Slides 5: Optimal Monetary Policy in the New Keynesian Model (following Galí 2007)

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1 Optimal Monetary Policy in the New Keynesian Model:

- We now address the question of how monetary policy should be conducted, using as a reference framework - the basic new Keynesian model
- First we characterize the model's efficient allocation.
- This is shown to correspond to the equilibrium allocation of the decentralized economy under monopolistic competition and flexible prices, once an appropriately chosen subsidy is in place.
- When prices are sticky, that allocation can be attained by means of a policy that fully stabilizes the price level.

1.1 The Efficient Allocation:

The problem facing a benevolent social planner seeking to maximize the representative household's welfare, given technology and preferences.

$$\max U(C_t; N_t)$$

or

$$\max U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right],$$

where:

$$C_t = \left[\int_0^1 C_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj \right]^{\frac{\varepsilon}{\varepsilon-1}}$$

subject to the production function $Y_t = A_t N_t^{1-\alpha}$ and the market clearing conditions $Y_t = C_t$.

Optimality conditions:

$$\frac{N_t^{\varphi}}{Y_t^{-\sigma}} = (1-\alpha) A_t N_t^{-\alpha}$$

or

$$-\frac{U_n(C_t; N_t)}{U_c(C_t; N_t)} = MPN_t$$

Moreover:

$$C_t\,(j) = C_t$$

1.2 Sources of Suboptimality of Equilibrium

1.2.1 Distortions unrelated to nominal rigidities: Monopolistic competition:

- Price is a markup ($\mathcal{M} = \varepsilon/(\varepsilon - 1)$) over marginal cost. Under flexible prices:

$$P_t = \mathcal{M}\psi_t$$

where

$$\psi_t = \partial(W_t N_t)/\partial Y_t = W_t/(\partial Y_t/\partial N_t)$$

So

$$P_t = \mathcal{M}\frac{W_t}{MPN_t}$$

or, given that the labour leisure decision holds:

$$-\frac{U_n(C_t; N_t)}{U_c(C_t; N_t)} = \frac{W_t}{P_t}$$

we have

$$-\frac{U_n(C_t; N_t)}{U_c(C_t; N_t)} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

$$\frac{N_t^{\varphi}}{Y_t^{-\sigma}} < (1-\alpha)A_t N_t^{-a}$$

Output will be smaller than the efficient level - natural given monopoly power of producers.

- Solution: employment subsidy τ:

$$P_t = \mathcal{M}(1-\tau)\frac{W_t}{MPN_t}$$

$$-\frac{U_n(C_t; N_t)}{U_c(C_t; N_t)} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

- Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = 1/\varepsilon$

1.2.2 Distortions driven by nominal rigidities:

1) Relative price distortions resulting from staggered price setting:

$$C_t(i) \neq C_t(j) \text{ if } P_t(i) \neq P_t(j)$$

- Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods.
- Accordingly, markups should be identical across firms/goods at all times.

2) Markup fluctuations

- Average markup- defined as the ratio of average price to average (assuming there is an employment subsidy that reduces marginal costs for firms)

$$\mathcal{M}_t \equiv \frac{P_t}{(1-\tau)\psi_t}$$

(assuming that the subsidy is optimal)

$$\mathcal{M}_t = \frac{\mathcal{M}P_t}{W_t/MPN_t}$$

So

$$\frac{W_t}{P_t} = \frac{\mathcal{M}}{\mathcal{M}_t} MPN_t \neq MPN_t$$

- Optimality requires that the average markup be stabilized at its frictionless level:

$$\mathcal{M}(1-\tau) = 1 \text{ and } \mathcal{M}_t = \mathcal{M}$$

1.3 Optimal Monetary Policy in the Basic NK Model

- Optimal employment subsidy and no markup fluctuations $\Rightarrow$ flexible price equilibrium allocation is efficient

$$\sum_{k=0}^{\infty} E_t \theta^k Q_{t.t+k} Y_{t+k,t} \left[P_t^* - (1-\tau)\mathcal{M}\psi_{t+k,t} \right] = 0$$

Flexible price allocation ($\theta = 0$)

$$P_t^* = (1-\tau)\mathcal{M}\psi_t$$

if $(1-\tau)\mathcal{M}$ =1

$$P_t^* = \psi_t$$

- Given that all firms face the same aggregate conditions, they will all set the same price => No inherited relative price distortions, i.e. $P(i) = P$ for all i

- Markup is constant and equal to 1

Equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

$$P_t = \psi_t = W_t/MPN_t$$

So

$$-\frac{U_n(C_t;N_t)}{U_c(C_t;N_t)} = MPN_t$$

Equilibrium under the Optimal Policy

- Flexible price allocation is optimal

$$y_t = y_t^n$$

so

$$\widetilde{y}_t = 0$$

Recalling the Phillips curve

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

So, optimal policy imply perfect price stabilization

$$\pi_t = 0$$

(all firms are charging the optimal markup, so there is no incentive to change prices)

- There is no policy trade-of between output and inflation: stabilizing prices also closes the output gap

Implementation: Some Candidate Interest Rate Rules

- The IS equation

$$\widetilde{y}_t = E_t\widetilde{y}_{t+1} - \sigma^{-1}(i_t - E_t\pi_{t+1} - r_t^n)$$

- *An Exogenous Interest Rate Rule*

$$i_t = r_t^n$$

- Equilibrium dynamics:

$$\widetilde{y}_t = E_t\widetilde{y}_{t+1} + \sigma^{-1}E_t\pi_{t+1}$$

and

$$\pi_t = \kappa\widetilde{y}_t + \beta E_t\pi_{t+1}$$

or

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = A_o \begin{bmatrix} E_t\widetilde{y}_{t+1} \\ E_t\pi_{t+1} \end{bmatrix}$$

where

$$A_o = \left[\begin{array}{cc} 1 & \sigma^{-1} \\ \kappa & \kappa\sigma^{-1} + \beta \end{array} \right]$$

- The solution $\widetilde{y}_t = \pi_t = 0$ for all t is not unique: one eigenvalue of A_o is strictly greater than one $=>$ indeterminacy (real and nominal). See, e.g. Blanchard and Kahn (1980).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_y \widetilde{y}_t + \phi_\pi \pi_t$$

- Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = A_o \begin{bmatrix} E_t \widetilde{y}_{t+1} \\ E_t \pi_{t+1} \end{bmatrix}$$

where

$$A_o = \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \begin{bmatrix} \sigma & 1 - \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

- Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

or

$$\phi_\pi + \kappa^{-1}(1 - \beta)\phi_y > 1$$

- Paradoxically, if this condition is satisfied both the output gap and inflation will be zero and, hence,

$$i_t = r_t^n$$

will hold ex-post.

- Quick proof: we have shown that the equilibrium is unique, then we can verify that $\widetilde{y}_t = \pi_t = 0$ (and thus $i_t = r_t^n$) is an equilibrium. So, it is the equilibrium.

- Thus, and in contrast with the case considered above (in which the equilibrium outcome $i_t = r_t^n$ was also taken to be the policy rule), it is the presence of a "threat" of a strong response by the monetary authority to an eventual deviation of the output gap and inflation from target that suffices to rule out any such deviation in equilibrium (ie guarantees determinacy).

