Unpublished Appendix for "On the Theory of Ethnic Conflict"

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1 Unpublished Appendix 1. Analysis of Model with open Conflict

1.1 Equilibrium group sizes

1.1.1 At node CC

At node CC in principle there could be passing from either group.

No passing from A to B We first show that members of group A never pass in CC equilibria. Define $\bar{n}_{CC}^{A}$ the hypothetical ex-post size that would make members of A indifferent between staying and passing. $\bar{n}_{CC}^{A}$ solves

$$(1-\Delta)\left(y_A + \frac{\alpha z}{\bar{n}_{CC}^{A}}\right) = (1-\Delta)\left((1-\phi)y_A + \frac{(1-\alpha)z}{1-\bar{n}_{CC}^{A}}\right)$$

so

$$\bar{n}_{CC}^{A} = \frac{-\left[(1-\alpha)z - \phi y_A\right] \pm \sqrt{\left[(1-\alpha)z - \phi y_A\right]^2 + 4\alpha z \phi y_A}}{2\phi y_A}$$

It can be shown that the "-" root is always less than 0, while the "+" root is greater than 1 (using $\alpha > 0.5$). We conclude that there is never passing from A to B at the CC node.

Equilibrium passing from B to A Now define $\bar{n}_{CC}^{B}$ the ex-post size that makes members of B indifferent between staying and passing at node CC. The condition for $\bar{n}_{CC}^{B}$ is

$$(1-\Delta)\left(y_B + \frac{(1-\alpha)z}{1-\bar{n}_{CC}^{B}}\right) = (1-\Delta)\left((1-\phi)y_B + \frac{\alpha z}{\bar{n}_{CC}^{B}}\right)$$

so

$$\bar{n}_{CC}^{B} = \frac{(z+\phi y_B) \pm \sqrt{(z+\phi y_B)^2 - 4\alpha z \phi y_B}}{2\phi y_B}$$

While the "+" root is always greater than 1, the "-" root is always between 0 and 1. We conclude that $\bar{n}^B_{CC}$ is strictly between 0 and 1 and is given by:

$$\bar{n}^B_{CC} = \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z \phi y_B}}{2\phi y_B}.$$

Passing behavior in the CC node can then be summarized as follows. If $n < \bar{n}^B_{CC}$ there will be passing from B to A. If $n > \bar{n}^B_{CC}$ there will be no passing. Hence, if the node is CC we define $n'_{CC} = \max\{n, \bar{n}^B_{CC}\}$.

For future reference, we note that $\bar{n}^B_{CC}$ is an increasing and concave function of z, which passes through the origin and asymptotes to α for $z \to \infty$. Hence, n'_{CC} is a constant equal to n if $\alpha < n$ (which makes sense). Instead, if $\alpha > n$, n'_{CC} is a constant through n up to a "kink," and then it becomes increasing and concave, and converges to α. Assumptions we make below for a variety of reasons imply that we focus on cases where $\alpha > n$. The kink is at z^k defined by

$$n = \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z \phi y_B}}{2\phi y_B}$$

so

$$z^k = \frac{\phi y_b n(1-n)}{(\alpha - n)}$$

1.1.2 At other nodes

The analysis of equilibrium population size at nodes CP and PC is identical to the case of exploitation in the baseline model. In particular, at node CP we have $n'_{CP} = \max\{n, \min[1, \bar{n}^B_{CP}]\}$, where

$$\overline{n}^B_{CP} \equiv \frac{z}{\phi y_B}.$$

At node PC we have $n'_{PC} = \min\{n, \max[0, \bar{n}^A_{PC}]\}$, where

$$\overline{n}^A_{PC} \equiv 1 - \frac{z}{\phi y_A}.$$

Finally, at node PP we obviously have $n' = n$.

1.2 Group B's Decision

1.2.1 At node C

We can now look at B's strategic decisions. When A has played C, B has to choose between acquiescence and fight back. B chooses fight back if

$$(1-\Delta)\left[y_B + \frac{(1-\alpha)z}{1-n'_{CC}}\right] > (1-\delta)\max\left[y_B, (1-\phi)y_B + z\right]$$

or

$$1 - \frac{(1-\Delta)(1-\alpha)z}{\max\left[0, (1-\delta)(z-\phi y_B)\right] + (\Delta - \delta)y_B} < n'_{CC}.$$

We already know the RHS is constant at n, has a kink at z^k, and then is increasing and concave asymptoting to α.

