Slides 3: Macrofinance - Asset Pricing

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1 Asset pricing:

We have bonds, equities and capital in the model above, so have a candidate asset pricing model

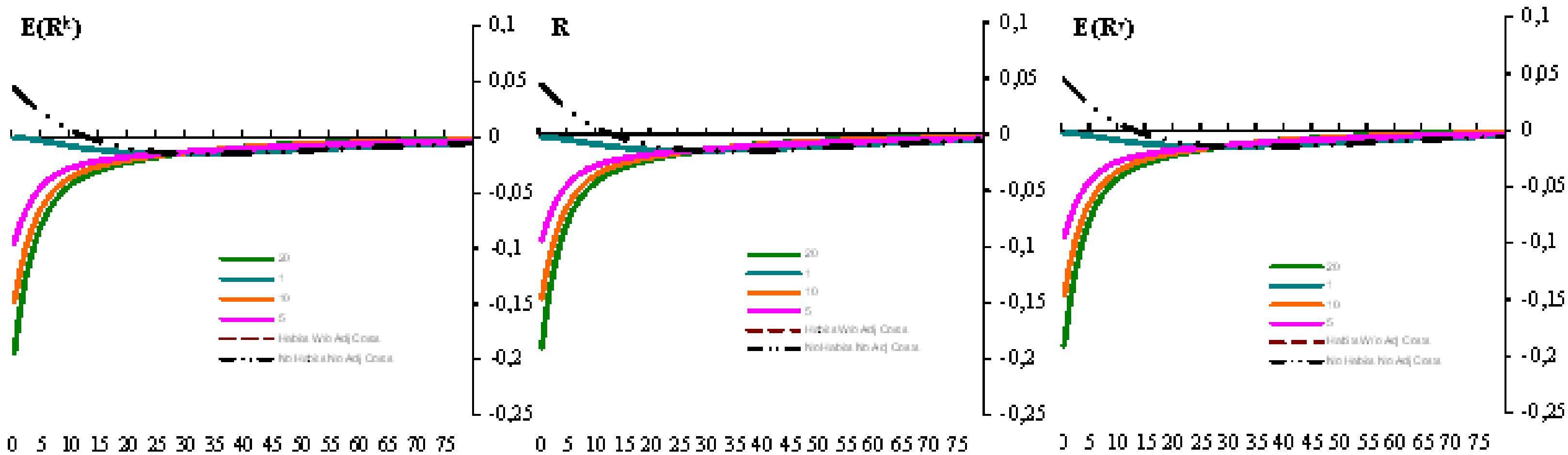
$$1 = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1} \right\}$$

$$1 = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R^y_{t+1} \right\}$$

$$\begin{aligned} 1 &= E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[\frac{\alpha \frac{Y_{a,t+1}}{K_{a,t}} + (1-\delta) + \chi\left(K_{t+1} - K_t\right)}{1 + \chi\left(K_t - K_{t-1}\right)} \right] \right\} \\ &= E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R^k_{t+1} \right\} \end{aligned}$$

- Numerical solution (Note: have to advise the program that $R_{t+1}$is known in period t- how?)

- In expectation, we have



- IRF: with capital adjustment cost, interest rates are counter-cyclical, without adjustment cost interest rates are pro-cyclical - Why?

- Without cost of adjusting capital, in order to smooth consumption, interest rates increase and so does investment. The return on capital is clearly larger given the increase in productivity.

- Capital adjustment costs reduce the return on capital and the level of investment.

- In the limit - when capital adjustment cost is infinite, what we have is an endowment economy in which agents consume everything that is produced, since nothing is invested. That is, the model is equivalent to a model without capital. In this case it is clear to see that interest rates have to fall following the shock. As agents are hit by a positive shock, they will increase their demand for savings, in order to postpone (smooth) consumption. But given that there is no saving vehicle, interest rates have to fall in order to discourage savings and induce agents to consume all new production.

- R_{t+1} and $E_t R^y_{t+1}$ are identical - Why?