Taylor-principle interpretation (Woodford (2000)):

- Consider a permanent increase in inflation of size d to occur (and assuming no permanent changes in the natural rate)
- in the long run

$$dy = \kappa^{-1}(1-\beta)d\pi$$

and

$$di = \phi_y dy + \phi_\pi d\pi$$

So

$$di = (\phi_y + \phi_\pi \kappa^{-1}(1-\beta))d\pi$$

- So the determinacy condition imply that

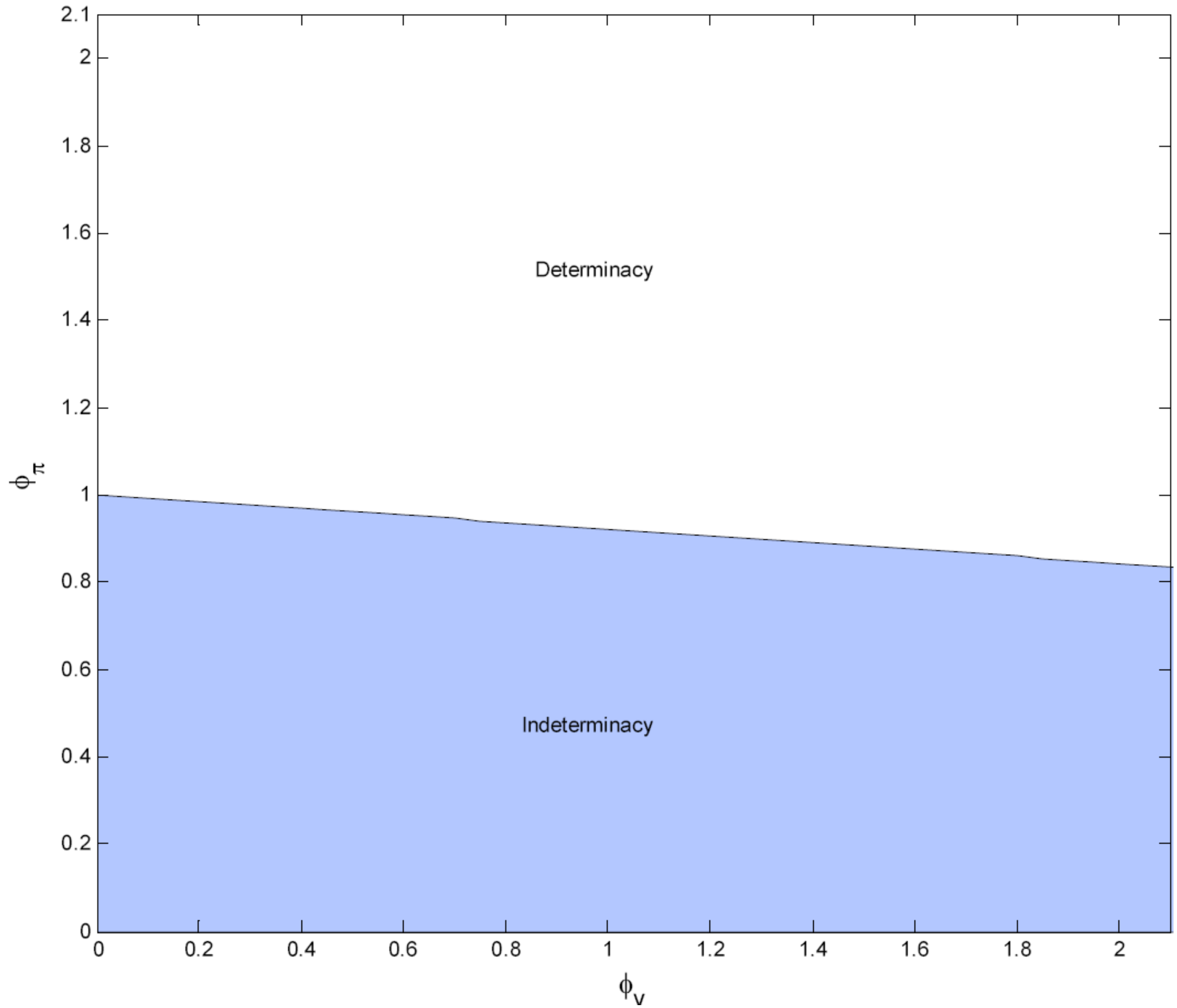
$$di/d\pi > 1$$

(eg, in the face of an inflationary shock, the real interest rate has to eventually rise to guarantee determinacy)

- Thus, the equilibrium will be unique under the proposed interest rate rule whenever ϕ_y and ϕ_π are sufficiently large to guarantee that the real rate eventually rises in the face of an increase in inflation (thus tending to counteract that increase and acting as a stabilizing force).

- The previous property is often referred to as the Taylor principle and, to the extent that it prevents the emergence of multiple equilibria, it is naturally viewed as a desirable feature of any interest rate rule.

Figure 4.1


Determinacy
Indeterminacy
ϕ_π
ϕ_v
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.1

Alternative policy rules that ensure price stability:

- Forward looking rule

$$i_t = r_t^n + \phi_y E_t \widetilde{y}_{t+1} + \phi_\pi E_t \pi_{t+1}$$

- have different determinacy conditions...
- These are all instrumental rule - ie, they specify how the policy instrument (i.e. nominal interest rate) should respond to shocks and changes in the observable variables
- Svensson (2003) $\Rightarrow$ Targeting rule
- in this case, the optimal targeting rule is

$$\pi_t = 0$$

- Directly specify the behavior of observable variable, and policy instruments should adjust to guarantee that the target is met.

Shortcomings of Optimal Rules

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

(i) the true model

(ii) true parameter values

(iii) realized shocks

- Alternative: "simple rules", i.e. rules that meet the following criteria:

(i) the policy instrument depends on observable variables only,

(ii) do not require knowledge of the true parameter values

(iii) ideally, they approximate optimal rule across different models

Examples of simple rules:

- Taylor rule:

$$i_t = \rho + \phi_y \hat{y}_t + \phi_\pi \pi_t$$

- Constant money growth rule

$$\Delta m_t = 0$$

To use this rule, need to specify, apart from the IS and the Phillips Curve, the money demand rule.

- But in general simple rules are not optimal $\Rightarrow$ create output and inflation volatility

Table 4.1: Evaluation of Simple Monetary Policy Rules

	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

where σ_ζ is the standard deviations of the money demand shock, and ρ_ζ is the persistence of such shock

- Table 4.1 shows the implied standard deviations of the output gap and (annualized) inflation, both expressed in percent terms, as well as the welfare losses resulting from the associated deviations from the efficient allocation, expressed as a fraction of steady state consumption.

- The first column corresponds to the calibration proposed by Taylor (1993) as a good approximation to the interest rate policy of the Fed during the Greenspan years.

- Versions of the rule that involve a systematic response to output variations generate larger fluctuations in the output gap and inflation and, hence, larger welfare losses.

- Those losses are moderate under Taylor's original calibration, but they become substantial (close to 2 percent of steady state consumption) when the output coefficient is set to unity.

- Secondly, the smallest welfare losses are attained when the monetary authority responds to changes in inflation only, and these become smaller as the strength of that response increases.

- $=>$ In the context of the basic new Keynesian model considered here, a simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.

Exercise: Assume that $U_t = E_t \sum\limits_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right]$, $Y_t = A_t N_t^{1-\alpha}$, *and* $\log(A_t) = a_t \sim AR(1)$

-Calculate the natural rate of output (as in week 5)

-Specify an interest rate rule that rule that mimics the optimal allocation

-Assuming $\sigma = 1$, $\varphi = 1$, $\theta = 2/3$, $\rho_a = 0.9$, $\alpha = 1/3$, $\varepsilon = 6$ *and* $\sigma_\varepsilon = 0.01$, *replicate (using Dynare or Reds and Solds) the output gap and inflation volatility in Table 4.1 for the cases in which the central bank is following a simple Taylor rule.of the form*

$$\hat{\imath}_t = \phi_y \hat{y}_t + \phi_\pi \pi_t$$

-For the case in which a Taylor rule is specified as

$$\hat{\imath}_t = \phi_y \hat{y}_t + \phi_\pi \pi_t$$

and $\rho_a = 0$ *calculate the the output gap and inflation volatility (as in Table 4.1) for the cases in which (i)* $\phi_\pi = 1.5$ *and* $\phi_y = -0.1$ *and (ii)* $\phi_\pi = 0.5$ *and* $\phi_y = 0$. *Explain the outcomes.*

But how are the welfare losses in Table 4.1 calculated?