If $\phi \leq (\Delta - \delta)/(1-\delta)$ the left side as a function of z starts at 1 and decreases monotonically (though with a kink at $z = \phi y_B$) asymptoting to $1-(1-\Delta)(1-\alpha)/(1-\delta)$. Since this is always greater than α the left side and right side never intersect so B never responds to C with C.

If $\phi > (\Delta-\delta)/(1-\delta)$ the left side as a function of z starts at 1 and decreases until $z = \phi y_B$, after which it turns increasing, asymptoting (from below) to $1-(1-\Delta)(1-\alpha)/(1-\delta)$.

Assumption: $\alpha \geq 2n - n^2$

With this assumption, the kink in LHS is always to the right of z^k. Note that this assumption implies $\alpha > n$. We make this assumption exclusively for convenience so as not to have to consider too many different cases. However the assumption does not affect the qualitative results in any significant way. In particular the discussion in the text is entirely unaffected.

The assumption implies that we can ignore the horizontal segment of the RHS. We can then reformulate the problem by saying that B responds with C if

$$1 - \frac{(1-\Delta)(1-\alpha)z}{\max\left[0,(1-\delta)(z-\phi y_B)\right] + (\Delta-\delta)y_B} < \bar{n}^B_{CC}.$$

This equation has either two solutions or no solution. The solution exists if, at the kink $z = \phi y_B$ the right hand side exceeds the left hand side, or

$$1 - \frac{(1-\Delta)(1-\alpha)\phi y_B}{(\Delta-\delta)y_B} < \frac{(\phi y_B + \phi y_B) - \sqrt{(\phi y_B + \phi y_B)^2 - 4\alpha\phi y_B \phi y_B}}{2\phi y_B}$$

or

$$\phi > \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}$$

Now note that the condition $\phi > \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}$ also implies $\phi > \frac{(\Delta-\delta)}{(1-\delta)}$.

Now we look at the two solutions. The first one is the solution to:

$$1 - \frac{(1-\Delta)(1-\alpha)z}{(\Delta-\delta)y_B} = \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z\phi y_B}}{2\phi y_B}$$

Define

$$q = \frac{(1-\Delta)(1-\alpha)}{(\Delta-\delta)}$$

then the solution is

$$z = \frac{(1-\alpha)y_B + q y_B \phi}{(q^2\phi + q)} \equiv z^l_{B,CC}$$

The other solution is implicitly given by

$$1 - \frac{(1-\Delta)(1-\alpha)z}{(1-\delta)(z-\phi y_B) + (\Delta-\delta)y_B} = \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z\phi y_B}}{2\phi y_B}, \tag{1}$$

and we denote it by $z^h_{B,CC}$.

In conclusion, if $\alpha \geq 2n - n^2$ then group B responds to C with C if and only if:

$$\phi > \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta - \delta)}{(1 - \Delta)}$$

and

$$z \in (z^l_{B,CC}, z^h_{B,CC}).$$

The subscript "B, CC" is a mnemonic for "B responds to C with C," and the superscripts l and h denote lower- and upper-bound values.

It is important for future reference to characterize the behavior of $z^l_{B,CC}$ and $z^h_{B,CC}$ as functions of ϕ.

By construction, we have $z^l_{B,CC} = z^h_{B,CC} = \phi y_B$ when $\phi = \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta-\delta)}{(1-\Delta)}$.

$z^l_{B,CC}$ is strictly increasing in ϕ and converges to y_B/q.

$z^h_{B,CC}$ is strictly increasing in ϕ and grows without bound.