1.1 Solution method based on linearized model:

- $R_{t+1}$is known in period t : bonds are risk-free - i.e. they deliver a unity of consumption in each period, the first equation can be written as:

$$1 = R_{t+1} E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} \right\}$$

whereas, for the case of equities

$$1 = E_t \left\{ \beta \frac{\Lambda_{t+1}}{\Lambda_t} R^y_{t+1} \right\}$$

- So, can we compute the equity risk premium?

- Defining the stochastic discount factor as $M_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t}$, a first order Taylor expansion of the above equations lead to:

$$0 = E_t \hat{m}_{t+1} + E_t \hat{r}^y_{t+1}$$

$$0 = E_t \hat{m}_{t+1} + \hat{r}_{t+1}$$

- (Note that the same expression arises in logs, as compared to log-deviations from steady state)
- But, risk premia is all about comovements of returns with economic conditions: investors dislike to have poor returns when they need extra cash! Linear model ignores this fact.

1.2 Log-normal approximation (finance approach)

- If z is a normal variable

$$E(e^z) = e^{E(z)+\frac{1}{2}var(z)}$$

  or

$$\log E(Z) = E(z) + \frac{1}{2}var(z)$$

- So, taking logs of the risk-free rate equation

$$0 = \log(R_{t+1}) + \log E_t \{M_{t+1}\}$$

or,

$$r_{t+1} = -\log E_t \{M_{t+1}\}$$

- If the stochastic discount factor is conditionally log-normal

$$\log E_t(M_{t+1}) = E_t(m_{t+1}) + \frac{1}{2} var_t(m_{t+1})$$

- And, thus

$$r_{t+1} = -E_t(m_{t+1}) - \frac{1}{2} var_t(m_{t+1})$$

- *Can we find an macroeconomic interpretation for the term $var_t(m_{t+1})$?*

- Taking logs of the asset pricing equation for equity returns

$$0 = \log \left[E_t \left\{ M_{t+1} R^y_{t+1} \right\} \right]$$

- If equity returns and the stochastic discount are jointly log-normal

$$\begin{aligned} \log E_t(M_{t+1} R^y_{t+1}) &= E_t(m_{t+1}) + E_t(r^y_{t+1}) + \frac{1}{2} var_t(m_{t+1}) \\ &\quad + cov_t(m_{t+1}, r^y_{t+1}) + \frac{1}{2} var_t(r^y_{t+1}) \end{aligned}$$

- So

$$\begin{aligned} E_t r^y_{t+1} &= -E_t(m_{t+1}) \\ &\quad - \frac{1}{2} var_t(m_{t+1}) - cov_t(m_{t+1}, r^y_{t+1}) - \frac{1}{2} var_t(r^y_{t+1}) \end{aligned}$$

- We can compute the equity risk premium

$$E_t r^y_{t+1} - r_{t+1} + \frac{1}{2} var_t(r^y_{t+1}) = -cov_t(m_{t+1}, r^y_{t+1})$$

- An asset with high covariance has its high return when consumption is high, i.e., when the marginal utility of consumption is low. This asset is not a good hedge against the uncertainties about consumption growth, so it commands a large premium. Moreover, a more volatile stochastic discount factor and/or a more volatile equity return will increase the magnitude of the equity risk premium.

- Note that the variance term on the right-hand side is a Jensen's inequality term that arises from taking logs of returns.

- *But can we make these distributional assumptions in a general equilibrium model?*

1.3 Second order approximation

- Taking a second order taylor expansion of

$$R_{t+1}^{-1} = \beta E_t \left\{ \frac{\Lambda_{t+1}}{\Lambda_t} \right\}$$

- expanding the left-hand-side is

$$(R_{t+1})^{-1} \simeq (\overline{R})^{-1} - (\overline{R})^{-2}(R_{t+1} - \overline{R}) + (\overline{R})^{-3}(R_{t+1} - \overline{R})^2$$

- Note that the equations do not hold with equality, given that this is an approximations and terms of order higher than 2 are dismissed.

