Information Aggregation in Financial Markets with Career Concerns*

Amil Dasgupta and Andrea Prat
London School of Economics and CEPR

October 2004; Revised December 2007

Forthcoming in the *Journal of Economic Theory*

Abstract

What are the equilibrium features of a dynamic financial market in which traders care about their reputation for ability? We modify a standard sequential trading model to include traders with career concerns. We show that this market cannot be informationally efficient: there is no equilibrium in which prices converge to the true value, even after an infinite sequence of trades. We characterize the most revealing equilibrium of this game and show that an increase in the strength of the traders' reputational concerns has a negative effect on the extent of information that can be revealed in equilibrium but a positive effect on market liquidity.

Keywords: Financial Equilibrium; Career Concerns; Information Cascades

*We are grateful to an Associate Editor and an anonymous referee for helpful comments. We thank Yacine Ait-Sahalia, Patrick Bolton, James Dow, Satoshi Kowanishi, Pete Kyle, Stephen Morris, Hyun Shin, Raj Singh, and Dimitri Vayanos for helpful discussions, and seminar participants at Bocconi, CeFiMS, CEPR-Gerzensee meetings, Heidelberg, Helsinki, IIES-Stockholm, LSE, Munich, NYU, Norwegian School of Economics, Nuremberg, Pompeu Fabra, Princeton, Toronto, Warwick Business School, WFA 2006 Meetings, and Yale for comments. Dasgupta thanks the EPSRC for financial support via grant GR/S83975/01. An earlier version of this paper was circulated under the title "Asset Price Dynamics When Traders Care About Reputation." E-mail: a.dasgupta@lse.ac.uk, a.prat@lse.ac.uk

1 Introduction

The substantial increase in the institutional ownership of corporate equity around the world in recent decades has underscored the importance of studying the effects of institutional trade on asset prices.[1] Institutions, and their employees, may be guided by incentives not fully captured by standard models in finance. For example, consider the case of US mutual funds which make up a significant proportion of institutional investors in US equity markets. An important body of empirical work highlights the fact that mutual funds (e.g. Chevalier and Ellison [8]) and their employees (Chevalier and Ellison [9]) both face *career concerns*: they are interested in enhancing their *reputation* with their respective principals and sometimes indulge in perverse actions (e.g. excessive risk taking) in order to achieve this. Given the importance of institutions in equity markets, it is plausible to expect that such behavior may affect equilibrium quantities in these markets. What are the equilibrium features of a market in which a large proportion of traders care about their reputation?

While a growing body of literature examines the effects of agency conflicts on asset pricing, the explicit modeling of reputation in financial markets is in its infancy.[2] Dasgupta and Prat [11] present a two-period micro-founded model of career concerns in financial markets to examine the effect of reputation in enhancing trading volume. However, that analysis is done for a static market: each asset is traded only once.

In this paper, in contrast, we study a multi-period sequential trade market in which some traders care about their reputations. We show that the equilibrium properties of this market are very different from those of standard markets. In particular, we show that the presence of career concerned traders limits the extent to which equilibrium prices can aggregate the private information of market participants. The endogenous limits on the informativeness of trades and prices that we derive have important implications for the liquidity and volatility of assets traded by institutions. We delineate these implications and relate them to the strength of institutional career concerns. This paper, therefore, provides a foundation for linking the incentives of delegated financial traders and the equilibrium properties of markets in which they trade over time.

[1] On the New York Stock Exchange the percentage of outstanding corporate equity held by institutional investors increased from 7.2% in 1950 to 49.8% in 2002 (NYSE Factbook 2003). Allen [1] presents persuasive arguments for the importance of financial institutions to asset pricing.

[2] For example, Allen and Gorton [2], Dow and Gorton [15], and He and Krishnamurthy [17] examine the asset pricing implications of non-reputational agency conflicts. Reputational concerns are implicit in the contractual forms assumed in the general equilibrium models of Cuoco and Kaniel [10] and Vayanos [30].

1.1 Summary of Results

We present the most parsimonious model that captures the essence of our arguments. Much of our model is standard. We present a T-period sequential trade market for a single (Arrow) asset where all transactions occur via uninformed *market makers* who are risk neutral and competitive (following Glosten and Milgrom [16] and Kyle [18]) and quote bid and ask prices to reflect the informational content of order flow. In addition there is a large group of liquidity-driven *noise traders* who trade for exogenous reasons that are unrelated to the liquidation value of the asset.

Our only innovation is that we introduce a large group of reputationally-concerned traders (whom we call *fund managers*), who trade on behalf of other (inactive) investors. These traders receive a payoff that depends both on the direct profits they produce and on the reputation that they earn with their principals.[3] Their reputation is determined endogenously by Bayesian investors, in a way that will be described shortly.

The fund managers can be of two types (smart or dumb) and receive informative signals about the asset liquidation value, where the precision depends on their (unknown) type. In each trading round either a randomly selected fund manager or a noise trader interact with the market maker. The asset payoff is realized at time T and all payments are made.

At time T, every fund manager is evaluated on the basis of all available information, with the exception of the agent's private signal. This implies that each investor can observe the liquidation of the asset and the portfolio choice of his own agent.[4] This assumption is plausible for relatively sophisticated investors, such as corporate pension plans, investment banks, insurance companies, and hedge fund clients. It may instead be an unrealistic requirement for retail mutual fund investors, who typically have limited knowledge of their fund's portfolio composition.[5]

We present the following results.

1. We begin with an impossibility result. We show that, in this market of career-concerned traders, *prices never converge to true liquidation value, even after an infinite sequence of trades.* If fund managers trade according to their private signal, the price evolves to incorporate such private information. Over time, the price should converge to the true liquidation value. However, as the uncertainty over the liquidation value is resolved,

[3] The principals may be line managers at mutual fund companies with oversight over the particular fund manager's activities, or, directly, the investors who have placed their funds with the company.

[4] For all our core results, it is irrelevant whether the investor observes the portfolio choice of other fund manager besides his.

[5] See Prat [23] for a discussion of the role of portfolio disclosure in delegated portfolio management with career concerns.

two things happen. First, the fund managers have less opportunity to make trading profits because the price is close to the liquidation value. The expected profit for a fund manager who trades according to his signal is always positive, but it tends to zero as the price becomes more precise. Second, taking a "contrarian" position (e.g. selling when the price has been going up) starts to carry an endogenous reputational cost: with high probability, the trade will turn out to be incorrect and the fund manager will "look dumb" in the eyes of (rational) principals. Because of the combination of these two effects, if the price becomes sufficiently precise fund managers begin to behave in a conformist way: their trade stops reflecting their private information. From then on, there is no information aggregation whatsoever and the price stays constant.

2. We then investigate how much private information can be revealed by equilibrium trades despite the presence of career concerns. We do this by characterizing the most revealing trading strategies that can be sustained in equilibrium. We show that, as long as the price leaves sufficient uncertainty about liquidation values, sincere trade can be supported in equilibrium. Thus, each manager's signal can be fully revealed via his trade. However, as uncertainty is resolved, equilibrium trade becomes partially or completely uninformative.

   We show a number of monotonicity results, which relate the strength of career concerns with the extent of information revelation. For every price level, the amount of information revealed in equilibrium is decreasing in the importance of career concerns. We also consider the maximal and minimal ranges of equilibrium prices that can support completely sincere and fully conformist trading respectively, and characterize how such price ranges evolve with the importance of career concerns.

3. We consider the impact of career concerns on other core financial market variables: market liquidity and price volatility. We show that increasing career concerns increases liquidity and decreases volatility. Thus, increased institutional presence in a market decreases price informativeness, but has potentially beneficial impacts via liquidity and volatility. Our analysis provides theoretical underpinnings for a number of recent papers on herding in financial markets, which are discussed in the conclusion.

4. Finally, we examine a number of natural extensions of the model. The baseline model is presented with a binary asset liquidation value. We show that our impossibility result extends to richer payoff spaces. Further, in our baseline model we assumed that fund managers were unaware of their type. We extend the model to demonstrate that as long as self-knowledge is not too accurate, our main conclusion remains valid.

1.2 Related Literature

This paper brings together two influential strands of the literature. The first strand concerns the theory of dynamic financial markets with asymmetrically informed traders (Glosten and Milgrom [16] and Kyle [18]). The second strand focusses on the analysis of career concerns in sequential investment decision-making (Scharfstein and Stein [24]). Models in the first strand consider a full-fledged financial market with endogenously determined prices but do not allow traders to have career concerns. Models in the second strand do the exact opposite: they analyze the role of reputational concerns in a partial equilibrium setting, where prices are exogenously fixed.

In the first strand, Glosten and Milgrom [16] have shown that in dynamic financial markets the price must tend to the true liquidation value in the long term. More recently, Avery and Zemsky [4] have shown that statistical information cascades à la Banerjee [5] and Bikhchandani, Hirshleifer, and Welch [6] are impossible in such a market.[6] After every investment decision, the price adjusts to reflect the expected value of the asset based on information revealed by past trades. Thus, traders with private information stand to make a profit by trading according to their signals. But by doing so, they release additional private information into the public domain. In the long run, the market achieves informational efficiency.

In the second strand, Scharfstein and Stein [24] have shown that managers who care about their reputation for ability may choose to ignore relevant private information and instead mimic past investment decisions of other managers.[7] This is because a manager who possesses "contrarian" information (for instance he observes a negative signal for an asset that has experienced price growth) jeopardizes his reputation if he decides to trade according to his signal. Scharfstein and Stein's analysis is carried out in partial equilibrium: prices play no informational role in such an analysis. For a general analysis of this class of partial equilibrium models see Ottaviani and Sorensen [21].

[6] A word of caution is in order here. There is almost universal agreement in the literature on the meaning of a *cascade*, which is the definition we have used above (an equilibrium event in which information gets trapped, and agents' actions no longer reveal any of their valuable private information). However, there is little agreement on the definition of the term *herds* (for example, substantively different definitions are used by Avery and Zemsky [4], Smith and Sorensen [28], and Chari and Kehoe [7]). In the interest of clarity, throughout this paper we shall restrict attention to cascades *only*.

Under additional assumptions, Avery and Zemsky [4] show that a form of herd behaviour may occur in the presence of prices. However, in all versions of their model cascades are absent and prices always converge to true liquidation value (Avery and Zemsky Proposition 2). Recently, Park and Sabourian [22] have explored generalizations of the necessary conditions for herds in Avery and Zemsky's model. As in Avery and Zemsky, however, cascades cannot arise in their model.

[7] Other more recent papers in this strand include, for example, Avery and Chevalier [3] and Trueman [29].

Our results provide a clean theoretical link between the two types of economies represented in these two strands of the literature. On the one hand, in "Glosten-Milgrom type" economies, prices play an informational role and agents are motivated purely by trading profits. In such economies, agents always utilize their information and prices always converge to true liquidation value in the long run. On the other hand, in "Scharfstein-Stein type" economies prices are *assumed* to play no informational role and agents care only about ex post reputation for ability. In such economies, agents engage in conformist behavior in order to enhance their reputation. Our central observation is that if traders care even slightly about reputation in a Glosten-Milgrom type economy, then prices can play only a *limited* informational role. In order to converge to true value, prices must get close enough to true value. But when this happens, profits become unimportant, and reputational concerns become predominant. Then, the Glosten-Milgrom economy metamorphoses into a Scharfstein-Stein economy. But in the latter, conformism arises, and thus prices cannot incorporate further information.

In addition, by studying career concerns in financial equilibrium, we are able to study the effects of micro-founded reputation-driven conformism on financial market quantities (prices, informational efficiency, trade patterns, liquidity, and volatility), which leads to relevant predictions on observable market variables, as discussed above.

Other authors (Lee [19] and Chari and Kehoe [7]) have argued that information cascades can occur when prices are endogenous. However, their arguments hinge on a market breakdown: trade stops altogether.[8] Instead, in our model cascades occur in a functioning financial market with trade.

The rest of the paper is organized as follows. In the next section we present the model. Section 3 demonstrates the impossibility of full information aggregation. Section 4 characterizes the relationship between the importance of career concerns and the extent of equilibrium information aggregation. Extensions are examined in section 5. Section 6 concludes.

2 The Model

The economy lasts T discrete periods: $1, 2, ...T$. Trade can occur in periods $1, 2...T-1$. The market trades an Arrow security, which has equiprobable liquidation value $v = 0$ or 1, which is revealed at time T.

In practice, the asset could be a bond with maturity date T with a serious possibility of default. It could also be the common stock of a company which is expected to make an

[8] In Lee's [19] model the existence of a transaction cost to trading can prevent traders with relatively inaccurate signals from trading, thus trapping private information in an illiquid market. In Chari and Kehoe [7], traders have the option of exiting the market (by making an outside investment) and may in equilibrium find it optimal to exit before further information arrives, thus, again, trapping private information.

announcement of great importance (earnings, merger, etc.) at time T: all traders know that the announcement will occur but they may have different information on the content of the announcement.

There are a large number of fund managers and noise traders. At each period $t \in \{1, 2, ..., T-1\}$ either a fund manager or a noise trader enters the market with probabilities $1-\delta$ and $\delta \in (0,1)$ respectively. The traders interact with a market maker, and can issue market orders (a_t) to buy ($a_t = 1$) one unit or sell ($a_t = 0$) one unit of the asset. The market maker posts ask (p_t^a) and bid (p_t^b) prices at which he will sell or buy one unit of the asset respectively. As is standard in the literature (Glosten and Milgrom [16], Kyle [18]), we assume that the market maker is risk-neutral competitive, and thus the quoted prices will be equal to expected value of v conditional on the order history.

Denote the history of observed orders at the beginning of period t (not including the order at time t) by h_t. Let $p_t = E(v|h_t)$, $p_t^a = E(v|h_t,\text{buy})$, $p_t^b = E(v|h_t,\text{sell})$. Note that at any time t, p_t plays a dual role: on the one hand it is the *most recent transaction price*; on the other, it represents the *public belief* about v at the beginning of period t. We shall, therefore, refer to p_t below interchangeably as the "price" or the "public belief", depending on the context.

The fund manager can be of two types: $\theta \in \{b, g\}$ with $\Pr(\theta = g) = \gamma \in (0,1)$. The type is independent of v. If at time t a fund manager appears, he receives a signal $s_t \in \{0,1\}$ with distribution

$$\Pr(s_t = v|v, \theta) = \sigma_\theta,$$

where

$$\frac{1}{2} \leq \sigma_b < \sigma_g \leq 1.$$

Fund managers do not know their type. Noise traders buy or sell a unit with equal probability independent of v.