1.2.2 At node P

Group B's decision if A has played P is isomorphic to A's decision in the baseline model. In particular, B plays C if

$$1 - n'_{PC} < \frac{(1-\delta)z}{\delta y_B + z},$$

or

$$\max\{1-n, \min[1, \frac{z}{\phi y_A}]\} < \frac{(1-\delta)z}{\delta y_B + z}$$

The left side starts out flat at $1-n$, then it increases linearly, then it becomes flat again at 1. The right side increases from 0 in concave fashion and converges to $(1-\delta)$. The two sides either never intersect or they intersect twice, once in the flat part of the left side and one in the increasing part of the left side. They intersect if and only if $n - \delta > \delta y_B/(\phi y_A)$

In conclusion, (i) if $\phi \leq \delta y_B/(y_A(n-\delta))$ then B never responds to P with C; (ii) if $\phi > \delta y_B/(y_A(n-\delta))$ then B responds to P with C if $z^l_{B,PC} < z < z^h_{B,PC}$, where

$$z^l_{B,PC} = \frac{(1-n)\delta y_B}{n-\delta}$$
$$z^h_{B,PC} = \phi y_A(1-\delta) - \delta y_B,$$

which, not surprisingly, describes a triangle much like the one in figure 1.

1.3 Decision by A

We can now examine the behavior of A. There are four cases, depending on the combination of actions chosen by B.

1.3.1 When B plays C at both nodes

If B plays C at both nodes, A plays C if

$$(1-\delta)y_A < (1-\Delta)\left(y_A + \frac{\alpha z}{n'_{CC}}\right)$$

$$\max\left[n, \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z\phi y_B}}{2\phi y_B}\right] < \frac{\alpha z}{(\Delta-\delta)\, y_A}$$

(note that if B plays C at P we know that $n'_{PC} > 0$, so we don't need to worry about the case where all the As pass.) There always is one and only one solution to this equation. Call this solution $z^l_{AC,CC}$ (for "A plays C when B plays C at both nodes"). The solution is either the solution to

$$\begin{aligned} n &= \frac{\alpha z}{(\Delta-\delta)\, y_A} \\ z &= \frac{(\Delta-\delta)\, y_A n}{\alpha} \end{aligned} \tag{2}$$

or the solution to

$$\begin{aligned} \frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z\phi y_B}}{2\phi y_B} &= \frac{\alpha z}{(\Delta-\delta)\, y_A} \\ z &= \frac{(\Delta-\delta)\, y_A\left[\phi y_B - (\Delta-\delta)\, y_A\right]}{\left[\phi y_B\alpha - (\Delta-\delta)\, y_A\right]} \end{aligned} \tag{3}$$

The solution to (2) is the "global" solution if it lies to the left of the kink in the LHS, z^k,or if

$$\begin{aligned} \frac{(\Delta-\delta)\, y_A n}{\alpha} &< \frac{\phi y_B n(1-n)}{(\alpha-n)} \\ \frac{(\alpha-n)\,(\Delta-\delta)\, y_A}{\alpha y_B(1-n)} &< \phi \equiv \phi_{C,CC} \end{aligned}$$

Therefore: we conclude that, for $\phi \le \phi_{C,CC}$, A plays C if z exceeds the expression in (3), while if $\phi > \phi_{C,CC}$ A plays C if z exceeds the expression in (2).[1]

Now we characterize how $z^l_{AC,CC}$ varies with ϕ. It can be shown that this begins at $(\Delta-\delta)y_A$ for $\phi = 0$ and decreases over the interval $[0, \phi_{C,CC})$, after which it becomes constant at $(\Delta-\delta)y_A n/\alpha$.

[1] Note that the solution in (3) exists if and only if z

$$\phi \le \frac{(\Delta-\delta)y_A}{y_B}.$$

However, this is always greater than $\phi_{C,CC}$ so this constraint is never binding.

1.3.2 When B plays P and C

If B plays P at node P and C at node C, A plays C if

$$y_A + z < (1-\Delta)\left(y_A + \frac{\alpha z}{n'_{CC}}\right) \tag{4}$$
$$n'_{CC} < \frac{(1-\Delta)\alpha z}{z+\Delta y_A}.$$

The right hand side starts at 0 and then increases, converging to $(1-\Delta)\alpha$. Since as we know RHS asymptotes to α, there are either 0 or two solutions for z. There are no solutions if $(1-\Delta)\alpha < n$. If instead $(1-\Delta)\alpha \geq n$ there still could be either two or no solution.