- Alternatively, we can express this expansion as

$$(R_{t+1})^{-1} = (\overline{R})^{-1} - (\overline{R})^{-2}(R_{t+1} - \overline{R}) + (\overline{R})^{-3}(R_{t+1} - \overline{R})^2 + o(\|\widetilde{\xi}\|^3)$$

- $o(\|\widehat{\xi}\|^3)$ are terms of order higher than 2

- and the right-hand-side:

$$E_t\left[M_{t+1}\right] = \overline{M} + E_t\left[M_{t+1} - \overline{M}\right] + o(\|\widetilde{\xi}\|^3).$$

- But remember, if you want to express variables in log deviations from steady state

$$(X - \overline{X}) = \overline{X}\hat{x} + \frac{1}{2}\overline{X}\hat{x}^2 + o(\|\xi\|^3)$$

- *Why? Just expand a logarithm function*

- So,

$$\hat{r}_{t+1} = -E_t\left[\hat{m}_{t+1} + \frac{1}{2}(\hat{m}_{t+1})^2\right] + \frac{1}{2}(\hat{r}_{t+1})^2 + o(\|\xi\|^3)$$

- Alternatively, we can drop the term $o(\|\xi\|^3)$ and write the above expression using an approximation sign $\simeq$ instead of the equality sign $=$;

- Squaring the above equation, we have

$$(\hat{r}_{t+1})^2 \simeq [E_t\hat{m}_{t+1}]^2$$

- Hence, the expression for the risk-free rate is

$$\hat{r}_{t+1} \simeq -E_t[\hat{m}_{t+1}] - \frac{1}{2}var_t(\hat{m}_{t+1}). \tag{1}$$

- The second-order approximation method and the log-normal approximation lead to the same expression for the risk-free rate - but if equity returns and the stochastic discount are jointly log-normal the equation above holds with equality.

- If we derive a second order approximation for the equity returns, we can show that

$$E_t\left[\hat{r}^y_{t+1}\right] - \hat{r}_{t+1} + \frac{1}{2}var_t(\hat{r}^y_{t+1}) \simeq -cov_t(\hat{m}_{t+1}, \hat{r}^y_{t+1}). \tag{2}$$

or

$$E_t\left[r^y_{t+1}\right] - r_{t+1} + \frac{1}{2}var_t(r^y_{t+1}) \simeq -cov_t(m_{t+1}, r^y_{t+1}). \tag{3}$$

- where $\frac{1}{2}var_t(r^y_{t+1})$ is called the Jensen´s inequality term

2 CCAPM

The finance approach:

- Starting from the equation

$$P = E(MX)$$

- where P is the price of an asset (a stock), X is a stochastic payoff

$$P = E(MX) = E(M)E(X) + cov(M, X)$$

- We also know that in the presence of a risk free asset, the covariance term will drop out and then

$$1 = E(M)E(R_f)$$

so that $R_f = \frac{1}{E(M)}$ generating the required expression.

- So we can show that

$$P = \frac{E(X)}{R_f} + cov(M, X)$$

where R_f is the risk free rate. Interpret the two terms:

- The first term is a standard present discounted value formula.
- The second term is a risk adjustment. An asset whose payoff covaries positively with the SDF has its price raised and viceversa.
- Substitute back for M from a first order utility maximization condition

$$P = \frac{E(X)}{R_f} + \frac{cov(U'(C_{t+1}), X_{t+1})}{U'(C_t)}$$

- Marginal utility declines as C rises. Thus, an asset's price is lowered if its payoff covaries positively with consumption.

- Why is the covariance between M and X more important than the variance of X?

- The investor objective is to smooth consumption fluctuations. The investor does not care about other volatilities or covariances, other than in the way they might affect consumption volatility.

- Consider then what happens when an investor buys a little more ϵ of payoff X. Then

$$\sigma^2_\varepsilon(C + \epsilon X) = \sigma^2_\varepsilon(C) + 2\epsilon cov(C, X) + \epsilon^2 \sigma^2_\varepsilon(X)$$

For small (marginal) portfolio changes, ϵ^2 goes to zero faster than ϵ and, therefore, $cov(C, X)$ has a larger impact on consumption volatility than $\sigma^2_\varepsilon(X)$.

