Slides 4: The New Keynesian Model (following Galí 2007)

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November 2009

1 The New Keynesian Model

- widely used for monetary policy analysis
- framework that can help us understand the links between monetary policy and the aggregate performance of an economy:
  - understand how interest rate decisions end up affecting the various measures of an economy's performance, i.e. the transmission mechanism of monetary policy
  - understand what should be the objectives of monetary policy and how the latter should be conducted in order to attain those objectives

- RBC model - perfect competition, frictionless markets and no monetary sector
- Classical monetary model - money is neutral and the Friedman rule is optimal
  - Money neutrality: monetary policy (ie changes in money supply or nominal interest rates) have no effect on real variables.
  - Friedman rule: the opportunity cost of holding money faced by private agents should equal the social cost of creating additional fiat money$\Rightarrow$Thus nominal rates of interest should be zero.
  - In practice, this means that the central bank should seek a rate of deflation equal to the real interest rate on government bonds and other safe assets, in order to make the nominal interest rate zero.

- But at odds with
  - empirical evidence (money has short run real effects)
  - and monetary policy practice (rates are not kept at zero)
- The canonical New Keynesian model: RBC model, usually abstracting from capital accumulation, with some non-classical features

1.1 Key elements in a New Keynesian model

- **Monopolistic competition**: The prices of goods and inputs are set by private economic agents in order to maximize their objectives, as opposed to being determined by an anonymous Walrasian auctioneer seeking to clear all (competitive) markets at once.

- **Nominal rigidities**: Firms are subject to some constraints on the frequency with which they can adjust the prices of the goods and services they sell (*Calvo price setting*). Alternatively, firms may face some costs of adjusting those prices (*Rotemberg price setting*). The same kind of friction applies to workers in the presence of sticky wages.

- **Short run non-neutrality of monetary policy**: As a consequence of the presence of nominal rigidities, changes in short term nominal interest rates (whether chosen directly by the central bank or induced by changes in the money supply) lead to variations in real interest rates (given that real money balances and expected inflation do not move proportionally).

- The latter bring about changes in consumption and investment
- and since firms find it optimal to adjust the quantity of goods supplied to the new level of demand, output and employment also change
- In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium.

- We will use Gali's book and lecture notes
- First, let's revise the classical model with and without money in the utility
- Then introduce monopolistic competition and nominal rigidities - the canonical NK model

1.2 Classical model - Recalling the RBC model but abstracting from capital

Households

Representative household solves

$$\max U(C_t; N_t)$$

or

$$\max U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right],$$

subject to

$$C_t P_t + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

Optimality conditions

1. Labour leisure decision

$$-\frac{U_n(C_t; N_t)}{U_c(C_t; N_t)} = \frac{W_t}{P_t}$$

2. Intertemporal decision

$$Q_t = \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right)$$

Or in log-linear terms, given the specific utility function (here we adopt Gali's notation that lower case variables are logs and variables with hats are log deviations from steady state)

$$w_t - p_t = \sigma c_t + \varphi n_t$$

and

$$c_t = E_t c_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho)$$

where $i_t \equiv -\log Q_t$ is the nominal interest rate and $\rho \equiv -\log \beta$ is the discount rate (and the steady state value of real interest rates)

If we want to include money in our model we can add an ad-hoc money demand equation

$$m_t - p_t = y_t - \eta i_t$$

Firms

- Profit maximization:

$$\max P_t Y_t - W_t N_t$$

subject to Technology

$$Y_t = A_t N_t^{1-\alpha}$$

taking the price and wage as given (perfect competition)

- Optimality condition:

$$\frac{W_t}{P_t} = MPN_t = (1-\alpha)A_t N_t^{-\alpha}$$

- In log-linear terms

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Equilibrium

- Goods market clearing

$$y_t = c_t$$

- Labor market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha)$$

- Asset market clearing:

$$y_t = E_t y_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho)$$

- Aggregate production relationship:

$$y_t = a_t + (1 - \alpha) n_t$$

- Implied equilibrium values for real variables

$$n_t = \psi_{na} a_t + \delta_n$$

$$y_t = \psi_{ya} a_t + \delta_y$$

$$r_t \equiv i_t - E_t \pi_{t+1} = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1}$$

$$\omega_t \equiv w_t - p_t = \psi_{wa} E_t \Delta a_{t+1} + \log(1-\alpha)$$

where

$\psi_{ya} = \frac{\varphi+1}{\sigma+\varphi+\alpha(1-\sigma)}$; $\psi_{na} = \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$; $\psi_{wa} = \frac{\varphi+\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$

and

$\delta_y = \frac{(1-\alpha)\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$; $\delta_n = \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$

$$
\begin{aligned}
n_t &= \psi_{na} a_t + \delta_n \\
y_t &= \psi_{ya} a_t + \delta_y \\
r_t &= \rho + \sigma \psi_{ya} E_t \Delta a_{t+1} \\
\omega_t &= \psi_{wa} E_t \Delta a_{t+1} + \log(1 - \alpha)
\end{aligned}
$$

- real variables determined independently of monetary policy (neutrality)
- optimal policy: indeterminate.
- specification of monetary policy needed to determine nominal variables

A Model with Money in the Utility Function

- Preferences

$$\max U(C_t; \frac{M_t}{P_t}; N_t)$$

- Budget constraint

$$C_t P_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t - T_t$$

or

$$C_t P_t + Q_t \mathcal{A}_{t+1} + (1 - Q_t) M_t \leq \mathcal{A}_t + W_t N_t - T_t$$

- where $\mathcal{A}_t = B_{t-1} + M_{t-1}$

$$1 - Q_t \simeq 1 - \exp(-i_t) \simeq i_t$$

- Interpretation: $i_t \Rightarrow$ opportunity cost of holding money

- Optimality conditions
- Apart from the ones previously derived, we can derive the endogenous money demand:

$$\frac{U_m}{U_c} = 1 - Q_t \simeq i_t$$

- Two cases:
  - utility separable in real balances $=>$ neutrality
  - utility non-separable in real balances (e.g. $U_{cm} > 0$) $=>$ non-neutrality [But in these models, short run non-neutrality is often accompanied by a contra-factual long-run non-neutrality]

Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

Social Planner's problem

$$\max U(C_t; \frac{M_t}{P_t}; N_t)$$

subject to technology and market clearing

$$Y_t = A_t N_t^{1-\alpha}$$

$$Y_t = C_t$$

- FOCs

$$\frac{U_n}{U_c} = MPN_t = (1-\alpha)A_t N_t^{-\alpha}$$

and

$$U_m = 0$$

- No constraint on the supply of money
- In the decentralized equilibrium

$$\frac{U_m}{U_c} = 1 - \exp(-i_t)$$

- Optimal policy (Friedman rule):

$$i_t = 0 \ \forall t.$$

$$i_t = 0 \ \forall t.$$

- Intuition: marginal cost of printing money zero, so that should be its opportunity costs
- Implied average inflation:

$$\pi = -\rho < 0$$

- optimal policy should produce deflation

Evidence on Money, Output, and Prices:

- The Long Run - money neutrality
- Short Run Effects of Monetary Policy Shocks
  - (i) persistent effects on real variables
  - (ii) slow adjustment of aggregate price level
  - (iii) liquidity effect (M2 displays a persistent decline in the face of the rise in the federal funds rate. Fed needs to reduce the amount of money in circulation to bring about an increase in nominal rate)
- Micro Evidence on Price-setting Behavior: significant price and wage rigidities (median duration of 3 to 4 quarters)

$\Rightarrow$*Failure of Classical Monetary Models*

- A Baseline Model with Nominal Rigidities
  - monopolistic competition
  - sticky prices (staggered price setting)
  - competitive labor markets, closed economy, no capital accumulation

Households

$$\max U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right],$$

where:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

(need to assume that firms produce differentiated products in order to characterize monopolistic competitive markets -> which in turns are necessary to have firms being price setters)

subject to

$$\int_0^1 C_t(j)\, P_t(j)\, dj + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

Optimality conditions

1. Optimal allocation of expenditures

$$\max C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$

s.t.

$$\int_0^1 P_t(j) C_t(j) = Z_t$$

The first order condition is

$$C_t(j) = (\lambda_t P_t(j))^{-\varepsilon} C_t$$

where λ_t is the Lagrange multiplier associated with the constraint.

Substituting in the definition of the consumption index

$$\lambda_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{-1}{1-\varepsilon}}$$

So, if we define the price level such that

$$\int_0^1 P(j) C\,(j)\, dj = P_t C_t$$

we have

$$\int_0^1 P(j) C_t\,(j) = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj \right]^{\frac{1}{1-\varepsilon}} C_t = P_t C_t$$

So

$$P_t = \left[\int_0^1 P_t(j)^{1-\varepsilon} dj\right]^{\frac{1}{1-\varepsilon}} = \lambda_t^{-1}$$

Therefore, the demand equation is

$$C_t\,(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t$$

Other optimality conditions

2. Labour leisure decision

$$-\frac{U_n(C_t;N_t)}{U_c(C_t;N_t)} = \frac{W_t}{P_t}$$

3. Intertemporal decision

$$Q_t = \beta E_t \left(\frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right)$$

Or in log-linear terms

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t \\ c_t &= E_t c_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho) \end{aligned}$$

Again, if we want to include money in our model we can add an ad-hoc money demand equation

$$m_t - p_t = y_t - \eta i_t$$

(necessary?! this equation pins down the relationship between money and interest rates, but if the central bank controls the later directly...)

Firms

- Continuum of firms, indexed by i
- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

- Probability of being able to reset price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- θ [0; 1] : index of price stickiness
- Implied average price duration $1/(1 - \theta)$

Optimal Price Setting

A firm re-optimizing in period t will choose the price P^* that maximizes the current market value of the profits generated while that price remains effective. Probability that this price will be effected at period k is θ^k

$$\max \sum_k E_t \theta^k Q_{t.t+k} \left[P_t^* Y_{t+k,t} - \Psi(Y_{t+k,t}) \right] = 0$$

where:

- $Q_{t.t+k}$ is the stochastic discount factor, given by $\beta^k \frac{U_{c,t+k}}{U_{c,t}} \frac{P_t}{P_{t+k}}$ (households own firms, and discount given their rate of marginal utility. When marginal utility in a given period is high relative to today, future profits are more valuable in utility terms, so firms are more patient)

- $Y_{t+k,t}(j) = \left(\frac{P_t^*)}{P_{t+k}}\right)^{-\varepsilon} C_{t+k}$ (monopolistic competitive firm knows the form of it's demand)

- $\Psi(Y_{t+k,t})$ is the total cost (which in this case is given by $W_t N_t(i)$)

Optimality condition

$$\sum_k E_t \theta^k Q_{t.t+k} Y_{t+k,t} \left[P_t^* - \mathcal{M} \psi_{t+k,t} \right] = 0$$

where:

- $\mathcal{M} = \frac{\varepsilon}{1-\varepsilon}$ and
- $\psi_{t+k,t} = \boldsymbol{\Psi}'_{t+k,t}$ is the marginal cost at period $t+k$ of firms that change their price at period t

$$\psi_{t+k,t} = W_{t+k} \left(Y_{t+k,t} \right)^{\alpha/1-\alpha} \left(A_{t+k} \right)^{-1/1-\alpha}$$

- Note: average marginal cost is

$$\psi_{t+k} = W_{t+k} \left(Y_{t+k}\right)^{\alpha/(1-\alpha)} \left(A_{t+k}\right)^{-1/(1-\alpha)}$$

So, given that

$$Y_{t+k,t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon} Y_{t+k}$$

we have

$$\psi_{t+k,t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon\alpha/(1-\alpha)} \psi_{t+k}$$

The log-linear pricing equation:

Given that in steady state $P^* = P = \psi\mathcal{M}$ and $Q_k = \beta^k$, a Taylor expansion of the pricing equations implies

$$\sum_k YE_t(\theta\beta)^k \left[p_t^* - \log(\psi_{t+k,t}) - \mu\right] = 0$$

where $\mu \equiv \log(\mathcal{M})$. Alternatively, we can express it in terms of the average marginal cost

$$\sum_k YE_t(\theta\beta)^k \left[p_t^* - \Theta(\log(\psi_{t+k}) - p_{t+k}) - p_{t+k} - \Theta\mu\right] = 0$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

Therefore, defining the real marginal cost as $mc_t \equiv \log(\psi_t) - p_t$ and given that in steady state $mc = -\mu$

$$p_t^* = (1 - \beta\theta) \sum_k E_t(\theta\beta)^k \left[\Theta \widehat{mc}_{t+k} + p_{t+k}\right]$$

or

$$p_t^* - p_t = (1 - \beta\theta)\Theta\widehat{mc}_t + \theta\beta E_t(p_{t+1}^* - p_t)$$

Aggregate price dynamics

$$P_t = \left[\theta \left(P_{t-1}\right)^{1-\varepsilon} + (1-\theta)\, P_t^{*1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$$

Defining $\pi \equiv \log P_t / P_{t-1}$ and log-linerizing around the zero-inflation steady state

$$\pi_t = (1-\theta)\left(p_t^* - p_{t-1}\right)$$

Turning again to the pricing equation we can write

$$\pi_t = \lambda \widehat{mc}_t + \beta E_t \pi_{t+1}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}\Theta$

The log linear Phillips curve

In order to write the above pricing equation in terms of output, we need to derive some conditions for the real marginal cost and market clearing