When there are two solutions one solves

$$n = \frac{(1-\Delta)\alpha z}{z+\Delta y_A}$$

and the solution is

$$z = \frac{\Delta y_A n}{(1-\Delta)\alpha - n} \equiv z^l_{AC,PC},$$

where the subscript stands for "A plays C when B plays P and C."

The other solution solves

$$\frac{(z+\phi y_B) - \sqrt{(z+\phi y_B)^2 - 4\alpha z \phi y_B}}{2\phi y_B} = \frac{(1-\Delta)\alpha z}{z+\Delta y_A}$$

so

$$\begin{aligned} z &= \frac{-\left\{(1+\Delta)\Delta y_A + (1-\Delta)\left[(1-\Delta)\alpha - 1\right]\phi y_B\right\} +}{2\Delta} \\ &\quad + \frac{\sqrt{\left\{(1+\Delta)\Delta y_A + (1-\Delta)\left[(1-\Delta)\alpha - 1\right]\phi y_B\right\}^2 - 4\Delta^2 y_A\left[\Delta y_A - (1-\Delta)\phi y_B\right]}}{2\Delta} \\ &\equiv z^h_{AC,PC}. \end{aligned}$$

The condition for having two solutions is that $z^l_{AC,PC}$ is to the left of the kink in the left side of (4), z^k. I.e. the condition is

$$\frac{\Delta y_A n}{(1-\Delta)\alpha - n} < \frac{\phi y_b n(1-n)}{(\alpha - n)}$$
$$\phi > \frac{\Delta y_A(\alpha - n)}{\left[(1-\Delta)\alpha - n\right] y_b(1-n)} \equiv \phi_{C,PC} \tag{5}$$

In conclusion, A plays C when B plays P, C if and only if $(1-\Delta)\alpha \geq n$, (5) is satisfied, and $z^l_{AC,PC} < z < z^h_{AC,PC}$.[2]

As functions of ϕ, the two bounds share the same value $z^l_{AC,PC}$ at $\phi_{C,PC}$. As ϕ increases beyond this value, $z^l_{AC,PC}$ is constant while $z^h_{AC,PC}$ increases without bound.

[2] Note that the slope condition for a solution to the last equation exist is that the right hand side is steeper at the origin. But this is satisfied whenever $\phi \geq \phi_{C,PC}$, which in turn holds when $(1-\Delta)\alpha < n$. We focus on this case below.

1.3.3 When B plays C and P

If B plays C at node P and P at node C, A plays C if

$$(1-\delta)y_A < (1-\delta)\left(y_A + \frac{z}{n'_{CP}}\right),$$

so A always plays C in this case.

1.3.4 When B plays P and P

If B always plays P (this is the case of the baseline model) A plays C if

$$y_A + z < (1-\delta)\left(y_A + \frac{z}{n'_{CP}}\right)$$

$$\max\{n, \min[1, \frac{z}{\phi y_B}]\} < \frac{(1-\delta)z}{z+\delta y_A}$$

As in the benchmark model, we focus on the case where $(1-\delta) > n$. Then if $\phi \leq \delta y_A/(y_B\,(1-n-\delta))$ A never plays C, while if $\phi > \delta y_A/(y_B\,(1-n-\delta))$ A plays C if $\frac{\delta y_A n}{(1-\delta)-n} < z < (1-\delta)\phi y_B - \delta y_A$.This is of course the same condition as in the baseline model of exploitation.

1.4 Regions where different equilibria prevail

We now bring it all together and characterize the regions of the parameter space where the various equilibria obtain.

1.4.1 Equilibria of type CC

Equilibria where both groups play C occur in two scenarios. (i) B plays C at both nodes, and A plays C. Or (ii) B plays P at node P and C at node C, and A plays C. To limit the number of cases we focus on situations where $(1-\Delta)\alpha < n$. As we have seen this means that A never fights a war of choice: faced with a choice of peace and open conflict, it always chooses peace. Open conflict only arises when B plays C in both nodes, i.e. when A's choice is between being exploited and being the exploiter. Again this assumption only serves to reduce the number of appendix pages, without materially affecting the insights of the model.

Assumption: $(1-\Delta)\alpha < n$

Let's collect the conditions for CC to happen.