The profit obtained by the trader at time t is defined by:

$$\pi_t\left(a_t, p_t^a, p_t^b, v\right) = \begin{cases} v - p_t^a & \text{if } a_t = 1 \\ p_t^b - v & \text{if } a_t = 0 \end{cases}$$

If a fund manager traded at time t, his actions are observed at time T. Principals (e.g., line managers in the fund management firm) form a posterior belief about the manager's type based upon all observables, namely the whole history of trades and prices (h_T) and the realized liquidation value (v). We call this the manager's reputation and define it to be:

$$r_t(h_T, v) = \Pr(\theta_t = g|h_T, v).$$

The fund manager at time t receives utility

$$u_t = \beta\pi_t + (1-\beta)r_t,$$

where $1-\beta \in (0,1)$ measures the importance of career concerns.[9] A game Γ is defined by the values of five parameters $(\beta, \sigma_b, \sigma_g, \gamma, \delta)$.

Let $\alpha^t_{s_t}(h_t)$ be the probability that the manager plays $a_t = 1$ given history h_t and signal realization s_t. A perfect Bayesian equilibrium of the game is a collection $\{\alpha^t_{s_t}(h_t)\}_{t=1}^{T-1}$ for every possible history h_t and signal realization s_t, satisfying the standard definition.

Finally, in a given PBE of the game, at a given time t, and for a given history h_t, we denote by $\Delta\pi^t_{s_t}$ the expected excess profit for the manager who has observed signal s_t from buying rather than selling. Similarly, denote the expected excess reputation by $\Delta r^t_{s_t}$ and the expected excess overall utility by $\Delta u^t_{s_t}$. Also, denote the private expectation of the manager about v after observing history h_t and his signal s_t by $v^t_{s_t}$. In our subsequent discussion, we will often hold time and history constant, and denote these simply by $\Delta\pi_s, \Delta r_s, \Delta u_s$ and v_s respectively.

Our model departs from Glosten and Milgrom's [16] only in that our informed traders – the fund managers – care about reputation as well as profit. If we set $\beta = 1$, our model becomes a special case of Glosten-Milgrom, and all their results apply as stated.

3 The Impossibility of Full Revelation

If there are no career concerns ($\beta = 1$), equilibrium behavior is *sincere* (Glosten and Milgrom [16], Avery and Zemsky [4]): for any t, h_t, $\alpha^t_1(h_t) = 1$ and $\alpha^t_0(h_t) = 0$. The presence of noise traders ensures that $p^a_t < v^1_t$ and $p^b_t > v^0_t$, which makes it optimal for the purely profit-motivated trader to buy if $s_t = 1$ and to sell if $s_t = 0$. Thus, each trader trades according to his information ($a_t = s_t$).[10] This means that prices impound information and $p_t \to v$ as $t \to \infty$. However, equilibrium behavior is very different when $\beta < 1$. We can now state our main result:

Proposition 1 *For any game Γ with $\beta < 1$, there exists $\underline{p} \in \left(0, \frac{1}{2}\right)$ such that in any equilibrium of the game, at all times, $p_t \in \left(\underline{p}, 1-\underline{p}\right)$.*

[9] Our qualitative results hold for a much larger class of payoff functions: $u_t = \beta\chi(\pi_t) + (1-\beta)R(r_t)$ where χ and R are increasing and piecewise continuous functions. Such an extension increases algebra without adding to intuition. See Dasgupta and Prat [12] for details.

[10] See Park and Sabourian [22] for a general characterization of necessary and sufficient conditions for non-herding in sequential trade models without career concerns.

For an equilibrium to be informative, the actions of traders who receive signal $s = 1$ must differ at least probabilistically from the actions of traders who receive signal $s = 0$, i.e., α_1 must be different from α_0. Following a given history (h_t), and a corresponding price (p_t), informative equilibrium strategies at t must, therefore, satisfy either $\alpha_1^t(h_t) > \alpha_0^t(h_t)$ or $\alpha_1^t(h_t) < \alpha_0^t(h_t)$. Our proof shows that, when prices are sufficiently extreme, neither of these is possible. The proof of the result (as well as those of all subsequent results) is in the appendix. Here, we provide some intuition for why the result is true. In the discussion that follows, we fix an arbitrary time (t) and history (h_t), and thus suppress the time and history dependence of α_1 and α_0.

There are three crucial (endogenous) properties of our financial market that drive our results. The first property is that profit motives always encourage traders to trade sincerely. Private information is valuable, and in the presence of noise, prices reflect only part of this information. It always enhances the profits of traders to follow their private information. The second property is that when transaction prices, and therefore public beliefs, indicate that some liquidation value (say, $v = x$) of the asset is sufficiently more likely than the other, the reputational incentives of a career-concerned fund manager encourage him to act in a manner that will make the principal believe that the manager received the signal that is more likely to arise when $v = x$. This enhances the manager's reputation, because types are differentiated by their relative information precision. Finally, the third property is that when prices become sufficiently extreme, and thus sufficiently precise, trading profits become small because the beliefs of informed and uninformed traders converge.

We shall now argue that a combination of two or more of these ingredients rule out equilibria where, when prices are high or low enough, it is possible to have either $\alpha_1 > \alpha_0$ or $\alpha_1 < \alpha_0$.

First, consider the case in which $\alpha_1 > \alpha_0$. It is easy to see that the combination of the second and third properties rule out informative equilibria of this type for high enough or low enough prices. In an equilibrium with $\alpha_1 > \alpha_0$, when the principal sees a manager buy, he attaches high probability to the manager having received signal 1. This enhances the reputation of the manager, if, ex post, the liquidation value turns out to be 1. If instead the liquidation value turns out to be 0, the manager's reputation suffers. Consider a manager who has received signal 1 and suppose that transaction prices p get very small (we loosely write "$p \to 0$"). The third property implies trading profit becomes small ($\Delta\pi_1 \to 0$) and has a small impact on trading decisions. However, if $p \to 0$ in an informative equilibrium, it becomes very likely that $v = 0$. Thus, the second property implies that the manager's reputational incentives will encourage him to take the action that will make the principal believe that he has received signal 0. Thus, from a reputational perspective, this manager must prefer to sell instead of buy ($\Delta r_1 < 0$). As profit motivations diminish and reputational motivations become one-sided, eventually the latter dominates the former ($\Delta u_1 < 0$) and the manager ignores his private information: $\alpha_1 = 0$. Thus, for sufficiently extreme p, it cannot be the case that $\alpha_1 > \alpha_0$.

Consider next the case in which $\alpha_1 < \alpha_0$. The combination of the first two properties rules out this type of equilibrium when transaction prices are sufficiently extreme. In an equilibrium with $\alpha_1 < \alpha_0$, when the principal sees a manager buy, he attaches high probability to the manager having received signal 0. This enhances the reputation of the manager if, ex post, the liquidation value turns out to be 0. Consider a manager who has received signal 1. The first property implies that profit motivations drive this manager to buy ($\Delta\pi_1 > 0$). However, since $\alpha_1 < \alpha_0$, it must be the case that $\alpha_1 < 1$. This implies that $\Delta u_1 \leq 0$, which can only arise if the reputational value of buying is strictly lower than the reputational value of selling ($\Delta r_1 < 0$). Now suppose that $p \to 0$. In an informative equilibrium, this means that it is very likely that $v = 0$. The second property now implies that the manager can gain reputationally by signalling that he received $s = 0$, which he can do only by *buying*! Thus, it must be reputationally advantageous for him to buy rather than sell ($\Delta r_1 > 0$) for low enough public beliefs, contradicting our conclusion above. Thus, for sufficiently extreme p, it cannot be the case that $\alpha_1 < \alpha_0$.

Thus, the only possible equilibrium actions for sufficiently extreme prices involves $\alpha_1 = \alpha_0$. But since these actions are uninformative, such trades do not move the price further.

The price bounds identified in Proposition 1 are independent of history and time, and therefore of the length of the game T. This is because, while equilibrium strategies can in general be time and history dependent, we have shown that if prices ever attain our bounds, the continuation equilibrium is unique, independent of history and time, and dictates complete conformism.[11]

What happens to the price in the long-run, i.e., as $T \to \infty$? To simplify our exposition, it is useful to briefly augment our notation: Let p'_t be the realized transaction price at t, and let h'_t be the history of orders up to and including t (so that $p'_t = p_{t+1}$ and $h'_t = h_{t+1}$). It is clear that transaction prices $\{p'_t\}_{t\geq 1}$ forms a non-negative martingale with respect to $\{h'_t\}$: $p'_t = E(v|h'_t)$, $p'_{t+1} = E\left(v|h'_{t+1}\right)$, and thus $E\left(p'_{t+1}|h'_t\right) = E\left(E\left(v|h'_{t+1}\right)|h'_t\right) = E(v|h'_t) = p'_t$. Thus, by a standard Martingale convergence theorem (see, for example, Shiryaev [25], Chapter 7), the sequence $\{p'_t\}_{t\geq 1}$ converges almost surely to a random variable, p'_∞.[12] This

[11] Needless to say, while are results are formally valid for all T, they are more *interesting* for large T. For sufficiently small T the range of possible transaction prices in the game *without* career concerns ($\beta = 1$) may lie within the bounds identified in Proposition 1.

[12] More formally, we would need to define a filtration with respect to which $\{p'_t\}_{t\geq 1}$ is a martingale. Fix a game Γ and an equilibrium ε. For $t = 1$, define, x_1 to be a random variable which takes the value b (buy) with

implies that transaction prices must "settle down" in the long run. However, note that $Var(v|p'_t)$ is bounded below by $\underline{p}\left(1-\underline{p}\right)$. Thus, it is possible that transaction prices will be "trapped" close to $\underline{p}$ when $v=1$ or close to $1-\underline{p}$ when $v=0$.

For our main result to have economic significance, it is necessary the non-revelation region of prices to be non-trivial. How large is the non-revelation region? This issue is addressed in the following result:

Proposition 2 *For any $\underline{p} \in \left(0, \frac{1}{3}\right)$, there exists an open and non-empty set of games Γ such that in any equilibrium of those games, at all times, $p_t \in \left(\underline{p}, 1-\underline{p}\right)$.*

This proposition establishes that the non-revelation region is non-trivial. For any $\underline{p} < \frac{1}{3}$, there exist a positive measure of games (the space of games is the space on which the parameters $(\beta, \sigma_b, \sigma_g, \gamma, \delta)$ are defined) in which transaction prices can never be lower than $\underline{p}$ or higher than $1-\underline{p}$. The $\underline{p} < \frac{1}{3}$ bound comes from the worst-case scenario for information revelation, namely when career concerns are very important ($\beta \to 0$), smart managers are very smart ($\sigma_g \to 1$), dumb managers are very dumb ($\sigma_b \to \frac{1}{2}$), and most managers are dumb ($\gamma \to 0$).

Our result bears a connection to Ottaviani and Sorensen [21], who provide a general analysis of reputational cheap talk in partial equilibrium and show that full information transmission is generically impossible. Our aim and analysis is different, however, since our reputation model is embedded in a financial market and our experts have both a profit motive as well as a reputation motive. As the discussion to date makes clear, our results are driven by the interaction of these two motives.

4 Career Concerns and the Sincerity of Equilibrium Trades

Full information aggregation fails in our model because the endogenous reputational incentives of delegated portfolio managers prevent them from using their private information in choosing their trades for sufficiently extreme prices. In the analysis that follows, we characterize the maximum extent to which career concerned traders can utilize their private information in equilibrium. In other words, we identify and characterize the *most revealing* trading that can arise in equilibrium. This characterization can be found in section 4.1. We

probability q_1 and s (sell) with probability $1-q_1$, where q_1 is uniquely determined by Γ and ε. For any $t \geq 2$, define x_t to be a random variable which takes the value b with probability q_t and s with probability $1-q_t$, where q_t is uniquely determined by the realized sequence $\{x_s; 1 \leq s < t\}$, Γ, and ε. Now let $F_t = \sigma_t(X_s; 1 \leq s \leq t)$, i.e., the sigma-algebra *generated* by the process $\{x_t\}_{t \geq 1}$, and let $p'_t = E(v|F_t)$. Then $\{F_t\}_{t \geq 1}$ is a *filtration*, and the sequence $\{p'_t, F_t\}_{t \geq 1}$ is a non-negative martingale.

then show in section 4.2 that there exists a range of prices for which the most revealing equilibrium trade can be *sincere*, i.e., fully reveal the signals of managers. Next, in section 4.3, we characterize how the most revealing equilibrium varies as a function of the importance of career concerns. Finally, in section 4.4, we consider the implications of these comparative statics results for financial market variables.

While we have demonstrated our main impossibility result across the full spectrum of potential perfect Bayesian equilibria, for our comparative statics results we focus only on "non-perverse" equilibria with $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$ for all t and h_t. These are the only reasonable equilibria in a financial context. Other "perverse" equilibria feature strictly negative bid-ask spreads along the equilibrium path, which are very unrealistic in financial markets.[13]

4.1 The Most Revealing Equilibrium

We first demonstrate that there exists an equilibrium in which managers always play non-perversely, i.e., $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$ for all t and h_t.

Proposition 3 *There exists an equilibrium in which, for every time $t = 1, ...T$ and every history h_t, $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$.*

Let E denote the (non-empty) set of non-perverse equilibria . Suppose there exists an equilibrium $\bar{e}$ such that, for any t, any h_t and any other equilibrium $e \in E$, we have

$$\alpha_1^t(h_t, \bar{e}) \geq \alpha_1^t(h_t, e) \text{ and } \alpha_0^t(h_t, \bar{e}) \leq \alpha_0^t(h_t, e).$$

If such an equilibrium exists, we call it a *most revealing equilibrium.* In most revealing equilibria, managers maximally condition their trades on their valuable private information.[14]

Denote the excess benefit to the manager who observes signal s_t at t by $\Delta u_{s_t}\left(\alpha_0^t, \alpha_1^t, p_t\right)$.[15] We now characterize most revealing equilibria as follows.

[13] A negative bid-ask spread would create an instantaneous risk-free arbitrage opportunity.This opportunity cannot be exploited in a Glosten-Milgrom setup like ours because there are no agents who can buy and sell at the same time. One could conceivably rule out perverse equilibria by adding uninformed short-lived arbitrageurs to the model. However, this would substantially complicate the model without generating additional insights on information aggregation. In addition, such a modification would take us further away from the well-known baseline model of sequential trade in the absence of career concerns, against which we currently benchmark our results.

[14] It is intrinsically difficult to compare equilibria in terms of overall "informativeness". The more information is revealed at time t, the less information is left to reveal at time $t+1$. Our most revealing equilibrium is the non-perverse equilibrium that reveals information in the "fastest way". Namely, at any time t, there exists no other non-perverse equilibrium that reveals more information at time t. If the goal of the social planner is information revelation, and the planner has a sufficiently high discount rate, this is the equilibrium the planner would prefer at any time t.