- Goods market clearing

$$y_t = c_t$$

- Labour market clearing

$$
\begin{aligned}
N_t &= \int_0^1 N_t(j)\,dj = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{1/(1-\alpha)} dj \\
&= \left(\frac{Y_t}{A_t}\right)^{1/(1-\alpha)} \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon/(1-\alpha)} dj
\end{aligned}
$$

or given that to a first order approximation the price dispersion term $\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon/(1-\alpha)} dj$, is eliminated (See Gali, ch.3 Appendix)

$$
(1-\alpha)n_t = y_t - a_t
$$

- The real marginal cost

$$MC_t = \frac{W_t}{P_t} \frac{Y_t^{\alpha/(1-\alpha)}}{(1-\alpha) A_t^{1/(1-\alpha)}}$$

or

$$mc_t = w_t - p_t + (1-\alpha)^{-1}(\alpha y_t - a_t) - \log(1-\alpha)$$

And recalling the labour leisure decision:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

and market clearing, we can write

$$mc_t = \left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t - \left(\frac{\varphi + 1}{1-\alpha}\right) a_t - \log(1-\alpha)$$

The log linear Phillips curve

$$\lambda^{-1}\pi_t = \left[\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right) y_t - \left(\frac{\varphi+1}{1-\alpha}\right) a_t - \log(1-\alpha) + \mu\right] + \lambda^{-1}\beta E_t \pi_{t+1}$$

With flexible prices $\lambda^{-1} \to 0$ (denoted with a superscript n)

$$y_t^n = \psi_{ya} a_t - \delta_y$$

where $\psi_{ya} = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)^{-1} \left(\frac{\varphi+1}{1-\alpha}\right)$ and $\delta_y = \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)^{-1} (\mu - \log(1-\alpha))$

- So, we can write the Phillips curve in terms of the output gap $\widetilde{y}_t = y_t - y_t^n$

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

where $\kappa = \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$

Exercise: Derive PC under indexation - Problem 3.4 in Gali

- for simplicity, assume $\omega = 1$
- hint 1: modify P_t^* accordingly in the maximization problem

$$\max \sum_k E_t \theta^k Q_{t.t+k} \left[P_t^* Y_{t+k,t} - \Psi(Y_{t+k,t}) \right] = 0$$

- hint 2: use linearization around steady state

The system of equilibrium conditions

1)PC curve

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

2)IS curve

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n)$$

where the natural rate of interest is defined as $r_t^n = \rho + \sigma \psi_{ya} E_t \Delta a_{t+1}$

3)A monetary policy rule!

1.3 Equilibrium under a Simple Interest Rate Rule

Policy rule 1 (PR1)

$$i_t = \rho + \phi_y \widetilde{y}_t + \phi_\pi \pi_t + v_t$$

- where v_t is exogenous monetary policy shock with zero mean.
- Equilibrium Dynamics: combining (PC), (IS), and (PR1)Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = A_o \begin{bmatrix} E_t \widetilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix} + B_o(\widehat{r}_t^n - v_t)$$

where

$$A_o = \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}; \text{ and } B_o = \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

where $\Omega = \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

- Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

- System is determine is the eigenvalues of A_o are inside the unity circle

- Eigenvalues satisfy:

$$\det |A_o - \lambda I|$$

or

$$\lambda_1 + \lambda_2 = \mathrm{tr}|A_o|$$

and

$$\lambda_1 \lambda_2 = \det |A_o|$$

- So, the solution can be obtained by constructing

$$\lambda^2 - \mathrm{tr}|A_o|\lambda + \det |A_o| = 0$$

- Moreover, if the characteristic polynomials satisfy $\lambda^2 + a_1\lambda + a_0 = 0$, then the eigenvalue are inside the unit circle if

$$|a_0| < 1$$

and

$$|a_1| < 1 + a_0$$

- So in this case, the conditions are

$$|\det|A_o|| < 1$$

and

$$|-\mathrm{tr}|A_o|| < 1 + \det|A_o|$$

- Determinant

$$\det|A_o| = \beta\frac{\sigma}{\sigma+\phi_y+\kappa\phi_\pi}$$

which is clearly between 0 and 1 if both coefficients ϕ_y and ϕ_π are positive

- Trace

$$\mathrm{tr}|A_o| = \frac{\sigma+\kappa+\beta(\sigma+\phi_y)}{\sigma+\phi_y+\kappa\phi_\pi}$$

is clearly positive if if both coefficients are positive.

- So the relevant condition is

$$\frac{\sigma + \kappa + \beta(\sigma + \phi_y)}{\sigma + \phi_y + \kappa\phi_\pi} < 1 + \frac{\beta\sigma}{\sigma + \phi_y + \kappa\phi_\pi}$$

or

$$\frac{\sigma + \kappa + \beta\phi_y}{\sigma + \phi_y + \kappa\phi_\pi} < 1$$

or

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

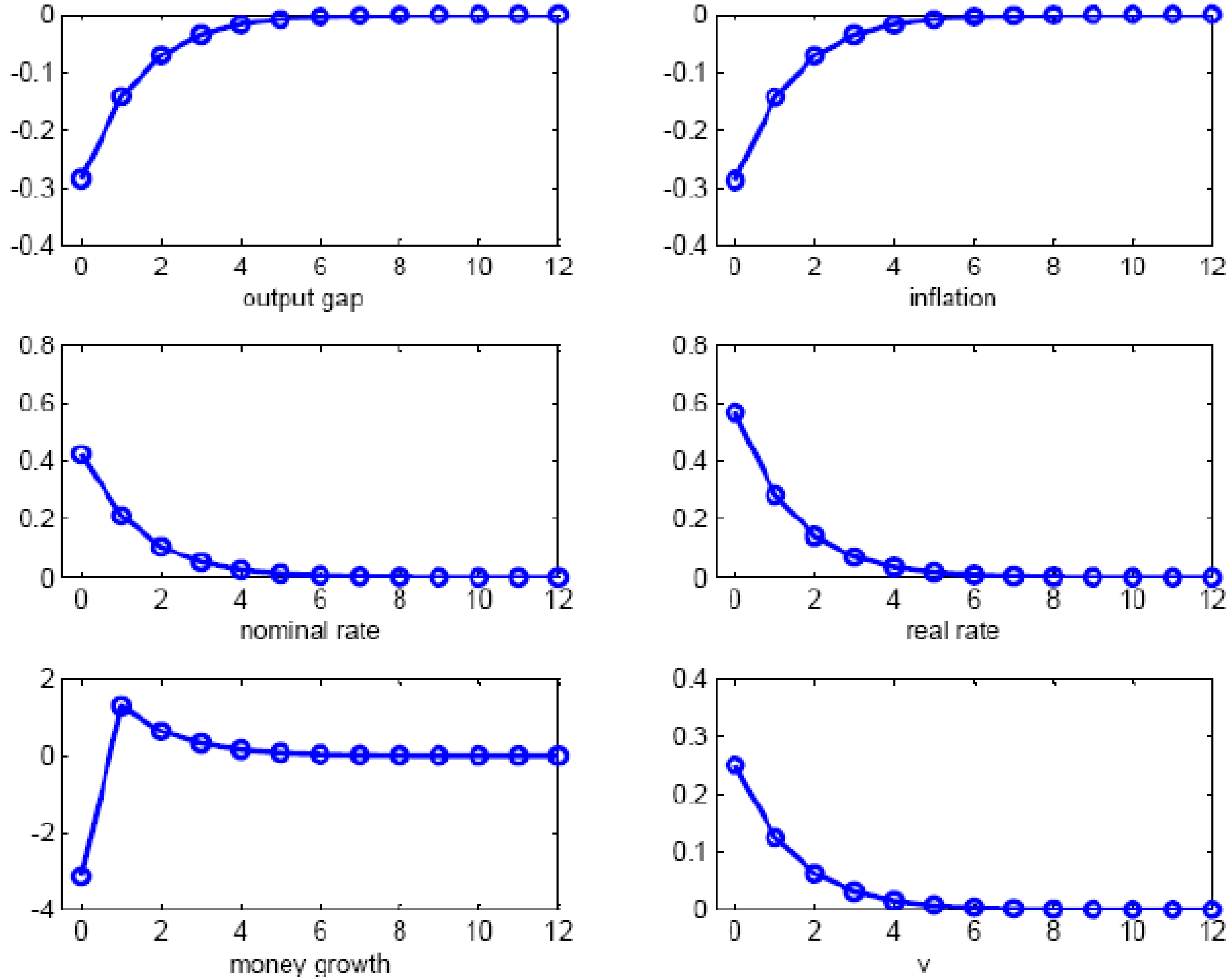
- (this is a sufficient condition if both coefficients are positive)
- E.g. $\phi_y = 0 => \phi_\pi > 1$
- Economics: real interest rate has to increase after shock in order to (further explanation later) - Taylor principle

