Slides 3b: Macrofinance - cont.

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1 Habit formation and time-variation in the risk premium

- Risk aversion in a model with habits varies over time
- $$\frac{-Cu_{CC}}{u_C} = \frac{\sigma}{S_t},$$
- So excess equity returns may vary over time.
- But can our solution method capture such fluctuations?

Homework:

Consider the following model:

$$M_{t+1} = \beta \left(\frac{S_{t+1} C_{t+1}}{S_t C_t} \right)^{-\sigma} = \beta \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma}$$

where consumption follows an AR(1) process (i.e. we have an endowment economy):

$$c_{t+1} = (1-\phi)\bar{c} + \phi c_t + \varepsilon_{t+1}$$

a) Using second-order perthubation methods, compute the average equity risk premium (unconditional mean or stochastic steady state in Dynare) when $\beta = 0.99$, $\phi = 0.9$, $\sigma = 2$, $h = 0.8$ and a $\sigma_\varepsilon = 0.01$.

b) Compute the dynamics of the equity risk premium following a shock.

c) Analytically show that a second-order approximation to the risk premium can be written as

$$E_t(\hat{r}^y_{t+1} - \hat{r}_{t+1}) + \frac{1}{2}var_t(\hat{r}^y_{t+1})^2 = \frac{\sigma}{(1-h)}cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + o(\|\widehat{\xi}\|^3)$$

where $o(\|\widehat{\xi}\|^3)$ *represents terms of order higher than 2.*

- Therefore, if we consider a homoskedastic stochastic process, the risk premium is **constant** and depends on σ, h, $var(r^y)$ and $cov(c, r^r)$.

c) If instead of using a second-order approximation we use a "log-normal approximation" would we obtain a constant risk premium?

- So in order to get time varying risk premium using perthubation techniques, one need to go beyond second-order.

- But if the solution method allows for this time variation, what determines its cyclical properties?

- In a habits model, risk aversion is countercyclical. In the data, risk premium is countercyclical. So can habits explain the cyclical properties of risk premia?

- (Following De Paoli and Zabczyk (2008) - DPZ hereafter), let excess consumption C_t^e and the surplus ratio S_t be defined as

$$C_t^e := C_t - hX_t$$

and

$$S_t := \frac{C_t - hX_t}{C_t} = \frac{C_t^e}{C_t}.$$

- The coefficient of consumption risk aversion is countercyclical. To see this, note that

$$\eta_t = \sigma \frac{C_t}{(C_t - hX_t)}$$

and so:

$$\frac{\partial \eta_t}{\partial C_t} = -\sigma \frac{hX_t}{(C_t - hX_t)^2} \leq 0.$$

- A second order approximation to the first order conditions implies

$$rp_t \approx \sigma\, cov_t(\hat{c}^e_{t+1}, \hat{r}^r_{t+1}).$$

- The risk premium is proportional to the *excess consumption* relative risk aversion coefficient σ and the conditional covariance of returns $\hat{r}^y_{t+1}$ with excess consumption $\hat{c}^e_{t+1}$.

- Stein's lemma postulates that, if X and Y are jointly normally distributed, then

$$cov(g(X), Y) = E[g'(X)]cov(X, Y).$$

- Therefore, under the assumption that $\hat{c}_{t+1}$ and $\hat{r}^y_{t+1}$ are jointly normally distributed, we can express $cov_t(c^e_{t+1}, r^r_{t+1})$ as

$$cov_t(\hat{c}^e_{t+1}, r^r_{t+1}) = \kappa_t \, cov_t \left(\hat{c}_{t+1}, \hat{r}^y_{t+1}\right)$$

where

$$\kappa_t \equiv E_t(\partial \hat{c}^e_{t+1} / \partial \hat{c}_{t+1}) = E_t(1/S_{t+1}).$$

- Agents' expectations about the surplus ratio matter because they affect the covariance of excess consumption and returns.

- So, the risk premium can be written as

$$rp_t \approx \eta_t \, cov_t(\hat{c}_{t+1}, \hat{r}^r_{t+1}) \, E_t \frac{S_t}{S_{t+1}}.$$

- The risk premium is determined by the coefficient of risk aversion η_t, the covariance of consumption and returns as well as expectations about the growth of the surplus ratio.

- Importantly, if agents' expectations of the future improve following a bad shock (or deteriorate following a good shock) then the risk premium can be pro-cyclical even though the risk aversion coefficient is countercyclical.

- Assuming:

$$X_t := (1-\phi)C_{t-1} + \phi X_{t-1} \tag{1}$$

$$\text{and } \hat{c}_t = \rho \hat{c}_{t-1} + \varepsilon_t. \tag{2}$$

- DPZ shows that in order to have a countercyclical premium one needs persistent shocks and slow-moving habits

$$\partial \kappa_t / \partial \varepsilon_t = E_t S_{t+1}^{-2} C_{t+1}^{-1} h(1-\phi) \left(C_t - \rho \sum_{s=0}^{+\infty} \phi^s C_{t-s} \right). \tag{3}$$

- consider a bad shock which pushes down the level of consumption -> If the shock is temporary and households very quickly change their habits, then next period they will be used to a lower level of consumption, while actual consumption will tend to revert back to its previous (higher) level.