[15] The proof of Proposition 3 shows that it is without loss of generality to write $\Delta u_{s_t}(h_t)$ as $\Delta u_{s_t}\left(\alpha_0^t, \alpha_1^t, p_t\right)$.

Proposition 4 *The most revealing equilibrium exists and is unique. In the most revealing equilibrium, play depends on history only through the price. It can be expressed as follows: If* $p_t \leq \frac{1}{2}$*, then*

$$\begin{aligned} \alpha_0^t(h_t) &= 0 \\ \alpha_1^t(h_t) &= \bar{\alpha}(p_t) \end{aligned}$$

where

$$\bar{\alpha}(p) \equiv \begin{cases} 1 & \text{if } \Delta u_1(\alpha_0 = 0, \alpha_1 = 1, p) \geq 0 \\ 0 & \text{if } \Delta u_1(\alpha_0 = 0, \alpha_1, p) < 0,\ \forall \alpha_1 \\ \max\{\alpha_1 | \Delta u_1(\alpha_0 = 0, \alpha_1, p) = 0\} & \text{otherwise} \end{cases}$$

If $p_t \geq \frac{1}{2}$*, then*

$$\begin{aligned} \alpha_0^t(h_t) &= \underline{\alpha}(p_t) \\ \alpha_1^t(h_t) &= 1 \end{aligned}$$

where

$$\underline{\alpha}(p) \equiv \begin{cases} 0 & \text{if } \Delta u_0(\alpha_0 = 0, \alpha_1 = 1, p) \leq 0 \\ 1 & \text{if } \Delta u_0(\alpha_0, \alpha_1 = 1, p) > 0,\ \forall \alpha_0 \\ \min\{\alpha_0 | \Delta u_0(\alpha_0, \alpha_1 = 1, p) = 0\} & \text{otherwise} \end{cases}$$

We now proceed to document properties of the most revealing equilibrium of our game. We first show that the most revealing equilibrium can be sincere for some range of prices. We then consider how the most revealing equilibrium changes as a function of the importance of career concerns.

4.2 Sincere Trading

In the absence of career concerns (when $\beta = 1$), sincere trading, i.e., trading which completely reveals individual signals, is the unique equilibrium outcome of our game (Glosten and Milgrom [16], Avery and Zemsky [4]). Our main result implies that in the presence of career concerns ($\beta < 1$) sincere trading cannot be sustained at all possible prices. We now consider whether, despite the presence of career concerns, sincere trading can be sustained for *some* prices. We show that if the price is sufficiently close to $\frac{1}{2}$, the most revealing equilibrium is sincere. Let $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$.

Proposition 5 *In the most revealing equilibrium, if* $p_t \in (1-\sigma, \sigma)$ *then the fund manager at* t *trades sincerely.*

The intuition for this result is straightforward. If $p_t \in (1-\sigma, \sigma)$, we have that $v_0^t < \frac{1}{2} < v_1^t$. The manager thinks that the high state is more likely if and only if he has a positive signal. It is easy to see that this implies that, if investors expect sincere play, a manager with a positive signal should indeed buy and one with a negative signal should indeed sell.

4.3 How Career Concerns Affect the Informativeness of Trade

How does the informativeness of financial market trade vary with the incentive structure faced by its traders? Having identified the most revealing equilibrium in Proposition 4, we now proceed to characterize how such an equilibrium evolves as a function of β.

We restrict attention to $p_t \leq \frac{1}{2}$. All results for $p_t \geq \frac{1}{2}$ are symmetric. Proposition 4 tells us that for $p_t \leq \frac{1}{2}$, trading strategies in the most revealing equilibrium at some price p_t, can be characterized as follows: $(\alpha_1(h_t) = \bar{\alpha}(p_t), \alpha_0(h_t) = 0)$. We show how $\bar{\alpha}(p_t)$ varies as a function of β. To emphasize the reliance on β we henceforth write $\bar{\alpha}(\beta, p_t)$ for $\bar{\alpha}(p_t)$.

Proposition 6 *For all $\beta'' > \beta'$:*
(i) $\bar{\alpha}(\beta'', p_t) \geq \bar{\alpha}(\beta', p_t)$;
(ii) if $\bar{\alpha}(\beta', p_t) \in (0,1)$, $\bar{\alpha}(\beta'', p_t) > \bar{\alpha}(\beta', p_t)$;

For a given vector of parameters, the most revealing equilibrium for price p_t can be non-informative, partially informative, or fully informative. A decrease in the strength of career concerns (an increase in β) will weakly improve the informativeness of the most revealing equilibrium. If the equilibrium is partially informative, it will strictly improve it.

To obtain intuition for this result, consider a given $p_t \leq \frac{1}{2}$ and let $\beta = \beta'$. Since profit motivations always drive manager towards sincere trading (i.e., $\Delta\pi_1 > 0$), if a manager with $s_t = 1$ is exactly indifferent between buying and selling (i.e., $\Delta u_1 = 0$) at a given p_t, under most revealing equilibrium strategy $\bar{\alpha}(\beta', p_t) < 1$, it must be the case that buying is reputationally costly for this manager (i.e., $\Delta r_1 < 0$). Now, increasing β (say, to β''), thus skewing incentives away from reputation, must make the manager strictly prefer to buy instead of sell under the proposed equilibrium strategies. Thus, $\bar{\alpha}(\beta', p_t)$ can no longer be the most revealing equilibrium strategy at p_t with $\beta = \beta''$. There are two possibilities: either $\Delta u_1 = 0$ for a strictly larger (interior) equilibrium strategy $\bar{\alpha}(\beta'', p_t)$, or $\Delta u_1 > 0$ for all $\alpha_1 > \bar{\alpha}(\beta', p_t)$, in which case sincere trading is an equilibrium and thus $\bar{\alpha}(\beta'', p_t) = 1$.

We can also derive monotone comparative statics on the relevant boundaries of the equilibrium regions. Again, we restrict attention to $p \leq \frac{1}{2}$(statements for $p \geq \frac{1}{2}$ are analogous), and define:

$$p_{\min}(\beta) = \sup\{p_t : \bar{\alpha}(\beta, p_t) = 0\}$$
$$p_{\max}(\beta) = \inf\{p_t : \bar{\alpha}(\beta, p_t) = 1\}$$

The first bound, $p_{\min}$, is the highest price at which a non-informative equilibrium can be sustained. The second, $p_{\max}$, is the lowest price with a sincere equilibrium.[16] We can now state:

Proposition 7 *(i) If $\beta'' > \beta'$, then $p_{\min}(\beta'') \leq p_{\min}(\beta')$ and $p_{\max}(\beta'') \leq p_{\max}(\beta')$;*
(ii) $\lim_{\beta \to 1} p_{\max}(\beta) = \lim_{\beta \to 1} p_{\min}(\beta) = 0$*;*
(iii) $\lim_{\beta \to 0} p_{\max}(\beta) = \lim_{\beta \to 0} p_{\min}(\beta) = 1 - \sigma$.

The intuition of this result builds directly on Proposition 6, which showed that at any given p_t increasing β cannot decrease the amount of information revealed in the most revealing equilibrium. Thus, increasing β can neither decrease the size of the sincere pricing region, $(p_{\max}(\beta), \frac{1}{2}]$, nor increase the size of the conformist region, $[0, p_{\min}(\beta))$.

As career concerns vanish (point ii), play becomes sincere for all prices, which confirms that Glosten and Milgrom [16] can be seen as a limit case of the present set-up. Point (iii) states that, as career concerns become more important, play is sincere only if $p_t > 1 - \sigma$, which shows that the bound identified in Proposition 5 is tight.

4.4 Career Concerns and Financial Market Variables

One can study how the importance of career concerns affect other standard financial market variables. In this section, we consider widely used measured of liquidity, volatility, and trade predictability.

The bid-ask spread is the difference between the ask price and the bid price $(p_t^a - p_t^b)$, and it is a commonly used measure of market illiquidity. Price volatility can be defined as variance of the price of the asset at time $t+1$ given the price at t: $Var[p_{t+1}|p_t]$. Finally, trade-predictability is the ability to predict the sign of the trade at time t based on public information. We measure it by $\frac{1}{Var(a_t|p_t)}$.

We show that each of these variables is monotonically related to the importance of career concerns:

Proposition 8 *For any given p, in the most revealing equilibrium, the bid-ask spread and price volatility are non-decreasing, and trade predictability is non-increasing, in β.*

Increasing β weakens career concerns. Thus, stronger career concerns make markets more liquid and less volatile, and makes trades more predictable. In sequential trade models with risk-neutral and competitive market makers, the bid-ask spread, and thus illiquidity, arises

[16] The other two conceivable bounds are uninteresting. The lowest price with a non-informative equilibrium is always zero and the highest price (given that $p \leq \frac{1}{2}$) with a sincere equilibrium is always $\frac{1}{2}$.

out of adverse selection. The more the informed traders (fund managers) utilize their private information in their trades, therefore, the higher the bid-ask spread, and thus the greater the amount of information revealed in equilibrium. Proposition 6 shows that the higher is β (i.e., the less important are career concerns) the more informative the trades of fund managers. Thus, increasing β increases the bid-ask spread. For the same informational reason, career concerns also make market prices and trades more predictable. As less information is revealed, the price is more stable.

The results of sections 4.3 and 4.4 provide theoretical underpinning to a number of existing empirical findings, and also suggest new avenues for empirical work. We discuss the connection to the empirical literature in the conclusion.

5 Extensions

5.1 A More General Set-Up

The baseline model was presented for a simple binary structure where liquidation values could take only two possible values. We extend it here to a generic discrete set of possible values $V \in \Re$. Denote the maximum and minimum possible values of $v \in V$ by $v_{\max}$ and $v_{\min}$. The ex ante distribution of v is determined by any arbitrary probability mass function.

Each fund manager of type $\theta \in \{b, g\}$ (with $\Pr(\theta = g) = \gamma$ as before) receives a signal distributed according to $\Pr(s = 1|v, \theta) = \sigma_{v,\theta}$, with the following properties:

A1 Full support: $\sigma_{v,\theta} \in (0, 1)$ for all θ and v.

A2 Monotone Likelihood Ratio Property (MLRP): For every pair of liquidation values $v'' > v'$,

$$\frac{\sigma_{v'',g}}{\sigma_{v'',b}} > \frac{\sigma_{v',g}}{\sigma_{v',b}}.$$

A3 Informativeness: $\sigma_{v,\theta}$ increasing in v for all θ, and $\sigma_{v_{\max},g} > \sigma_{v_{\max},b}$ and $\sigma_{v_{\min},g} < \sigma_{v_{\min},b}$.

The first assumption (A1) is crucial. It implies that the signal is never fully informative: for all s and v: $\Pr(v|s) < 1$. If a manager knows he has the truth, he would follow his signal even if all his predecessors had traded in the opposite direction.

If there are no career concerns ($\beta = 1$) there exists a fully informative equilibrium (see Avery and Zemsky [4]). We show that:

Proposition 9 *For $\beta < 1$, there exists no equilibrium for which $\lim_{t\to\infty} p_t = v$ for more than one liquidation value v.*

The intuition parallels the case with binary liquidation values, and we therefore provide only a concise summary here. Assumptions A2 and A3 guarantee that for all but possibly one liquidation value, either $\sigma_{v,g} > \sigma_{v,b}$ or $\sigma_{v,g} < \sigma_{v,b}$. For each such v we show that there cannot exist informative equilibria when p is close enough to v. Consider the possibility that there is an informative equilibrium with $\alpha_1 > \alpha_0$. As $p \to v$, profits become unimportant, and the manager finds it desirable to indicate via her action that she has received a particular reputation-enhancing signal. If, for example, v is such that $\sigma_{v,g} < \sigma_{v,b}$ it is better for the manager to sell, which indicates that she was likely to have received signal 0. But since this is true even for a manager with $s = 1$, we must have $\alpha_1 = 0$, and thus the equilibrium cannot be informative. Alternatively, consider the possibility that there is an informative equilibrium with $\alpha_1 < \alpha_0$. Then, as we have argued earlier in the main model, the manager with $s = 1$ must always prefer to sell from a reputational perspective: $\Delta r_1 < 0$. Suppose again that $p \to v$ where $\sigma_{v,g} < \sigma_{v,b}$. Then, the manager must find it reputationally beneficial to indicate that she has received $s = 0$, but can only do this (since $\alpha_1 < \alpha_0$) by buying, which contradicts the fact that $\Delta r_1 < 0$. Thus for p close enough to v there cannot be informative equilibria with $\alpha_1 < \alpha_0$.

5.2 Self-Knowledge

The baseline analysis was carried out under the assumption that the manager did not know his type. In this section we show that the central economic message of our baseline model is robust to the presence of self-knowledge.

We now let each fund manager receive two signals: the now familiar s_t, and a new signal z_t, with $\Pr(z_t = \theta|\theta) = \rho \in \left(\frac{1}{2}, 1\right)$. The rest of the model is exactly as in the main analysis.

Denote by α_{sz} the probability with which a manager who has received liquidation value signal s and self knowledge signal z chooses to buy. It is easy to see that, in the absence of career concerns ($\beta = 1$) – and provided that the proportion of noise traders is sufficiently high – the game has a unique equilibrium: $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$. This equilibrium has the property that trades fully reveal the liquidation value signal s. We demonstrate that, for sufficiently extreme prices, no such equilibrium can exist when $\beta < 1$. We also rule out the possibility of the existence of an *informationally equivalent* "perverse" equilibrium with $\alpha_{1g} = \alpha_{1b} = 0$ and $\alpha_{0b} = \alpha_{0g} = 1$.

Proposition 10 *There exists a threshold $\bar{p} \in (0, 1)$, such that if $p < \bar{p}$ or $p > 1 - \bar{p}$, there is neither an equilibrium with $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$ nor an equilibrium with $\alpha_{1g} = \alpha_{1b} = 0$ and $\alpha_{0b} = \alpha_{0g} = 1$.*

The intuition behind this result is simple: in order for equilibrium trades to fully reveal the liquidation value signal (s_t) it is necessary for the manager to not condition his behavior on the self-knowledge signal (z_t). But if the manager does not condition his behavior on his self-knowledge signal, the inference process is identical to the baseline model, and thus full-revelation cannot occur.

Propositions 10 implies that allowing for career concerns "slows down" the rate of information aggregation via prices compared to the case with no career concerns: it is no longer possible that the trades of fund managers will reveal their signals in each period once prices are sufficiently extreme.