1.4 Effect of Monetary Policy shock

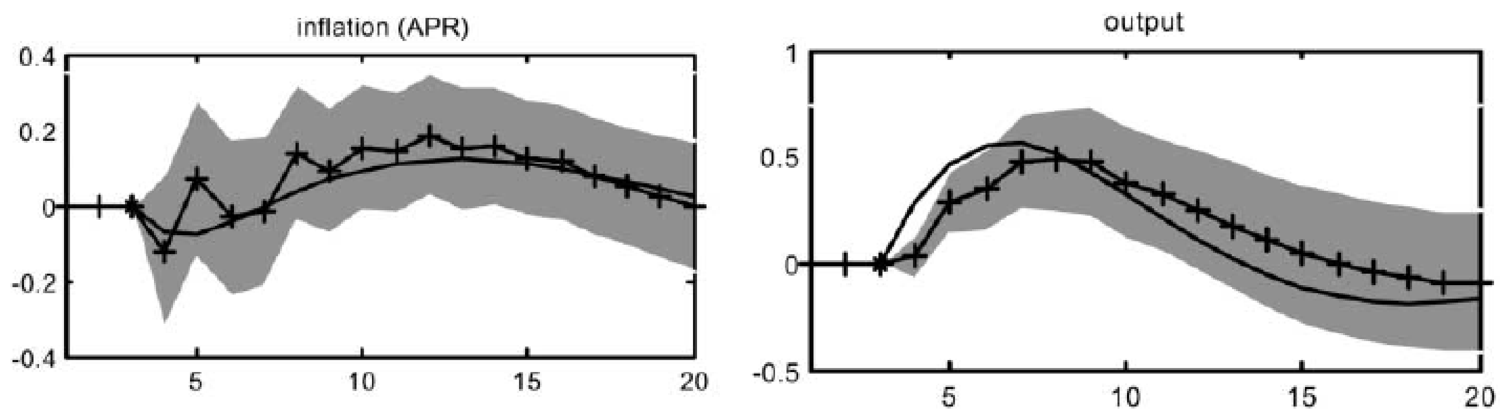
Shock process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

- if prices were flexible, the increase in nominal rate, which tend to put downward pressure on demand, would be accompanied by a fall in prices and an increase expected inflation, leaving real rates unchanged. As a result real activity (and real money balances) would be unchanged (Classical vertical aggregate supply)
- But given that prices do not adjust, the fall in demand will be accompanied by a fall in activity (NK upward sloping aggregate supply)


output gap
inflation
nominal rate
real rate
money growth
v

- A simple New Keynesian Model does not generate enough persistence.
- State-of-art New Keynesian model: **Christiano, Eichenbaum, Evans (JPE, 2005)**
- Reason: real marginal cost responds quickly to Monetary Policy
- CEE add some assumptions that prevents mc from responding quickly
  - nominal wage stickiness
  - variable capital utilization
- Other new assumptions
  - habit formation in consumption
  - wage and price indexation



CEE (2005)

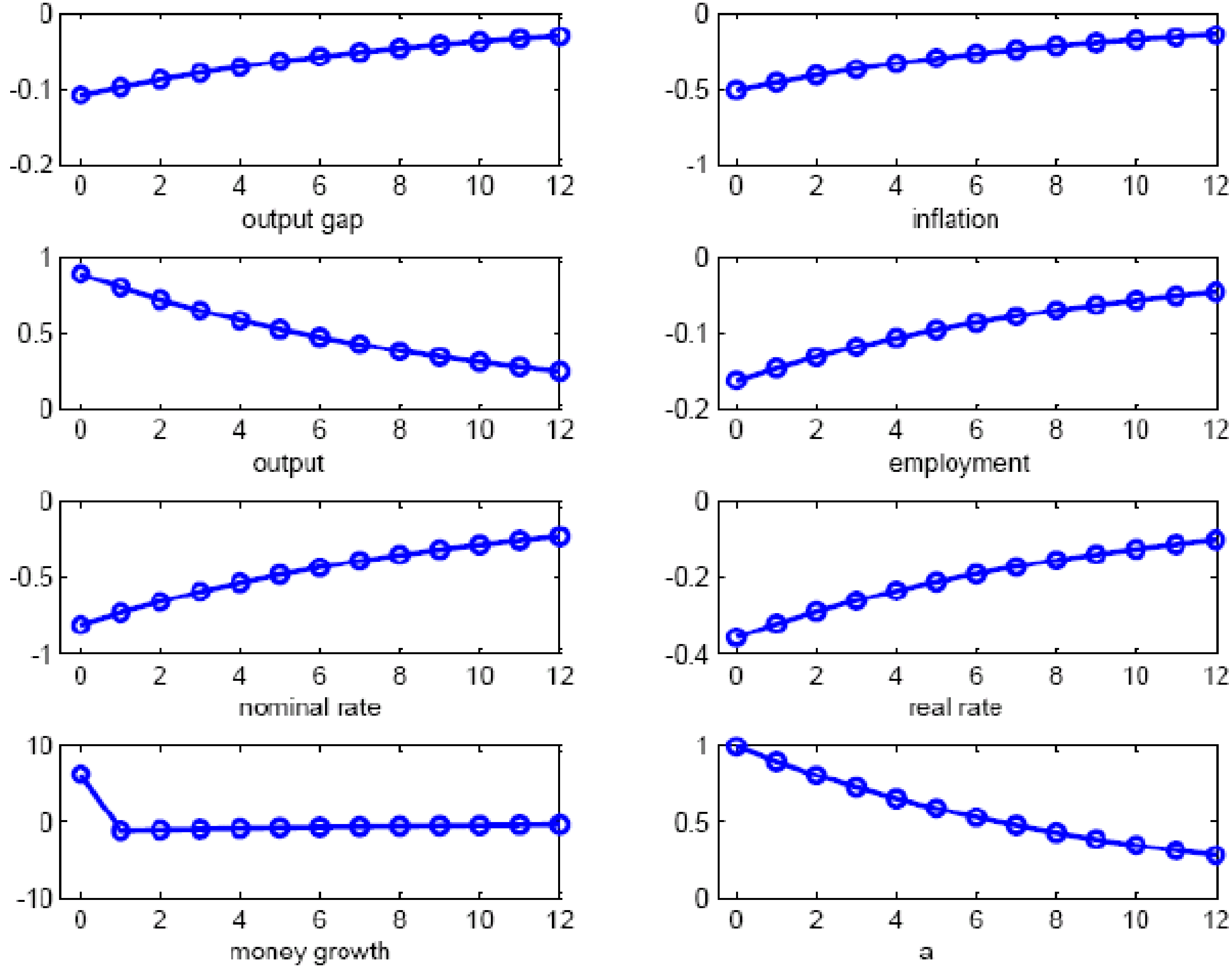
- solid line: model IRF; line with $+$: VAR IRF; gray areas: 95% confidence interval
- VAR identification assumption: variables does not respond contemporaneously to MP shock
- Model fits data fairly well.