2.1 Implications of Model of Lecture 1 for risk premia

- With a CRRA utility function σ denoting the coefficient of risk aversion, the risk free rate is

$$\hat{r}_{t+1} = \sigma E_t \Delta \hat{c}_{t+1} - \frac{\sigma^2}{2} var_t(\Delta \hat{c}_{t+1}). \tag{4}$$

- the return on equity would be

$$E_t \left[\hat{r}^y_{t+1}\right] + \frac{1}{2} var_t(\hat{r}^y_{t+1}) = \sigma E_t \Delta \hat{c}_{t+1} + \sigma cov_t \left(\Delta \hat{c}_{t+1}, \hat{r}^y_{t+1}\right) - \frac{\sigma^2}{2} var_t(\Delta \hat{c}_{t+1}). \tag{5}$$

- and the risk premium

$$rp_t \equiv E_t\left[\hat{r}^y_{t+1}\right] - \hat{r}_{t+1} + \frac{1}{2}var_t(\hat{r}^y_{t+1}) \simeq \sigma cov_t\left(\Delta \hat{c}_{t+1}, \hat{r}^y_{t+1}\right)$$

- Expected excess (log) return of asset i is determined by covariance of asset return with consumption.

- note that we can arrive at this equation using both log-normal or 2nd order approximations.

Linear/log-normal method (Lettau (2003))

- Uses linear equilibrium conditions for the macro variables and then substitute these in non-linear asset pricing equations

- Use linearized RBC model, in which unexpected consumption growth and asset returns are linear functions of stochastic shocks to technology

$$rp_t = \sigma cov_t \left(\eta_{cz}\varepsilon_{t+1}, \eta_{rz}\varepsilon_{t+1}\right) = \sigma\eta_{cz}\eta_{rz}\sigma_\varepsilon^2$$

- Can this model explain observed facts?

2.2 Facts

1. Average real rate of return on stocks is high. 7.6% in US quarterly data post-war. Comparable rates for European countries.

2. Average rate of return on riskless assets low. US Treasuries have yielded 0.8%.

3. Stock returns volatile: s.d. 15.5% in US.

4. Ex post s.d. on riskless rate 1.8%, and more than half is foreseeable (predictable inflation and nominal rates), so actual risk even less.

5. Consumption growth very smooth: s.d. of non-durables 1.1%

2.3 The equity premium puzzle

$$rp_t = \sigma cov_t \left(\Delta \hat{c}_{t+1}, \hat{r}^y_{t+1} \right)$$

or

$$rp_t = \sigma \eta_{cz} \eta_{rz} \sigma^2_\varepsilon$$

- We cannot explain why the return on stocks so much higher than the return on the safe asset (facts 1 and 2)
- Data measures in fact 5 reveal the equity premium puzzle: the σ needed to satisfy the equation are extremely large in all countries of the world, sometimes exceeding 100. At plausible σ values the risk premium is too small to explain the equity premium.

2.4 The risk free rate puzzle

with CRRA utility

$$\hat{r}_{t+1} = \sigma E_t \Delta \hat{c}_{t+1} - \frac{\sigma_\varepsilon^2}{2} var_t(\hat{c}_{t+1}). \tag{6}$$

- Suppose we were to accept the very high values of σ needed to resolve the equity premium puzzle.
- Problem 1) a GE model would generate a very small variance for consumption.

$$r_{t+1} = -\log\beta + \sigma E_t \Delta \hat{c}_{t+1} - \frac{\sigma^2}{2} var_t(\hat{c}_{t+1}). \tag{7}$$

- Problem 2) because consumption growth is on average positive, and the variance of consumption is small, the last two terms in the equation above will give a very large number.

- If then we choose the discount factor to reconcile this big number with the observed risk-free rates, we come up with $\beta > 1$, i.e., negative rate of time preference.