Under additional assumptions, it is possible to make stronger statements about the equilibrium set with self-knowledge. For example, if we are willing to restrict parameters such that $\Pr(\theta = g|s = v) > \Pr(\theta = g|z = g)$, then it is possible to show that for sufficiently extreme prices there is never any equilibrium in which $\alpha_{1g} \geq \alpha_{1b} \geq \alpha_{0b} \geq \alpha_{0g}$ with at least one strict inequality.[17] The intuition is as follows: A manager can signal the quality of his type either by making the (ex post) correct trade or by taking an action that reveals that he received a positive signal about his own type. For a given history, these two actions may not be identical. As long as the parameters of the model are such that the manager's reputation is helped more by revealing that he received the ex post correct signal about asset payoffs rather than by revealing that he received a good signal about his type, our baseline results go through.

5.3 Informed individual traders

It is also possible to introduce informed non-career concerned (individual) traders into our model. Informed individuals devoid of career concerns would always trade sincerely, and thus, in the presence of such traders, prices would eventually converge to true value. Thus, transaction prices need no longer be contained within the band identified in Proposition 1.

However, the qualitative properties of managerial behavior remain unchanged: at sufficiently extreme prices, career concerned managers would ignore their own information. Thus, convergence to true value would be determined solely by the trading activity of individual traders, and would thus be much slower than in a market without career concerns.

We can relate the rate of price convergence as a function of the proportion of individual traders. Fix a game Γ, and replace a fraction τ of fund managers by informed individual traders without career concerns. These individual traders have the same precision of signals as fund managers on average: i.e., receive signals s_t where $\Pr(s_t = v|v) = \sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$. We can now state the following result ($\underline{p}$ is the the bound identified in Proposition 1):

[17] This analysis is available upon request from the authors.

Proposition 11 *Consider any price $p' < \underline{p}$. For any length of game T, there exists $\tau' > 0$, such that, for any $\tau < \tau'$ the price never goes below p'.*

If the transaction price ever escapes the range identified in Proposition 1, then further price convergence is solely determined by the proportion of individual traders. As this proportion gets small, the speed of price convergence goes to zero.

5.4 Other Extensions

It is possible to extend our model in a variety of other directions. As we have noted earlier, our qualitative results go through for a richer class of payoff functions $\beta\chi(\pi_t) + (1-\beta)R(r_t)$, where χ and R are increasing and piecewise continuous functions. The extension to such payoff functions increases algebraic complexity without adding to the intuition behind our results.

In addition, instead of having managers derive utility from their absolute reputation, we could allow them to care about reputation *relative* to their peers. For example, we could redefine manager t's reputational payoff by $R_t(r_1, ..., r_T)$, where r_i represents the realized reputation of manager i, with $\frac{\partial R_t}{\partial r_t} > 0$ and $\frac{\partial R_t}{\partial r_\tau} < 0$ for $\tau \neq t$. Even in this more complex case, it is possible to show that sincere trading cannot be sustained as an equilibrium. To see why, imagine that we are in an equilibrium with sincere trade, and consider the incentives of the last manager. Since this manager's actions cannot affect the principal's beliefs about his peers, he is just like a manager in our baseline model. He will conform for sufficiently extreme prices.

Finally, it is possible to micro-found the utility function assumed in this paper. This, and the other extensions alluded to in this subsection, are discussed in greater detail in Dasgupta and Prat [12].

6 Conclusion

The central message of this paper is that we should expect the presence of traders with reputational concerns to affect the equilibrium properties of financial markets in which they trade over time. In particular, we have shown that stronger career concerns necessarily lead to more conformist behavior among traders, less precise information aggregation through prices, and better market liquidity. This paper creates a link between two sets of variables: the incentive structure faced by traders and the properties of asset markets. As both sets of variables are potentially measurable, our comparative statics results lead to clear-cut testable predictions.

In particular, our analysis provides theoretical underpinnings for a number of empirical findings. First, in all equilibria of our game, institutional investors exhibit conformist trading at some prices. Such conformism introduces high serial correlation in institutional trade. This prediction provides a theoretical rationale for the results of Sias [26]. Sias examines the quarterly SEC 13-F reports of US institutional money managers from 1983 to 1997 and finds a strong positive relationship between the fraction of institutions buying individual stocks over adjacent quarters, consistent with money managers herding behind each other's trades.[18] In addition, our results indicate the extent of institutional conformism (e.g., measured by the informativeness of their trading strategies or by the range of prices over which they herd) is linked to the incentive structure of their traders. This prediction finds indirect support in the work of Massa and Patgiri [20]. Massa and Patgiri study data on US mutual funds for the period 1994-2003, and quantify the extent of profit-based incentives in the contracts of fund managers. They find that those managers who receive higher profit-based incentives (i.e., have higher β in our setting) exhibit less conformism. Finally, some of our predictions point to potentially interesting new empirical exercises. For example, it would be interesting to examine whether there is a relationship between the incentives of money managers and the liquidity and volatility of the stocks they trade. Our stylized model predicts that, ceteris paribus, career concerned fund managers will increase liquidity and reduce volatility of the assets they trade.

Our model is stylized. We believe that it is important to build richer and more realistic models of dynamic financial markets with career concerned traders. The increasing importance of professional money managers in financial markets make such extensions topical. Our results establish a benchmark against which such future findings can be understood.

7 Appendix

Proof of Proposition 1: We first characterize some crucial properties of beliefs:

Lemma 1 *Let* $v^t_{s_t} = \Pr(v = 1|h_t, s_t)$. *Then,*

(a) $v^t_{s_t}$ *is strictly increasing and continuous in* p_t;

(b) $v^t_{s_t} = 1\,(0)$ *if* $p_t = 1\,(0)$;

(c) $v^t_1 > v^t_0$ *if* $p_t \in (0,1)$

[18] This finding is complemented by Dasgupta, Prat, and Verardo [13] who examine SEC 13-F reports from 1983 to 2003 and find (amongst other things) that institutional traders, taken as a whole, exhibit conformist trading patterns. They excessively buy (sell) stocks that have been persistently bought (sold) by their peers consecutively over a period of 5 or more quarters. See also Dennis and Strickland [14]. Sias [27] provides a recent survey and reconciliation of the growing literature on momentum trading and herding by institutional traders.

Proof. By Bayes' rule,

$$v_{s_t}^t = \frac{p_t \Pr(s_t|v=1)}{p_t \Pr(s_t|v=1) + (1-p_t)\Pr(s_t|v=0)}$$

Now parts (a) and (b) follow immediately. To see part (c) note that

$$v_1^t = \frac{p_t}{p_t + (1-p_t)\frac{\Pr(s_t=1|v=0)}{\Pr(s_t=1|v=1)}} = \frac{p_t}{p_t + (1-p_t)\frac{1-\sigma}{\sigma}}$$

where $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$. Similarly

$$v_0^t = \frac{p_t}{p_t + (1-p_t)\frac{\sigma}{1-\sigma}}$$

Since $\sigma_g > \sigma_b > \frac{1}{2}$, $\sigma > \frac{1}{2}$. Thus $\frac{1-\sigma}{\sigma} < 1 < \frac{\sigma}{1-\sigma}$. This then implies $v_1^t > v_0^t$ which completes the proof of the lemma. ■

Given that the action set of every manager is binary, it is easy to see that the game has at least one PBE. Focus on time t. Suppose that given history h_t (with price p_t), equilibrium play dictates strategy $\alpha_0^t(h_t)$ and $\alpha_1^t(h_t)$.

There are three cases: $\alpha_0^t(h_t) < \alpha_1^t(h_t)$, $\alpha_0^t(h_t) > \alpha_1^t(h_t)$, and $\alpha_0^t(h_t) = \alpha_1^t(h_t)$. We shall identify a lower bound and an upper bound to price such that the first two cases are impossible if the price is above the upper bound or below the lower bound. As will be apparent, these two bounds are independent of time and history. The third case denotes uninformative play on the part of the manager at time t. Note that, if at a certain time t the price goes above the upper bound or below the lower bound, uninformative play guarantees that the price will not change in the next round; hence, play will be uninformative from then on.

In the remainder of the proof, we hold history and time constant. For simplicity, therefore, we drop the history and time arguments (e.g. $\alpha_0^t(h_t)$ *become* α_0*).*

For any (α_1, α_0) we can compute the following quantities. The bid and ask prices as follows:

$$\begin{aligned} p_a &= \frac{\delta\frac{1}{2} + (1-\delta)\left[\alpha_1\sigma + \alpha_0(1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[\alpha_1\Sigma_t + \alpha_0(1-\Sigma_t)\right]}p \\ p_b &= \frac{\delta\frac{1}{2} + (1-\delta)\left[(1-\alpha_1)\sigma + (1-\alpha_0)(1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[(1-\alpha_1)\Sigma_t + (1-\alpha_0)(1-\Sigma_t)\right]}p, \end{aligned}$$

where $\sigma = \gamma\sigma_g + (1-\gamma)\sigma_b$, and $\Sigma = p\sigma + (1-p)(1-\sigma)$.

The manager's equilibrium strategy fully determines investors' beliefs (the beliefs do not depend on history or price directly – they only depend on history and price through α_1 and

α_0):[19]

$$
\begin{aligned}
r\left(a=1,v=1\right) &= \frac{\alpha_1\sigma_g+\alpha_0\left(1-\sigma_g\right)}{\alpha_1\sigma+\alpha_0\left(1-\sigma\right)}\gamma \\
r\left(a=1,v=0\right) &= \frac{\alpha_1\left(1-\sigma_g\right)+\alpha_0\sigma_g}{\alpha_1\left(1-\sigma\right)+\alpha_0\sigma}\gamma \\
r\left(a=0,v=1\right) &= \frac{\left(1-\alpha_1\right)\sigma_g+\left(1-\alpha_0\right)\left(1-\sigma_g\right)}{\left(1-\alpha_1\right)\sigma+\left(1-\alpha_0\right)\left(1-\sigma\right)}\gamma \\
r\left(a=0,v=0\right) &= \frac{\left(1-\alpha_1\right)\left(1-\sigma_g\right)+\left(1-\alpha_0\right)\sigma_g}{\left(1-\alpha_1\right)\left(1-\sigma\right)+\left(1-\alpha_0\right)\sigma}\gamma
\end{aligned}
$$

Suppose the manager observes signal $s=1$. The difference in his expected payoff if he plays $a=1$ instead of $a=0$ can be denoted with

$$\Delta u_1=\beta\Delta\pi_1+\left(1-\beta\right)\Delta r_1,$$

where the profit component is

$$\Delta\pi_1=\left(v_1-p_a\right)-\left(p_b-v_1\right)$$

and the reputational component is

$$\Delta r_1=v_1\left(r\left(a=1,v=1\right)-r\left(a=0,v=1\right)\right)+\left(1-v_1\right)\left(r\left(a=1,v=0\right)-r\left(a=0,v=0\right)\right)$$

Case with $\alpha_0<\alpha_1$: Suppose first that $\alpha_0<\alpha_1$. We first show that in all equilibria either the manager with the high signal or the manager with the low signal play a pure strategy.

Lemma 2 *There are no mixed strategy equilibria in which $0<\alpha_0\leq\alpha_1<1$ for any t.*

Proof. Consider a putative equilibrium in which $1>\alpha_0>0$, i.e. the agent at time t who receives signal zero is exactly indifferent between buying and selling. We will show that in this equilibrium, it must be the case that the agent who receives signal 1 at time t must strictly prefer to buy rather than sell. Consider the expected profit difference between buying and selling: $\Delta\pi_s$. This can be written as

$$v_s\left(\left(1-p_a\right)-\left(p_b-1\right)\right)+\left(1-v_s\right)\left(\left(0-p_a\right)-\left(p_b-0\right)\right)$$

Since $\left(1-p_a\right)-\left(p_b-1\right)>0>\left(0-p_a\right)-\left(p_b-0\right)$, and by Lemma 1 $v_1>v_0$, it is clear that $\Delta\pi_1>\Delta\pi_0$.

[19] This key property of beliefs in our game is due to the assumption that investors observe the liquidation value v and that the managers' signals are mutually independent given v.

Now consider the expected reputational payoff difference between buying and selling: Δr_{s_t}. This can be expressed as:

$$v_s[r\left(a=1,v=1\right)-r(a=0,v=1)]+\left(1-v_s\right)\left[r\left(a=1,v=0\right)-r\left(a=0,v=0\right)\right]$$

Notice that $r\left(a=1,v=1\right)\geq r(a=0,v=1)$. To see why note that

$$\frac{\alpha_1\sigma_g+\alpha_0\left(1-\sigma_g\right)}{\alpha_1\sigma+\alpha_0\left(1-\sigma\right)}<\frac{\left(1-\alpha_1\right)\sigma_g+\left(1-\alpha_0\right)\left(1-\sigma_g\right)}{\left(1-\alpha_1\right)\sigma+\left(1-\alpha_0\right)\left(1-\sigma\right)}\Rightarrow\left(\sigma_g-\sigma\right)\left(\alpha_1-\alpha_0\right)<0$$

which is a contradiction since $\sigma_g-\sigma>0$ and $\alpha_1-\alpha_0\geq 0$. A similar argument establishes that $r\left(a=1,v=0\right)\leq r\left(a=0,v=0\right)$. Thus,

$$r\left(a=1,v=1\right)-r(a=0,v=1)\geq 0\geq r\left(a=1,v=0\right)-r\left(a=0,v=0\right).$$

Given Lemma 1, we know that $v_1>v_0$, and thus $\Delta r_1\geq\Delta r_0$.