Approximating welfare:

$$\max U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right],$$

Deriving a second order approximation of the utility function = welfare measure = - loss function

$$\begin{aligned} U(C_t, N_t) &\simeq \bar{U} + \bar{U}_c(C_t - \bar{C}) + \bar{U}_n(N_t - \bar{N}) \\ &\quad + \frac{1}{2}\bar{U}_{cc}(C_t - \bar{C})^2 + \frac{1}{2}\bar{U}_{nn}(N_t - \bar{N})^2 + \bar{U}_{cn}(C_t - \bar{C})(N_t - \bar{N}) \end{aligned}$$

Given that

$$\begin{aligned} \bar{U}_{cn} &= 0; \bar{U}_c = \bar{C}^{-\sigma}; \bar{U}_n = \bar{N}^{\varphi}; \\ \bar{U}_{cc} &= -\sigma U_c^{-1}\bar{C}; \bar{U}_n = \varphi U_n \bar{N}^{-1} \end{aligned}$$

and, the labour leisure decision holds:

$$-\frac{\bar{U}_n}{\bar{U}_c} = \frac{\bar{W}}{\bar{P}}$$

Moreover, from the price setting

$$\bar{P} = \mathcal{M}\frac{\bar{W}}{\overline{\overline{MPN}}}$$

and given the production function is $Y_t = A_t N_t^{1-\alpha}$ and the market clearing conditions is $Y_t = C_t$

$$\overline{MPN} = (1-\alpha)\bar{A}\bar{N}_t^{-a} = (1-\alpha)\frac{\bar{Y}}{\bar{N}} = (1-\alpha)\frac{\bar{C}}{\bar{N}}$$

So:

$$\bar{U}_n\bar{N} = -\frac{(1-\alpha)}{\mathcal{M}(1-\tau)}\bar{U}_c\bar{C}$$

Denoting $\widehat{x}_t = \log(X_t/X)$, we know that

$$\frac{X_t - \bar{X}}{X} \simeq \widehat{x}_t + \frac{1}{2}\widehat{x}_t^2$$

we can write

$$U(C_t, N_t) \simeq \bar{U} + \bar{U}_c\bar{C}\left[\widehat{y}_t - \frac{1-\alpha}{\mathcal{M}(1-\tau)}\widehat{n}_t + \frac{1}{2}(1-\sigma)\widehat{y}_t^2 - \frac{1}{2}\frac{(1+\varphi)(1-\alpha)}{\mathcal{M}(1-\tau)}\widehat{n}_t^2\right]$$

As shown in the appendix of Gali ch.3/4

$$(1-\alpha)\widehat{n}_t \simeq \widehat{y}_t - a_t + \frac{\varepsilon}{2\Theta}var(p(i))$$

where $\Theta = \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

Assuming the optimal subsidy $\mathcal{M}(1-\tau) = 1$:

$$\frac{U(C_t, N_t) - \bar{U}}{\bar{U}_c \bar{C}} \simeq -\frac{1}{2}\left[\frac{\varepsilon(1-\alpha+\alpha\varepsilon)}{1-\alpha} var(p(i)) + \frac{\sigma+\varphi+\alpha(1-\sigma)}{1-\alpha}\widetilde{y}_t^2\right]$$

where $\widetilde{y}_t = y_t - y_t^n$ and $y_t^n = \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)} a_t$.

Moreover, as shown in Woodford chapter 6

$$\sum_{k=0}^{\infty} \beta^k var(p_t(i)) = \lambda^{-1} \sum_{k=0}^{\infty} \beta^k \pi_t^2$$

So, welfare losses resulting from deviations from the efficient allocation, expressed as a fraction of steady state consumption (or output), can be expressed as

$$\mathcal{W} \equiv -E_0 \sum_{k=0}^{\infty} \beta^k \frac{U(C_t, N_t) - \bar{U}}{\bar{U}_c \bar{C}} \simeq \frac{1}{2\lambda} E_0 \sum_{k=0}^{\infty} \beta^k \left[\varepsilon \pi_t^2 + \kappa \tilde{y}_t^2\right]$$

Or, on average, the welfare loss will be given by

$$\mathcal{L} = \frac{1}{2\lambda} \left[\varepsilon var(\pi_t) + \kappa var(\tilde{y}_t) \right]$$

1.4 Optimal policy: traditional representation - but here in a microfounded model

$$\min \frac{1}{2\lambda} E_0 \sum_{k=0}^{\infty} \beta^k \left[\varepsilon \pi_t^2 + \kappa \tilde{y}_t^2\right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

F.O.C.

$$\lambda_t - \lambda_{t-1} = \varepsilon \pi_t$$

$$\lambda_t = -\kappa \widetilde{y}_t$$

So

$$\varepsilon \pi_t + \kappa \Delta \widetilde{y}_t = 0$$

In this case the first best can be achieved because stabilizing inflation also closes the output gap.

Exceptions to this rule:

- Markup or cost-push shocks
- Distorted steady state (absence of optimal subsidy)
- Wage stickyness
- Open economy

1.5 The case of markup shocks (Woodford (2003))

- Shocks to the markup of firms or shocks that generate an increase in inflation at the same time than reducing output (eg oil price shocks)

- with $\alpha = 0$

$$\pi_t = \frac{\kappa}{\varphi + \sigma}\left[(\varphi + \sigma)y_t - (1 + \varphi)a_t + \mu_t\right] + \beta E_t \pi_{t+1}$$

or

$$\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}$$

where

$$y_t^n = c + \frac{(1 + \varphi)a_t}{(\varphi + \sigma)} - \frac{\mu_t}{(\varphi + \sigma)} \tag{1}$$

- The policy problem of the central bank is

$$\min E_0 \sum_{k=0}^{\infty} \beta^k \left[\varepsilon \left(\pi_t\right)^2 + \kappa \left(\tilde{y}_t^w\right)^2\right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

- but the welfare relevant output gap is $\tilde{y}_t^w = y_t - y_t^t$, where

$$y_t^t = c + \frac{(1+\varphi)a_t}{(\varphi+\sigma)} \neq y_t^n \tag{2}$$

- (c is a constant that is irrelevant because it also enters in y_t)
- F.O.C.

$$\lambda_t - \lambda_{t-1} = \varepsilon \pi_t$$

$$\kappa \lambda_t = -\kappa \widetilde{y}_t^w$$

- Which can be represented by the following "targeting rule"

$$\varepsilon\pi_t + \Delta(\widetilde{y}_t - (\varphi+\sigma)^{-1}\mu_t) = 0$$

- So there is a trade off between stabilizing inflation and closing the output gap

- Positive markup shocks increase inflation without affecting the welfare relevant output gap.

1.6 The case of distorted steady state (Woodford and Benigno (2003))

- here fiscal shocks rather than productivity shocks are considered
- The Phillips curve is given by (exercise will guide you through deriving it)

$$\pi_t = \frac{\kappa}{\varphi + \sigma} [(\varphi + \sigma) y_t - \sigma g_t] + \beta E_t \pi_{t+1}$$

or

$$\pi_t = \kappa \tilde{y}_t + \beta E_t \pi_{t+1}$$

where

$$y_t^n = c + \frac{\sigma g_t}{(\varphi + \sigma)} \tag{3}$$

- Again, the policymaker's problem can be illustrated by the relative weight of inflation with respect to output in the loss function and by the difference between y_t^t and y_t^n

$$\min E_0 \sum_{k=0}^{\infty} \beta^k \left[\varepsilon \left(\pi_t\right)^2 + \kappa \left(\tilde{y}_t^w\right)^2\right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t + \beta E_t \pi_{t+1}$$

- but the welfare relevant output gap is $\tilde{y}_t^w = y_t - y_t^t$, where

$$y_t^t = \frac{d\sigma g_t}{(\varphi + \sigma)} \neq y_t^n \tag{4}$$

- where $d = (\bar{\mu}\varphi + \sigma)(\bar{\mu}\varphi + \sigma + (\bar{\mu} - 1))^{-1}$.
- and $\overline{\mu} \equiv \frac{1}{(1-\tau)\mathcal{M}}$

- So, unless the steady state is efficient (ie $(1-\tau)\mathcal{M}=1 \Rightarrow \bar{\mu}=1 \Rightarrow d=1$), there is a trade-off between stabilizing the output gap and inflation

- *(can you derive the loss function with an efficient steady state and fiscal shocks and show that the flexible price allocation is efficient?)*

- Optimal targeting rule is

$$\varepsilon\pi_t + \Delta(\widetilde{y}_t - (d-1)/(\varphi+\sigma)g_t) = 0$$

- In this case, a shock can affect the welfare relevant output gap without affecting inflation. If the central bank wants to stabilize the output gap, it will have to accept changes in inflation.

