{(1-\tau)Ak_t^\alpha(l_t - \underline{l})^\beta - k_{t+1}} + \frac{\alpha\delta}{k_{t+1}} = 0, \quad (9)$$

$$\frac{\beta(1-\tau)Ak_t^\alpha(l_t - \underline{l})^{\beta-1}}{(1-\tau)Ak_t^\alpha(l_t - \underline{l})^\beta - k_{t+1}} - \frac{1+\delta}{l_t} = 0. \quad (10)$$

Equation (9) describes the optimal reinvestment of earnings. The first term is the marginal utility loss from diminished current consumption from an extra unit of investment, while the second term is the marginal utility gain from the extra output that investment will deliver next period. First order condition (10) determines the optimal current employment level l_t . Here the trade-off is that an extra worker has a positive marginal impact on current output (first term) but also a negative marginal impact on the share of other workers both in the current period and in the next period, both of which effects are captured in the second term.

This system is easy to solve and yields:

$$\begin{aligned} l_t &= \frac{1+\delta}{1+\delta-\beta(1+\alpha\delta)} \underline{l} \equiv l_{coop}, \\ k_{t+1} &= \frac{\alpha\delta}{1+\alpha\delta} (1-\tau) A k_t^\alpha (l_t - \underline{l})^\beta. \end{aligned} \tag{11}$$

The first of these two equations shows that, when expecting a constant labour input in the next period, cooperatives choose a constant labour input in the current period. This both verifies our conjecture and defines the equilibrium cooperative size, l_{coop} . The second equation characterizes the investment policy of cooperatives. This policy inherits the conventional proportionality to current income associated with log utility. Plugging in the form of $l_t = l_{coop}$, we obtain the capital accumulation equation for a single cooperative:

$$k_{t+1} = \frac{\alpha\delta}{1+\alpha\delta} (1-\tau) A \left(\frac{\beta(1+\alpha\delta)}{1+\delta-\beta(1+\alpha\delta)} \underline{l} \right)^\beta k_t^\alpha, \tag{12}$$

which has the same qualitative features as those derived for the capital accumulation process of individuals in the capitalist economy. We define k_{coop}^* the steady state cooperative capital implied by (12). For later reference we also define $\mathcal{U}(k_{it})$ as the maximized value of (8). It is trivial (but important) to see that $\mathcal{U}(k_{it})$ is an increasing function: workers prefer joining incumbents with larger capital stocks.

To move now to a full characterization of the dynamics of the economy, as well as to complete the argument that the equilibrium sketched thus far exists, we must now consider the possibility of entry. It is easy to see that, in the equilibrium we are constructing, the allocation of labour to an entrant and the entrant's investment policy must maximize the objective

$$\log \left(\frac{F(0, l) - k}{l} \right) + \delta \log \left(\frac{\tau A k^\alpha (l_{coop} - l)^\beta}{l} \right). \tag{13}$$

Note that this problem is time invariant, so both entry size and the utility afforded to a young worker who helps forming a new cooperative are also time invariant. To facilitate the discussion

of dynamics we label \mathcal{L}_e the size of an entrant, \mathcal{K}_e its investment policy, and \mathcal{U}_e the utility experienced by a worker joining an entrant.

At any time t , it may conceivably be the case that $\mathcal{U}_e > \mathcal{U}(k_{it})$ for some incumbents i with sufficiently low capital stock. In this case, these incumbents will not be able to attract any workers and will have to exit. We define as $I_t^+ \subseteq I_t$ the set of incumbents at time t such that $\mathcal{U}_e \leq \mathcal{U}(k_{it})$. We can think of I_t^+ as the set of *viable* incumbents. As we will soon see, the key assumption we need to make to insure the existence of a cooperative equilibrium fulfilling put operational criteria is that $\mathcal{U}_e \leq \mathcal{U}(k_{coop}^*)$. In other words, an incumbent endowed with the steady state level of capital is viable. We refer to this as Assumption 1.²⁶

Define

$$N_{coop} \equiv \frac{L}{l_{coop}}$$

as the measure of incumbent cooperatives consistent with full employment when each cooperative operates at its optimal size l_{coop} . The dynamics of the economy, as well as the further assumptions (if any) required to establish the existence of the equilibrium, are slightly different in the case in which N_{coop} is smaller or larger than the initial endowment of viable cooperatives, $|I_0^+|$, where we use $|x|$ for the measure of set x .

Case 1: $N_{coop} \leq |I_0^+|$

In this case the N_{coop} incumbents with the largest capital stocks will scoop up all the workers in the economy at time 0, and each of them will employ l_{coop} workers. The $|I_0| - N_{coop}$ coops with the *smallest* capital stock (including some viable ones) will exit. No entry will occur as all continuing incumbents afford workers more utility. Moving to period 1, there are no non-viable incumbents. Those incumbents which had capital stock less than k_{coop}^* have experienced capital growth, so they are a fortiori viable in period 1. Even those incumbents which started in period 0 with capital in excess of k_{coop}^* are still viable in light of Assumption 1. Furthermore, since the existing viable incumbents are exactly N_{coop} , there are no workers left out and forced to create

²⁶This is an assumption that $F(0, l)$ is not too productive. If $F(0, l) = BG(l)$ one can always choose B low enough that Assumption 1 is verified.

a new cooperative. Hence, there is neither entry nor exit, and the same is true in all subsequent periods. Hence, each coop's capital stock evolves according to (12), and eventually, *the entire measure N_{coop} of cooperatives converge to the identical steady state level k_{coop}^** . Note that no further assumptions on $F(0, l)$ were required.

Case 2: $N_{coop} > |I_0^+|$

In this case the economy is not initially endowed with a measure of viable incumbents sufficient to absorb the entire young-worker population. Hence, while each viable incumbent will be assigned l_{coop} workers, there will have to be entry to employ the remaining $L - |I_0^+|l_{coop}$ workers. To fully describe the dynamics and establish existence of the equilibrium we then need further restrictions on $F(0, l)$. The first restriction (Assumption 2) is that the size of entrants is no less than the size of incumbents, or $\mathcal{L}_e \geq l_{coop}$.²⁷ The second restriction (Assumption 3) is that entrants become viable incumbents in the period after entry, or $\mathcal{U}(\mathcal{K}_e) \geq \mathcal{U}_e$.²⁸

With these assumptions, consider first the special case in which $\mathcal{L}_e = l_{coop}$. In this case there will be exactly a measure $N_{coop} - |I_0^+|$ of entrants at time 0. From there, just as in Case 1, there is no further entry or exit, and each coop once again converges to the capital stock k_{coop}^* . If instead $\mathcal{L}_e > l_{coop}$, the size of period-0 entrants will drop to l_{coop} in period 1, necessitating a further round of entry in that period to insure full-employment. This pattern of residual entry and subsequent shrinkage will continue until the measure of entrants shrinks to 0. From then on no further entry or exit occurs and once again we converge to a steady state with N_{coop} identical cooperatives, all

²⁷It may seem counter-intuitive to have entrants which are larger than incumbents, but our intuitions are based on observations of capitalist economies. There is no empirical basis to form a prior on whether entering cooperatives would be larger or smaller than incumbent ones.

²⁸A sufficient condition for Assumption 2 is that $F(0, l) = B(l - \underline{l}_e)^\gamma$, $\gamma \in (0, (1 + \alpha)/(1 + \alpha\delta))$, and $\underline{l}_e \geq [1 + \delta - \gamma(1 + \alpha\delta)] / [1 + \delta - \beta(1 + \alpha\delta)] \underline{l}$. This can be verified by substituting these assumptions into (13) and solving the maximization problem. If $\gamma = \beta$ and $\underline{l}_e = \underline{l}$ then $\mathcal{L}_e = l_{coop}$. As regards Assumption 3, if $F(0, l) = BG(l)$ one can always find a B small enough that the assumption is verified. Notice that since \mathcal{L}_e does not depend on B there is no possible tension between Assumptions 1, 2, and 3.

with capital k_{coop}^* .^{29,30}

5.3 Comparison

In this section, we compare economic performance in the two models along two dimensions: (i) static organization of production and efficiency, and (ii) capital accumulation. We also show how differences in steady state output can be exactly decomposed into two terms reflecting differences in these dimensions. We also include a quantitative comparison as a prelude to the subsequent quantitative section, which uses more realistic preferences.

5.3.1 Firm Size and Static Efficiency

Consider the choices of a planner whose intention is to make the economy *statically efficient*, i.e. to maximize aggregate output *for a given aggregate stock of capital K* . Because of the concavity of the production function, the planner will distribute the capital and labor endowments equally across whatever number of production units she chooses to have, so her problem is equivalent to identifying the optimal firm size.³¹ In this subsection we identify this statically-efficient firm size, and compare it to firm sizes in the capitalist and cooperative economies. Of course the welfare significance of static efficiency is limited, because overall efficiency also depends on the amount of capital in the economy, which in turn depends on dynamic considerations (which we take up in the next sub-section). Still, in our quantitative exercises differences in static efficiency turn

²⁹To understand why Assumption 2 is needed consider the consequences of entrants having scale smaller than l_{coop} . These entrants would have to *grow* in size to satisfy the conjectured equilibrium property that all incumbents are allocated l_{coop} workers. But this is clearly incompatible with the labor resource constraint, because there are not enough workers in the economy to allow all entrants to grow to size l_{coop} . Similarly, a violation of Assumption 3 would imply re-exit at time 1 of cooperatives which entered at time 0, but this violates the equilibrium requirement that all incumbents have membership l_{coop} for $t > 0$.

³⁰In the text we have implicitly assumed that incumbent firms do not have access to technology $F(0, l)$. This is clearly immaterial for Case 1. In Case 2 one could wonder whether non-viable incumbents might be able to avoid exit by switching to the labor-only technology. The answer is no, as any young worker joining an incumbent “inherits” the incumbent’s stock of former workers and is thus subject to the sharing rule. She is thus always better off striking out with a new venture.

³¹Aggregate output is $NF(K/N, L/N)$, where N is the number of production units the planner chooses to have. Using $l = L/N$ this rewrites as $L/lF(lK/L, l)$, and static efficiency is achieved by maximizing this with respect to l .

out to play an important role.

Using our functional assumption, the statically-efficient firm size is the solution to

$$\max_l \quad A \frac{(l - \underline{l})^\beta}{l^{1-\alpha}} K^\alpha L^{1-\alpha}.$$

As a result, the aggregate variables K and L do not affect the maximisation problem, and we can define

$$Z(l) = A \frac{(l - \underline{l})^\beta}{l^{1-\alpha}}$$

as a measure of static efficiency associated with any arbitrary firm size. Indeed, the social planner's objective is simply to maximise $Z(l)$ with respect to l . The larger $Z(l)$, the more statically efficient the economy. The socially optimal firm size trades off the following considerations: smaller firms allows the economy to spread variable inputs across more units, thereby reducing the impact of diminishing returns to variable inputs. On the other hand, the larger the measure of firms, the larger the amount of labor “wasted” because of the fixed cost \underline{l} .