(i) B plays C at node P

$$\phi > \frac{\delta y_B}{y_A\,(n-\delta)}$$

$$z^l_{B,PC} < z < z^h_{B,PC}$$

(ii) B plays C at node C

$$\phi > \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}$$

$$z^l_{B,CC} < z < z^h_{B,CC}$$

(iii) A plays C when B plays C, C

$$z^l_{AC,CC} < z$$

Therefore we observe open conflict if and only if

$$\max\left\{z^l_{B,PC}, z^l_{B,CC}, z^l_{AC,CC}\right\} < z < \min\left\{z^h_{B,PC}, z^h_{B,CC}\right\}$$
$$\max\left\{\frac{\delta y_B}{y_A\left(n-\delta\right)} \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta-\delta)}{(1-\Delta)}\right\} < \phi.$$

Note that this region *always* exists. To see this, notice that both $z^h_{B,PC}$ and $z^h_{B,CC}$ grow without bound with ϕ, while all of $z^l_{B,PC}$, $z^l_{B,CC}$, and $z^l_{AC,CC}$ converge to finite constants.

An increase in y_A increases both $z^h_{B,PC}$ and $z^l_{AC,CC}$. The former reflects that in a possible PC equilibrium there will be less passing from A to B. This makes B more aggressive for a larger set of values of z, forcing A to choose C more often. The increase in $z^l_{AC,CC}$ reflects the fact that a higher y_A makes open conflict more costly for A, and thus increases the set of zs such that A is willing to let itself be exploited by B. Hence, an increase in the wealth of the stronger groups shifts the conflict region "to the right."

An increase in y_B increases $z^l_{B,PC}$ (exploiting A becomes more costly for B), $z^l_{B,CC}$ (open conflict becomes more costly), On the other hand an increase in y_B reduces $z^h_{B,PC}$ (again, the cost of exploiting A are greater), and have ambiguous effects on $z^h_{B,CC}$. Hence, the lower bound of the conflict region unambiguously increases, while the upper bound could either fall or increase.

An increase in n decreases $z^l_{B,PC}$ (exploiting A becomes more attractive when A is larger) and increases $z^l_{AC,CC}$ (A becomes more likely to acquiesce to being exploited). Hence the effect of n is ambiguous.

1.4.2 Equilibria of type CP

These equilibria emerge in two sets of circumstances. (i) When B plays P at both nodes, and A decides to exploit; and (ii) when B responds to P with C, and to C with P (we know that A always plays C in this case).

(i) B plays PP, A plays C (i.i) B plays P at P

$$\phi \leq \frac{\delta y_B}{y_A\left(n-\delta\right)}$$
$$\text{OR } z \leq z^l_{B,PC} = \frac{(1-n)\delta y_B}{n-\delta}$$
$$\text{OR } z \geq z^h_{B,PC} = \phi y_A(1-\delta) - \delta y_B,$$

(i.ii) B plays P at C

$$\phi \leq \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta-\delta)}{(1-\Delta)}$$
$$\text{OR } z \leq z^l_{B,CC}$$
$$\text{OR } z \geq z^h_{B,CC}$$

(i.iii) A plays C when B plays PP

$$\phi > \frac{\delta y_A}{y_B(1-n-\delta)}$$
$$z^l_{AC,PP} \equiv \frac{\delta y_A n}{(1-\delta)-n} < z < (1-\delta)\phi y_B - \delta y_A \equiv z^h_{AC,PP}$$

Now notice that when all the other conditions in this case are satisfied, then $z^h_{B,CC} > z^h_{AC,PP}$. The reason is simple. If $z > z^h_{B,PC}$ it means that under CP the entire B population passes. But then it cannot be optimal for A to choose CP over PP. Hence this region is of the form

$$\frac{\delta y_A}{y_B(1-n-\delta)} < \phi \leq \min\left\{\frac{\delta y_B}{y_A(n-\delta)}, \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}\right\}$$
$$z \in \left[z^l_{AC,PP}, \min\left\{z^l_{B,PC}, z^l_{B,CC}\right\}\right]$$

(ii) B plays CP, A plays C (ii.i) B plays C at P

$$\phi > \frac{\delta y_B}{y_A(n-\delta)}$$
$$z^l_{B,PC} < z < z^h_{B,PC}$$

(ii.ii) B plays P at C

$$\phi \leq \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}$$
$$\text{OR } z \leq z^l_{B,CC}$$
$$\text{OR } z \geq z^h_{B,CC}$$