- Intuition: when the steady state is distorted, beyond its goals of stabilizing prices, the central bank want to improve production (that is inefficient because of monopolistic competition).

1.7 Central bank behavior when there are policy trade-offs

- We have seen that in a presence of cost-push (or markup shocks) there are monetary policy trade-off
  - That is, there are short run deviations between the natural (y^n) and efficient (y^t) levels of output.
- But how will the central bank behave in this case?

The policy problem of the central bank is

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon \left(\pi_t\right)^2 + \kappa \left(\tilde{y}_t^w\right)^2\right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t^w + \beta E_t \pi_{t+1} + u_t$$

where

$$u_t = \widetilde{y}_t - \widetilde{y}_t^w = y_t^t - y_t^n$$

Time variations in the gap between the efficient and natural levels of output–reflected in fluctuations in u_t*–generate a trade-off for the monetary authority, since they make it impossible to attain simultaneously zero inflation and an efficient level of activity*

We have abstracted from the fact that:

- The forward-looking nature of the constraint in the policy problem, requires that we specify the extent to which the central bank can credibly commit in advance to future policy actions.

- As will be clear below, the reason is that by committing to some future policies the central bank is able to influence expectations in a way that improves its short-run trade-offs.

- We can characterize the optimal monetary policy under two alternative assumptions regarding the central bank's ability to commit to future policies.

1.7.1 Optimal policy under commitment

- central bank which is assumed to be able to commit, with full credibility, to a policy plan.
- In the context of our model such a plan consists of a specification of the desired levels of inflation and the output gap at all possible dates and states of nature, current and future.
- More specifically, the monetary authority is assumed to choose a state-contingent sequence of $\{\pi_t, \tilde{y}_t^w\}_{t=0}^{\infty}$ that minimizes the loss function subject to the Phillips curve;

- Optimal plan:

$$
\begin{aligned}
& \min E_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon \left(\pi_t\right)^2 + \kappa \left(\tilde{y}_t^w\right)^2 - \lambda_t(\pi_t - \kappa \widetilde{y}_t^w + \beta E_t \pi_{t+1} + u_t)\right] \\
= \; & E_0\{....\beta^{t-1}\left[\varepsilon \left(\pi_{t-1}\right)^2 + \kappa \left(\tilde{y}_{t-1}^w\right)^2 - \lambda_t(\pi_{t-1} - \kappa \widetilde{y}_{t-1}^w + \beta \pi_t + u_t)\right] \\
& + \beta^t \left[\varepsilon \left(\pi_t\right)^2 + \kappa \left(\tilde{y}_t^w\right)^2 - \lambda_t(\pi_t - \kappa \widetilde{y}_t^w + \beta \pi_{t+1} + u_t)\right] + ...\}
\end{aligned}
$$

- Implicitly, that´s what we were doing before
- F.O.C

$$\lambda_t - \lambda_{t-1} = \varepsilon \pi_t$$

$$\kappa \lambda_t = -\kappa \widetilde{y}_t^w$$

- which should hold for $t = 0, 1, 2...$ and $\lambda_{-1} = 0$ given that the constraint at $t = -1$ is not relevant for a central bank starting to optimize at period 0
- Which can be represented by the following "targeting rule"

$$\varepsilon \pi_t + \Delta \widetilde{y}_t^w = 0$$

or

$$\varepsilon (p_t - p_{t-1}) + (\widetilde{y}_t^w - \widetilde{y}_{t-1}^w) = 0$$

- Iterating backwards

$$\varepsilon(p_t - p_{-1}) + (\widetilde{y}_t^w - \widetilde{y}_{-1}^w) = 0$$

but from the second F.O.C

$$\widetilde{y}_{-1}^w = 0$$

So the rule can be written as

$$\varepsilon(p_t - p_{-1}) + \widetilde{y}_t^w = 0$$

or

$$\widetilde{y}_t^w = -\varepsilon p_t$$

if we normalize the initial price to zero

- under the optimal policy with commitment the central bank sets the sign and size of the output gap in proportion to the deviations of the price level from its initial value

- We can write the Phillips curve as

$$p_t = ap_{t-1} + a\beta E_t p_{t+1} + au_t$$

where $a = \frac{1}{1+\beta+\kappa\varepsilon}$

- The solution for this differential equation can be written in state-space representation

$$p_t = \delta p_{t-1} + \delta_1 u_t$$

price level targeting!

- Then we can derive a similar expression for the output gap

$$\widetilde{y}_t^w = \delta \widetilde{y}_{t-1}^w - \varepsilon \delta_1 u_t$$

where $\delta \in (0, 1)$ and δ_1 are specified in Galí Ch.5.

1.7.2 Optimal Discretionary Policy

- The central bank treats the problem described above as one of sequential optimization
- It makes whatever decision is optimal each period without committing itself to any future actions.
- More specifically, each period the monetary authority is assumed to choose $(\pi_t, \tilde{y}_t^w)$ in order to minimize the period losses

$$\min \left[\varepsilon\left(\pi_t\right)^2 + \kappa\left(\tilde{y}_t^w\right)^2\right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t^w + v_t$$

where the term $v_t = \beta E_t \pi_{t+1} + u_t$ is taken as given by the monetary authority

- $v_t = \beta E_t \pi_{t+1} + u_t$ is taken as given since:
  - u_t is exogenous and
  - $E_t \pi_{t+1}$ is a function of expectations about future output gaps (as well as future u_t's) which, by assumption, cannot be currently influenced by the policymaker.

- The optimality condition for the problem above is given by

$$\widetilde{y}_t^w = -\varepsilon \pi_t$$

- The previous condition has a simple interpretation: in the face of inflationary pressures resulting from a cost-push shock the central bank must respond by driving output below its efficient level–thus creating a negative output gap–, with the objective of dampening the rise in inflation.

- The central bank carries out such a "leaning against the wind" policy up to the point where this condition is satisfied.

- We can write the phillips curve as

$$\pi_t = a_1 \beta E_t \pi_{t+1} + a_1 u_t$$

where $a_1 = \frac{1}{1+\kappa\varepsilon}$

- The solution for this differential equation can be written in state-space representation

$$\pi_t = \frac{\kappa}{\varepsilon}\delta_2 u_t$$

- (Note on consistency between expectation and the behavior of central bank)

- Then we can derive a similar expression for the output gap

$$\widetilde{y}_t^w = -\kappa\delta_2 u_t$$

  where δ_2 is specified in Galí Ch.5

- Thus, under the optimal discretionary policy, the central bank lets the output gap and inflation deviate from their targets in proportion to the current value of the cost-push shock.

- In the case of discretionary policy, both the output gap and inflation return to their zero initial value once the shock has vanished (i.e. one period after the shock).
- By contrast, under the optimal policy with commitment the deviations in the output gap and inflation from target persist well beyond the life of the shock, i.e. they display endogenous or intrinsic persistence.
- *Given that a zero inflation, zero output gap outcome is feasible once the shock has vanished, why does the central bank find it optimal to maintain a persistently negative output gap and inflation?*
- The reason is simple: by committing to such a response, the central bank manages to improve the output gap/inflation trade-off in the period when the shock occurs.
- In the case illustrated in Figure 5.1 it lowers the initial impact of the cost-push shock on inflation (relative to the discretionary case), while incurring smaller output gap losses in the same period.

- This is possible because of the forward-looking nature of inflation:

$$\pi_t = \kappa \widetilde{y}_t^w + \kappa \sum_{k=0}^{\infty} \beta^k E_t \widetilde{y}_{t+k}^w + u_t$$

- Hence, we see that the central can offset the inflationary impact of a cost push shock by lowering the current output gap $\widetilde{y}_t^w$, but also by committing to lower future output gaps.