The firm size l_{eff} (for “efficient”) which maximizes $Z(l)$ is easily derived from the first order condition, and the verdict on static efficiency is as follows:

$$l_{eff} = l_{cap} > l_{coop},$$

(where the last inequality is proved in Appendix A.5). Hence, the capitalist economy is statically efficient, but the cooperative economy features firm sizes which are inefficiently small.

There are two reasons why cooperatives are inefficiently small. First, unlike the social planner, cooperatives take their *current* capital stock as given. When they consider adding extra workers they only perceive the impact on the average product of labor. Instead, the social planner also takes into account that an extra worker increases the marginal product of capital, and that he can therefore counter the decline in the marginal product of labor by reallocating some extra capital to the production unit. The same happens in the capitalist economy, because extra workers

induce the firm to rent extra capital.

The second reason why cooperatives are inefficiently small (in a static sense) is the existence of the sharing rule. An extra worker today is an extra claimant to the payments that will accrue to old workers tomorrow. This is why the firm size in the cooperative economy is decreasing in the weight agents give to old-age consumption, δ .^{32,33}

5.3.2 Capital Accumulation and Dynamic Efficiency

Aggregating the capitalist law of motion (7) over individuals and the cooperative law of motion (12) over cooperatives, and making the appropriate substitutions, we easily see that both economies have laws of motion for the aggregate capital stock K_t of the form

$$K_{t+1} = sF(K_t, L),$$

with the corresponding aggregate savings rates

$$s_{cap} = \frac{\delta}{1 + \delta}(1 - \alpha), \quad (14)$$

and

$$s_{coop} = \frac{\alpha\delta}{1 + \alpha\delta}(1 - \tau). \quad (15)$$

It is worth discussing the qualitative similarities and differences between these two saving rates.

In both economies, a higher preference for the future increases the saving rate – which is hardly surprising. However, a higher elasticity of output to capital reduces the saving rate in the

³²We can confirm these intuitions by considering the case $\alpha = \delta = 0$, i.e. when labor is the only input and agents discount old age completely. In this case, we can readily check that $l_{coop} = l_{cap} = l_{eff} = \underline{l}/(1 - \beta)$. Firm size in all scenarios depends exclusively on how rapidly diminishing returns to labor set in (the more so, the smaller the firm size). If $\delta = 0$ but $\alpha > 0$, firm size in the cooperative economy is still $\underline{l}/(1 - \beta)$, which maximizes firm output per worker keeping firm capital constant, but the efficient and capitalist firm size is the larger expression we have derived above, and is increasing in α . Finally, if $\alpha = 0$ but $\delta > 0$ the efficient size is $\underline{l}/(1 - \beta)$, but the cooperative size drops to $\underline{l}(1 + \delta)/(1 + \delta - \beta)$.

³³An additional known reason why cooperative size may be inefficiently small is worker heterogeneity in the presence of strictly egalitarian pay rules (Farrell and Scotchmer, 1988; Levin and Tadelis (2005)).

capitalist economy, while it increases it in the cooperative economy. In the capitalist economy all savings are financed out of labor income, so a larger capital share reduces resources available for saving. In the cooperative economy, the share of income received by young workers is $1 - \tau$, independent of α . However, workers in the cooperative economy internalize the concavity of the production function. The less steeply the marginal product of capital declines with the capital stock (i.e. the higher is α) the more they wish to invest.³⁴

It is well established that capitalist economies can exhibit dynamic inefficiency, in the sense that a reduction in saving can improve the consumption and hence the welfare of all generations. This of course applies to the capitalist version of the economy studied here. But can the cooperative economy also be dynamically inefficient?

The standard analysis of dynamic efficiency begins by establishing a *golden rule* level of the capital stock (or, equivalently, of the saving rate) which maximizes total consumption (the sum of the consumption of the young and of the old) subject to enough output being reinvested to keep the total capital stock constant. In our context this problem can be stated as

$$\max_{K,N} c^Y + c^O$$

subject to

$$NF\left(\frac{K}{N}, \frac{L}{N}\right) = L(c^Y + c^O) + K.$$

Now it is clear that, for any K , the optimal N in the problem just stated must be the output-maximizing one which we identified in the previous subsection. Using this and maximizing with respect to K we find the familiar Cobb-Douglas golden rule $K = \alpha Y$. It follows from comparison with (15) that *the cooperative economy can never be dynamically inefficient* as $s_{coop} < \alpha$. (Comparison with (14) confirms that the capitalist economy can be.)

³⁴The following stylized version of the problems faced by workers in the two economies further clarifies this point. In the capitalist economy workers essentially maximize $\log(w - k) + \delta \log(rk)$, which as is well known means that r , and hence α , is irrelevant to the chosen level of k , since income and substitution effect cancel each other out. Instead, if workers maximize $\log(w - k) + \delta \log(Ak^\alpha)$ the solution will directly depend on α and indeed it is clear that the term $\alpha\delta$ will be critical. Outside of the log case (e.g. in the next section), α affects individual saving decisions in the capitalist economy as well, indirectly through r .

The intuition is closely linked to our discussion of saving in the two economies in the earlier part of this subsection. As is (now) well understood, in the capitalist economy the potential dynamic inefficiency is due to a pecuniary externality: the young do not internalize the fact that by increasing saving they lower the return to capital for everyone.³⁵ In contrast, as we have seen, young cooperative members fully take into account the consequences of their accumulation decision on the marginal product of capital, and this prevents them from over-accumulating.³⁶

5.3.3 Steady State Output

If an economy with our Cobb-Douglas technology features a steady state in which all firms are identical and operate with constant inputs k^* and l^* , then steady-state aggregate output per worker can be written as

$$\frac{Y^*}{L} = (s^*)^{\frac{\alpha}{1-\alpha}} (Z^*)^{\frac{1}{1-\alpha}}, \quad (16)$$

where Z is the measure of *static efficiency* we derived in Section 5.3.1, and $s^* = K^*/Y^*$ is the saving rate in steady state.³⁷

The interpretation is straightforward after the discussions in the last two subsections. An economy's steady-state output per worker is driven by two factors: how efficiently it produces in a static sense, and how much it saves.

Under our log-utility assumption both the capitalist and the cooperative economy have constant saving rates and constant firm sizes l , the latter implying constant Z s. We have seen that we cannot sign the difference between s_{coop} and s_{cap} , and that $Z_{coop} \leq Z_{cap}$. Despite this disad-

³⁵Acemoglu (2009, pp. 338-339) discusses the evolution of thinking about the sources of dynamic inefficiency in OLG economies.

³⁶It is well known that introducing a pay-as-you-go social security system in a capitalist OLG economy can reduce excess savings and, depending on the quantitative strength of this effect, lessen the risk of dynamic inefficiency. Since our cooperatives operate an internal pay-as-you go system, it may be tempting to interpret our finding that they are dynamically efficient as arising from the same mechanism. But this would be inaccurate: in the capitalist economy the reduction in savings occurs simply because the existence of the system reduces the young workers' perceived need (and income available) to save, and thus depends on the size of the social-security tax. In the cooperative economy, as discussed, dynamic efficiency arises from the internalization of the effect of investment on the rate of return, and is independent of the size of the transfer τ .

³⁷To see this write aggregate output as $Y^* = \frac{L}{l} F(k^*, l^*)$. The capital input of each individual firm is given by: $k^* = \frac{l^*}{L} K^* = \frac{l^*}{L} s^* Y^*$. Plugging this into Y^* and using the functional form for F we get the decomposition in the text.

Table 1: Calibrated parameters

Concept	Parameter	Target	Data	Value for $\sigma = 1$	Value for $\sigma = 2$
Capital share	α	rK/Y	0.33	0.33	0.33
Variable labor elasticity	β	\underline{l}/l_{cap}	0.18	0.55	0.55
Discount rate	δ	K/Y	3/25	0.22	0.13
Sharing Rule	τ	Max U		0.12	0.15

Note: σ is the elasticity of intertemporal substitution in preferences over consumption paths (see Equation (17)), so $\sigma = 1$ is the log-utility case.

vantage, because cooperative economies could potentially save at a higher rate, which economy has a higher output per worker is a quantitative matter.³⁸

5.3.4 Quantification

In this subsection we calibrate the log-utility economy for a first set of quantitative insights on the comparison between capitalist and cooperative economies. In the next section we quantify an example with more realistic preferences (but no closed-form solutions).

We do not observe a cooperative-based economy but we do observe economies which are broadly organized according to capitalist principles. Hence, we calibrate the parameters of the model so that the capitalist economy in steady-state matches corresponding moments of the US economy in recent decades. Table 1 summarises the values chosen for the parameters of the model in the column titled “Value for $\sigma = 1$ ” (σ being defined later as the intertemporal elasticity of substitution in consumption, and hence $\sigma = 1$ being the log case). The parameter α maps as usual into the share of capital in national income, which we set at 0.33 following standard practice in the quantitative macroeconomic literature. Given α , a choice of β in (6) uniquely determines the share of fixed labour in total firm employment, \underline{l}/l_{cap} . We match this to the average share of non-production workers in the economy between January 1964 and April 2020, from the Bureau of Labor Statistics.³⁹ Finally, given α a choice of δ uniquely determines the saving rate in the

³⁸The extension of the model to allow for population growth, presented in Appendix A.11, uncovers a further source of inefficiency in the cooperative economy. All incumbent firms operate with the optimal number of workers, so at every generation the extra workers who join the economy because of population growth are forced to create a new coop, which operates with zero capital. Hence, there is a permanent heterogeneity in the capital stock of cooperatives, which is itself inefficient for the economy as a whole.