Putting these together, we have $\Delta u_1>\Delta u_0=0$. Thus, if $0<\alpha_0<1$, then $\alpha_1=1$. An identical argument establishes that if $0<\alpha_1<1$, then $\alpha_0=0$. ■

At a given price p, consider $\Delta\pi_s$, the profit incentives of an agent who has received s to buy vs sell:

$$2v_s(p)-p_a(p)-p_b(p)$$

Since $p_a\geq 0$ and $p_b\geq 0$, and $v_0(p)<v_1(p)=\frac{\sigma}{\Sigma}p$, it is immediate that $\Delta\pi_s$ is bounded above by:

$$2\frac{\sigma}{\Sigma}p$$

At the same price p consider Δr_s the reputational incentives of this agent to buy vs sell:

$$v_s\left(\frac{\alpha_1\sigma_g+\alpha_0\left(1-\sigma_g\right)}{\alpha_1\sigma+\alpha_0\left(1-\sigma\right)}-\frac{\left(1-\alpha_1\right)\sigma_g+\left(1-\alpha_0\right)\left(1-\sigma_g\right)}{\left(1-\alpha_1\right)\sigma+\left(1-\alpha_0\right)\left(1-\sigma\right)}\right)\gamma+$$
$$\left(1-v_s\right)\left(\frac{\alpha_1\left(1-\sigma_g\right)+\alpha_0\sigma_g}{\alpha_1\left(1-\sigma\right)+\alpha_0\sigma}-\frac{\left(1-\alpha_1\right)\left(1-\sigma_g\right)+\left(1-\alpha_0\right)\sigma_g}{\left(1-\alpha_1\right)\left(1-\sigma\right)+\left(1-\alpha_0\right)\sigma}\right)\gamma$$

Lemma 2 allows us to restrict attention to cases where either $\alpha_1=1>\alpha_0\geq 0$ or $1\geq\alpha_1>\alpha_0=0$. It is then not difficult to see that Δr_s is bounded above by

$$v_s\left(\frac{\sigma_g}{\sigma}-\frac{1-\sigma_g}{1-\sigma}\right)\gamma+\left(1-v_s\right)\left(1-\frac{\sigma_g}{\sigma}\right)\gamma,$$

which, in turn, is bounded above by

$$\frac{\sigma}{\Sigma}p\left(\frac{\sigma_g}{\sigma}-\frac{1-\sigma_g}{1-\sigma}\right)\gamma+(1-\frac{\sigma}{\Sigma}p)\left(1-\frac{\sigma_g}{\sigma}\right)\gamma.$$

Thus, an upper bound on the expected utility difference enjoyed by this agent from buying vs selling at p is

$$\beta\left[2\frac{\sigma}{\Sigma}p\right]+(1-\beta)\left[\frac{\sigma}{\Sigma}p\left(\frac{\sigma_g}{\sigma}-\frac{1-\sigma_g}{1-\sigma}\right)\gamma+(1-\frac{\sigma}{\Sigma}p)\left(1-\frac{\sigma_g}{\sigma}\right)\gamma\right]$$

This is linear and increasing in $\frac{\sigma}{\Sigma}p = v_1(p)$, which in turn, is increasing in p. It crosses 0 exactly once at $p = \widehat{p}_1$ which is defined by:

$$v_1(\widehat{p}_1)=\frac{(1-\beta)\left(\frac{\sigma_g}{\sigma}-1\right)\gamma}{2\beta+(1-\beta)\left(\frac{\sigma_g}{\sigma}-\frac{1-\sigma_g}{1-\sigma}+\frac{\sigma_g}{\sigma}-1\right)\gamma}>0$$

Since $v_1(p) > 0$ if and only if $p > 0$, we know that $\widehat{p}_1 > 0$. Thus we have proved that if $p < \widehat{p}_1$ managers will sell regardless of their signals. A symmetric proof establishes that for $p > 1 - \widehat{p}_1$ managers will buy regardless of their signals. Thus, for $p < \widehat{p}_1$ and $p > 1 - \widehat{p}_1$ it cannot be the case that $\alpha_1 > \alpha_0$.

Case with $\alpha_0 > \alpha_1$: We now move on to the case where $\alpha_0 > \alpha_1$. As before, we define

$$\Delta u_1 = \beta\Delta\pi_1 + (1-\beta)\,\Delta r_1$$

If $\alpha_0 > \alpha_1$, a manager who observes $s = 1$ plays $a = 0$ with positive probability. It must be that $\Delta u_1 \leq 0$. As $\Delta\pi_1 > 0$, a necessary condition for the existence of such an equilibrium is that $\Delta r_1 < 0$. We shall show that this condition cannot hold if p is sufficiently low.

As before,

$$\Delta r_1 = v_1\left(r(a=1,v=1)-r(a=0,v=1)\right)+(1-v_1)\left(r(a=1,v=0)-r(a=0,v=0)\right)$$

The necessary condition can thus be re-written as

$$\frac{v_1}{1-v_1} > -\frac{r(a=1,v=0)-r(a=0,v=0)}{r(a=1,v=1)-r(a=0,v=1)}$$

Let

$$\begin{aligned} a &= \alpha_1\sigma_g+\alpha_0\left(1-\sigma_g\right) & A &= \alpha_1\sigma+\alpha_0\left(1-\sigma\right)\\ b &= \alpha_1\left(1-\sigma_g\right)+\alpha_0\sigma_g & B &= \alpha_1\left(1-\sigma\right)+\alpha_0\sigma \end{aligned}$$

Note that

$$\begin{aligned} \frac{1}{\gamma}\left(r\left(a=1,v=1\right)-r\left(a=0,v=1\right)\right) &= \frac{a}{A}-\frac{1-a}{1-A}=\frac{a-A}{A\left(1-A\right)}=\frac{\left(\alpha_1-\alpha_0\right)\left(\sigma_g-\sigma\right)}{A\left(1-A\right)}\\ \frac{1}{\gamma}\left(r\left(a=1,v=0\right)-r\left(a=0,v=0\right)\right) &= \frac{b}{B}-\frac{1-b}{1-B}=\frac{b-B}{B\left(1-B\right)}=\frac{-\left(\alpha_1-\alpha_0\right)\left(\sigma_g-\sigma\right)}{B\left(1-B\right)} \end{aligned}$$

The necessary condition becomes

$$\frac{v_1}{1-v_1} > \frac{A\left(1-A\right)}{B\left(1-B\right)} \equiv K \tag{1}$$

We are interested in the lower bound $\inf_{\alpha_1<\alpha_0} K$. If it is strictly greater than zero, then for p low enough the necessary condition (1) cannot be satisfied.

Lemma 3 $\inf_{\alpha_1<\alpha_0} K = \frac{1-\sigma}{\sigma}$.

Proof. First, assume that the infimum is reached at an interior point: $0 < \alpha_1 < \alpha_0 < 1$. As K is twice differentiable, such point satisfies the two first-order conditions

$$\begin{aligned}\frac{\partial}{\partial \alpha_0}\frac{A(1-A)}{B(1-B)} &= 0\\ \frac{\partial}{\partial \alpha_1}\frac{A(1-A)}{B(1-B)} &= 0\end{aligned}$$

These can be expressed as

$$\begin{aligned}\sigma(1-2A)B(1-B) &= (1-\sigma)(1-2B)A(1-A)\\ (1-\sigma)(1-2A)B(1-B) &= \sigma(1-2B)A(1-A)\end{aligned}$$

Since$\frac{\sigma}{1-\sigma} > \frac{1-\sigma}{\sigma}$ and $A, B \in (0,1)$, the only way these two hold together is if $A = B = \frac{1}{2}$ which is impossible since $\alpha_1 \neq \alpha_0$.

Consider instead the corner solution: $0 = \alpha_1 < \alpha_0 \leq 1$. Now $A = \alpha_0(1-\sigma)$ and $B = \alpha_0\sigma$. Thus,

$$K = \frac{\alpha_0(1-\sigma)(1-\alpha_0(1-\sigma))}{\alpha_0\sigma(1-\alpha_0\sigma)} = \frac{1-\sigma}{\sigma}\frac{1-\alpha_0(1-\sigma)}{1-\alpha_0\sigma}$$

Since $\sigma > \frac{1}{2}$ this is clearly increasing in α_0. Thus, the infimum can be obtained by taking

$$\lim_{\alpha_0\to 0} K = \lim_{\alpha_0\to 0}\frac{1-\sigma}{\sigma}\frac{1-\alpha_0(1-\sigma)}{1-\alpha_0\sigma} = \frac{1-\sigma}{\sigma}$$

The other potential corner solution is obtained by: $0 \leq \alpha_1 < \alpha_0 = 1$. Now $A = \alpha_1\sigma + 1 - \sigma$ and $B = \alpha_1(1-\sigma) + \sigma$. Thus,

$$K = \frac{\sigma}{1-\sigma}\frac{(\alpha_1\sigma + 1 - \sigma)}{(\alpha_1(1-\sigma)+\sigma)}$$

which is minimized for $\alpha_1 \to 0$, with value 1. Hence, the infimum is $\frac{1-\sigma}{\sigma}$. ■

Thus, there exists $\widehat{p}_2 > 0$ such that for $p < \widehat{p}_2$ the necessary condition for $\alpha_0 > \alpha_1$ fails. A corresponding upper bound of $1 - \widehat{p}_2$ follows from a symmetric proof. Thus, we have shown that for $p < \widehat{p}_2$ and $p > 1 - \widehat{p}_2$ it is not possible to have $\alpha_0 > \alpha_1$.

Price bounds: We have now shown that there exists $\widehat{p}_1 > 0$ and $\widehat{p}_2 > 0$ such that for $p \notin [\widehat{p}_1, 1-\widehat{p}_1]$ it is not possible to have $\alpha_1 > \alpha_0$ and for $p \notin [\widehat{p}_2, 1-\widehat{p}_2]$ it is not possible to have $\alpha_1 < \alpha_0$. Now define $\widehat{p} = \min(\widehat{p}_1, \widehat{p}_2)$. Thus for $p \notin [\widehat{p}, 1-\widehat{p}]$ it is not possible to have $\alpha_1 \neq \alpha_0$.

In order to compute the lowest possible transaction price that can potentially be reached, we compute the bid-price at $\widehat{p}$ under the assumption that play is sincere and that there are no noise traders. This is given by

$$\underline{p} \equiv \Pr[v = 1|\widehat{p}, s = 0] = \frac{(1-\sigma)\widehat{p}}{\sigma(1-\widehat{p}) + (1-\sigma)\widehat{p}}$$

A price below $\underline{p}$ can never be reached, because it would imply an informative trade following a transaction price (public belief) of $p = \widehat{p}$ or lower. An upper bound on prices of $1-\underline{p}$ follows by symmetry. Note that for $\delta > 0$ (i.e., with noise traders) transaction prices of $\underline{p}$ and $1-\underline{p}$ are never actually reached. Thus, for all t, $p_t \in (\underline{p}, 1-\underline{p})$.

Proof of Proposition 2: For the case where $\alpha_1 > \alpha_0$ note that

$$\lim_{\beta \to 0, \sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} v_1(\widehat{p}_1) = \frac{1}{3}.$$

It is easy to see that in this situation, since $\sigma \to \frac{1}{2}$, $\lim \underline{p} = \frac{1}{3}$ as well. By continuity (of Δu over all the parameters of the game as well as p), one sees that for any $\underline{p} < \frac{1}{3}$ there is a set of parameter values with positive measure such that for prices below $\underline{p}$ there are no non-perverse equilibria in all games with parameters in that set.

For the case where $\alpha_1 < \alpha_0$, we find the maximal value of $\frac{1-\sigma}{\sigma}$, which is attained under the same limiting values used for the case with $\alpha_1 > \alpha_0$. Note that $\frac{v_1}{1-v_1} > \frac{1-\sigma}{\sigma}$ if and only if $v_1 > 1-\sigma$. $\lim_{\sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} \sigma = \min \sigma = \frac{1}{2}$. In this limit, there is no equilibrium with $\alpha_1 < \alpha_0$ if $v_1 \leq \frac{1}{2}$. Given that $\sigma \to \frac{1}{2}$, $v_1 \to p$. This shows that $\lim_{\sigma_g \to 1, \sigma_b \to \frac{1}{2}, \gamma \to 0} \widehat{p}_2 = \frac{1}{2}$. This would yield a boundary $\underline{p} = \frac{1}{2}$.

Comparing the perverse and the non-perverse case, we see that the lower boundary is $\underline{p} = \frac{1}{3}$.

Proof of Proposition 3: We begin by proving a technical result. For a given time t and a certain history h_t, consider the function

$$\Delta u_{s_t}^t(h_t) = \beta \Delta \pi_{s_t}^t(h_t) + (1-\beta)\Delta r_{s_t}^t(h_t),$$

where $\Delta\pi$ and Δr are defined as in the proof of Proposition 1. This function represents the expected payoff different for a manager at time t who receives signal s_t between playing 1 and 0. The analysis contained in the proof of Proposition 1 shows that the function satisfies the following properties:

- It depends on history h_t only through the price p_t and the strategies $\alpha_0^t(h_t)$ and $\alpha_1^t(h_t)$;
- It does not depend on strategies that will be used by managers after t;

- It is continuous in p_t, α_0^t, and α_1^t;
- If $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$, the function satisfies $\Delta u_1^t(h_t) > \Delta u_0^t(h_t)$ (this is because $\Delta\pi_1^t > \Delta\pi_0^t$ and, if , $\alpha_1^t \geq \alpha_0^t$, $\Delta r_1^t \geq \Delta r_0^t$).

From now on, we write

$$\Delta u_{s_t}^t(h_t) = \Delta u_{s_t}\left(\alpha_0^t, \alpha_1^t, p_t\right).$$

This function is well-defined except in the cases where $\alpha_0^t = \alpha_1^t = 0$ and $\alpha_0^t = \alpha_1^t = 1$. We extend the function by defining:

$$\Delta u_{s_t}(0,0,p_t) = \lim_{\alpha_1^t \to 0} \Delta u_{s_t}\left(0, \alpha_1^t, p_t\right) \quad \text{and} \quad \Delta u_{s_t}(1,1,p_t) = \lim_{\alpha_0^t \to 1} \Delta u_{s_t}\left(\alpha_0^t, 1, p_t\right) \qquad (2)$$

We now have that:

Lemma 4 *For every p, there exists a pair (α_0, α_1) such that $\alpha_0 \leq \alpha_1$ and one of the following statements is true:*

(a) $\Delta u_0(0,0,p) < \Delta u_1(0,0,p) \leq 0$;

(b) For some $\alpha_1 \in (0,1)$, $\Delta u_0(0,\alpha_1,p) < 0 = \Delta u_1(0,\alpha_1,p)$;

(c) $\Delta u_0(0,1,p) \leq 0 \leq \Delta u_1(0,1,p)$ with at least one strict inequality;

(d) For some $\alpha_0 \in (0,1)$, $\Delta u_0(\alpha_0,1,p) = 0 < \Delta u_1(\alpha_0,1,p)$;

(e) $0 \leq \Delta u_0(1,1,p) < \Delta u_1(1,1,p)$.

Proof of Lemma 4: Suppose (a), (c), and (e) are false. Then, the falsity of (a) implies that $\Delta u_1(0,0,p) > 0$; the falsity of (e) implies that $\Delta u_0(1,1,p) < 0$; the falsity of (c) implies that either (i) $\Delta u_0(0,1,p) > 0$ or (ii) $\Delta u_1(0,1,p) < 0$ (both cannot simultaneously occur since $\Delta u_1(0,1,p) > \Delta u_0(0,1,p)$). If (i) is true, $\Delta u_0(0,1,p) > 0$ and $\Delta u_0(1,1,p) < 0$, which, by continuity in α_0 implies that there exists $\alpha_0 \in (0,1)$ such that $\Delta u_0(\alpha_0,1,p) = 0$. This, in turn, implies that $\Delta u_1(\alpha_0,1,p) > 0$ (since $\Delta u_1 > \Delta u_0$) and thus statement (d) is true. If, (ii) is true, then $\Delta u_1(0,1,p) < 0$ and $\Delta u_1(0,0,p) > 0$, which by continuity in α_1 implies that there exists $\alpha_1 \in (0,1)$ such that $\Delta u_1(0,\alpha_1,p) = 0$. Then, statement (b) is true. This concludes the proof of lemma 4.