- If credible, such "promises" will bring about a downward adjustment in the sequence of expectations $E_t \widetilde{y}_{t+k}^w$ for $k = 1; 2; 3, ..$(or, equivalently, a reduction in inflation expectations)

- As a result, and in response to a positive realization of the cost-push shock ut, the central bank may achieve any given level of current inflation t with a smaller decline in the output gap

- The output gap/inflation trade-off is improved by the possibility of commitment

Figure 5.1: Optimal Responses to a Transitory Cost-Push Shock

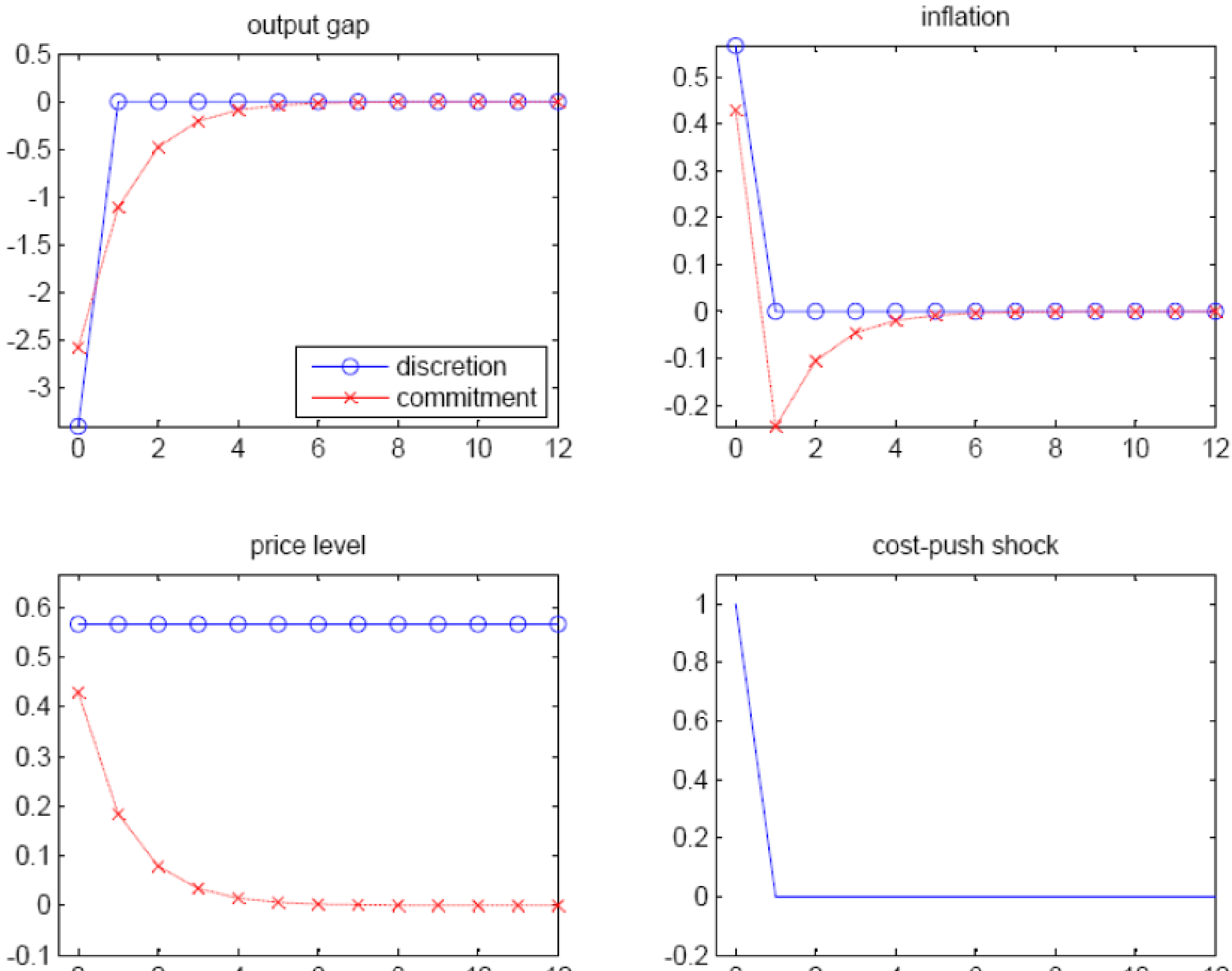
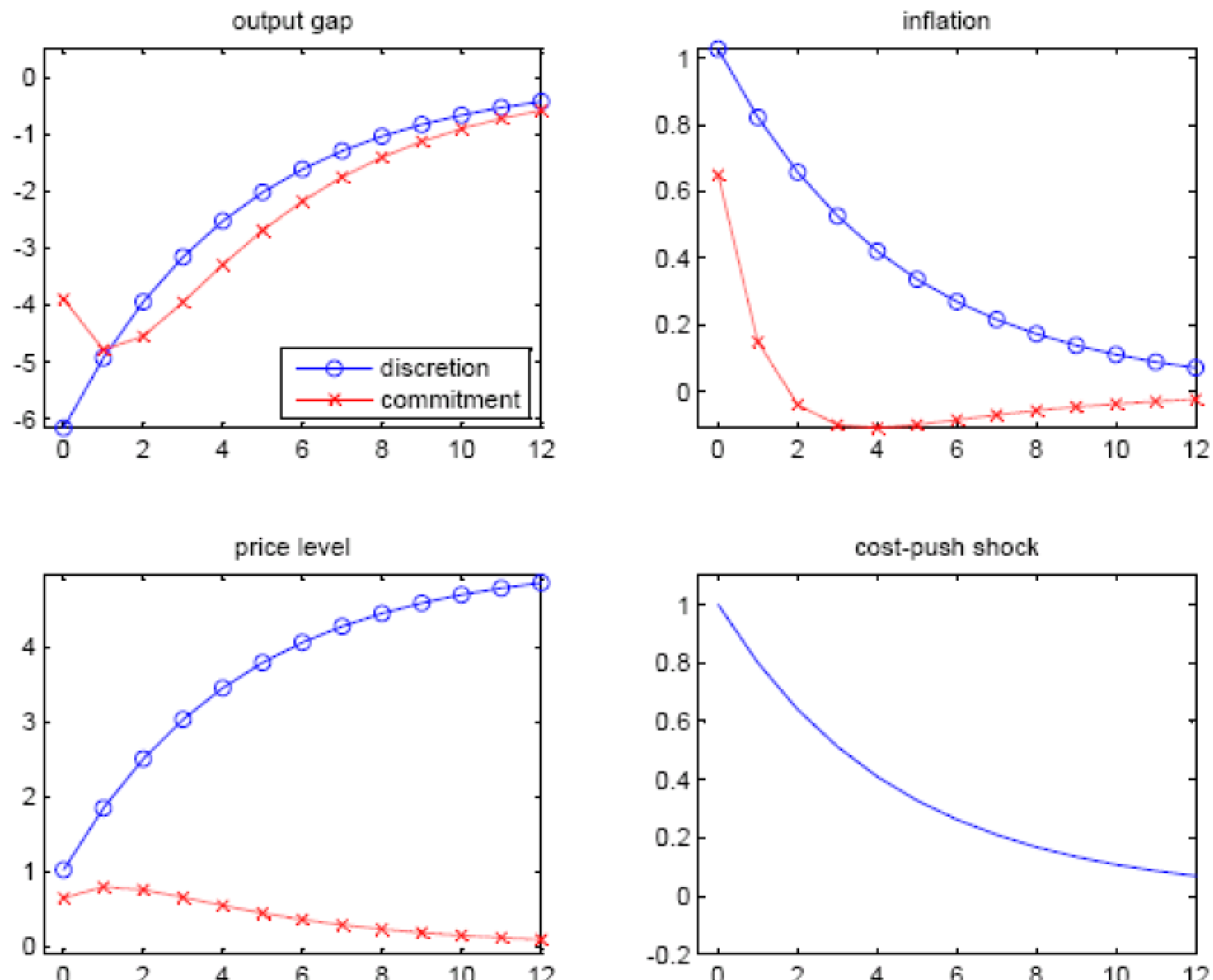


Figure 5.2 : Optimal Responses to a Persistent Cost Push Shock



- Given the convexity of the loss function in inflation and output gap deviations, the dampening of those deviations in the period of the shock brings about an improvement in overall welfare relative to the case of discretion

- This is because the implied benefits are not offset by the (relatively small) losses generated by the deviations in subsequent periods (and which are absent in the discretionary case).