- So a high degree of risk aversion does not appear to be the solution to the puzzle

2.5 Stock market volatility puzzle

- Why are stock returns so volatile? (facts 3 and 4)
- Lettau (2003):

$$rp_t = \sigma \eta_{cz} \eta_{rz} \sigma_{\varepsilon}^2 \tag{8}$$

- Preferences and elasticities are constant (deep parameters) and model is homoskedastic - risk premium is constant!
- A constant risk premium is not consistent with empirical evidence
- From the definition of equity returns:

$$R_{t+1}^y = \frac{V_{t+1} + D_{t+1}}{V_t}$$

- or in log-deviations from steady state

$$\hat{r}^y_{t+1} = \rho \hat{v}_{t+1} + (1-\rho)\hat{d}_{t+1} - \hat{v}_t$$

- where $\rho = 1/(1+\mu)$ and $\mu = \bar{D}/\bar{V}$

- [Campbell show formulas in logs and not log-deviations from steady state]

$$\begin{aligned} r^y_{t+1} &= k + \rho v_{t+1} + (1-\rho) d_{t+1} - v_t \\ \text{where } k &= -\log\rho - (1-\rho)\log(\rho^{-1} - 1) \end{aligned}$$

- Solve now this approximation formula forward, impose the terminal condition $\lim_{j\to\infty} \rho^j v_{t+j} = 0$ (no rational bubbles) take expectations and subtract the current dividend to get:

$$v_t - d_t = \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j (\Delta d_{i,t+1+j} - r^y_{t+1+j})$$

- Recall that this is an approximation to a definition, so it must hold as an accounting identity (unless the approximation is not good). It says that a high asset price-to-dividend ratio must be accompanied either by high future expected dividends or by low future expected rates of return.

- or using the definition of the risk premium

$$\begin{aligned} v_t - d_t &= \frac{k}{1-\rho} + E_t \sum_{j=0}^{\infty} \rho^j \Delta d_{i,t+1+j} \\ &\quad - E_t \sum_{j=0}^{\infty} (rp_{t+j} + r_{t+1+j}) \end{aligned}$$

- Empirical literature [often referred as *return predictability literature*] suggests that a model with constant risk premia is not good at explaining stock market volatility.

- The standard model derives all volatility from changing dividend growth (and long-run consumption growth) and real interest rates. But long-run forecasts of these variable are more or less constant (in the short run they may vary).

- In the data excess stock return forecasts are seem to drive stock market volatility. So there appears to be need of models that imply changing risk premium - if the risk premium varies during the cycle excess returns will vary too and this might drive volatility

Tentative Solutions for Asset Pricing Puzzles:

- Change in preferences:
  - Long run risk models (Bansal and Yaron, 2004)
  - Habit Persistence (Campbell and Cochrane, 1999)
- Heterogeneous Agent Models (Constantinides and Duffie, 1996)
- Rare Events (Barro, 2005)

2.6 Long Run Risk

- For CRRA, σ measures simultaneously intertemporal substitution, risk aversion and precautionary savings.
- But risk aversion describes aversion to consumption substitution across states of nature and is meaningful even in an atemporal setting
- while EIS describes willingness to substitute consumption over time and is meaningful even in a deterministic setting.
- E-Z preferences break this link and maintain recursivity:

$$U_t = \left\{ (1-\beta)\, C_t^{\frac{1-\sigma}{\theta}} + \beta \left(E_t \left[U_{t+1}^{1-\sigma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\sigma}}$$

where $\theta := \frac{1-\sigma}{1-1/\epsilon}$, $\epsilon = EIS$, $\sigma = RRA$.

- E-Z (1989) derive the Euler equation (for equities) associated with this problem

$$E_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\epsilon} \right]^{\theta} R_{a,t+1}^{-(1-\theta)} R_{t+1}^{y} \right\} = 1$$

- R_a is the return on the market porfolio.

- Market porfolio is a hypothetical portfolio containing every security available to investors in a given market in amounts proportional to their market values.