Hence

$$\frac{\delta y_B}{y_A(n-\delta)} < \phi \leq \frac{1}{\sqrt{1-\alpha}}\frac{(\Delta-\delta)}{(1-\Delta)}$$
$$z \in \left[z^l_{B,PC}, z^l_{B,CC}\right] \cup \left[z^h_{B,CC}, z^h_{B,PC}\right]$$

An increase in y_A increases $z^h_{B,PC}$. It also increases $z^l_{AC,PP}$ because an increase in y_A makes it more expensive for A to exploit B. Hence, the "bottom" corridor of the CP region narrows, while the "top" corridor may either narrow or widen. The reason for the difference in results with the baseline model is that now when y_A increases there is less passing from A to B when B exploits A. This makes B more likely to respond to P with C, and may force A to preemptively play C more often.

An increase in y_B increases $z^l_{B,PC}$ and $z^l_{B,CC}$, has ambiguous effects on $z^h_{B,CC}$, and reduces $z^h_{B,PC}$. Hence, the lower bound of the CP region unambiguously increases, while the upper bound may increase or decrease.

An increase in n decreases $z^l_{B,PC}$ and increases $z^l_{AC,PP}$ (lower benefits of exploitation by A). Hence the effect of n is ambiguous.

1.4.3 Equilibria of type PC

This equilibrium emerges only if B plays C at both nodes, and A prefers being exploited than engaging in open conflict. (We already know that when B plays C and O, A always plays C: better to exploit than being exploited.)

(i) B plays C at P

$$\phi > \frac{\delta y_B}{y_A (n-\delta)}$$
$$z^l_{B,PC} < z < z^h_{B,PC}$$

(ii) B plays C at C

$$\phi > \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta-\delta)}{(1-\Delta)}$$
$$z^l_{B,CC} < z < z^h_{B,CC}$$

(iii) A plays P when B plays C and C

$$z^l_{AC,CC} \geq z.$$

Hence

$$\max\left\{z^l_{B,PC}, z^l_{B,CC}\right\} < z < \min\left\{z^l_{AC,CC}, z^h_{B,PC}, z^h_{B,CC}\right\}$$
$$\max\left\{\frac{\delta y_B}{y_A (n-\delta)} \frac{1}{\sqrt{1-\alpha}} \frac{(\Delta-\delta)}{(1-\Delta)}\right\} < \phi.$$

This converges to a sort of corridor, when it exists, whose upper bound is the limit of $z^l_{AC,CC}$ and the lower bound is the limit of either $z^l_{B,PC}$ or $z^l_{B,CC}$.

An increase in y_A increases both $z^h_{B,PC}$ and $z^l_{AC,CC}$. Hence, an increase in the wealth of the stronger groups unambiguously increases the region where the weaker group exploits the richer group.

An increase in y_B increases $z^l_{B,PC}$ and $z^l_{B,CC}$, has ambiguous effects on $z^h_{B,CC}$, and reduces $z^h_{B,PC}$. Hence, the lower bound of the PC region unambiguously increases, while the upper bound could either fall or increase.

An increase in n decreases $z^l_{B,PC}$ and increases.$z^l_{AC,CC}$ Hence an increase in n unambiguously increases the size of the PC region.

2 Unpublished Appendix 2: Leaders and Followers

The literature on ethnic conflict has emphasized the unequal gains from ethnic competition. Leaders of ethnic groups stand to gain large amounts of wealth and power, so their behavior is easy to explain. But what about the masses? On this question there is some disagreement.

Some point out that the unequal distribution of material benefits does not imply that there is no benefit for the masses (or, at least, no *expected* material benefit). The elite may share (or promise to share) enough of the cake as to make participation or acquiescence to other group's exploitation worthwhile even for the foot soldiers. In the case of ethnic politics this will take the form of public-sector jobs, handouts, subsidies, location of public projects and infrastructure, or a law-enforcement system skewed in favor of coethnics. Bates and Posner, among others, take this view and provide many examples. In the case of open conflict the evidence is less systematic, but it seems clear that the masses of followers tend to enjoy freedom to loot, which could be a significant reward. Another benefit is that followers may use the open conflict to eliminate creditors, or take over property like land, cattle, and housing, which used to belong to members of the losing group. And of course there is the expectation of further benefits from the group's political control of the state once the conflict is over.