³⁹Specifically we use the series PRODUCTION AND NONSUPERVISORY EMPLOYEES (CES0500000006)

capitalist economy, and in turn this saving rate equals the capital-output ratio in steady state, for which we use the standard value from the quantitative macro literature, which is 3 (with an adjustment for a putative 25-year duration of a model period.) The parameter τ is unique to the cooperative economy and thus cannot be calibrated on any kind of data. Hence, we select the value of τ that maximises steady-state lifetime utility of the representative consumer in the cooperative economy.⁴⁰ Simple calculations show that this value is:

$$\tau = \frac{\delta}{1 + \delta}(1 - \alpha).$$

(This happens to also be the saving rate in the capitalist economy - but we do not have a compelling intuition for this coincidence.)⁴¹

The implications of this calibration are presented in Table 2. First, cooperatives are only around a third as large as capitalist firms, or $l_{coop}/l_{cap} = 0.35$. This large size difference implies a significant disparity in static efficiency: $(Z_{coop}/Z_{cap})^{1/(1-\alpha)} = 0.78$. Second, the cooperative economy also saves half as much as the capitalist one, as we have $s_{cap} = 0.12$ and $s_{coop} = 0.06$. (Note that the capitalist economy is dynamically efficient). Hence, the contribution of saving to the output gap is $(s_{coop}/s_{cap})^{\frac{\alpha}{1-\alpha}} = 0.71$. When combined, the static inefficiency and the lower saving rate of the cooperative economy imply that steady state output per worker is 55% of steady state output per worker in the capitalist economy.

We can also evaluate the welfare consequences of a transition to a cooperative economy. In particular, we can compute the total amount of consumption which a worker in the cooperative economy would need to be given to obtain the same utility as in the capitalist economy. This calculation is made on the assumption that the worker is free to allocate this transfer as she

divided by the series ALL EMPLOYEES (CES0500000001) from the "Employment, Hours, and Earnings from the Current Employment Statistics survey (National)"

⁴⁰In the version of the model with endogenous τ presented in the Appendix, there is a continuum of values of τ which can be sustained in equilibrium. The implicit assumption in our calibration is thus that the first generation of young who set the value of τ do so with steady-state welfare in mind.

⁴¹The fixed cost l cancels out in all the ratios of capitalist-to-cooperative outcomes we wish to present, so it does not need to be calibrated here. Similarly, there is no need to choose values for the size of the population L and the productivity factor A .

Table 2: Numerical results

	Value for $\sigma = 1$	Value for $\sigma = 2$
$\left(\frac{Z_{coop}}{Z_{cap}}\right)^{\frac{1}{1-\alpha}}$	0.78	0.69
$\left(\frac{s_{coop}}{s_{cap}}\right)^{\frac{\alpha}{1-\alpha}}$	0.71	1.05
$\left(\frac{Y_{coop}}{Y_{cap}}\right)$	0.55	0.73

Note: σ is the elasticity of intertemporal substitution in preferences over consumption paths (see Equation (17); $\sigma = 1$ is the log-utility case).

wishes over her lifetime. In steady state, this welfare loss as a percentage of GDP in the coop economy is 51% (in the current log case).⁴²

6 An Example with Numerical Computations

The assumption of log preferences in the previous section was extremely useful in deriving a closed-form characterisation of the equilibrium, and analytical formulas to compare steady states in the cooperative and capitalist economies. However, most macroeconomic applications use

$$U(c^Y, c^O) = \frac{(c^Y)^{1-\sigma}}{1-\sigma} + \delta \frac{(c^O)^{1-\sigma}}{1-\sigma}, \quad (17)$$

with an elasticity of intertemporal substitution σ closer to 2. It turns out that, if σ is exactly 2, we can still produce analytical solutions for the capitalist steady state, which is extremely useful for calibration purposes. Hence, this is the case we study in this section.

The competitive equilibrium in the capitalist economy is unaffected by the assumption on preferences, which only affects the saving rule. As a result, the derivation of equilibrium prices and number of firms in the previous section is still valid. Importantly, this implies that firm size

⁴²We can also compute the dynamic path of output and welfare following the introduction of the cooperative organisation of production. Suppose that we begin with a capitalist economy initially in steady-state. At some initial period, all capital is seized and distributed equally to $N = L/l_{coop}$ cooperatives. Old agents in the initial period receive a share τ of output. From then on, the economy evolves according to our model of cooperative economy. Under our baseline calibration, relative output of the cooperative economy steadily and gradually declines towards its steady state level, while the compensation required by the current generation to be as well off as in the steady state of the capitalist economy steadily rises towards the steady state.

is still constant and takes the same value derived above, l_{cap} . Among other things this means that α and β do not need to be re-calibrated.

In Appendix A.6 we study the consumption-saving decision of young workers in the capitalist economy. We show that the capitalist economy converges to a steady state, and, for the case $\sigma = 2$, the aggregate steady state saving rate is

$$s_{cap}^* = \frac{4(1-\alpha)}{\left(\left(\frac{\alpha}{\delta(1-\alpha)}\right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)}\right)^{1/2}\right)^2}.$$

As in the previous example, $s^* = K^*/Y^*$, and as we already have a calibration for α , this equation can be used to re-calibrate δ , as reported in the last column of Table 1.

While we have closed form characterizations of the (steady state of the) capitalist economy, for the cooperative economy we must proceed numerically. We begin as before with the problem of an incumbent. In particular, we use policy function iteration to find a (numerical) fixed point for the mapping $\mathcal{L}(k)$ which solves problem (4). This is done using the already calibrated α , β , and δ , as well as normalized values for \underline{l} , A and L . (Appendix A.7 shows that comparison of steady state values among the two economies is independent of \underline{l} . That A and L can be normalized is obvious.) We also re-calibrate τ , to maximise, as before, the steady-state lifetime utility of the representative agent under the new preferences (and the new value of δ).

The policies $\mathcal{L}(k)$ and $\mathcal{K}(k)$ implied by our calibration are plotted in Figures 1 and 2. $\mathcal{L}(k)$ is decreasing in k , while $\mathcal{K}(k)$ is increasing and concave. This last property implies that there exists a steady state for the cooperative economy in which all coops have the same capital stock k^* , the same membership $\mathcal{L}(k^*)$, and their measure is $L/\mathcal{L}(k^*)$.⁴³

Given the existence of a steady state where identical incumbents maximize the unconstrained-cooperative problem, and given the allocation criterion $\mathcal{L}(k)$ and the investment function $\mathcal{K}(k)$, sufficient conditions for convergence to this steady state can be identified using a reasoning similar

⁴³The existence of this steady state is established only numerically via the numerical properties of the policy function. Its uniqueness is not established in any formal sense. All we can say is that our policy function iteration converges to the same fixed point from a wide variety of initial guesses we have attempted.

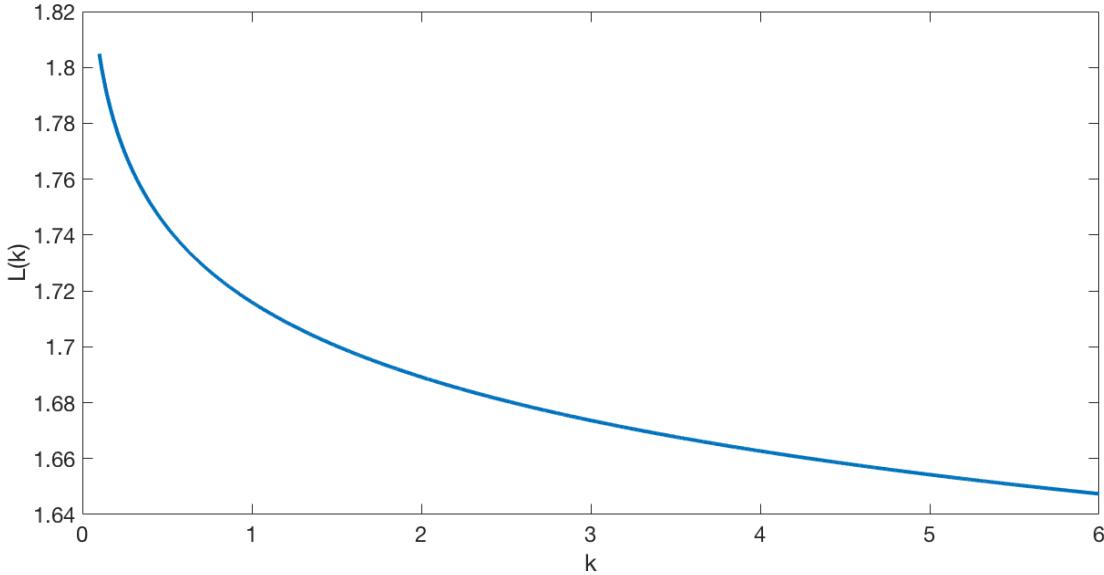


Figure 1: Cooperative labour input as a function of initial capital stock – numerical solution to problem (4).

to the one we used in Section 5.2. In particular, if (i) all initial incumbents start with a capital stock $k_{i0} \leq k^*$; (ii) in any period, entrants' worker allocation l_e and optimal investment k_e satisfy (a) $l_e \geq \mathcal{L}(k_e)$ and (b) $k_e \leq k^*$; and (iii) one-period-old coops are viable; then every coop's capital stock grows over time towards the steady-state level, while every coop's labour input decreases over time towards the steady-state, generating entry but no exit.

Recall now that that decomposition (16) is valid for any economy featuring a steady state with identical firm sizes, and is thus still valid – with the same interpretation – in the current example. The terms of the decomposition are reported in the last column of Table 2. With the alternative choice of preferences the cooperative economy features an even stronger bias towards small firms, meaning that its static inefficiency cause an even greater disadvantage relative to the capitalist economy: the term in Z drops to 0.69. On the other hand, the higher elasticity of intertemporal substitution boosts the relative savings rate of the cooperative economy, which is now 10 per cent higher than the capitalist one (resulting in a 5 per cent higher term in s). As a consequence of this latter feature, the relative output of the cooperative economy rises to 0.73. Welfare is correspondingly much less impacted than in the log case: the welfare loss from moving

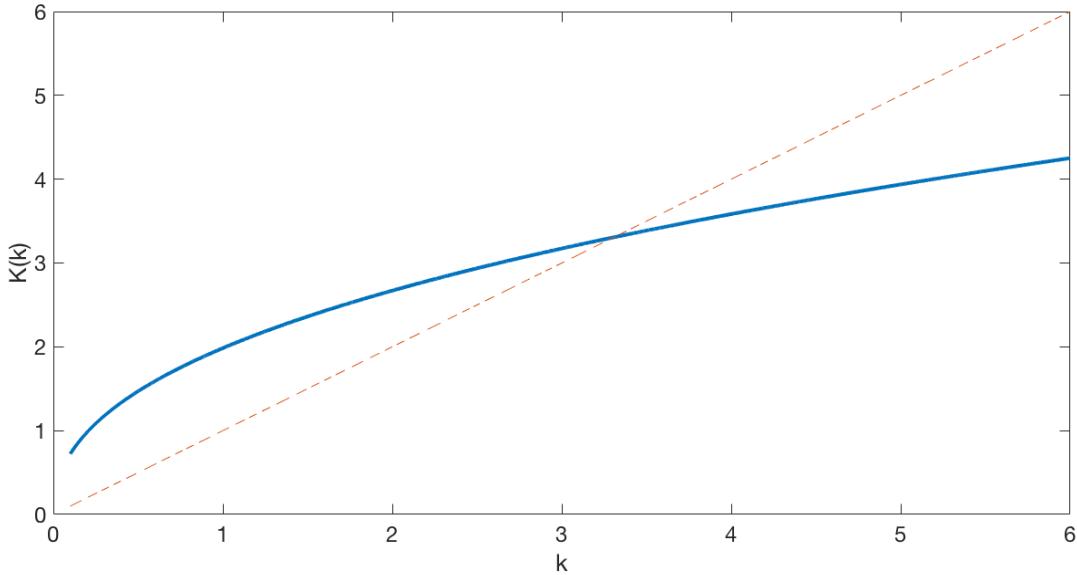


Figure 2: Cooperative capital investment as a function of initial capital stock – numerical solution to problem (4).

to a coop economy is now 28% of the coop economy’s GDP.