Let $\Gamma^t(h_t)$ denote the subgame that begins at time t after history h_t. We will prove existence by backward induction. First, we shall prove that for all h_T, the last-period subgame $\Gamma(h_T)$ has a continuation equilibrium which satisfies $\alpha_1^T(h_T) \geq \alpha_0^T(h_T)$. Second, we shall prove that if all subgames that begin at time t have a continuation equilibrium satisfying $\alpha_1^t(h_t) \geq \alpha_0^t(h_t)$, then all subgames that begin at time $t-1$ have a continuation equilibrium satisfying $\alpha_1^{t-1}(h_{t-1}) \geq \alpha_0^{t-1}(h_{t-1})$.

For the first step of the induction argument, assume a history h_T and consider the last stage. By lemma 4, we see that the continuation game has an equilibrium that satisfies

$\alpha_1^T(h_T) \geq \alpha_0^T(h_T)$. Note that, if the equilibrium involves pooling, then off-equilibrium trades are assumed to be sincere: this is implied by the way in which $\Delta u_{s_t}(0,0,p_t)$ and $\Delta u_{s_t}(1,1,p_t)$ are defined above in condition (2).

For the second step, fix time t and suppose that for every history h_t there is a continuation equilibrium where for all $\tau \geq t$ and all h_τ, $\alpha_1^\tau(h_\tau) \geq \alpha_0^\tau(h_\tau)$. Now consider manager $t-1$. By lemma 4, we see that there exists $\alpha_1^{t-1} \geq \alpha_0^{t-1}$ such that one of conditions (a), (b), (c), (d), or (e) is satisfied. For each of those cases the pair $(\alpha_0^{t-1}, \alpha_1^{t-1})$ is a best response for player $t-1$.

Proof of Proposition 4: The proof is in three parts. First, we show that there exists a non-perverse equilibrium where, for every time $t = 1, ...T$ and every history h_t, if h_t is such that $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$, then $\alpha_0^t(h_t) = 0$ $\left(\alpha_1^t(h_t) = 1\right)$. This is done via Lemma 5. Next, we show that in *any* non-perverse equilibrium, if, for some t and h_t, equilibrium strategies satisfy $\alpha_1^t(h_t) > \alpha_0^t(h_t)$, then $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$ implies $\alpha_0^t(h_t) = 0$ $\left(\alpha_1^t(h_t) = 1\right)$. This is done via Lemma 6. Finally, we demonstrate the existence and uniqueness of the most revealing equilibrium.

Define the function $\Delta u_{s_t}^t(h_t)$ as in the proof of Proposition 3.

Lemma 5 *There exists a non-perverse equilibrium where, for every time $t = 1, ...T$ and every history h_t, if h_t is such that $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$, then $\alpha_0^t(h_t) = 0$ $\left(\alpha_1^t(h_t) = 1\right)$.*

Proof of Lemma 5: We first state a straightforward refinement of Lemma 4.

Claim 1 *Consider the five statements contained in Lemma 4.*

For every $p \leq \frac{1}{2}$, there exists a pair (α_0, α_1) such that $\alpha_0 \leq \alpha_1$ and one of statements (a), (b), or (c) is true.

For every $p \geq \frac{1}{2}$, there exists a pair (α_0, α_1) such that $\alpha_0 \leq \alpha_1$ and one of statements (c), (d), or (e) is true.

Proof of Claim 1: We prove the case for $p \leq \frac{1}{2}$. The proof of the other case is symmetric. First we show that $\Delta u_0(0, \alpha_1, p) < 0$ for all $p \leq \frac{1}{2}$ and all $\alpha_1 \geq 0$. This follows from the facts that for $\alpha_1 \geq \alpha_0$, $\Delta\pi_0(0, \alpha_1, p) < 0$ for all p, and $\Delta r_0(0, \alpha_1, p) \leq 0$ for $p \leq \frac{1}{2}$. The first is obvious at this point. For the second, note that with $\alpha_1 \geq \alpha_0$ it is easy to show that $\Delta r_0(\alpha_0, \alpha_1, p)$ is non-decreasing in p. Thus, if we could show that $\Delta r_0\left(0, \alpha_1, \frac{1}{2}\right) \leq 0$ for all $\alpha_1 \geq 0$ then we would be done. For $p = \frac{1}{2}$, $v_0(p) = 1 - \sigma$. We can now write:

$$\begin{aligned}
\Delta r_0\left(0, \alpha_1, \frac{1}{2}\right) &= (1-\sigma)\left(\frac{\sigma_g}{\sigma} - \frac{(1-\alpha_1)\,\sigma_g + 1 - \sigma_g}{(1-\alpha_1)\,\sigma + 1 - \sigma}\right)\gamma + \sigma\left(\frac{1-\sigma_g}{1-\sigma} - \frac{(1-\alpha_1)(1-\sigma_g) + \sigma_g}{(1-\alpha_1)(1-\sigma) + \sigma}\right)\gamma \\
&= \frac{(\alpha_1 - 1 + \sigma\alpha_1(\sigma - 1))(\sigma_g - \sigma)(2\sigma - 1)}{(1 - \sigma\alpha_1)(1 + \sigma\alpha_1 - \alpha_1)(1-\sigma)\,\sigma}\gamma \leq 0, \text{ since } \alpha_1 \leq 1 \text{ and } \frac{1}{2} < \sigma < 1.
\end{aligned}$$

Thus, for $p \leq \frac{1}{2}$, $\Delta u_0(0, \alpha_1, p) < 0$ for $\alpha_1 \geq 0$. If, in addition, $\Delta u_1(0, 0, p) \leq 0$ then statement (a) is true. If, on the other hand, if $\Delta u_1(0, 0, p) > 0$, then by definition, this means that $\lim_{\alpha_1 \to 0} \Delta u_1(0, \alpha_1, p) > 0$. Continuity in α_1 implies that either (A) $\Delta u_1(0, 1, p) \geq 0$ in which case, since we have shown that $\Delta u_0(0, 1, p) < 0$, statement (c) is true; or (B) there exists $\alpha_1 > 0$ such that $\Delta u_1(0, \alpha_1, p) = 0$, in which case, statement (b) is true. The proof of the claim for $p \geq \frac{1}{2}$ is symmetric. This concludes the proof of Claim 1.

Let $\Gamma^t(h_t)$ denote the subgame that begins at time t after history h_t. We will prove existence by backward induction. First, we shall prove that for all h_T, the last-period subgame $\Gamma^t(h_T)$ has a continuation equilibrium which satisfies the property in Lemma 5. Second, we shall prove that if all subgames that begin at time t have a continuation equilibrium satisfying the property in Lemma 5, then all subgames that begin at time $t-1$ have a continuation equilibrium satisfying the property in Lemma 5.

For the first step of the induction argument, assume a history h_T and consider the last stage. By Claim 1, we see that the continuation game has an equilibrium that satisfies the property in Lemma 5. Note that, if the equilibrium involves pooling, then off-equilibrium trades are assumed to be sincere: this is implied by the way in which $\Delta u_{s_t}(0, 0, p_t)$ and $\Delta u_{s_t}(1, 1, p_t)$ are defined above.

For the second step, fix time t and suppose that for every history h_t there is a continuation equilibrium where for all $\tau \geq t$ and all h_τ, the property in Lemma 5 is satisfied. Now consider manager $t-1$. By Claim 1, we see that there exists $\alpha_1^{t-1} \geq \alpha_0^{t-1}$ such that if $p_{t-1}(h_{t-1}) \leq \frac{1}{2}$ one of conditions (a), (b), or (c) is satisfied, and if $p_{t-1}(h_{t-1}) \geq \frac{1}{2}$ one of conditions (c), (d), or (e) is satisfied. For each of those cases the pair $(\alpha_0^{t-1}, \alpha_1^{t-1})$ is a best response for player $t-1$. This concludes the proof of Lemma 5.

Next, we show that in *any* non-perverse equilibrium, if, for some t and h_t, equilibrium strategies satisfy $\alpha_1^t(h_t) > \alpha_0^t(h_t)$, then $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$ implies $\alpha_0^t(h_t) = 0$ $\left(\alpha_1^t(h_t) = 1\right)$.

Lemma 6 *If, in any equilibrium of* Γ*, for some* t *and* h_t*, equilibrium strategies satisfy* $\alpha_1^t(h_t) > \alpha_0^t(h_t)$*, then* $p_t \leq \frac{1}{2}$ $\left(p_t \geq \frac{1}{2}\right)$ *implies* $\alpha_0^t(h_t) = 0$ $\left(\alpha_1^t(h_t) = 1\right)$*.*

Proof of Lemma 6: Consider an equilibrium of the game Γ and suppose that for some manager t and some history h_t the equilibrium strategy satisfies $\alpha_1^t(h_t) > \alpha_0^t(h_t)$. For simplicity drop the t subscript and the h_t argument. Suppose first that $p_t \leq \frac{1}{2}$. Consider the manager who has received signal $s = 0$. Suppose that $\alpha_0 = \alpha > 0$. Lemma 2 and the assumption that $\alpha_1 > \alpha_0$ imply that $\alpha_1 = 1$. It is clear at this point that $\Delta \pi_0(\alpha, 1, p) < 0$ for all p. We now show that for $p \leq \frac{1}{2}$, $\Delta r_0(\alpha, 1, p) \leq 0$. It is easy to check that $\Delta r_0(\alpha, 1, p)$ is increasing in p. Thus, if $\Delta r_0\left(\alpha, 1, \frac{1}{2}\right) \leq 0$, then $\Delta r_0(\alpha, 1, p) \leq 0$ for all $p \leq \frac{1}{2}$. Let $p = \frac{1}{2}$. Thus,

$v_0(p) = 1 - \sigma$. Using the values of r computed earlier, we now write:

$$\begin{aligned} \Delta r_0 \left(\alpha, 1, \frac{1}{2}\right) &= (1-\sigma)\left(\frac{\sigma_g + \alpha(1-\sigma_g)}{\sigma + \alpha(1-\sigma)} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + \sigma\left(\frac{1-\sigma_g+\alpha\sigma_g}{1-\sigma+\alpha\sigma} - \frac{\sigma_g}{\sigma}\right)\gamma \\ &= -\frac{(\sigma_g - \sigma)(2\sigma - 1)(1-\alpha)}{[\sigma + \alpha(1-\sigma)][1-\sigma+\alpha\sigma]}\gamma < 0, \end{aligned}$$

which implies $\Delta u_0 (\alpha, 1, p) < 0$, and thus $\alpha = 0$, a contradiction. This concludes the proof of Lemma 6.

Finally, we demonstrate the existence and uniqueness of the most-revealing equilibrium. Lemmas 2 and 6 taken together imply that the only non-perverse equilibrium in which for $p_t \leq \frac{1}{2}$ it is possible that $\alpha_0^t(h_t) > 0$ is the pooling equilibrium with $\alpha_0^t(h_t) = \alpha_1^t(h_t) = 1$. Such an equilibrium reveals no information about private signals. Thus, when looking for the most revealing equilibrium, we can ignore this equilibrium. We are now ready to prove Proposition 4.

Proceed by backward induction. Consider the last period, T. We prove the case for $p_t \leq \frac{1}{2}$. The proof for $p_t \geq \frac{1}{2}$ is analogous. By the argument above, we can set $\alpha_0^T = 0$. (By Lemma 5 we know that there exists an equilibrium with $\alpha_0^T = 0$.) What is the largest α_1^T that can be achieved? This is given by

$$\alpha_1^T = \begin{cases} 1 & \text{if } \Delta u_1(\alpha_0^T = 0, \alpha_1^T = 1, p_T) \geq 0 \\ 0 & \text{if } \Delta u_1(\alpha_0^T = 0, \alpha_1^T, p_T) < 0, \ \forall \alpha_1^T \\ \max\{\alpha_1^T | \Delta u_1(\alpha_0^T = 0, \alpha_1^T, p_T) = 0\} & \text{otherwise} \end{cases}$$

which exists and is unique.

Now suppose that for every $\tau \geq t$ if $p_\tau \leq \frac{1}{2}$ the strategy is given by $\alpha_1^\tau(h_\tau) = \bar{\alpha}(p)$ and $\alpha_0^\tau(h_\tau) = 0$ (and conversely when $p_\tau \geq \frac{1}{2}$). Suppose that $p_{t-1} \leq \frac{1}{2}$. Thus, again, $\alpha_0^{t-1} = 0$. Define

$$\alpha_1^{t-1} = \begin{cases} 1 & \text{if } \Delta u_1(\alpha_0^{t-1} = 0, \alpha_1^{t-1} = 1, p_{t-1}) \geq 0 \\ 0 & \text{if } \Delta u_1(\alpha_0^{t-1} = 0, \alpha_1^{t-1}, p_{t-1}) < 0, \ \forall \alpha_1^{t-1} \\ \max\{\alpha_1^{t-1} | \Delta u_1(\alpha_0^{t-1} = 0, \alpha_1^{t-1}, p_{t-1}) = 0\} & \text{otherwise} \end{cases}$$

This is a best-response for manager $t-1$ and it is easy to see that no other α_1^{t-1} greater than this can be a best response for manager $t-1$.■

Proof of Proposition 5: Note that $1 - \sigma < \frac{1}{2} < \sigma$. Given the definition of the most revealing equilibrium, we need only to show that $\bar{\alpha}(p_t) = 1$ for $p_t \in \left(1-\sigma, \frac{1}{2}\right]$, and $\underline{\alpha}(p_t) = 0$ for $p_t \in \left[\frac{1}{2}, \sigma\right)$. We show the first. The second result follows by symmetry. For the fund manager with $s_t = 1$, it is clear that $\Delta\pi_1(\alpha_0 = 0, \alpha_1 = 1, p_t) > 0$ for all p_t. Next consider

$\Delta r_1(\alpha_0 = 0, \alpha_1 = 1, p_t)$. This can written is:

$$\begin{aligned} & v_1^t \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma + (1 - v_1^t) \left(\frac{1-\sigma_g}{1-\sigma} - \frac{\sigma_g}{\sigma} \right) \gamma \\ = \quad & (2v_1^t - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma \end{aligned}$$

which is positive if $v_1^t > \frac{1}{2}$. Since $v_1^t = \frac{\sigma p_t}{\sigma p_t + (1-\sigma)(1-p_t)}$, $v_1^t > \frac{1}{2}$ if $p_t > 1 - \sigma$. Thus, for $p_t \in \left(1 - \sigma, \frac{1}{2}\right]$, $\alpha_1 = 1$ is a best response for the manager who observes $s_t = 1$, and therefore $\bar{\alpha}(p_t) = 1$ for $p_t \in \left(1 - \sigma, \frac{1}{2}\right]$.