- The **SDF** is

$$\hat{m}_{t+1} = -\frac{\theta}{\epsilon}\Delta\hat{c}_{t+1} + (\theta - 1)\hat{r}_{a,t+1}$$

- So the **risk free rate** is

$$r_{t+1} = \frac{\theta}{\epsilon}[E_t(\Delta\hat{c}_{t+1}) - \frac{\theta}{\epsilon}\frac{1}{2}var(\hat{c}_{t+1})] + (\theta - 1)\frac{1}{2}var(\hat{r}_{a,t+1})$$

- **Return on the market portfolio**:
- Eg: in a general equilibrium endowment setting, the market portfolio would yield consumption as dividend - that would be identical to the equity return. So, in this case $R_a = R^y$, and

$$E_t \left\{ \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\epsilon} R_{a,t+1} \right]^{\theta} \right\} = 1$$

$$\begin{aligned} E(\hat{r}_{a,t+1}) + \frac{1}{2} var(\hat{r}_{a,t+1}) &= \frac{\theta}{\epsilon} E(\Delta \hat{c}_{t+1}) - \left(\frac{\theta}{\epsilon} \right)^2 \frac{1}{2} var(\hat{c}_{t+1}) \\ &\quad + \frac{\theta}{\epsilon} \frac{1}{2} cov(\Delta \hat{c}_{t+1}, \hat{r}_{a,t+1}) + (1-\theta) \frac{1}{2} var(\hat{r}_{a,t+1}) \end{aligned}$$

- Special case - CCAPM ($\theta = 1$ and $\sigma = 1/\epsilon$)

- Thus, the risk premium on the market portfolio

$$E(\hat{r}_{a,t+1} - r_{t+1}) + \frac{1}{2} var(\hat{r}_{a,t+1}) = \frac{\theta}{\epsilon}\frac{1}{2} cov(\Delta \hat{c}_{t+1}, \hat{r}_{a,t+1}) + (1-\theta) var(\hat{r}_{a,t+1})$$

- We know that the term $cov(\Delta \hat{c}_{t+1}, r_{a,t+1})$ cannot account for the observed premium

- EZ impose restrictions that guarantees that investors explicitly fear adverse movements in expected growth and economic volatility. In particular they consider the case of $\sigma > 1/\epsilon$. Noting that

$$\theta = 1 - \frac{\sigma - 1/\epsilon}{1 - 1/\epsilon}$$

- $\theta < 1$, then premium augmented by $var(r_{a,t+1})$

- Problem 1) the contribution of the covariance term falls
- Problem 2) Risk free rate
- Problem 3) reverse engineering?

3 Barro's (2005) model of rare events

Mehra and Prescott (1985): for reasonable parameter values

- equity premium too low
- risk-free real rate too high
- Next period's growth rate can be *bad* or *good* – note the symmetry

Rietz (1988):

- can have reasonable parameters if we introduce a third *really bad* state

Barro (2005) version

$$U_t = \mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \frac{C_{t+j}^{1-\sigma}}{1-\sigma}$$

Endowment economy:

$$C_t = A_t$$

$$\log A_{t+1} = \log A_t + g + u_{t+1} + v_{t+1} + w_{t+1}$$

$g =$ growth-rate

$u_{t+1} =$ standard shock (normally distributed)

$v_{t+1} =$ rare negative event that contracts A_{t+1} by a proportion b:

$$v_{t+1} = \begin{cases} 0 & \text{probability} \quad e^{-p} \\ -b & \text{probability} \quad (1 - e^{-p}) \end{cases}$$

$w_{t+1} =$ end of the world ($A_{t+1} = 0$), occurs with frequency q

- Risk-free rate

$$r = -\log\beta + \sigma g - \frac{\sigma^2 \sigma_\varepsilon^2}{2} + q - \log\left(e^{-p} + (1 - e^{-p})e^{\theta b}\right) \qquad (9)$$

- Risk premium:

$$rp \approx \sigma\sigma_\varepsilon^2 + p(1 - e^{-b})(e^{\sigma b} - 1) \qquad (10)$$

- A rise in p or b
  - lowers the risk-free rate
  - raises the expected return on equity.