7 Comparative Statics

In this section we explore the dependence of relative output and welfare to changes in some of the parameters of the model. These exercises can be interpreted as robustness checks on the benchmark numerical results of the previous section or, perhaps more usefully, as numerical comparative-static results for our model of cooperatives.

When varying the parameters of technology or preferences, a choice needs to be made about whether to hold τ constant at its benchmark level, or allow it to vary so that, for each configuration of parameters, the inter-generational transfer is always the one that maximizes steady state welfare. Because both strategies are defensible, we do both.

In Figure 3 we present a series of plots showing the numerical dependence of the output ratio $\frac{Y_{coop}^*}{Y_{cap}^*}$, as well as its two components, on the parameters α , β , γ , *without* changing the value of τ . We also of course look at the impact of different values of τ holding the other parameters

constant. In Appendix A.8 we show the analogous sensitivity graphs for α , β , and γ when we re-calculate the value of τ as the other parameters vary. The qualitative patterns are virtually identical and even quantitatively the sensitivities are quite similar to those in Figure 3. Hence, the commentary which follows applies almost equally well to comparative statics with and without re-optimization.

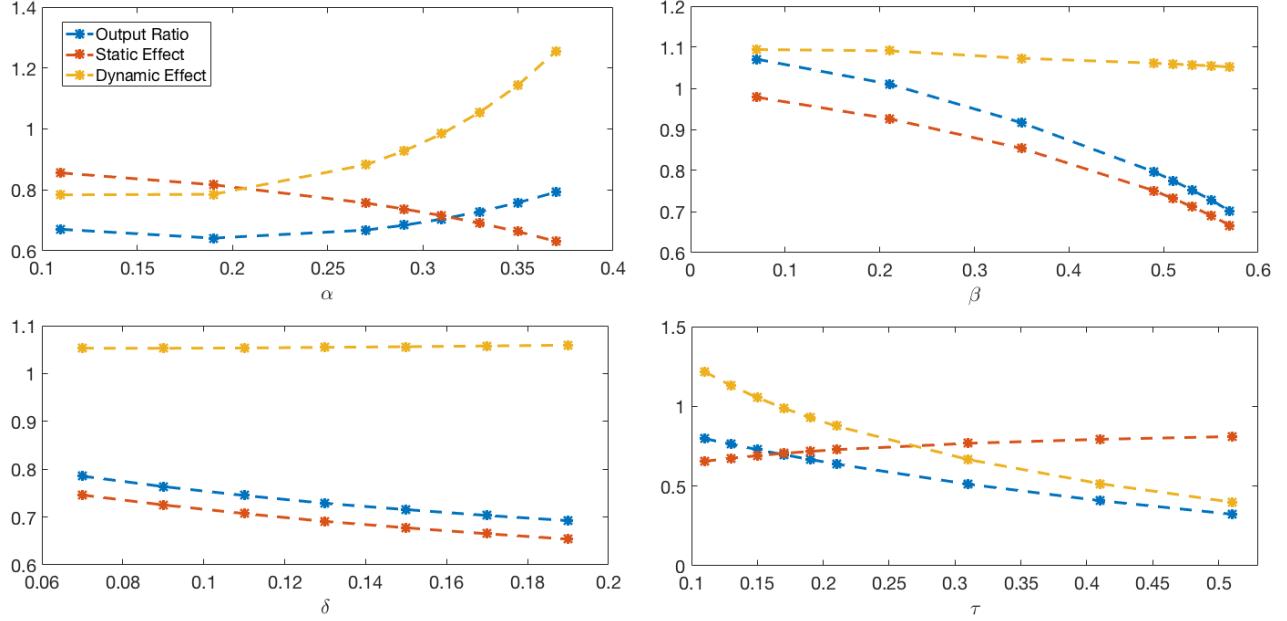


Figure 3: Steady-state output ratio $\frac{Y_{coop}^*}{Y_{cap}^*}$ and its static $\left(\left(\frac{Z_{coop}}{Z_{cap}}\right)^{\frac{1}{1-\alpha}}\right)$ and dynamic $\left(\left(\frac{s_{coop}}{s_{cap}}\right)^{\frac{\alpha}{1-\alpha}}\right)$ components, as functions of the model's parameters.

The top-left plots reveal that relative steady state cooperative output is first decreasing and then increasing in the elasticity of output to capital α . Looking at the two sub-components reveals why: static efficiency steadily declines with α , while the saving rate increases with it - for the reasons we discussed in Section 5.3. Clearly the former effect dominates at low level of the output elasticity of capital, while the latter dominates for larger values.

The top-right panel shows relative cooperative output to be monotonically decreasing in the elasticity to variable labour, β . Inspection of equations (6) and (11) shows that, as β increases, firm size increases in both economies, but proportionally more so in the capitalist (and statistically efficient) economy. This leads to an exacerbation of the static inefficiency of cooperative

economies. Quantitatively this is clearly the main driver of the decline of the output ratio with β , though the graph shows that the relative saving rate is also slightly decreasing in this parameter. Another interesting feature of this panel is that it confirms that there actually exist *combinations of parameter values such that steady state output in the cooperative economy is higher than in the capitalist economy*. In this particular case, this happens when the static inefficiency is minimized (through a very low value of β) so that the entire difference in incomes is due to the higher saving rate of the cooperative economy.

The static inefficiency also dominates the dependence of relative output on the discount factor δ . As seen in Section 5.3.1, the more importance workers give to the future, the more they wish to limit current employment. This negative effect is quantitatively much stronger than the positive effect of δ on relative saving, which goes in the opposite direction.

Finally, a larger share of output devoted to former workers, τ , directly reduces the cooperative economy's saving rate, leading again to a reduction in relative cooperative-economy output. This is despite the effect that an increase in τ improves somewhat the cooperative economy's static allocative efficiency.

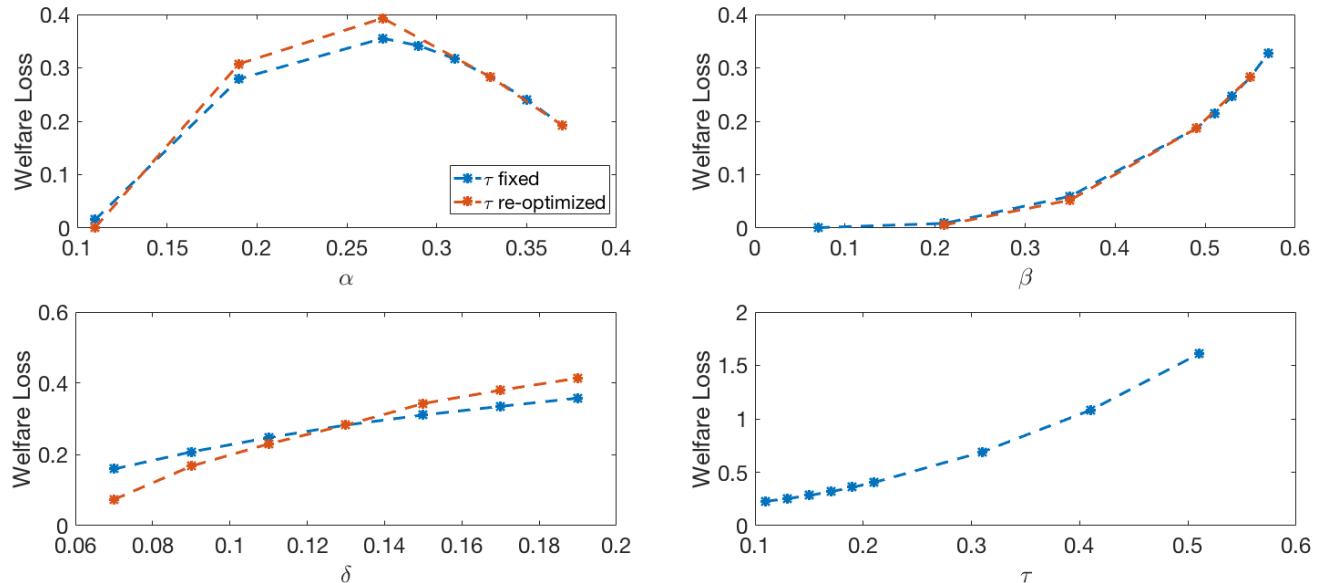


Figure 4: Amount of consumption to be given to agents in the cooperative economy to equalize their utility to agents in the capitalist economy, in steady state.

The corresponding sensitivity plots for the welfare loss from moving to a cooperative economy are presented, both with and without re-calculation of τ , in Figure 4. Qualitatively the welfare losses tend to be mirror-images of the output-ratio graphs in Figure 3: the lower the relative output of the cooperative economy, the larger the welfare loss to adopting this growth model. Quantitatively however the welfare losses are quite sensitive to parameter values. For example, agents are virtually indifferent between the two economies if α is very small, even though there is a significant difference in GDP. This is due to the very poor consumption-smoothing properties of capitalism when the capital share is very small. The strong dependence of the welfare loss on τ is also likely related to consumption smoothing (this time in the coop economy).⁴⁴

8 Conclusions

In light of the current crisis in the perceived legitimacy of corporation-based capitalism it is important to investigate the macroeconomic consequences of alternative institutional arrangements for the production of goods and services. This paper has taken a first step towards developing a theoretical and quantitative framework towards this goal, with a particular focus on worker cooperatives as the engine of economic activity. We have also provided quantitative examples of comparisons of macroeconomic outcomes under corporation-based capitalism and under labor-management.

Much more work needs to be done for a proper qualitative and quantitative comparison of capitalist economies and cooperative-based economies. In the rest of these Conclusions we outline the agenda for future research.