- Summary:
- Commitment involves history dependence.
- This manages private-sector expectations in a favorable way
- The policy trade-off is improved

1.8 The importance of inflation expectations

- How does policy design depend on how inflation expectations are formed?
- Over the past year, globally, inflation has risen markedly given the effect of increases in oil and energy prices.
- This increase in inflation was accompanied by an increase in inflation expectations. Either measured by surveys or market measures such as the ones derived using inflation-linked bonds.

- There could be a concern that some will interpret this as evidence that central banks have lost concentration on their main target.

- But the increase in inflation expectations may not indicate a loss of confidence in the inflation target.

- Instead, it may be the result of households and firms learning that the world is in the midst of a period of substantial commodity price volatility which cannot be fully stabilized by monetary authorities.

- So the rise in expectations could indicate that households and firms are behaving rationally and paying attention to macroeconomic developments rather than losing confidence in the monetary framework.

- In reality it is very hard for us to know which of these hypotheses lies behind an increase in inflation expectations.

- But what would be the consequence of a de-anchoring of inflation expectations?

- $\Rightarrow$ the more expectations move, the more central banks need to increase policy rates in order to stabilize inflation.

- In the scenario that the public lose confidence policymaker´s commitment to the inflation target, such policymakers would need to be particularly aggressive in their policy response in order to demonstrate to those who have lost confidence that they were very wrong to do so.

- Let´s consider the economy's response to a cost push shock for 3 different ways of forming expectations.
- First let´s consider the case in which expectations are always equal to the inflation target.
- Perhaps this would approximates what would happen if agents had experienced a very long period of stable inflation, and had come to believe that inflation would never wander from the target.

The system of equilibrium conditions:

1)PC curve

$$\pi_t = \kappa \widetilde{y}_t^w + \beta \pi_t^e + u_t$$

2)IS curve

$$\widetilde{y}_t^w = E_t \widetilde{y}_{t+1}^w - \sigma^{-1}(i_t - \pi_t^e - r_t^t)$$

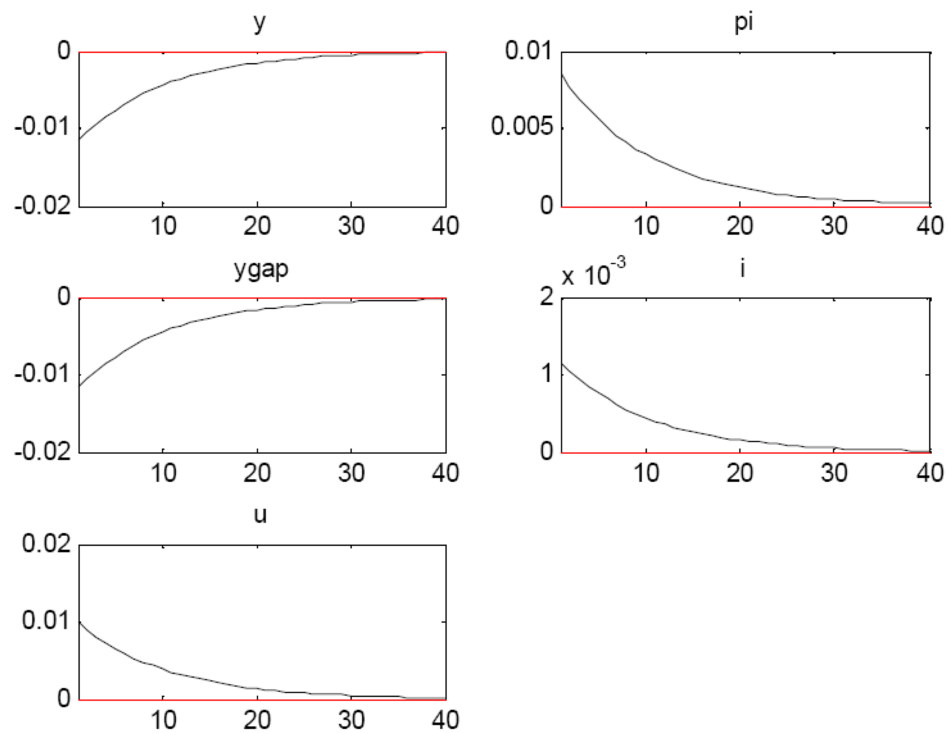
where the efficient rate of interest is defined as $r_t^t = \rho + \sigma \psi_{ya} E_t \Delta y_{t+1}^t$

3)A monetary policy rule - *simple Taylor rule:*

$$\hat{\imath}_t = \phi_y \widetilde{y}_t^w + \phi_\pi \pi_t$$

4) **Inflation expectations are "stuck"**

$$\pi_t^e = \pi^e = 0$$



THEORETICAL MOMENTS

VARIABLE	STD.	VARIANCE
y	0.0340	0.0012
pi	0.0195	0.0004
yt	0.0229	0.0005
rt	0.0023	0.0000
ygap	0.0267	0.0007
i	0.0034	0.0000
a	0.0229	0.0005
pie	0.0000	0.0000
u	0.0229	0.0005

- Now, let´s look at the case in which agents know the model and form expectations ‘rationally’.

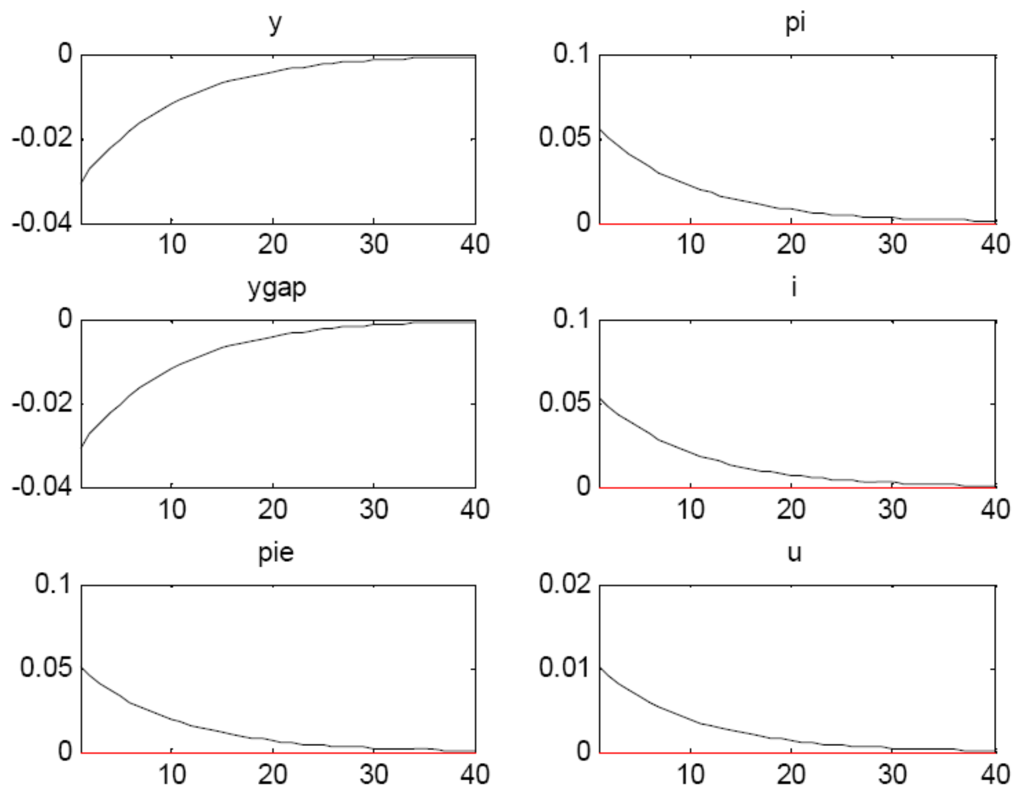
- The system of equilibrium conditions - as before but