1.5 Effect of Productivity shock

Shock process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- productivity boost increases potential output -> activity slack (or negative output gap) -> deflationary
- lower nominal rates, as prescribed by the policy rule, and higher money growth, as a consequence of the money demand equation



output gap
inflation
output
employment
nominal rate
real rate
money growth
a

1.6 Equilibrium under an exogenous money growth process

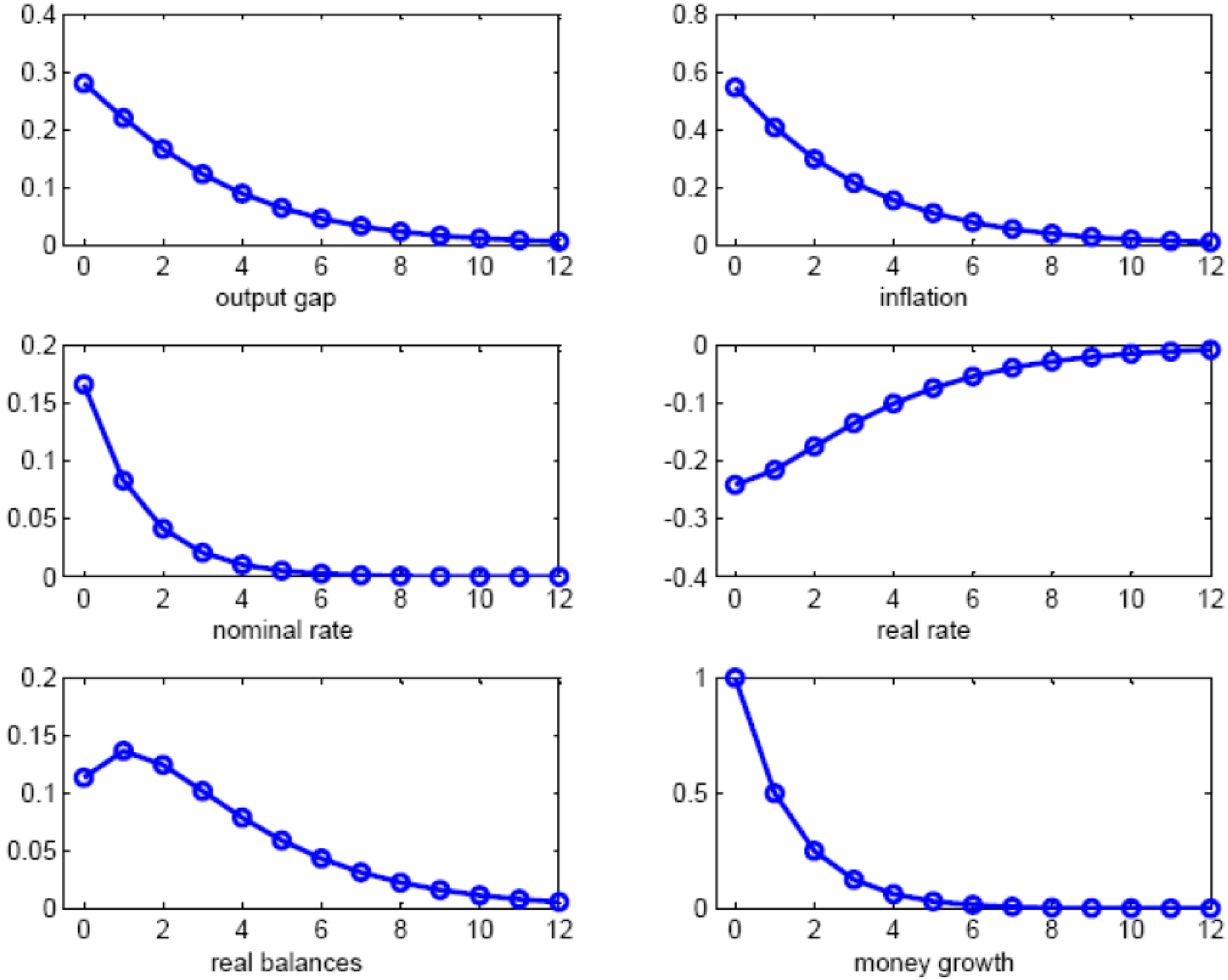
Policy rule 2 (PR2)

$$\Delta m_t = \rho_m \Delta m_{t-\mathbf{1}} + \varepsilon_t^m$$

To simulate the dynamic effects of monetary and productivity shock under these rules you also need to specify a money demand equation