Our cooperative economy differs from the capitalist economy in the following main respects: (i) there is a non-wage mechanism which assigns workers to firms in a manner that is collectively rational and yet decentralized; (ii) former workers retain rights to the distribution of the cooper-

⁴⁴For the avoidance of confusion, there is no contradiction between the fact that the optimal τ in the cooperative economy (0.15) is not the τ which minimizes the welfare loss to moving to a cooperative economy. The objective functions are conceptually completely different.

ative's income; (iii) investment decisions are made by worker collectives to maximize the lifetime utility of current workers; (iv) the capital used in production by each cooperative is the result of past cooperative investments from retained earnings.

We don't think there is much scope to investigate alternatives to (i) if the productive units in our economy must continue to be recognizable as worker cooperatives. Indeed we think of the conceptualization of the worker-allocation mechanism in a cooperative economy as one of the key contributions of the paper. Similarly, dropping (ii) while leaving (iii) in place would trivially lead to an economy with zero investment.

What happens if we drop (iii), i.e. return the investment decision to the individuals? We explore this below in Appendix A.10. There, young workers save in the form of capital, and cooperatives rent capital from old individuals. We retrieve a classic result: the cooperative economy becomes isomorphic to the capitalist one. This is because the rental rate on capital is the marginal product of capital, so young workers are the residual claimants of the same share of income as in the capitalist version.⁴⁵

This leaves us with (iv), and it is here that a truly important and fruitful alternative could potentially lie. In particular, it would be useful to investigate the consequences of opening up a market on which cooperatives could rent capital from each other. We have noted earlier that one reason for the inefficiently small size of cooperatives is that they take their capital stock as given. The existence of a rental market for capital might therefore lead to different decisions. Unfortunately, extending our framework to feature an inter-cooperative rental market for capital is challenging, as the worker-allocation rule for each cooperative would have to depend on the indefinite future history of rental rates. Hence, we leave this task for future work.

However, the true, long-term payoff of this research agenda will only come from much richer qualitative and quantitative descriptions of the economy. A more complex demographic structure is only a minor aspect of this quest. Introducing realistic distortions to the capitalist economy (monopoly power, monopsony power, short-termism in decision-making, etc.) would put the

⁴⁵The insight that, in a perfectly competitive market, it makes no difference whether capital hires labour or labour hires capital is often attributed to Samuelson (1957).

comparison of efficiency and production on a more even playing field. Considerations of externalities (e.g. pollution) would similarly be informative on the relative welfare properties of the two systems. Most important of all, introducing realistic sources of heterogeneity (in skills, in initial wealth, in access to schooling and high-return assets) would allow to compare corporation-based capitalism and cooperative-based alternatives not only on their implications for aggregate productivity but also on their implications for income and wealth inequality. Since it is aversion to the consequences of extreme inequality which has fostered much of the current push back against capitalism, it is essential that efficiency losses associated with a cooperative-based system (if any) be evaluated against the likely benefits in terms of lower inequality. We hope that our paper will prove to be a first step on this (long) road.

A Appendix

A.1 Appearances of “worker cooperatives” in digitized books

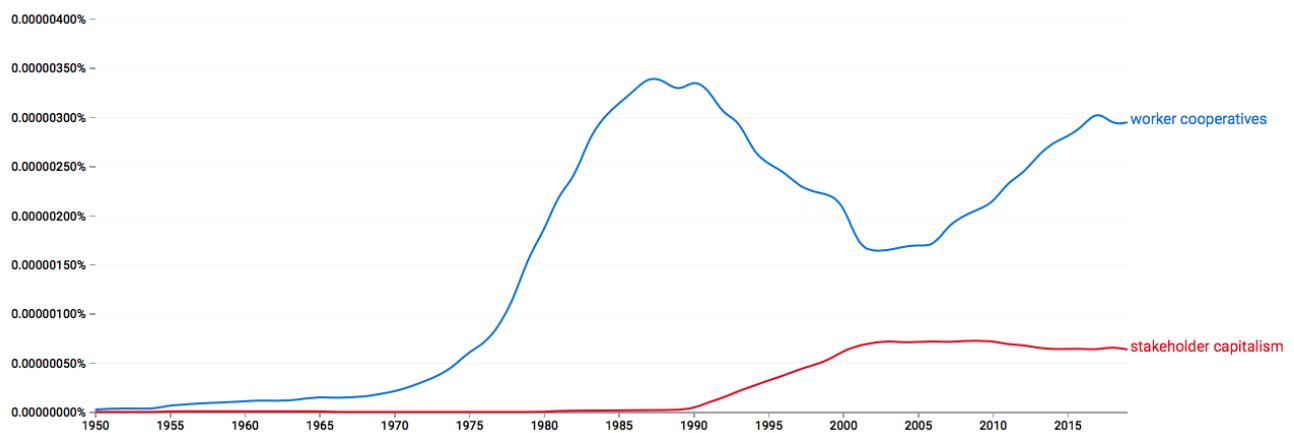


Figure 5: Frequency of the (case-insensitive) bigrams “worker cooperatives” and “stakeholder capitalism” among all bigrams contained in the sample of English-language books digitized by Google, by date of publication.⁴⁶

Figure 5 shows an increasing trend in the occurrence of the phrase “worker cooperatives” among newly published digitized books since the mid-2000s. For reference, we also added “stakeholder capitalism”. We do not interpret the figure as showing that worker cooperatives are necessarily a more prominent alternative than stakeholder capitalism, as other terms are probably used to refer to the concept. However, cooperatives are certainly as prominent nowadays as they were in the heydays of work on the subject, i.e. the 1980s.

A.2 Proof that (4) holds under the Refinement of the Equilibrium Concept

For $k > 0$ and $l > 0$, denote:

$$\mathcal{U}(k, l) = \max_{k'} U\left(\frac{(1-\tau)F(k, l) - k'}{l}, \frac{\tau F(k', \mathcal{L}(k'))}{l}\right).$$

The result we wish to prove is that

$$\mathcal{L}(k) \in \arg \max_l \mathcal{U}(k, l).$$

We will prove this by contradiction.

Consider an incumbent $\bar{i} \in I_t$, and suppose that $\mathcal{L}(k_{\bar{i}t})$ does *not* coincide with the argmax. Condition (1) implies that if $l \leq \mathcal{L}(k_{it})$ then $\mathcal{U}(k_{it}, l) \leq \mathcal{U}(k_{it}, \mathcal{L}(k_{it}))$, so the argmax must be strictly greater than $\mathcal{L}(k_{\bar{i}t})$.

Now apply conditions (2) and (3) to $j = \bar{i}$ and $i \in I_{t+1}$ an arbitrary cooperative. If there is a feasible reallocation in which \bar{i} is allocated $l > \mathcal{L}(k_{\bar{i}t})$ workers and i fewer workers than in the original allocation \mathbb{L}_i , then either:

$$\mathcal{U}(k_{\bar{i}t}, l) < \mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})),$$

⁴⁶The figure was generated using the Google Ngram Viewer (<https://books.google.com/ngrams>). Details on the corpus are presented by Michel et al (2011).

or:

$$\mathcal{U}(k_{it}, \mathbb{L}_i) \geq \mathcal{U}(k_{\bar{i}t}, l).$$

Since $\mathcal{L}(k_{\bar{i}t})$ is strictly less than the argmax, there must exist $l > \mathcal{L}(k_{\bar{i}t})$ such that the first condition is violated. It follows that the second condition must hold for any other cooperative $i \in I_{t+1}$. In particular, since we take l such that $\mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})) < \mathcal{U}(k_{\bar{i}t}, l)$, it follows that:

$$\forall i \neq \bar{i} \in I_{t+1}, \quad \mathcal{U}(k_{it}, \mathbb{L}_i) > \mathcal{U}(k_{\bar{i}t}, \mathcal{L}(k_{\bar{i}t})).$$

So any incumbent that is not allocated its optimal labour input must be the cooperative that provides the lowest utility level to its workers among all cooperatives in the economy. But being such a cooperative depends not only on that cooperative's own capital stock, but on the capital stock of all other cooperatives. This then contradicts the premise that the allocation of workers to incumbent cooperatives depends exclusively on each cooperative's own initial capital.

A.3 Algorithm to solve for the equilibrium

In practice, we solve for equilibria as follows. The first step is to obtain the equilibrium choices of labour input and capital investment of a cooperative:

$$(\mathcal{L}(k), \mathcal{K}(k)) \in \arg \max_{l, k'} U\left(\frac{(1-\tau)F(k, l) - k'}{l}, \frac{\tau F(k', \mathcal{L}(k'))}{l}\right),$$

as well as the optimal capital investment level of an entering cooperative with an arbitrary labour input l :

$$\mathbf{K}(l) \in \arg \max_{k'} U\left(\frac{(1-\tau)F(0, l) - k'}{l}, \frac{\tau F(k', \mathcal{L}(k'))}{l}\right).$$

The value of this problem is denoted $\mathcal{U}_0(l)$. Then, given any initial distribution of capital $\{k_{i0}\}_{i \in I_0}$, one can construct the growth path of the economy and check for feasibility.

Specifically, in each period, all incumbents $i \in I_t$ are allocated $\mathcal{L}(k_{it})$ workers. If $\int_{I_t} \mathcal{L}(k_{it}) di > L$, feasibility is violated so the initial distribution cannot lead to an equilibrium satisfying the

requirement. If $\int_{I_t} \mathcal{L}(k_{it}) di = L$, then $I_{t+1} = I_t$ and $k_{i,t+1} = \mathcal{K}(k_{it})$. If $\int_{I_t} \mathcal{L}(k_{it}) di < L$, define:

$$l^* \in \arg \max_l \mathcal{U}_0(l).$$

Then a set of entrants E_t of measure $|E_t| = \frac{L - \int_{I_t} \mathcal{L}(k_{it}) di}{l^*}$ is created. That is, new cooperatives are created with $l_i = l^*$ workers. Finally, $I_{t+1} = I_t \cup E_t$ where E_t is the set of newly created cooperatives, and for $i \in I_t$, $k_{i,t+1} = \mathcal{K}(k_{it})$, while for $i \in E_t$, $k_{i,t+1} = \mathbf{K}(l^*)$.