- **Inflation expectations are rational**

$$\pi_t^e = E_t \pi_{t+1}$$

So the phillips curve is forward looking

$$\pi_t = \kappa \widetilde{y}_t^w + \beta E_t \pi_{t+1} + u_t$$



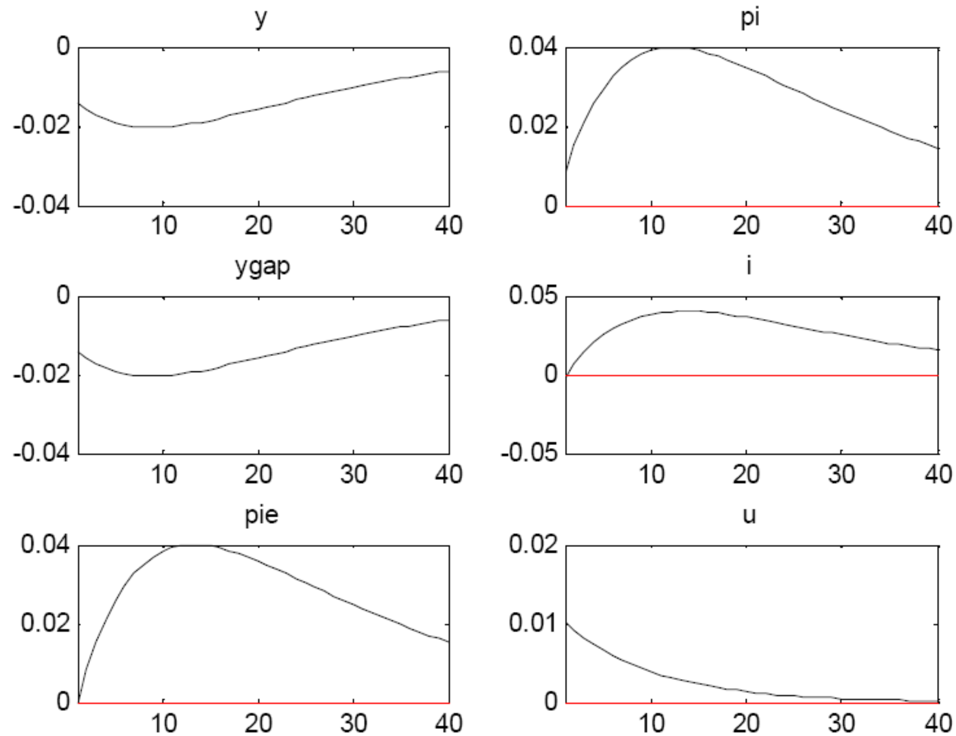
VARIABLE	STD.	VARIANCE
y	0.0734	0.0054
pi	0.1285	0.0165
yt	0.0229	0.0005
rt	0.0023	0.0000
ygap	0.0701	0.0049
i	0.1227	0.0151
a	0.0229	0.0005
pie	0.1157	0.0134
u	0.0229	0.0005

- Finally, let´s assume people use a simple 'random walk' 'heuristic' whereby expected inflation is always equal to last period's realized value.
- This might be a realistic alternative if we think agents expectations have been 'de-anchored' and they no longer think the inflation target is guiding monetary policy.
- We can also think about the case of inertial inflation expectation (Brazilian experience)
- **Inflation expectations are de-anchored**

$$\pi_t^e = \pi_{t-1}$$

So the phillips curve is backward looking

$$\pi_t = \kappa \widetilde{y}_t^w + \beta \pi_{t-1} + u_t$$



THEORETICAL MOMENTS

VARIABLE	STD.	VARIANCE
y	0.0992	0.0098
pi	0.1922	0.0370
yt	0.0229	0.0005
rt	0.0023	0.0000
ygap	0.0968	0.0094
i	0.1944	0.0378
a	0.0229	0.0005
pie	0.1922	0.0370
u	0.0229	0.0005

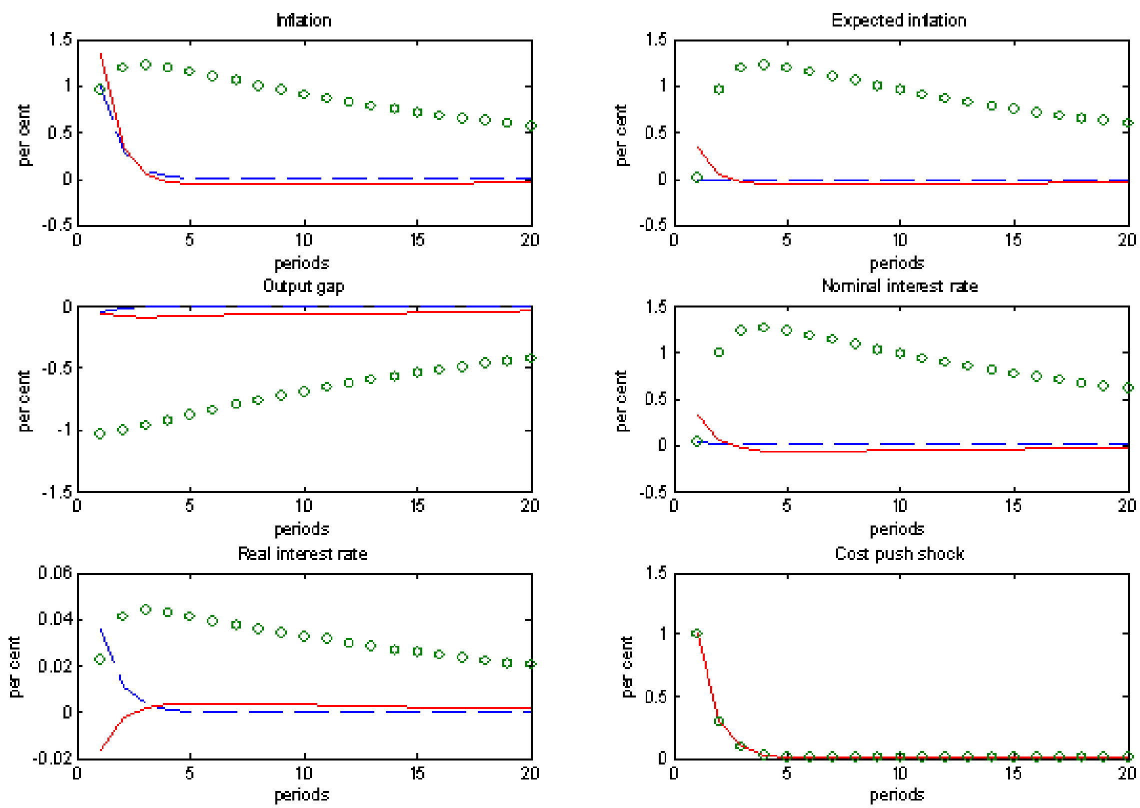
- Optimal policy under the different "heuristics"
- Can you guess what is the optimal policy under "stuck" expectations? (equal to the one under discretion, the difference is that now expectations will indeed be constant)
- The case of rational expectation is the case of commitment
- And the case of de-anchored expectation the problem is

$$\min E_0 \sum_{k=0}^{\infty} \beta^k \left[\varepsilon \left(\pi_t \right)^2 + \kappa \left(\tilde{y}_t^w \right)^2 \right]$$

s.t

$$\pi_t = \kappa \widetilde{y}_t^w + \beta \pi_{t-1} + u_t$$

- The figure below plots the economy's response to a cost push shock under the **optimal policy** for 3 different ways of forming expectations (Nikolov and Yates (2008)).
- The solid red (rational expectations) and the dashed blue (expectations equal to the target) lines are fairly close to one another.
- This is because the optimal policy is very good at stabilizing the expectations of 'rational' consumers and therefore their behavior is not very far from that of a consumer with fixed expectations.
- Nevertheless, there are small differences. Inflation rises by more and output falls by a larger amount when expectations are fully rational. Policy is tightened more aggressively too as measured by nominal short rates.
- In contrast, the economy with de-anchored expectations (the green circles) is very different.