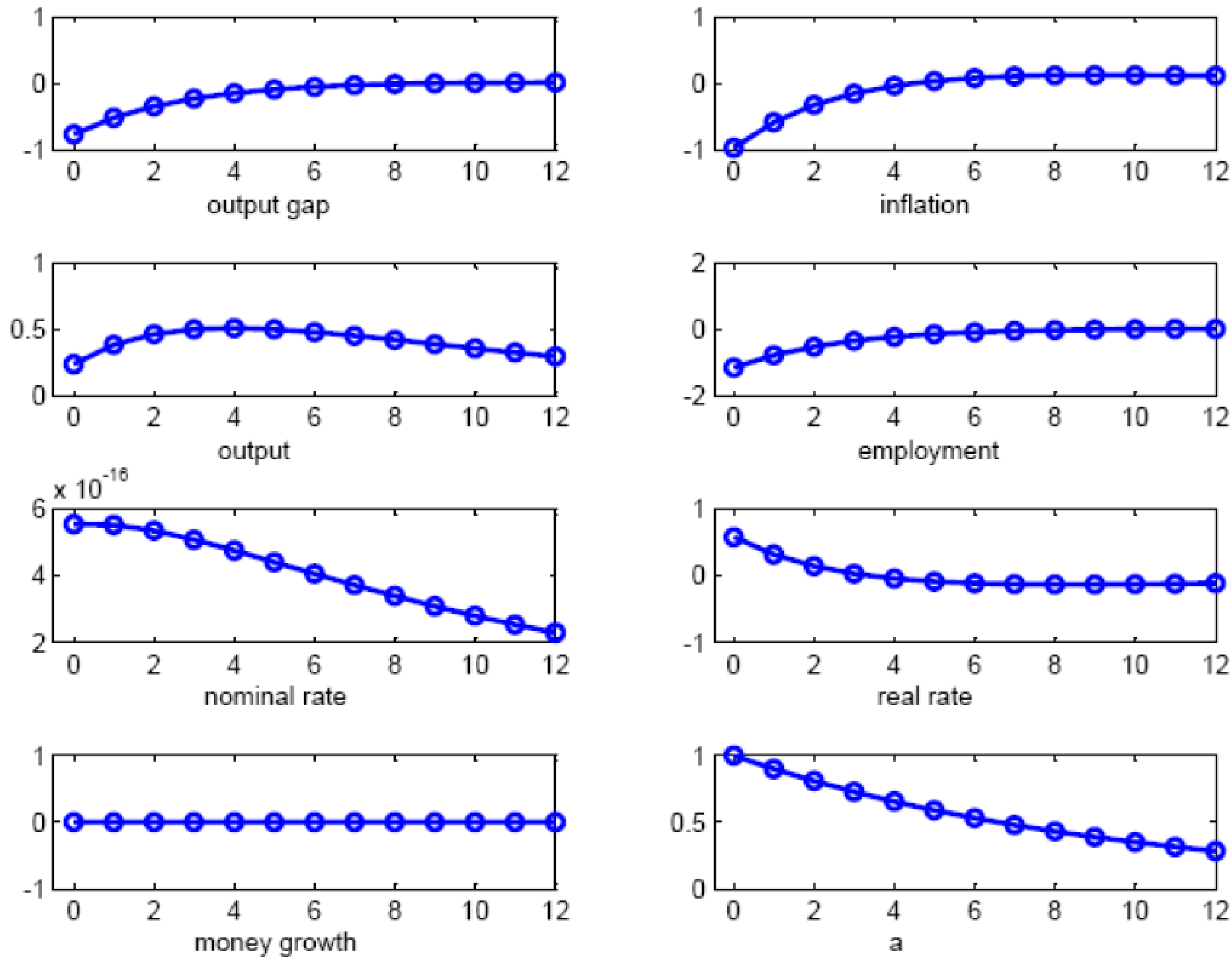
- the effect of a negative ε_t^m shock is similar to an increase in v_t

Figure 3.3: Effects of a Monetary Policy Shock (Money Growth Rule)



- **Productivity shock:** under 'money rule' nominal interest rate practically does not move so fall in inflation relative to output is bigger than under 'interest rate rule'

Figure 3.4: Effects of a Technology Shock (Money Growth Rule)



1.7 Money-supply vs interest rate rule

- Determinacy of REE:
- REE always determinate under money-supply rule
- REE indeterminate under exogenous interest-rate rule
- REE determinate under π targeting interest-rate rule when ,$\phi_\pi > 1$ (Taylor principle)
- In this sense, money supply is a stronger nominal anchor
- In practice, many Central Banks use interest rates as their policy instruments. Why?

1.8 Money-supply vs interest rate rule

- In practice, money demand shock is very volatile and difficult to observe.
- When there is financial innovation, money demand curve shifts.
- We can model period utility as

$$U(C_t) + U\left(\zeta_t, \frac{M_t}{P_t}\right) - V(N_t)$$

- where ζ_t is money demand shock.
- Then log-linearized money demand is

$$\hat{m}_t - \hat{p}_t = \eta_y \hat{y}_t - \eta_i \hat{\imath}_t + \zeta_t$$

- ζ_t unobservable and volatile: a reason for choosing interest-rate as policy instrument

1.8.1 Empirical estimates of interest rate rules

- Time path of (US) nominal interest rate can be well explained by a simple feedback rule

- Original 'Taylor rule' (Taylor, 1993)

$$i_t = 0.04 + 0.5\widetilde{y}_t + 1.5(\pi_t - 0.02)$$

- 0.04:interest rate in LR; 0.02: inflation target; $\widetilde{y}_t$: output gap.