Framework became popular:

- Christian Julliard and Anisha Ghosh (2008) ‘Can rare events explain the equity premium puzzle?’ - answer No
- Jessica Wachter (2008) ‘Can time-varying risk of rare disasters explain aggregate stock market volatility?’ - answer Yes
- Xavier Gabaix and Emmanuel Farhi (2008) ‘Rare Disasters and Exchange Rates’ - answer Yes
- Application to term premium less successful (because the assumption that consumption growth is serially uncorrelated implies a term structure that is flat)

4 Heterogeneous Agent Models

Constantinides and Duffie (JPE, 1996): show how heterogeneity can affect asset prices - idiosyncratic risk rises in recession generating a high equity premium.

$$E_t \left\{ \beta R_{t+1} \left(\frac{C_{it+1}}{C_{it}} \right)^{-\sigma} \right\} = 1 = E_t \left\{ \beta R_{t+1} \left(\frac{\delta_{it+1} C_{t+1}}{\delta_{it} C_t} \right)^{-\sigma} \right\}$$

Depending on the properties of

$$\log \frac{C_{it+1}/C_{t+1}}{C_{it}/C_t} = \log \frac{\delta_{it+1}}{\delta_{it}}$$

we get countercyclical risk premia, and if the variance of consumption is high enough we can get a higher equity premium.

- But have to introduce some market incompleteness in order to allow for idiosyncratic risk
- Lucas (1990), Ketterer & Marcet (1989) and Marcet & Singleton(1990), and Telmer (1992) have posed models with two agents where individual endowment processes are not perfectly correlated.
- They explore the nature of the equilibrium allocations and prices under various market structures that include trading on some assets and find that they are very similar to the ones obtained in complete market settings.

5 Habit formation

Habit formation means that today's consumption influences tomorrow's marginal utility of consumption because a habit is formed. Let C_t be today's consumption and X_t the time-varying habit level.

- In general, the utility function is $U(C_t, X_t)$, with $U_{CX} > 0$.
- Two modelling devices have been used, either a CRRA utility function of C_t/X_t or of $C_t - X_t$.
- X_t is either 'internal habit', i.e., depends on the individual's consumption or "catching up with the Joneses" in which X_t is per capita aggregate consumption.
- Current utility could react fast to changes in consumption, e.g., $X_t = hC_{t-1}$, or it could react gradually, with X_t a weighted average of past consumption levels.

- As has been demonstrated by Campbell and Cochrane (1999) in the context of endowment economies, the equity premium puzzle can be reconciled by a modification of the utility function.
- Why?
- 1) It increases risk aversion
- 2) risk aversion becomes state-contingent
- Higher curvature in the utility -> more risk aversion.
- Coefficient of relative risk aversion ->sensitivity (or elasticity) in this curvature to changes in consumption

$$-\frac{\partial Uc}{\partial C}\frac{C}{U_C} \equiv -\frac{CU_{CC}}{U_C}$$

- Habits:

$$U_t = \sum_{j=0}^{\infty} \beta^j \frac{\left(C_{t+j} - X_{t+j}\right)^{1-\sigma} - 1}{1-\sigma}$$

Define

$$S_t \equiv \frac{C_t - X_t}{C_t}$$

The local coefficient of relative risk aversion now is

$$\frac{-CU_{CC}}{U_C} = \frac{\sigma}{S_t},$$

so if S_t is low (the individual closer to his habit level) risk aversion is higher.

- Suppose $X_t = hC_{t-1}$, $h = 0.8$ and there is no growth. In steady state $\bar{S} = 0.2$.

- Even with $\sigma = 1$ consumption risk aversion is 5.

- Because agents actually dislike deviations of consumption from the habit level, for any given fluctuation in consumption, the equity risk premia will be higher.