A.4 Derivation of capitalist equilibrium with log utility

Conditional factor demands from individual firms take the form:

$$k(r_t, w_t) = \left[A \left(\frac{\alpha}{r_t} \right)^{1-\beta} \left(\frac{\beta}{w_t} \right)^\beta \right]^{\frac{1}{1-\alpha-\beta}},$$

$$l(r_t, w_t) = \underline{l} + \left[A \left(\frac{\alpha}{r_t} \right)^\alpha \left(\frac{\beta}{w_t} \right)^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}},$$

while profits write:

$$\pi(r_t, w_t) = (1 - \alpha - \beta) \left[A \left(\frac{\alpha}{r_t} \right)^\alpha \left(\frac{\beta}{w_t} \right)^\beta \right]^{\frac{1}{1-\alpha-\beta}} - w_t \underline{l}.$$

As a result, we can solve the system of equilibrium conditions to derive:

$$r(\kappa_t) = A \frac{\alpha}{(1-\alpha)^{1-\alpha}} \beta^\beta \left(\frac{1-\alpha-\beta}{\underline{l}} \right)^{1-\alpha-\beta} \kappa_t^{\alpha-1},$$

$$w(\kappa_t) = A(1-\alpha)^\alpha \beta^\beta \left(\frac{1-\alpha-\beta}{\underline{l}} \right)^{1-\alpha-\beta} \kappa_t^\alpha,$$

$$N(\kappa_t) \equiv N = \frac{1-\alpha-\beta}{1-\alpha} \left(\frac{L}{\underline{l}} \right).$$

Note that the number of firms is constant over time and hence independent of the size of the capital stock, or equivalently, capitalist firms have the constant size given in equation (6).

The solution to the Young's consumption-saving problem leads to the well known log-utility

saving rule

$$\kappa_{t+1} = \frac{\delta}{1+\delta} w_t.$$

Substituting from the equations above this delivers the capital accumulation equation (7).

A.5 Proof that cooperatives are smaller than capitalist firms

Capitalist firms have $\frac{1-\alpha}{1-\alpha-\beta} l$ employees, while cooperatives have $\frac{1+\delta}{1+\delta-\beta(1+\delta\alpha)} l$ workers. Since $\alpha \in (0, 1)$, it must be the case that:

$$1 + \delta \geq (1 - \alpha)(1 + \delta\alpha).$$

It follows that $\frac{1}{1-\alpha} \geq \frac{1+\delta\alpha}{1+\delta}$, which implies that:

$$1 - \frac{\beta}{1 - \alpha} \leq 1 - \frac{\beta(1 + \delta\alpha)}{1 + \delta}.$$

Therefore:

$$\frac{1 - \alpha}{1 - \alpha - \beta} l \geq \frac{1 + \delta}{1 + \delta - \beta(1 + \delta\alpha)} l.$$

A.6 Capitalist dynamics with IES = 2

Solving the consumption-saving problem of young agents yields the following saving rule:

$$\kappa_{t+1} = \frac{\delta^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}}}{1 + \delta^{\frac{1}{\sigma}} r_{t+1}^{\frac{1-\sigma}{\sigma}}} w_t.$$

As a result, capital accumulation dynamics are characterised by the following equation:

$$\begin{aligned} & \left[1 + \delta^{-\frac{1}{\sigma}} \left(A \frac{\alpha}{(1 - \alpha)^{1-\alpha}} \beta^\beta \left(\frac{1 - \alpha - \beta}{l} \right)^{1-\alpha-\beta} \kappa_{t+1}^{\alpha-1} \right)^{-\frac{1-\sigma}{\sigma}} \right] \kappa_{t+1} \\ &= A(1 - \alpha)^\alpha \beta^\beta \left(\frac{1 - \alpha - \beta}{l} \right)^{1-\alpha-\beta} \kappa_t^\alpha. \end{aligned} \tag{18}$$

If $\sigma > 1$ and $\alpha \in (0, 1)$, equation (18) defines κ_{t+1} as an increasing and concave function of κ_t , with a first-order derivative which is infinite at 0 and vanishes at infinity.

To simplify notations, denote:

$$a = \delta^{-\frac{1}{\sigma}} \left(A \frac{\alpha}{(1-\alpha)^{1-\alpha}} \beta^\beta \left(\frac{1-\alpha-\beta}{l} \right)^{1-\alpha-\beta} \right)^{-\frac{1-\sigma}{\sigma}},$$

$$b = A(1-\alpha)^\alpha \beta^\beta \left(\frac{1-\alpha-\beta}{l} \right)^{1-\alpha-\beta},$$

$$\theta = \alpha + \frac{1-\alpha}{\sigma}.$$

Then equation (18) rewrites:

$$\kappa_{t+1} + a\kappa_{t+1}^\theta = b\kappa_t^\alpha,$$

or equivalently:

$$\kappa_{t+1} = f^{-1}(\kappa_t),$$

where $f(x) = \left(\frac{1}{b}\right)^{\frac{1}{\alpha}} \left(x + ax^\theta\right)^{\frac{1}{\alpha}}$ is strictly increasing and strictly convex on $(0, \infty)$, and satisfies:

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} f'(x) = 0,$$

$$\lim_{x \rightarrow \infty} f(x) = \lim_{x \rightarrow \infty} f'(x) = \infty,$$

if we restrict attention to the case $\sigma > 1$, so that $\theta \in (\frac{1}{\sigma}, 1)$.

Indeed, these properties are easily derived from differentiating twice, which yields:

$$f'(x) = \left(\frac{1}{b}\right)^{\frac{1}{\alpha}} (1 + a\theta x^{\theta-1}) (x + ax^\theta)^{\frac{1-\alpha}{\alpha}},$$

and:

$$f''(x) = \left(\frac{1}{b}\right)^{\frac{1}{\alpha}} (x + ax^\theta)^{\frac{1-\alpha}{\alpha}-1} \left[\frac{1-\alpha}{\alpha} + \theta(\theta-3 + \frac{2}{\alpha})ax^{\theta-1} + \theta\left(\frac{\theta}{\alpha}-1\right)(ax^{\theta-1})^2 \right],$$

where $\frac{\theta}{\alpha} - 1 = \frac{1-\alpha}{\alpha\sigma} > 0$, and $\theta - 3 + \frac{2}{\alpha} = (1-\alpha)\left(\frac{2-\alpha}{\alpha} + \frac{1}{\sigma}\right) > 0$.

It follows that capital accumulation follows standard dynamics with a unique strictly positive attractive steady-state.

In the special case where $\sigma = 2$, the steady-state capital stock per old worker takes a simple algebraic form:

$$\kappa^* = \left(\frac{\left(4A(1-\alpha)^\alpha \beta^{\beta} \left(\frac{1-\alpha-\beta}{\underline{l}} \right)^{1-\alpha-\beta} \right)^{1/2}}{\left(\frac{\alpha}{\delta(1-\alpha)} \right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)} \right)^{1/2}} \right)^{\frac{2}{1-\alpha}}.$$

It follows that the discount factor δ can still be identified from the targeting of the capital-output ratio:

$$\frac{K}{Y} = \frac{4(1-\alpha)}{\left(\left(\frac{\alpha}{\delta(1-\alpha)} \right)^{1/2} + \left(4 + \frac{\alpha}{\delta(1-\alpha)} \right)^{1/2} \right)^2}.$$

A.7 Fixed-cost Normalization

We use the following functional form for the production technology:

$$F_{\underline{l}}(k, l) = Ak^\alpha(l - \underline{l})^\beta.$$

In this appendix, we show that the specific value of the parameter \underline{l} does not affect the quantitative comparison between the two economies.

For any given \underline{l} , we can implement the following change of variables. Any quantity of labour l can be renormalized as:

$$\tilde{l} = \frac{l}{\underline{l}},$$

while any quantity of capital k can be renormalized as:

$$\tilde{k} = \underline{l}^{-\frac{\beta}{1-\alpha}} k.$$

It follows that the production output is also renormalized as:

$$F_{\underline{l}}(k, l) = \underline{l}^{\frac{\beta}{1-\alpha}} F_1(\tilde{k}, \tilde{l}).$$

In our model of capitalist economy, the problem of the firm is completely unchanged as long as the wage is suitably renormalized to:

$$\tilde{w} = \underline{l}^{1-\frac{\beta}{1-\alpha}} w.$$

The consumer's problem is unchanged either (note that each consumer now supplies $1/\underline{l}$ units of labour). The renormalization implies that consumption level c is to be renormalized as: $\tilde{c} = \underline{l}^{-\frac{\beta}{1-\alpha}} c$. Given that preferences are of the form:

$$U(c^Y, c^O) = \frac{(c^Y)^{1-\sigma}}{1-\sigma} + \delta \frac{(c^O)^{1-\sigma}}{1-\sigma},$$

the renormalization amounts to multiplying the utility function by a positive constant, thus does not affect choices. Note also that in the log case, the renormalization simply corresponds to adding a constant to the utility function, so the argument is also valid.

Similarly, in the cooperative model, consumption levels per consumer write:

$$c^Y = \underline{l}^{\frac{\beta}{1-\alpha}} \frac{(1-\tau)F_1(\tilde{k}, \tilde{l}) - \tilde{k}'}{\tilde{l}},$$

$$c^O = \underline{l}^{\frac{\beta}{1-\alpha}} \frac{\tau F_1(\tilde{k}', \tilde{l}')}{\tilde{l}}.$$

Therefore, choices are unaffected by the same renormalization. Since the normalization affects the levels of relevant quantities in the same way in the two models, no quantitative comparison is affected by the level of \underline{l} .

A.8 Sensitivity Analysis with τ recalculated

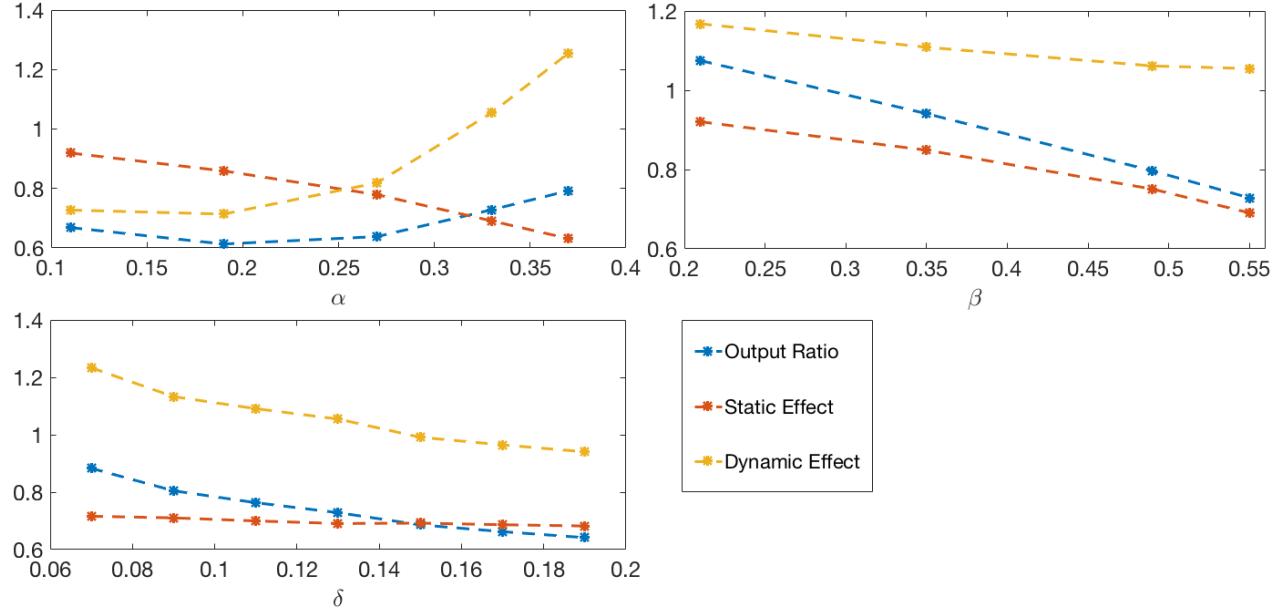


Figure 6: Steady-state output ratio $\frac{Y_{coop}^*}{Y_{cap}^*}$ and its static $((\frac{Z_{coop}}{Z_{cap}})^{\frac{1}{1-\alpha}})$ and dynamic $((\frac{s_{coop}}{s_{cap}})^{\frac{\alpha}{1-\alpha}})$ components, as functions of the model's parameters, with τ chosen to maximize steady state utility for each combination of parameters.