Other authors are more skeptical that the masses are in it for the material benefits. We have already seen that Horowtiz stresses individuals' self-esteem from seeing coethnics in positions of power. Others focus on elite manipulation of coethnic primordial feelings [e.g. Brass (1997), Woodward (1995), Glaeser (2005)] and/or information [e.g. de Figuereido and Weingast (1999)].

In this appendix we return to the baseline model of exploitation and sketch an extension where the masses follow for the material benefits. In this extension the stronger group, group A, has an elite of size ν, and the reminder $n-\nu$ are "the masses." The elite moves first and chooses between a C and a P action. If the elite chooses P the rest of the game is exactly as in the baseline model. If the elite chooses C there is a "spoil-sharing" rule which determines that a fraction β of the resources appropriated through conflict gets equally divided among the masses, while the remaining $1-\beta$ is divided equally among the elite. For simplicity we treat β as an exogenous parameter. Next, the masses decide whether to support the elite in the conflict decision or to abstain. If the masses abstain the outcome is once again the peace outcome, with no social costs and equal society-wide division of the country's resources. This captures the idea that the elite needs the support of the mass of its coethnics to implement an exploitation strategy. If the masses cooperate there is an exploitation equilibrium, in which members of the elite receive a per-capita fraction $(1-\beta)/\nu$ of the country's appropriable resources (net of exploitation costs) and members of the masses receive a fraction $\beta/(n'-\nu)$.[3] Finally, members of group B decide whether to pass or not. Realistically, we assume that members of group B who pass will be part of group A's masses, i.e. it is impossible to pass oneself as a member of A's elite.

The passing decision in this extended model leads to a solution for the equilibrium size of group A in the case of conflict similar to the baseline case, namely $n' = \max[n, \min(1, \nu + \beta\bar{n})]$, where $\bar{n}$ was defined in the baseline section as $z/(\phi y_B)$. Hence the only difference is that the passing threshold $\beta\overline{n}$ now depends on the spoil-sharing rule. Exploitation occurs if both elite and masses of group A are

[3] Once again because all members of the masses are identical their colllective decision would be the same under virtually any mechanism to aggregate preferences.

better off under conflict. This results in the following two conditions:

$$\frac{\beta}{(n' - \nu)}(1 - \delta)z + (1 - \delta)y_A > z + y_A, \tag{6}$$

for the masses, and

$$\frac{1 - \beta}{\nu}(1 - \delta)z + (1 - \delta)y_A > z + y_A, \tag{7}$$

for the elite. Note that (7) is always satisfied for ν sufficiently small, so we assume this constraint is never binding. In other words the elite of the dominant group always gains from ethnic politics, which seems realistic. The question is whether the masses go along.

From (6) the masses will go along if

$$n' < \nu + \frac{\beta(1 - \delta)z}{z + \delta y_A} = \nu + \beta\tilde{n}.$$

Once again this expression is closely reminiscent of the baseline model of Section **??**. Indeed as the size of the elite goes to 0, both the passing threshold and the conflict threshold converge to the baseline expressions, except that both thresholds are multiplied by β. Aside from this rescaling, therefore, all the comparative static results with respect to the baseline model parameters are exactly the same as in the homogenous case, and the model delivers the same messages.

3 Unpublished Appendix 3: Multiple Groups

So far we have focused on countries with only one (potential) ethnic cleavage. In many countries there are multiple politically-relevant ethnic groups. Furthermore, in many countries there are multiple dimensions along which ethnicity can become politically salient. For example there could be cross-cutting religious and skin-color cleavages, and the relevant dimension for group action could turn out to be religion (giving rise to groups that are heterogenous in skin color) or racial (giving rise to groups that are heterogenous in religion). In this section we sketch how our model could be extended to account for the multiplicity of potential ethnic categories.