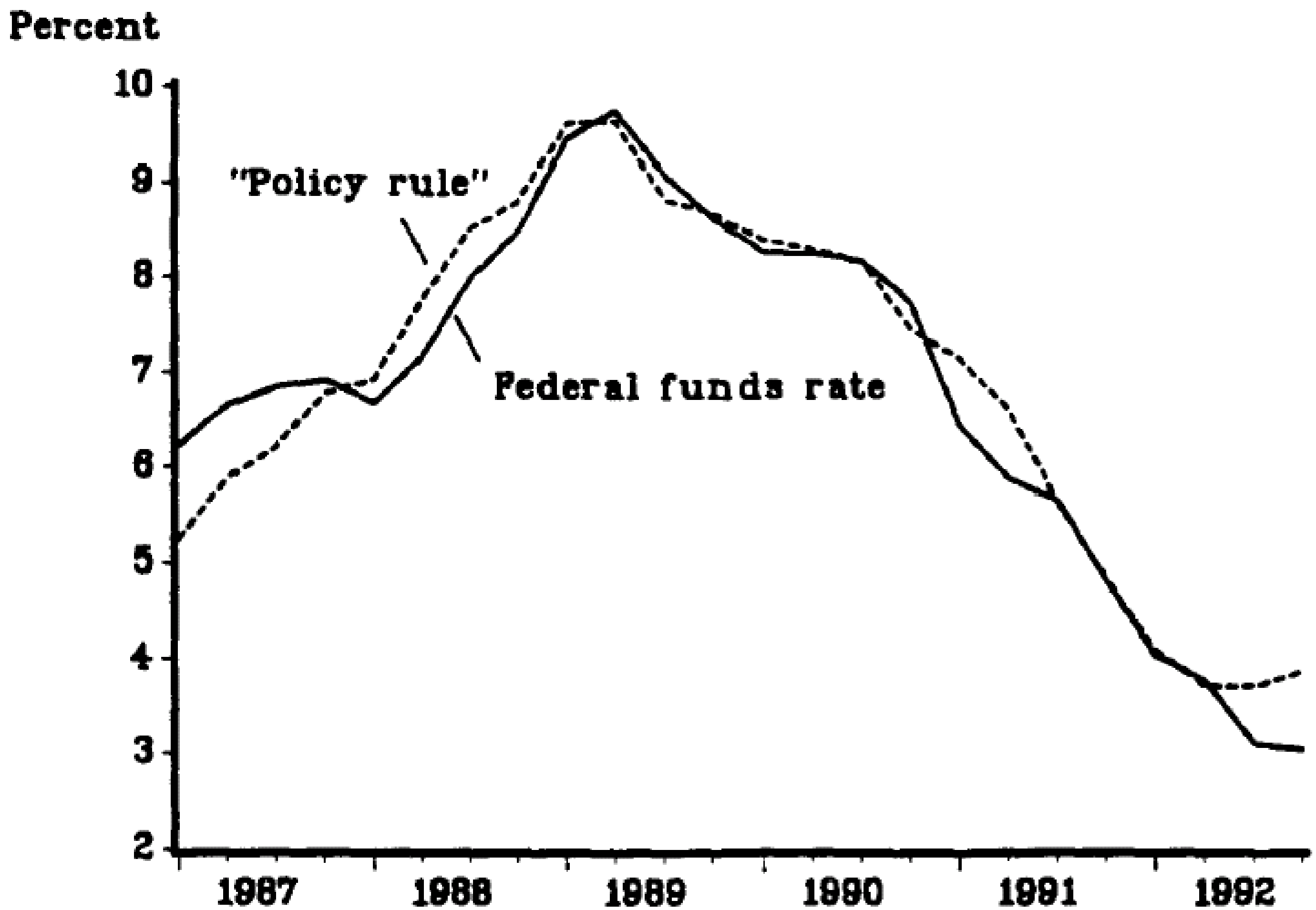


Figure 1. Federal funds rate and example policy rule.

1.8.2 Empirical estimates of interest rate rules (Taylor 1999)

- Does CB respond strongly enough to make REE determinate?
- For π-targeting rules, the condition for determinacy is the Taylor principle.
- Taylor (1999): US economy

	ϕ_π	ϕ_x
'60-'79	0.81	0.25
'87-'97	1.53	0.77

- After Volcker's chairmanship, the Fed increased ϕ_π.

1.9 Empirical estimates of interest rate rules - Clarida, Gali, Gertler (2000)

- A more complicated interest rate rule:

$$
\begin{aligned}
i_t &= (1-\rho)i_t^* + \rho i_{t-1} \\
i_t^* &= \rho + \phi_y \widetilde{y}_t + \phi_\pi \pi_t + v_t
\end{aligned}
$$

- i_t^*: target interest rate at time t; ρ: degree of partial adjustment

	ϕ_π	ϕ_x	ρ	$SD(\pi)$	$SD(x)$
60-79	0.93	0.27	0.68	1.48	1.83
79-96	2.15	0.93	0.79	0.96	1.49

- Aggressive MP after Volcker's chairmanship contributed to inflation stabilization

1.10 Empirical estimates of interest rate rules - Clarida, Gali, Gertler (1999)

- European countries:

	ϕ_π	ϕ_x	ρ
Germany (74-93)	1.31	0.25	0.91
UK (79-90)	0.98	0.19	0.92
France (83-89)	1.13	0.99	0.95
Italy (81-89)	0.9	0.22	0.95

UK

Table 6: Summary of reaction function estimates

Regime	Long-run inflation response	Long-run output gap response	Smoothing parameter	Ex-post real interest rate
1972–1976	0.12	0.57	0.27	–5.72%
1976–1979	0.62	0.11	0.62	–3.14%
1979–1987	0.38	0.16	0.37	4.66%
1987–1990[1]	0.00	0.45	0.52	5.76%
1992–1997	1.27	0.47	0.29	2.99%

1. German short–term interest rate enters rule with long-run coefficient 1.11.

1.11 Policy rules: summary

- Problem of determinacy
- Money supply vs interest rate: money-supply control may be subject to uncertainty about money demand shock.
- Empirical studies on interest-rate rules: In the 70s, violation of the Taylor principle might have made inflation volatile

1.12 Empirical study of NKPC: Gali and Gertler (1999)

- Estimation of Phillips curve does not work well when de-trended output is used as a proxy of the output gap

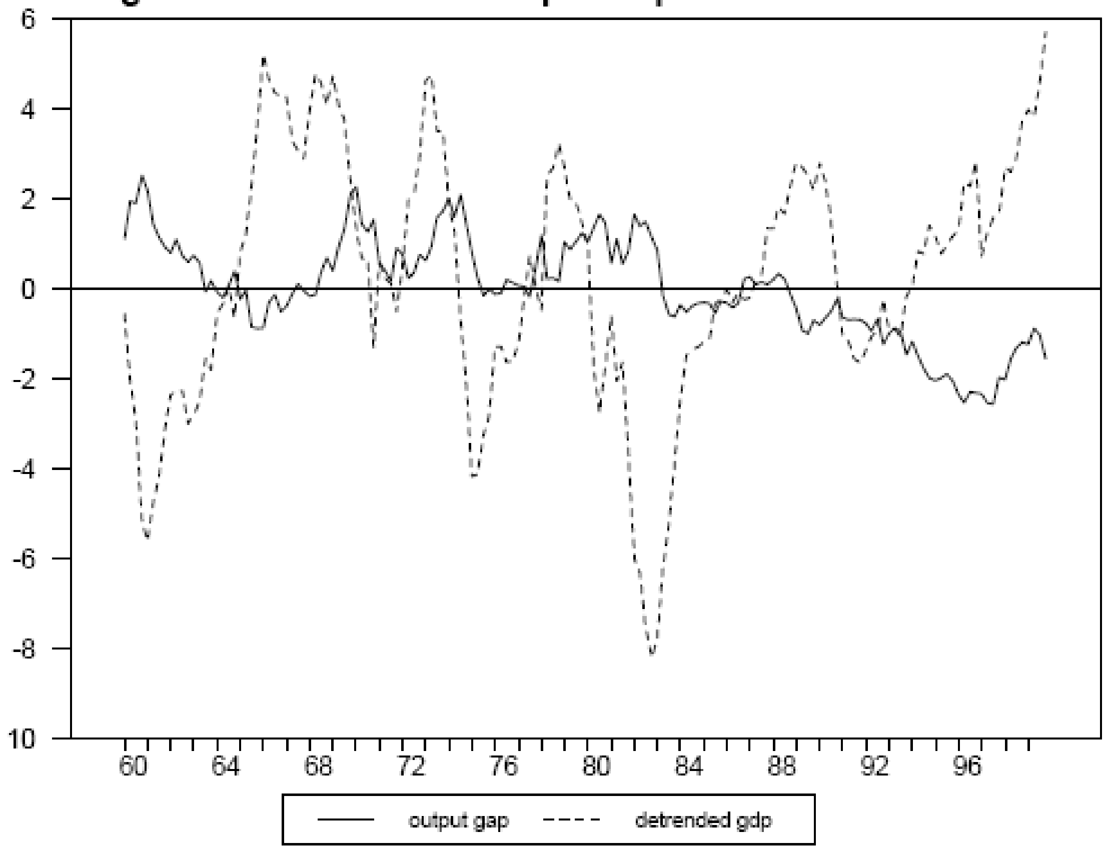
$$\pi_t = 0.988 E_t \pi_{t+1} + 0.016 \tilde{y}_t$$