- Hence, households hit by the negative shock have every reason to expect consumption next period to be high relative to the benchmark.

- Accordingly, even though risk aversion increases as a result of the bad shock, prospects of *good times* ahead make agents take on more risk and actually lead to a compression of premia.

- This is why temporary shocks and quickly adjusting habits translate into procyclical risk premia.

- Importance of persistent shocks and slow-moving habits -> both assumptions are in C&C 1999 - so their model does a good job matching observed variations in excess returns

1.1 Production Based Asset Pricing

Jermann (1998).

Homework:

a) compute the equility risk premium in the basic RBC model of Lecture 1

b) add consumption habits. What are the implications for the equity risk premium?

c) now add capital adjustment costs. Compute the equity risk premium and the volatitlity of the risk free rate.

d) now introduce endogenous labour markets.

- Compared with an endowment economy, a model with capital decreases the premium because now agents can adjust their capital stock to smooth consumption, decreasing the volatility of the stochastic discount factor

- With habit formation and no adjustment costs, the equity premium is even lower than in the no habit case because agents smooth consumption even more now and therefore the equity premium is even lower.

- But with adjustment costs and habit formation reasonable equity premia can be generated from the model. That it is not enough to have agents disliking consumption fluctuations, you need to prevent them from doing something about it

- So, when you introduce labor market flexibility, the equity risk premium also drops as agents can smooth consumption by adjusting their labor supply.

- Qualitatively, you should get a picture similar to Table D in DPSW

	Base case	No real rigidities	No capital adjustment costs	No labour habits	No consumption habits
χ^P	0	0	0	0	0
χ^C	0.82	0	0.82	0.82	0
χ^N	0.82	0	0.82	0	0.82
χ^K	0.3	30,000	30,000	0.30	0.30
ERP	2.43	0.03	0.04	0.58	0.60

(Note that labor habits introduce rigidities in the endogenous labor market)

1.2 Real yield curve

- we can derive a second-order approximation to the above equations. The FOC with respect to real bonds with maturity j implies

$$R^{br}_{j,t+1} = \left(E_t \left[\beta \frac{\Lambda_{t+j}}{\Lambda_t} \right] \right)^{-\frac{1}{j}} .$$

Approximating the above equation to second order and expressing the variables in log deviations from steady state, we have:

$$E\left[\hat{r}^{br}_{j,t+1}\right] \simeq -\frac{1}{j}\left(E\left[\hat{m}_{t,t+j}\right] + \frac{1}{2} var(\hat{m}_{t,t+j}) \right)$$

where $M_{t,t+j} \equiv \beta \frac{\Lambda_{t+j}}{\Lambda_t}$

- *Homework: show that in an endowment economy with a standard CRRA utility for any maturity the stochastic mean real yield will always be below the deterministic real yield*

- We can then examine the real term premium, the difference between the return on a longer-term real bond and the one-period real bond. The average yield spread between real bond of maturity j and one-period real bond is, therefore,

$$E\left[\hat{r}^{br}_{j,t+1} - \hat{r}^{br}_{1,t+1}\right] \simeq \frac{1}{2}\left(var(\hat{m}_{t+1}) - \frac{var(\hat{m}_{t,t+j})}{j}\right)$$

- Whether the real yield curve is upward or downward sloping will depend on whether the term on the right-hand side is positive or negative. If the growth rate of marginal utility is positively autocorrelated, such that the numerator $var(\hat{m}_{t,t+j})$ rises faster than j, then the yield curve is downward sloping.

- the profile of the term structure depends on whether uncertainty about future marginal utility (and hence the precautionary savings motive) is proportionally larger or smaller as maturity increases.

- To explore this further, consider first what would happen in the case where there are no consumption habits, so that marginal utility is a function of the level of consumption. If consumption growth is positively correlated, shocks in the growth rate are persistent. Uncertainty about levels of consumption grows rapidly, more rapidly than the denominator in the above equation, the maturity of the bond. This implies a downward-sloping real term structure – real long bonds are regarded as insurance, and carry a negative term premium.

- *Homework: prove the above statement for the term premium in a two period bond*

- This feature of the standard neoclassical growth model has been noted by den Haan (1995) and Lettau (2003), and this implication of positively correlated consumption growth (as found in the data) is incompatible with upward-sloping real and nominal term structures.

- That is, if a ‘bad’ shock is expected to be followed by other bad events, risk-averse investors appreciate locking-in today a given return in the future, and therefore longer-term bonds serve as a form of insurance. This points us to examine the autocorrelation of impulse responses of the stochastic discount factor.