A.9 Endogenous Sharing Rule

In this appendix, we present an extended version of our model of cooperatives in which we relax the assumption that old workers automatically receive a share of current revenues. Instead, in each period, current workers decide by a vote whether to implement a sharing-rule. We describe those sharing-rules that can be sustained as an equilibrium of the dynamic game played by the different generations of workers within a cooperative. In equilibrium, each generation expects to receive payments when old only if they agree to pay their old workers when young. Therefore agreement to a sharing-rule arises endogenously. Our approach follows closely that of Cooley and Soares (1999), who introduce endogenous pay-as-you-go social security systems in a general-equilibrium overlapping-generations model.

Specifically, we assume that, upon creation of a cooperative i , the initial workers choose a linear sharing rule $\tau_i \in [0, 1]$. Following generations of workers are not committed to abide by

the policy designed by their predecessors, but may vote only for or against its implementation. That is, in each following period, if cooperative i is assigned workers, those workers choose, once production has taken place, whether to distribute a share τ_i of revenues to the old workers or to keep all revenues. Then, the maintained share is split between investment and payment to the young workers as in the main text.

Our general equilibrium concept can be adapted to include decisions regarding the sharing-rule. We call a sharing-rule *sustainable* if its implementation in every period can be supported by trigger strategies such that every generation of workers in cooperative i chooses to implement the sharing rule τ_i as long as every previous generation has done so. If workers of cooperative i in period t deviate from τ_i , they expect to be punished by the following generation of workers and not to receive any payment as old. As a result, they have no incentive to invest at all. It follows that the sharing-rule τ_i is sustainable if, on path:

$$U\left(\frac{(1 - \tau_i)F(k_{i,t}, l_{i,t}) - k_{i,t+1}}{l_{i,t}}, \frac{\tau_i F(k_{i,t+1}, l_{i,t+1})}{l_{i,t}}\right) \geq U\left(\frac{F(k_{i,t}, l_{i,t})}{l_{i,t}}, 0\right). \quad (*)$$

An important consequence is that no sharing rule is sustainable in a cooperative that exits in equilibrium, and conversely, a cooperative with non-sustainable sharing rule immediately exits after its first period of existence⁴⁷.

Now, the definition of an equilibrium follows naturally from that of section 4.3. An equilibrium is characterised by a worker allocation mechanism, investment decisions, and sharing-rules. Investment decisions are required to be optimal subject to condition (*), taking as given the worker allocation mechanism. In turn, the worker allocation mechanism takes as input the set of incumbent cooperatives, their current capital stock and the sharing rule in place in each of them, and operates according to the same requirements as in the main text. In particular, when we consider reallocations, we take into account the potential creation of a new cooperative i with any arbitrary sharing rule τ_i . In this sense, sharing rules are chosen optimally upon the creation

⁴⁷This follows from noting that the last generation of workers before a cooperative exits would have no incentive to distribute revenues to old workers. In anticipation, the previous generation would have no incentive to implement a sharing rule either, and so on by backward induction.

of a cooperative.

As in section 4.4, we may impose restrictions in order to define an operational equilibrium concept. First, we require that an incumbent cooperative's allocation of workers depends only on its capital stock and its sharing rule. That is, given the sharing rule τ_i , there is a mapping $\mathcal{L}_{\tau_i}(k_{i,t})$ such that, on path, incumbent cooperative i is allocated $\mathcal{L}_{\tau_i}(k_{i,t})$ in any state in which its capital stock is $k_{i,t}$ and its sharing rule τ_i . Second, we impose that, given the mapping $\mathcal{L}_{\tau_i}(\cdot)$, the resulting optimal investment path implies that τ_i is a sustainable sharing rule. That is, condition $(*)$ is satisfied in every period, when every generation of workers invests as if τ_i was to be automatically implemented in every future period. This second restriction is in line with the limited rationality requirements that motivate our operational equilibrium concept in the main text, as workers do not need to anticipate the behaviour of every future generation when they invest, only that of the next generation when they vote on the implementation of the sharing rule. Importantly though, it limits the set of sharing rules that can be selected by the first generation of workers in a cooperative, since those must choose among those sharing rules that are indeed sustainable given the optimal investment path. If they do not choose such a sharing rule, they must understand it and the cooperative immediately exits. Then, as mentioned above, exit does not have to be ruled out for an operational equilibrium concept, but it may occur only to newly created cooperatives.

It is easy to establish under these restrictions that equation (4) is still valid for a sustainable sharing rule τ . If we further impose symmetry across cooperatives regarding their initial choice of sharing rule, the rest of our analysis applies without modification to this model with endogenous sharing rules. In particular, in the examples of sections 5 and 6, any sharing rule $\tau \in (0, 1)$ is sustainable. This result follows from the fact that the workers' utility goes to $-\infty$ if consumption goes to 0 in any period. Therefore condition $(*)$ has to hold.

A.10 Private Ownership of Capital

An important feature of the cooperative economy is that all investment decisions are made collectively by the current workers of each cooperative. In this section, we relax this assumption and consider an economy in which worker-managed firms rent capital from private individuals, who make their own investment decisions. After describing the model, we demonstrate that this alternative economy is isomorphic to the capitalist model.

We maintain the physical environment described in Section 2. We imagine an economy in which firms are self-managed, but individuals are still allowed to own capital. The final consumption good acts as the numéraire. The price of capital in period t is denoted r_t . Since saving decisions are made privately, worker-managed firms maximize the lifetime utility of their workers by providing them the largest possible income. That is, they choose productive inputs to solve:

$$m(r_t) = \max_{k,l} \frac{F(k,l) - r_t k}{l}. \quad (19)$$

Denote $k(r_t)$ and $l(r_t)$ the corresponding factor demands.

Given income m_t , a young individual in period t saves in the form of physical capital, which can be traded at price r_{t+1} in the following period. The optimal investment $\kappa(m_t, r_{t+1})$ solves:

$$\max_{\kappa} U(m_t - \kappa, r_{t+1}\kappa) \quad (20)$$

In equilibrium, the market for capital clears, while free-entry of cooperatives guarantees full-employment. Denote κ_t the stock of capital owned by each old individual⁴⁸ at the beginning of period t , and N_t the measure of operating cooperatives in period t . An equilibrium is characterized by the following system:

$$N_t l(r_t) = L, \quad (21)$$

$$N_t k(r_t) = L \kappa_t, \quad (22)$$

⁴⁸As in Section 3, all heterogeneity disappears after the first generation.

defining the equilibrium price of capital and measure of firms as a function of the state variable κ_t : $r_t = r(\kappa_t)$ and $N_t = N(\kappa_t)$. Plugging in $m_t = m[r(\kappa_t)]$ as defined by equation (19), the capital accumulation dynamics are characterized by:

$$\kappa_{t+1} = \kappa(m[r(\kappa_t)], r(\kappa_{t+1})).$$

We have the following result.

Proposition 1. *Equilibria of this alternative economy are isomorphic to the equilibria of the capitalist economy described in Section 3.*

Proof. Given an equilibrium of the alternative economy, a candidate equilibrium of the capitalist economy can be constructed by setting $w(\kappa_t) = m[r(\kappa_t)]$ as the wage, with all other notations translated transparently. It is immediately apparent that young individuals solve the same program. In addition, conditions (21) and (22) guarantee market clearing in the capitalist economy. The zero-profit condition in the capitalist economy holds since worker-managed firms effectively make no profit, distributing all surplus from production as labor income to their workers. As a result, it is sufficient to verify that solutions to problem (19) also maximize capitalist-firm profits when the wage is given by $w_t = m(r_t)$. In this case, for arbitrary inputs (k, l) , we must have:

$$\frac{F(k, l) - r_t k}{l} \leq w_t,$$

which rewrites:

$$F(k, l) - r_t k - w_t l \leq 0 = F(k(r_t), l(r_t)) - r_t k(r_t) - w_t l(r_t),$$

where the equality follows from $w_t = m(r_t)$. Since (k, l) are taken arbitrary, profits are indeed maximized by $[k(r_t), l(r_t)]$.

Reciprocally, given an equilibrium of the capitalist economy, we construct similarly an equilibrium of the alternative economy. As explained in the previous paragraph, translating the relevant

notations is straightforward. It is sufficient to verify that factor demands $[k(r_t, w_t), l(r_t, w_t)]$ are solutions to problem (19) when the equilibrium condition $\pi(r_t, w_t) = 0$ holds. To see this, observe that, for arbitrary (k, l) , we have:

$$\begin{aligned} \frac{F(k, l) - r_t k}{l} &= \frac{F(k, l) - r_t k - w_t l}{l} + w_t \\ &\leq 0 + w_t \\ &= \frac{F[k(r_t, w_t), l(r_t, w_t)] - r_t k(r_t, w_t) - w_t l(r_t, w_t)}{l(r_t, w_t)} + w_t \\ &= \frac{F[k(r_t, w_t), l(r_t, w_t)] - r_t k(r_t, w_t)}{l(r_t, w_t)}, \end{aligned}$$

establishing the result. \square

A.11 Population Growth

It is easy to incorporate (predictable) population growth in the general model of cooperatives. There are interesting differences in terms of how the capitalist and cooperative economies incorporate population growth. As an illustration, we revisit the benchmark example of Section 5 under the assumption that population grows over time: $L_{t+1} \geq L_t$, where L_t is the size of the cohort of young born in period t .