Proof of Proposition 6: For simplicity suppress the t subscript in what follows. Since we are considering $p \leq \frac{1}{2}$ we know that $\alpha_0 = 0$, and so we suppress α_0 in the proof. By the same token, since we are only concerned with the incentives of the manager with signal $s_t = 1$ we also suppress the s_t subscript. Finally, to emphasize the dependence on β, we include β explicitly as an argument of Δu_1. So, we write $\Delta u(\alpha_1, p, \beta)$ for Δu_1, and similarly for $\Delta \pi$ and Δr. It is easy to check that the function $\Delta u(\alpha, p, \beta)$ is continuous in α (for $\alpha > 0$).

Suppose first that $\bar{\alpha}\left(\beta', p_t\right) = 1$. This means that

$$\Delta u\left(1, p, \beta'\right) = \beta' \Delta \pi(1, p) + \left(1 - \beta'\right) \Delta r(1, p) \geq 0.$$

We know that $\Delta \pi > 0$ for all values. If $\Delta r\left(1, p, \beta'\right) \geq 0$, then it is immediate that

$$\beta'' \Delta \pi(1, p) + \left(1 - \beta''\right) \Delta r(1, p) \geq 0.$$

If instead $\Delta r\left(1, p, \beta'\right) < 0$, we see that

$$\beta'' \Delta \pi(1, p) + \left(1 - \beta''\right) \Delta r(1, p) > \beta' \Delta \pi(1, p) + \left(1 - \beta'\right) \Delta r(1, p) \geq 0.$$

In both cases, $\Delta u\left(1, p, \beta''\right) \geq 0$ and $\bar{\alpha}\left(\beta'', p_t\right) = 1$.

Next, assume that $\bar{\alpha}\left(\beta', p_t\right) \in (0, 1)$. It must then be that

$$\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta'\right) = \beta' \Delta \pi\left(\bar{\alpha}\left(\beta', p_t\right), p\right) + \left(1 - \beta'\right) \Delta r\left(\bar{\alpha}\left(\beta', p_t\right), p\right) = 0.$$

As $\Delta \pi > 0$, this implies that $\Delta r\left(\bar{\alpha}\left(\beta', p_t\right), p\right) < 0$. Hence,

$$\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta''\right) > \Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta'\right) = 0$$

As $\Delta u(\alpha, p, \beta)$ is continuous in α, at least one of the following statements must be true: (i) There exists $\alpha'' \in \left(\bar{\alpha}\left(\beta', p_t\right), 1\right)$ such that $\Delta u\left(\bar{\alpha}\left(\beta', p_t\right), p, \beta''\right) = 0$ (in which case there exists an informative equilibrium with $\alpha = \alpha''$); or (ii) $\Delta u\left(1, p, \beta''\right) \geq 0$ (in which case there exists a separating equilibrium). Either way, $\bar{\alpha}\left(\beta'', p_t\right) > \bar{\alpha}\left(\beta', p_t\right)$.

Proof of Proposition 7: Start with (i). If $\widehat{p} \in \{p_t : \bar{\alpha}(\beta, p_t) = 0\}$ then by definition $\Delta u_1(0, \widehat{p}, \beta) \leq 0$ and $\Delta u_1(\alpha, \widehat{p}, \beta) < 0$ for all $\alpha \in (0, 1]$. Hence, $p_{\min}(\beta)$ must satisfy the following two conditions: $\Delta u_1(0, p_{\min}(\beta), \beta) \leq 0$ and $\Delta u_1(\alpha, p_{\min}(\beta), \beta) \leq 0$ for all $\alpha \in (0, 1]$. Thus, $\Delta u_1(\alpha, p_{\min}(\beta), \beta) \leq 0$ for all $\alpha \in [0, 1]$.

Let $\beta = \beta''$. $\Delta u_1(\alpha, p_{\min}(\beta''), \beta'') \leq 0$ for all $\alpha \in [0, 1]$. Consider $\beta' < \beta''$. Since $\Delta\pi_1 > 0$ and $\Delta r_1 < 0$, $\Delta u_1(\alpha, p_{\min}(\beta''), \beta) < 0$ for all $\alpha \in [0, 1]$. But this implies that $p_{\min}(\beta'') \in \{p_t : \bar{\alpha}(\beta, p_t) = 0\}$. Thus, $p_{\min}(\beta'') \leq p_{\min}(\beta')$.

If $\widehat{p} \in \{p_t : \bar{\alpha}(\beta, p_t) = 1\}$ then by definition $\Delta u_1(1, \widehat{p}, \beta) \geq 0$. Hence, $p_{\max}(\beta)$ must satisfy $\Delta u_1(1, p_{\max}(\beta), \beta) = 0$. Let $\beta = \beta'$. Since Δu_1 is continuous in p and we know from Proposition 1 that $p_{\max}(\beta) > 0$, it must be the case that $\Delta u_1(1, p_{\max}(\beta'), \beta') = 0$, which implies that $\Delta\pi_1(1, p_{\max}(\beta')) > 0$ and $\Delta r_1(1, p_{\max}(\beta')) < 0$. Consider $\beta'' > \beta'$. It is now clear that $\Delta u_1(1, p_{\max}(\beta'), \beta'') > 0$, which means that $p_{\max}(\beta') \in \{p_t : \bar{\alpha}(\beta'', p_t) = 1\}$. Thus, $p_{\max}(\beta') \geq p_{\max}(\beta'')$.

For (ii), simply note that for all α and $p > 0$

$$\lim_{\beta \to 1} \Delta u_1(\alpha, p, \beta) = \Delta\pi_1(\alpha, p) > 0$$

Hence, for all $p > 0$,

$$\lim_{\beta \to 1} \bar{\alpha}(\beta, p) = 1,$$

which shows that $\lim_{\beta \to 1} p_{\max}(\beta) = \lim_{\beta \to 1} p_{\min}(\beta) = 0$.

For (iii), first we show that $\lim_{\beta \to 0} p_{\max}(\beta) = 1 - \sigma$. Note that

$$\begin{aligned}
\lim_{\beta \to 1} \Delta u_1(1, p, \beta) &= \Delta r_1(1, p) \\
&= v^1(p)\left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma}\right)\gamma + (1 - v^1(p))\left(\frac{1-\sigma_g}{1-\sigma} - \frac{\sigma_g}{\sigma}\right)\gamma
\end{aligned}$$

where $v^1(p) = \frac{\sigma p}{\sigma p + (1-\sigma)(1-p)}$. It is easy to see that $\Delta r_1(1, p) \geq 0$ if and only if $p \geq 1 - \sigma$.

Now consider $\lim_{\beta \to 0} p_{\min}(\beta)$. For any α, $\lim_{\beta \to 1} \Delta u_1(\alpha, p, \beta) = \Delta r_1(\alpha, p)$, where

$$\Delta r_1(\alpha, p) = v^1(p)\left(\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}\right)\gamma + (1 - v^1(p))\left(\frac{1-\sigma_g}{1-\sigma} - \frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma}\right)\gamma$$

We wish to find p such that $\Delta r_1(\alpha, p) < 0$ for all $\alpha > 0$ and $\Delta r_1(0, p) \leq 0$. We observe that $\Delta r_1(\alpha, p)$ is strictly increasing in p. Thus, for any α there exists a $p(\alpha)$ such that $\Delta r_1(\alpha, p) < 0$ if $p < p(\alpha)$. We compute $p(\alpha)$ for all α and minimize with respect to α. This gives $\lim_{\beta \to 0} p_{\min}(\beta)$.

For any α $p(\alpha)$ is defined by $\Delta r_1(\alpha, p(\alpha)) = 0$ which is equivalent to:

$$v_1(p(\alpha)) = \frac{1}{\frac{\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma}}{\frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma} - \frac{1-\sigma_g}{1-\sigma}} + 1}$$

Since $v_1(p)$ is increasing in p, we minimize $v_1(p(\alpha))$ with respect to α, which is equivalent to solving the following problem:

$$\max_{\alpha} \frac{\frac{\sigma_g}{\sigma} - \frac{(1-\alpha)\sigma_g+1-\sigma_g}{(1-\alpha)\sigma+1-\sigma}}{\frac{(1-\alpha)(1-\sigma_g)+\sigma_g}{(1-\alpha)(1-\sigma)+\sigma} - \frac{1-\sigma_g}{1-\sigma}}$$

Upon some simplification, this can be shown to be equivalent to:

$$\max_{\alpha} \frac{1-\sigma}{\sigma} \frac{1-\alpha(1-\sigma)}{1-\alpha\sigma}$$

The maximand is monotone increasing in α since $\sigma > \frac{1}{2}$, and thus

$$\min_{\alpha} v_1(p(\alpha)) = v_1(p(1)) = \frac{1}{2} \Rightarrow \min_{\alpha} p(\alpha) = p(1) = 1-\sigma$$

Thus, $\lim_{\beta\to 0} p_{\min}(\beta) = 1-\sigma$.

Proof of Proposition 8: *Bid-ask spread.* Focus on $p \in \left[0, \frac{1}{2}\right]$. Recall that in the most revealing equilibrium a manager with $s_t = 0$ sells and a manager with $s_t = 1$ buys with probability $\bar{\alpha}(\beta, p)$. The ask price and the bid price are

$$\begin{aligned} p_t^a &= \frac{\delta\frac{1}{2} + (1-\delta)\,\bar{\alpha}\sigma}{\delta\frac{1}{2} + (1-\delta)\,\bar{\alpha}\Sigma_t} p_t \\ p_t^b &= \frac{\delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\sigma + (1-\sigma)\right]}{\delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\Sigma_t + (1-\Sigma_t)\right]} p_t, \end{aligned}$$

It is easy to check that the former is increasing in $\bar{\alpha}$, which, by Proposition 6, is non-decreasing in β.

Price-volatility. Given p_t, the random variable p_{t+1}takes two values: p_t^a with probability $\Lambda = \delta\frac{1}{2} + (1-\delta)\,\bar{\alpha}\Sigma_t$ and p_t^b with probability $1-\Lambda = \delta\frac{1}{2} + (1-\delta)\left[(1-\bar{\alpha})\Sigma_t + (1-\Sigma_t)\right]$. The variance is

$$\begin{aligned} Var\left[p_{t+1}|p_t\right] &= \Lambda\left(p_t^a - p_t\right)^2 + (1-\Lambda)\left(p_t^b - p_t\right)^2 \\ &= \frac{\left(\bar{\alpha}\,(1-\delta)\,(\sigma-\Sigma_t)\right)^2}{\Lambda} p_t^2 + \frac{\left(-\bar{\alpha}\,(1-\delta)\,(\sigma-\Sigma_t)\right)^2}{1-\Lambda} p_t^2 \\ &= p_t^2\,(1-\delta)^2\,(\sigma-\Sigma_t)^2\,\frac{\bar{\alpha}^2}{\Lambda(1-\Lambda)} \end{aligned}$$

It is then easy to check that $Var\left[p_{t+1}|p_t\right]$ is increasing $\bar{\alpha}$, and hence non-decreasing in β.

Trade-predictability. This is immediate because

$$Var\left(a_t|p_t\right) = \Lambda\left(1-\Lambda\right)$$

and (for $p < \frac{1}{2}$) Λ is increasing in $\bar{\alpha}$. Thus, trade predictability, $\frac{1}{Var(a_t|p_t)}$, is decreasing in $\bar{\alpha}$, and thus non-increasing in β.

Proof of Proposition 9: Note that there is at most one v for which $\sigma_{v,g} = \sigma_{v,b}$. Denote this by v_{equal}, so that for $v < v_{equal}$, $\sigma_{v,g} < \sigma_{v,b}$ and for $v > v_{equal}$, $\sigma_{v,g} > \sigma_{v,b}$. Consider any arbitrary (true) liquidation value $v^* < v_{equal}$ (the case for $v^* > v_{equal}$ is symmetric and is omitted). Suppose for contradiction that the equilibrium is such that $\lim_{t\to\infty} p_t = v^*$. Namely, for every $\Pr[v^*|h_t]$, there must be an informative equilibrium with either $\alpha_1 > \alpha_0$ or $\alpha_0 > \alpha_1$.

Case with $\alpha_1 > \alpha_0$

First note that, reusing the notation of the baseline model, $\Delta\pi_1 > 0 > \Delta\pi_0$. In addition, for $v^* < v_{equal}$ and $\alpha_1 > \alpha_0$ it is easy to show that $r(v^*, 1) - r(v^*, 0) < 0$. Thus, since $\Delta\pi_0 < 0$, $r(v^*, 1) - r(v^*, 0) < 0$, and $r(v, 1) - r(v, 0)$ is bounded for all v, there exists an $\epsilon > 0$ such that for $\Pr(v = v^*|p_t) > 1 - \epsilon$, $\Delta u_0 < 0$ and thus $\alpha_0 = 0$.[20] Now for histories implying that $\Pr(v = v^*|p_t) > 1 - \epsilon$, we can set $\alpha_0 = 0$, and write:

$$r(v^*, 1) - r(v^*, 0) = \left(\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - \frac{\sigma_{v^*,g}(1 - \alpha_1) + 1 - \sigma_{v^*,g}}{\sigma_{v^*}(1 - \alpha_1) + 1 - \sigma_{v^*}} \right) \gamma$$

where $\sigma_{v^*} = \gamma\sigma_{v^*,g} + (1 - \gamma)\sigma_{v^*,b} > \sigma_{v^*,g}$. It follows that we can find a strictly negative upper bound for $r(v^*, 1) - r(v^*, 0)$ since

$$\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - \frac{\sigma_{v^*,g}(1 - \alpha_1) + 1 - \sigma_{v^*,g}}{\sigma_{v^*}(1 - \alpha_1) + 1 - \sigma_{v^*}} \leq \frac{\sigma_{v^*,g}}{\sigma_{v^*}} - 1 < 0$$

Now, consider the agent with $s = 1$. Given the boundedness of $\Delta\pi_1$ and $r(v, 1) - r(v, 0)$ for all v, we can write:

$$\lim_{\Pr(v=v^*|p_t)\to 1} \Delta u_1 = (1 - \beta)(r(v^*, 1) - r(v^*, 0)) \leq (1 - \beta)\left(\frac{\sigma_{v^*,g}}{\sigma_{v^*}} - 1 \right) \gamma < 0$$

Thus, $\alpha_1 = 0$, and for $\Pr(v = v^*|p_t)$ high enough, the equilibrium cannot be informative.