- $\tilde{y}_t$: de-trended output is the proxy for the output gap
- Because of this failure, the hybrid version has been popular:

$$\pi_t = \phi_b \pi_{t-1} + \phi_f E_t \pi_{t+1} + \kappa \tilde{y}_t$$

- Then the coefficient on $E_t \pi_{t+1}$ becomes insignificant!
- Looks more like the traditional backward looking Phillips curve (in which disinflation is costly).
- Reason for failure: de-trended output is a poor proxy of the output gap $y_t - y_t^n$

Figure 2. Model-Based Output Gap vs. Detrended GDP



source: Gali (2003) ‘New Perspectives on Monetary Policy, Inflation and the Business Cycle

1.13 Estimation of NKPC using real marginal cost

- The real marginal cost is the driving force of inflation

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t$$

- Real marginal cost for the case of constant returns ($\alpha = 0- > Y_t = A_t N_t$)
- Cost$= W_t N_t = W_t \left(\frac{Y_t}{A_t}\right)$ -> marginal cost$=\frac{W_t}{A_t}$
- Real marginal cost=Labour share

$$\frac{W_t}{P_t A_t} = \frac{W_t N_t}{P_t Y_t}$$

- Use labour share in estimation

$$\pi_t = 0.942 E_t \pi_{t+1} + 0.023 \widehat{mc}_t$$

- Coefficients have the right signs and significant.

Consideration of backward-looking term

- Estimation of the hybrid PC:

$$\pi_t = \phi_b \pi_{t-1} + \phi_f E_t \pi_{t+1} + \lambda \widehat{mc}_t$$

- Estimate of $\phi_b = 0.2 - 0.3$
- Forward $\phi_f = 0.6 - 0.75$
- Forward-looking term more important

1.14 Introducing fiscal shocks into NK Model

(the simplest specification)

Private sector aggregate budget constraint (in real terms):

$$Y_t = C_t + T_t$$

where T_t denotes lump sum taxes

Government resource constraint:

$$\tau_t Y_t = T_t$$

So economy-wide market clearing

$$(1 - \tau_t) Y_t = C_t$$

Let´s define the fiscal shock such that a positive shock is an expansionary policy.

So we can define

$$g_t \equiv \log(1 - \tau_t)$$

Such that a fall in taxes leads to an increase in g_t

We know that

$$g_t \equiv \log(1 - \tau_t) = \log C_t - \log Y_t = c_t - y_t$$

So:

$$y_t = c_t + g_t$$

Now let´s assume that there are only fiscal shocks (no productivity shock)

Production function

$$Y_t(j) = N_t(j)$$

Note that with constant returns there is no difference between average and firm specific marginal cost - before

$$\psi_{t+k,t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\epsilon\alpha/(1-\alpha)} \psi_{t+k}$$

So when $\alpha = 0$

$$\begin{aligned}
\psi_{t+k,t} &= \psi_{t+k} \\
&= W_{t+k}N_{t+k} \\
&= W_{t+k}Y_{t+k}
\end{aligned}$$

Marginal cost is W_t, so real marginal cost, in log terms

$$\begin{aligned} mc_t &= w_t - p_t = \sigma c_t + \varphi n_t \\ &= (\sigma + \varphi) y_t - \sigma g_t \end{aligned}$$

With flexible prices, marginal cost is constant and equal to $-\mu$

$$y_t^n = (\sigma + \varphi)^{-1}(-\mu + \sigma g_t)$$

Government expenditure crowds out private consumption $=>$ lower consumption imply higher marginal utility of consumption and thus larger labour supply for a given wage $=>$ for a constant price level, potential output is larger

The Euler equation

$$c_t = E_t c_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho)$$

becomes

$$y_t - g_t = E_t(y_{t+1} - g_{t+1}) - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho)$$

$$\begin{aligned} y_t - \frac{\sigma}{\sigma+\varphi} g_t &= E_t\left(y_{t+1} - \frac{\sigma}{\sigma+\varphi} g_{t+1}\right) - \sigma^{-1}(i_t - E_t \pi_{t+1} - \rho) \\ &\quad - \frac{\varphi}{\sigma+\varphi} E_t \Delta g_{t+1} \end{aligned}$$

or

$$\begin{aligned} \tilde{y}_t &= E_t \tilde{y}_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n) \\ &\quad \text{where } r_t^n = \rho - \frac{\varphi}{\sigma+\varphi} E_t \Delta g_{t+1} \end{aligned}$$

So, a fiscal shock (g_t) increases the natural interest rate.

1.14.1 Policy prescription

Can you already infer the policy prescription of a central bank that wants to maintain price stability?

The system of equilibrium conditions

1)PC curve

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

2)IS curve

$$\widetilde{y}_t = E_t \widetilde{y}_{t+1} - \sigma^{-1}(i_t - E_t \pi_{t+1} - r_t^n)$$

So, interest rate that is consistent with

$$\pi_t = 0 \text{ for all } t$$

and

$$\widetilde{y}_t = 0$$

would imply:

$$i_t = r_t^n$$

(let´s abstract from the problem of determinacy for the moment...)

Given that the natural rate of interest is defined as

$$r_t^n = \rho + \sigma\psi_{ya} E_t \Delta a_{t+1}$$

in the case of productivity shock, and

$$r_t^n = \rho - \frac{\varphi}{\sigma + \varphi} E_t \Delta g_{t+1}$$

in the case of fiscal shocks...

- The interest rate that is consistent with constant prices falls after an increase in a_t => so, the central bank that wants to maintain inflation at bail can cut rates
- Productivity shock increases potential output so it is a deflationary force

- The interest rate that is consistent with constant prices increases after an increase in g_t. $=>$ so, the central bank that wants to maintain inflation at bail have to increase rates
- Two effects of g_t
  - - the shock increases potential output and thus reduces inflationary pressures for any given level of production.
  - - the shock increases aggregate demand and create inflationary pressures
- The second effect dominates - fiscal shock normally modelled as demand shock.