- Help reconcile the equity premium puzzle

5.1 Campbell and Cochrane (1999) specification

Suppose that log consumption follows a random walk

$$\Delta c_{t+1} = g + \varepsilon_{t+1}$$

and that the habit evolves slowly over time, but S_t always remains positive. In logs (not log deviations from steady state):

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t)\varepsilon_{t+1}$$

The sensitivity of habits to consumption shocks is variable. Now note

$$M_{t+1} = \left(\beta \frac{U'(C_{t+1})}{U'(C_t)}\right) = \beta \left(\frac{S_{t+1}C_{t+1}}{S_t C_t}\right)^{-\sigma}.$$

Calculate partials, take logs, use the random walk on log consumption and the generating process for s_t to get the safe rate:

$$r_{t+1} = -\ln\beta + \gamma g - \sigma(1-\phi)(s_t - \bar{s}) - \frac{\sigma^2\sigma_c^2}{2}[\lambda(s_t)+1]^2$$

- the third term reflects intertemporal substitution and the last term precautionary savings. I
- Intertemporal substitution is governed by mean reversion in marginal utility, due to the mean reversion of s.
- This equation can be made to match the safe rate (and so avoid the risk-free interest rate puzzle) by having a ϕ close to 1, and $\lambda'(s) < 0$.
- Campbell and Cochrane choose $\lambda(s)$ to satisfy: a constant risk-free rate.
- They choose $\sigma = 2$, but get that $\bar{S} = 0.057$, i.e., 94% of consumption becomes habit. Then the effective risk aversion coefficient is $\sigma/\bar{S} = 2/.057 = 35$. This is what explains the equity premium.
- This explanation is achieved without causing a riskless rate puzzle.

The riskless rate in steady state

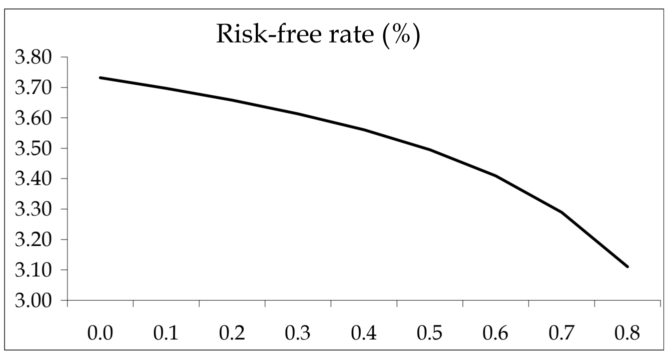
$$r = -\ln\beta + \sigma g - \left(\frac{\sigma}{\bar{S}}\right)^2 \frac{\sigma_c^2}{2}$$

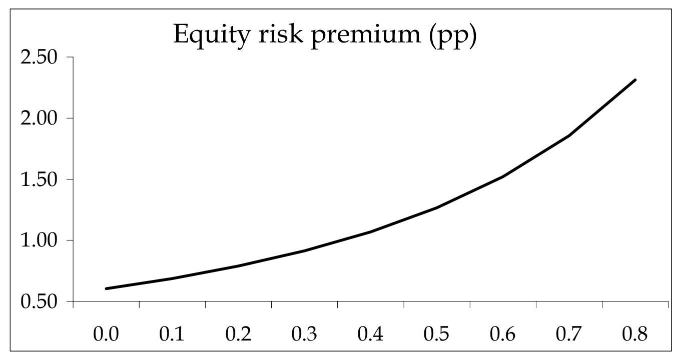
The important result is that in the second term, with $+$ in front, the coefficient is σ, whereas in the third, with $-$ in front, it is the much bigger $\sigma/\bar{S}$. So a low S can justify the low riskless rate.

- In De Paoli, Scott and Weeken (2007) (DPSW hereafter), in a model which assumes

$$U_t = \sum_{j=0}^{\infty} \beta^j \frac{\left(C_{t+j} - X_{t+j}\right)^{1-\sigma} - 1}{1 - \sigma}$$

and $X_t = hC_{t-1}$, varying h we get (rates are annualized):





$$\frac{-CU_{CC}}{U_C} = \frac{\sigma}{S_t},$$

- Also - risk aversion becomes state-contingent: agents are more risk averse during recessions than during booms.
- Potentially (not necessarily) this can introduce time-variation in the risk premium
- Solving the return volatility problem

- Problems with this solution:
- 1) habits by itself might just reduce consumption volatility -$>$ reducing premia
- 2) time variation in risk aversion might not be sufficient -$>$ the specification of the process for X_t (and its implication for S_t and $\lambda(s_t)$)is crucial!