Capitalist Economy.— The approach to solving for the equilibrium is standard and exactly the same as in the main text. Firm size remains constant at l_{cap} . As a result, the number of firms grows at the same rate as the size of the population. Individual capital accumulation dynamics satisfy:

$$\kappa_{t+1} = \frac{\delta}{1 + \delta} A (1 - \alpha)^\alpha \beta^\beta \left(\frac{1 - \alpha - \beta}{l} \right)^{1-\alpha-\beta} \left(\frac{L_{t-1}}{L_t} \kappa_t \right)^\alpha$$

Cooperative Economy.— Individual cooperatives behave exactly the same as in the main text. That is, they each have the same constant size l_{coop} and accumulate capital towards a steady state. It follows that population growth leads to the creation of new cooperatives in each period.

The new cooperatives start producing without capital, and accumulate capital slowly, according to the same dynamics as described in the main text.

As a consequence, while the price system forces homogeneity in the capitalist economy, heterogeneity across firms persists in the cooperative economy. In addition to generating inequality within generations (between workers of new coops and workers of older coops), this heterogeneity is an additional source of static inefficiency (recall that the static social planner would organize production across identical firms, due to the concavity of the production function).

References

Acemoglu, Daron (2009): *Introduction to Modern Economic Growth*, Princeton University Press.

Alchian Armen A. and Harold Demsetz (1972): “Production, Information Costs, and Economic Organization”, *The American Economic Review*, Dec., Vol. 62, No. 5 (Dec., 1972), pp. 777-795

Aoki, Masahiko (1980): ”A Model of the Firm as a Stockholder-Employee Cooperative Game,” *American Economic Review*, 70, 600-10.

Atkinson, Anthony B. (1972): Worker Management and the Modern Industrial Enterprise, *Quarterly Journal of Economics*, Vol. 87, No. 3, pp.375-392

Block, Fred L. (2018): *Capitalism: The Future of an Illusion*, University of California Press.

Bonin, John (1983): “Membership and Employment in an egalitarian Cooperative,” *Economica*, 51, 296-305, 1983

Bonin, J., and Puterman, L. (1987): *Economics of cooperation and the labour-managed economy*, Routledge.

Bonin, J. P., Jones, D. C., and Puterman, L. (1993): “Theoretical and empirical studies of producer cooperatives: will ever the twain meet?” *Journal of Economic Literature*, 31(3), 1290-1320.

Bradley, Keith, and Alan Gelb (1983): *Cooperation at Work, The Mondragon Experience*, Heinemann Educational Books.

Bureau of Labour Statistics: Employment, Hours, and Earnings from the Current Employment Statistics Survey (National).

Cass, Oren (2018): *The Once and Future Worker*, Encounter Books.

Collier, Paul (2018): *The Future of Capitalism*, Allen Lane.

Cooley, T. F., and J. Soares, 1999. A Positive Theory of Social Security Based on Reputation, *Journal of Political Economy*, 107(1), 135-160.

Domar, Evsey (1966): "The Soviet Collective Farm as a Producer Cooperative," *The American Economic Review*, Vol. 56, No. 4, Part 1, pp. 734-757.

Drèze, Jaques H. (1976): "Some Theory of Labor Management and Participation," *Econometrica*, 44, 1125-39.

Drèze, Jaques H. (1989): *Labour Management, Contracts, and Capital Markets*, Basil Blackwell.

Drèze and Greenberg (1980): "Hedonic Coalitions: Optimality and Stability," *Econometrica*, May, 1980, Vol. 48, No. 4 (May, 1980), pp. 987-100.

Farrell, Joseph and Suzanne Scotchmer (1988): "Partnerships," *The Quarterly Journal of Economics*, Vol. 103, No. 2 (May, 1988), pp. 279-297.

Fudenberg, Drew, David Levine and Eric Maskin (1994), "The Folk Theorem with Imperfect Public Information," *Econometrica*, Sep., 1994, Vol. 62, No. 5 (Sep., 1994), pp. 997-1039

Furubotn, Eric G. (1976): "The long-run analysis of the labor-managed firm: An alternative interpretation," *The American Economic Review*, 66(1), 104-123

Furubotn, Eric G. and Svetozar Peyovich (1973): "Property Rights, Economic Decentralization, and the Evolution of the Yugoslav Firm, 1965-1972", *The Journal of Law and Economics* Volume 16, Number 2.

Greenberg (1979): "Existence and Optimality of Equilibrium in Labor Managed Economies," *Review of Economic Studies*, 46, (1979), 419-433.

Hart, Oliver, and John Moore (1996): “The governance of exchanges: members’ cooperatives versus outside ownership,” *Oxford Review of Economic Policy*, Volume 12, Issue 4, Pages 53–69.

Hart, Oliver, and John Moore (1998): “Cooperatives vs. Outside Ownership,” NBER Working Paper 6421.

Hart, Oliver, and Luigi Zingales (2017): “Companies Should Maximize Shareholder Welfare Not Market Value,” *Journal of Law, Finance, and Accounting*.

Holmström, Bengt (1982): “Moral Hazard in Teams,” *The Bell Journal of Economics*, Vol. 13, No. 2, pp. 324-340.

Ichishi, Tatsuro (1977): “Coalition Structure in a Labor-Managed Economy,” *Econometrica*, 45, 2, 341-360.

Jones, D.C. (2018): “The economics of Participation and Employee Ownership (PEO): an assessment,” *Journal of Participation and Employee Ownership*, 1(1), 4-37.

Kandori, M., 1992. Repeated Games Played by Overlapping Generations of Players, Review of Economic Studies, 59(1), 81-92.

Kremer, Michael (1997): “Why are worker cooperatives so rare?” National Bureau of Economic Research Working Paper, No. w6118.

Laffont, Jean Jaques, and Michel Moreaux (1983): “The Nonexistence of a Free Entry Cournot Equilibrium in Labor-Managed Economies,” *Econometrica* Vol. 51, No. 2, pp. 455-462.

Lang, Kevin and Peter-John Gordon (1995): “Partnerships as Insurance Devices: Theory and Evidence,” *RAND Journal of Economics*, 26.4, 614-629.

Law, Peter (1977): “The Illyrian Firm and Fellner’s Union-Management Model,” *Journal of Economic Studies*, 4, 29-37.

Levin, Jonathan and Steve Tadelis (2005): “Profit Sharing and the Role of Professional Partnerships,” *The Quarterly Journal of Economics*, Volume 120, Issue 1, February 2005, Pages 131–171.

Magill, Michael, Quinzii, Martine, and Jean-Charles Rochet (2015): “A Theory of the Stakeholder Corporation,” *Econometrica*, Vol. 83, No. 5, 1685–1725.

Meade, James E, (1972). "The Theory of Labour-Managed Firms and of Profit Sharing," *Economic Journal*, Royal Economic Society, vol. 82(325), pages 402-428, Suplemen.

Meade, J.E. (1986): *Alternative Systems of Business Organization and of Workers' Remuneration*. Allen & Unwich.

Michel, Jean-Baptiste, Yuan Kui Shen, Aviva Presser Aiden, Adrian Veres, Matthew K. Gray, William Brockman, The Google Books Team, Joseph P. Pickett, Dale Hoiberg, Dan Clancy, Peter Norvig, Jon Orwant, Steven Pinker, Martin A. Nowak, and Erez Lieberman Aiden (2011): "Quantitative Analysis of Culture Using Millions of Digitized Books". *Science*, 331, 6014, pages 176-182.

Mirrlees, James A. "The Optimal Structure of Incentives and Authority within an Organization." *The Bell Journal of Economics* 7, no. 1 (1976): 105-31

Miyazaki, Hajime (1984): "Internal Bargaining, Labor Contracts, and a Marshallian Theory of the Firm," *American Economic Review*, 74, 381-93.

Miyazaki, Hajime, and Hugh M. Neary: "The Illyrian Firm Revisited," *The Bell Journal of Economics*, Spring, 1983, Vol. 14, No. 1, pp. 259-270;

Mulder, Catherine P. (2015): *Trascending Capitalism through Cooperative Practices*, Palgrave Macmillan.

Pencavel, J. (2013): *The economics of worker cooperatives*, Edward Elgar Publishing.

Putterman, L, (1982): "Some Behavioural Perspectives on the Dominance of Hierarchical over Democratic Forms of Enterprise," *Journal of Economic Behaviour and Organization*, 3, 139-160.

Radner, Roy (1986). "Repeated Partnership Games with Imperfect Monitoring and No Discounting." *Rev. Econ. Studies* 53 (January 1986): 43-57.

Radner, Roy, Roger Myerson and Eric Maskin (1986): "An Example of a Repeated Partnership Game with Discounting and with Uniformly Inefficient Equilibria," *The Review of Economic Studies* , Jan., 1986, Vol. 53, No. 1 (Jan., 1986), pp. 59- 69.

Ranis, Peter (2016): *Cooperatives Confront Capitalism*, ZED Books.

Rey, Patrick, and Jean Tirole (2007): "Financing and access in cooperatives," *International*

Journal of Industrial Organization, Volume 25, Issue 5, October 2007, Pages 1061-1088.

Samuelson, Paul (1957): "Wages and Interest: A Modern Dissection of Marxian Economic Models," *American Economic Review*, 47, 884-894.

Sapir André (1980): "A Growth Model for a Tenured-Labor-Managed Firm," *The Quarterly Journal of Economics*, Nov., 1980, Vol. 95, No. 3, pp. 387-402.

Sasaki, Hiroo, and Toda, Manabu (1996): "Two-Sided Matching Problems with Externalities," *Journal of economic theory*, 1996-07, Vol.70 (1), p.93-108

Sen, Amartya K. (1966) "Labour allocation in a cooperative enterprise," *The Review of Economic Studies*, 33(4), 361-371.

Steinherr, A. and Thisse, J.F. (1979), "Are Labor-Managers Really Perverse?" *Economics Letters*, Vol. 2 (1979), pp. 137-142.

Svejnar, Jan (1982): "On the theory of a participatory firm," *Journal of Economic Theory*, Volume 27, Issue 2, Pages 313-330,

Vanek, Jaroslav (1970): *The general theory of labor-managed market economies*, Cornell University Press.

Ward, Benjamin (1958): "The firm in Illyria: market syndicalism," *The American Economic Review*, 48 (4), 566-589.

Weitzman, Martin (1984): *The Share Economy*, Harvard University Press.

Weitzman, Martin (1985): "The Simple Macroeconomics of Profit Sharing," *American Economic Review*.

Whyte, William Foote, and Kathleen King Whyte (1988): *Making Mondragon, The Growth and Dynamics of the Worker Cooperative Complex*, ILR Press.