Suppose that there are I "ethnically homogenous" groups, in the sense that all the members of each groups have the same physical, religious, linguistic, and cultural features. In the example above with two skin colors and two religions $I = 4$. Each group is characterized by its per-capita income, y_i, and relative group size, n_i. Furthermore, each *pair* of groups i, j is characterized by a switching cost $\phi_{i,j}$, which is the cost of switching identity from i to j and j to i.

Assume next that there is a nonempty set of "potentially winning coalitions." A potentially winning coalition is a coalition of groups that has the capability of imposing an exploitation equilibrium on the groups who are not in it. Naturally a potentially winning coalition could be made of a single group. Denote by W both the set and the number of potentially winning coalitions. Next assume that there is a "natural order" among the potentially winning coalitions. A coalition in W gets to decide whether to impose an exploitation equilibrium only if none of the previous coalitions in the natural order has decided to exploit. If a coalition gets an opportunity to decide, it imposes an exploitation equilibrium if and only if all the groups in the coalition play the same action C. If any group in the coalition plays P the decision passes on to the next coalition in W.[4] There is an exploitation equilibrium if one of the coalitions in W decides to exploit the other groups. Without loss of generality coalitions in W are indexed by their natural order (i.e. coalition 1 is the first in the natural order, etc.)

If no coalition exploits, then each member of group i receives $y_i + z$, for every i. If coalition $t \in W$ exploits, then each member of group $i \in t$ receives $(1-\delta)(y_i + z/n')$, where $n' = \sum_{s \in t} n'_s$, i.e. n' is the ex-post sum of members of the groups in the exploiting coalition.[5] Each member of group $i \notin t$ receives $(1-\delta)y_i$ if he does not pass, and $(1-\delta)((1-\tilde{\phi}_{it})y_i + z/n')$, where $\tilde{\phi}_{it} = \min\{\phi_{is}, s \in t\}$. In other words members of each exploited group will pass, if they pass at all, into the group in the exploiting coalition that is less distant from them. In order to avoid possible indeterminacies we assume that passing also occurs sequentially among members of the exploited groups. In particular, the group whose members have most to gain from passing passes first. If passing makes residual members of this first group indifferent between passing or staying then there is no further passing. However if all the members of the first group have passed then the opportunity moves to members of the group with the highest gains from passing among the remaining exploited groups, etc. Notice that unlike in the baseline case there can be exploitation in equilibrium even when an entire group passes - provided there are sufficiently many remaining people in other exploited groups.

There is no way to solve this model in closed form, but it would be easy to do so numerically. One would start by computing for each group the value of the game if the decision reached coalition W, i.e. the last potentially winning coalition. This would depend on the decision of the members of coalition W to play C or P, as well as the amount of equilibrium passing (in case of conflict) from

[4] The "natural order" assumption is a simplification. It would be possible to use results in the literature on coalition formation to endogenize the order in which coalitions decide.

[5] Hence we assume that members of the exploiting coalition share equally. This could be extended by introducing a within-coalition bargaining stage.

the groups not in W. One would then move to coalition $W-1$ (the preceding one in the natural order) and compute for each group in $W-1$ the relative payoff of exploiting as members of $W-1$ or let the game move on to coalition W. One would then proceed recursively backward all the way to coalition 1 in W. Note that some coalitions in W may pass on the opportunity to exploit even if they prefer conflict to peace, if some of the groups in that coalition prefer to be part of a smaller exploiting coalition further down the natural order (and predict that the game will arrive to that coalition). Using this algorithm, for each vector of incomes y_i, ex-ante group sizes n_i, matrix of bilateral passing costs ϕ_{ij}, resource rents z, and exploitation cost δ one can predict whether an exploitation equilibrium will prevail.

Despite the elusiveness of general closed-form results, it should be easy at this point to see that the model will share qualitative features of the baseline model. For example, there will be no conflict if all the ϕ_{ij}s are close to zero, as all potentially winning coalitions will be infiltrated to the point of making exploitation pointless. By the same token, at least some of the ϕ_{ij}s need to be reasonably large to make exploitation worthwhile. Hence, the model still implies that ethnic distance is a key determinant of conflict. Similarly, very large values of z will trigger more passing and thus discourage conflict. The model will thus generate a similar inverted-U relation between z and conflict (holding constant the ethnic structure) as the baseline model.