Case with $\alpha_1 < \alpha_0$

In order to have an informative equilibrium, it must be the case that $\Delta u_1 \leq 0$ and since $\Delta\pi_1 > 0$ it must be the case that $\Delta r_1 < 0$. That is

$$\sum_v \Pr[v|h_t, s = 1](r(v, 1) - r(v, 0)) < 0.$$

Thus,

$$\Pr[v = v^*|h_t, s = 1](r(v^*, 1) - r(v^*, 0)) < \sum_{v \neq v^*} \Pr[v|h_t, s = 1](r(v, 0) - r(v, 1))$$

[20] Note that, as in the baseline model, we can rule out totally mixed equilibria with $\alpha_1 > \alpha_0$.

It is clear that for any $\alpha_1 < \alpha_0$, $r(v,1) - r(v,0) > 0$ if and only if $\sigma_{v,g} < \sigma_{v,b}$. In particular, for any $\alpha_1 < \alpha_0$ the maximum value of $r(v,0) - r(v,1)$ is attained at $v = v_{\max}$. This is because, for any given $\alpha_1 < \alpha_0$, $r(v,0)$ is increasing in v and $r(v,1)$ is decreasing in v. To see that (we omit the $r(v,0)$ case), note that

$$r(v,1) = \frac{\sigma_{v,g}\alpha_1 + (1-\sigma_{v,g})\alpha_0}{\sigma_v\alpha_1 + (1-\sigma_v)\alpha_0},$$

where, as before, $\sigma_v = \gamma\sigma_{v,g} + (1-\gamma)\sigma_{v,b}$. By A2, the ratio $\frac{\sigma_{v,g}}{\sigma_v}$ is increasing in v and the ratio $\frac{1-\sigma_{v,g}}{1-\sigma_v}$ is decreaing in v. As $\alpha_0 > \alpha_1$, this implies that the whole ratio $r(v,1)$ is decreasing in v.

Thus, the maximum value that can be taken by the right hand side of the above inequality is $\sum_{v \neq v^*} \Pr[v|h_t, s=1](r(v_{\max},0) - r(v_{\max},1))$. Hence, a necessary condition for $\Delta r_1 < 0$ at $v = v^*$ is

$$\Pr[v = v^*|h_t, s=1](r(v^*,1) - r(v^*,0)) < (1 - \Pr[v = v^*|h_t, s=1])(r(v_{\max},0) - r(v_{\max},1))$$

Which can be rewritten as follows:

$$\frac{r(v^*,1) - r(v^*,0)}{r(v_{\max},0) - r(v_{\max},1)} < \frac{1 - \Pr[v = v^*|h_t, s=1]}{\Pr[v = v^*|h_t, s=1]}$$

Define

$$E = -\frac{r(v^*,1) - r(v^*,0)}{r(v_{\max},1) - r(v_{\max},0)}$$

We shall show that $\inf_{\alpha_1<\alpha_0} E > 0$, which means that for histories implying $\Pr[v = v^*|h_t, s=1]$ high enough, the necessary condition must fail. The proof is a convoluted version of the relevant subcase of the proof of the main result. Define

$$b = \sigma_{v^*,g}\alpha_1 + (1-\sigma_{v^*,g})\alpha_0 \qquad B = \sigma_{v^*}\alpha_1 + (1-\sigma_{v^*})\alpha_0$$
$$a = \sigma_{v_{\max},g}\alpha_1 + (1-\sigma_{v_{\max},g})\alpha_0 \quad A = \sigma_{v_{\max}}\alpha_1 + (1-\sigma_{v_{\max}})\alpha_0$$

where $\sigma_{v^*} = \gamma\sigma_{v^*,g} + (1-\gamma)\sigma_{v^*,b}$, and similarly for $\sigma_{v_{\max}}$. Now it is easy to show that

$$E = \frac{\sigma_{v^*} - \sigma_{v^*,g}}{\sigma_{v_{\max},g} - \sigma_{v_{\max}}}\frac{A(1-A)}{B(1-B)}$$

Note that $\frac{\sigma_{v^*} - \sigma_{v^*,g}}{\sigma_{v_{\max},g} - \sigma_{v_{\max}}} > 0$ and independent of α_0, α_1. Thus, finding the infimum reduces to finding

$$\inf_{\alpha_1<\alpha_0} \frac{A(1-A)}{B(1-B)}$$

As before, interior solutions are ruled out by the facts that $\sigma_{v_{\max}} > \sigma_{v^*}$ and $A \neq B$. The remaining possibilities are that $\alpha_1 = 0$ and $\alpha_0 > 0$, in which case the infimum can be shown to be $\frac{1-\sigma_{v_{\max}}}{1-\sigma_{v^*}} > 0$ and $\alpha_1 < 1$ and $\alpha_0 = 1$, in which case the infimum can be shown to be $\frac{1-\sigma_{v_{\max}}}{1-\sigma_{v^*}} \frac{\sigma_{v_{\max}}}{\sigma_{v^*}} > 0$.

Proof of Proposition 10: We first show that there cannot be an equilibrium with $\alpha_{1g} = \alpha_{1b} = 1$ and $\alpha_{0b} = \alpha_{0g} = 0$, because, in such an equilibrium: $r(a=1, v=0) < \gamma < r(a=0, v=0)$. In general,

$$r(a=1, v=0) = \frac{1}{1 + \frac{1-\gamma}{\gamma} \frac{\sigma_b(1-\rho)\alpha_{0g} + \sigma_b\rho\alpha_{0b} + (1-\sigma_b)(1-\rho)\alpha_{1g} + (1-\sigma_b)\rho\alpha_{1b}}{\sigma_g\rho\alpha_{0g} + \sigma_g(1-\rho)\alpha_{0b} + (1-\sigma_g)\rho\alpha_{1g} + (1-\sigma_g)(1-\rho)\alpha_{1b}}}$$

Substituting in the equilibrium values of α_{sz} gives: $r(a=1, v=0) = \frac{1}{1+\frac{1-\gamma}{\gamma}\frac{1-\sigma_b}{1-\sigma_g}} < \gamma$. It follows that: $\lim_{p\to 0} \Delta u_{g1} = (1-\beta)\left[r(a=1, v=0) - r(a=0, v=0)\right] < 0$, contradicting $\alpha_{1g} = 1$.

Next we show that there cannot be an equilibrium with $\alpha_{1g} = \alpha_{1b} = 0$ and $\alpha_{0b} = \alpha_{0g} = 1$. Since $\alpha_{0g} = 1$, $\Delta u_{0g} \geq 0$. Since $\Delta\pi_{0g} < 0$, it must be the case that $\Delta r_{0g} > 0$. For sufficiently extreme p, this condition is violated, because $r(a=1, v=1) < \gamma < r(a=0, v=1)$ in this equilibrium. To see this, compute:

$$r(a=1, v=1) = \frac{1}{1 + \frac{1-\gamma}{\gamma} \frac{\sigma_b(1-\rho)\alpha_{1g} + \sigma_b\rho\alpha_{1b} + (1-\sigma_b)(1-\rho)\alpha_{0g} + (1-\sigma_b)\rho\alpha_{0b}}{\sigma_g\rho\alpha_{1g} + \sigma_g(1-\rho)\alpha_{1b} + (1-\sigma_g)\rho\alpha_{0g} + (1-\sigma_g)(1-\rho)\alpha_{0b}}}$$

Inserting the equilibrium values of α_{sz} we have: $r(a=1, v=1) = \frac{1}{1+\frac{1-\gamma}{\gamma}\frac{1-\sigma_b}{1-\sigma_g}} < \gamma$. For $p \to 1$, $\Delta r_{0g} \to r(a=1, v=1) - r(a=0, v=1) < 0$ which violates $\Delta r_{0g} > 0$.

Proof of Proposition 11: The revised proportions of noise traders, managers, and individuals are $\delta, (1-\delta)(1-\tau)$, and $(1-\delta)\tau$ respectively. Individuals are identical to managers without career concerns ($\beta = 1$), and therefore they trade sincerely. The presence of individual traders will change the bid and ask prices, but it will still be the case that $0 \leq p_a, p_b \leq 1$. In deriving the bounds on $\Delta\pi_s$ in the proof of Proposition 1, we utilized only the fact that $0 \leq p_a, p_b \leq 1$. These bounds are, therefore, unaffected. The bounds on Δr_s are also unaffected since they depend only on the public belief and on γ, σ_g, and σ_b. Thus, the threshold $\widehat{p}$ identified in Proposition 1 is unchanged: in any equilibrium, fund managers must sell (buy) if $p < \widehat{p}$ ($p > 1 - \widehat{p}$).

Suppose the true valuation is $v = 0$. With some speculators ($\tau > 0$), the price will converge to zero in the long term. Suppose that the price falls below the minimal transaction price identified in Proposition 1: $\underline{p}$. Since $\underline{p} < \widehat{p}$ (by construction), fund managers will sell in an uninformative manner. It is easy to see that the updating rule is

$$p_{t+1} = \begin{cases} \frac{\delta\frac{1}{2} + (1-\delta)\tau\sigma}{\delta\frac{1}{2} + (1-\delta)\tau\Sigma_t} p_t & \text{if } a_t = 1 \\ \frac{\delta\frac{1}{2} + (1-\delta)(1-\tau) + (1-\delta)\tau(1-\sigma)}{\delta\frac{1}{2} + (1-\delta)(1-\tau) + (1-\delta)\tau(1-\Sigma_t)} p_t & \text{if } a_t = 0 \end{cases}$$

Define

$$K(\tau) = \frac{\delta\frac{1}{2} + (1-\delta)(1-\tau) + (1-\delta)\tau(1-\sigma)}{\delta\frac{1}{2} + (1-\delta)(1-\tau) + (1-\delta)\tau(1-\Sigma_t)}$$

Note $\lim_{\tau\to 0} K(\tau) = 1$. Start at some $p_t \leq \underline{p}$. Suppose there is series of s sells. We can write

$$p_{t+s} = K(\tau)\, p_{t+s-1} = (K(\tau))^2\, p_{t+s-2} = ... = (K(\tau))^s\, p_t$$

Given two prices, $p' > p''$, below $\underline{p}$, the shortest number of periods it takes to go from p' to p'' is given by the lucky case where all noise traders and all speculators sell. In that case, it takes s periods where s is the smallest integer that solves $p'' \geq (K(\tau))^s\, p'$. It is easy to see that the solution s goes to infinity when $K(\tau) \to 1$.

References

[1] F. Allen, Do Financial Institutions Matter? J. Finance 56 (2001), 1165-1175.

[2] F. Allen, G. Gorton, Churning Bubbles, Rev. Econ. Stud. 60 (1993), 813-836.

[3] C. Avery, J. Chevalier, Herding Over the Career, Econ. Letters 63 (1999), 327-333.

[4] C. Avery, P. Zemsky, Multidimensional Uncertainty and Herd Behavior in Financial Markets, Amer. Econ. Rev. 88 (1998), 724-748.

[5] A. Banerjee, A Simple Model of Herd Behavior, Quart. J. Econ. 107 (1992), 797-817.

[6] S. Bikhchandani, D. Hirshleifer, I. Welch, A Theory of Fads, Fashion, Custom, and Cultural Change as Information Cascades, J. Polit. Economy 100 (1992), 992-1026.

[7] V. Chari, P. Kehoe, Financial Crises as Herds: Overturning the critiques, J. Econ. Theory 119 (2004), 19-33.

[8] J. Chevalier, G. Ellison, Risk Taking by Mutual Funds as a Response to Incentives, J. Polit. Economy 105 (1997), 1167-1200.

[9] J. Chevalier, G. Ellison, Career concerns of mutual fund managers, Quart. J. Econ. 114 (1999), 389-432.

[10] D. Cuoco, R. Kaniel, Equilibrium Prices in the presence of Delegated Portfolio Management, working paper, The University of Pennsylvania, 2007.

[11] A. Dasgupta, A. Prat, Financial Equilibrium with Career Concerns, Theoretical Economics 1 (2006), 67-93.

[12] A. Dasgupta, A. Prat, Asset Price Dynamics When Traders Care About Reputation, CEPR Discussion Paper 5372, 2005.

[13] A. Dasgupta, A. Prat, M. Verardo, Institutional Trade Persistence and Long-Term Equity Returns, CEPR Discussion Paper 6374, 2007.

[14] P. Dennis, D. Strickland, Who Blinks in Volatile Markets, Individuals or Institutions, J. Finance 57 (2002), 1923-1950.

[15] J. Dow, G. Gorton, Noise trading, delegated portfolio management, and economic welfare, J. Polit. Economy 105 (1997), 1024-1050.

[16] L. Glosten, P. Milgrom, Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders, J. Finan. Econ. 14 (1985), 71-100.

[17] Z. He, A. Krishnamurthy, Intermediation, Capital Immobility, and Asset Prices, working paper, Northwestern University, 2006.

[18] A. Kyle, Continuous Auctions and Insider Trading, Econometrica 53 (1985), 1315-1335.

[19] I. Lee, Market Crashes and Informational Avalanches, Rev. Econ. Stud. 65 (1998), 741-759.

[20] M. Massa, R. Patgiri, Compensation and Managerial Herding: Evidence from the Mutual Fund Industry, working paper, INSEAD, 2005.

[21] M. Ottaviani, P. Sorensen, Professional Advice, J. Econ. Theory 126 (2006), 120-142.

[22] A. Park, H. Sabourian, Herd behavior in efficient financial markets, working paper 249, University of Toronto, 2006.

[23] A. Prat, The Wrong Kind of Transparency, Amer. Econ. Rev. 95 (2005), 862-877.

[24] D. Scharfstein, J. Stein, Herd behavior and investment, Amer. Econ. Rev. 80 (1990), 465-479.

[25] A. N. Shiryaev, Probability, Springer, New York, 1996.

[26] R. Sias, Institutional Herding, Rev. Finan. Stud. 17 (2004) 165-206.

[27] R. Sias, Reconciliable Differences: Momentum Trading by Institutions, Finan. Rev. 42 (2007), 1-22.

[28] L. Smith, P. Sorensen, Pathological Outcomes of Observational Learning, Econometrica 68 (2000), 371-398.

[29] B. Trueman, Analyst Forecasts and Herding Behavior, Rev. Finan. Stud. 7 (1994), 97-124.

[30] D. Vayanos, Flight to Quality, Flight to Liquidity, and the Pricing of Risk, working paper, London School of Economics, 2003.