Inflation
Expected inflation
Output gap
Nominal interest rate
Real interest rate
Cost push shock
per cent
periods

- The cost push shock increases inflation, leading to persistently higher inflation expectations.

- The only way to bring inflation back to target in this model is to keep interest rates high for a long time, generating a long-lived recession which gradually pulls inflation back to target after 5-6 years.

- The outcomes for the economy are much worse under de-anchored expectations. This point is underscored by the next graph.

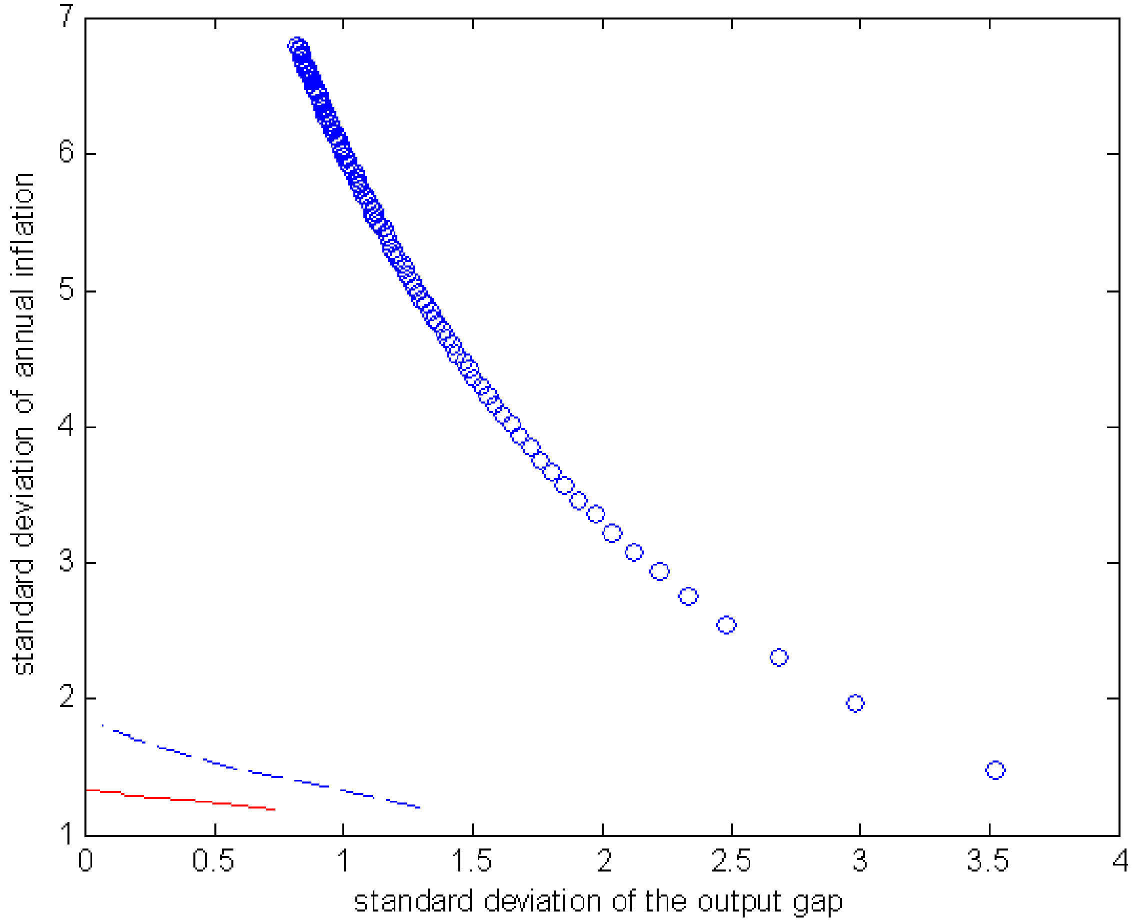
- Let´s suppose you delegate to a central bank and given it a loss function with certain weights on inflation and the output gap. This figure plots the combinations of inflation and output gap variability that are possible under the different expectations mechanisms.

- Each line is generated by varying the weight that policymakers put on real economy stabilization from 0.02 to 2.

- The solid red line was generated under the assumption that expectations are always equal to the target.

- The dashed blue line was computed under 'rational' expectations while the blue circles were generated under de-anchored expectations.

- What jumps out of this chart is how much greater is both inflation and output gap variability when expectations are de-anchored, regardless of policymakers' preferences.



7
6
5
4
3
2
1
0
0.5
1
1.5
2
2.5
3
3.5
4
standard deviation of annual inflation
standard deviation of the output gap

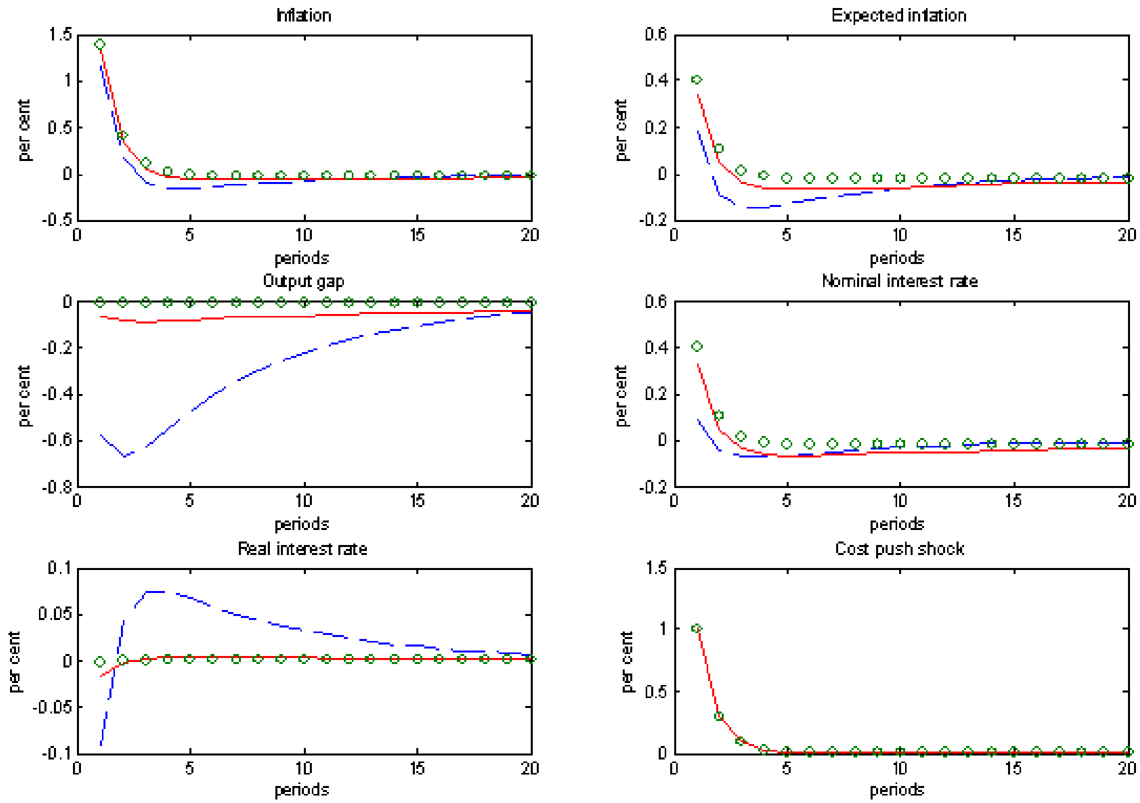
- In the figure bellow we assumed ‘rational expectations’ and examined how the reaction of the output gap and inflation to a cost push shock depended on the central bank’s preferences over output gap and inflation stability. (green circles – output gap weight $= 10$, red solid line – output gap weight $= 1$, dashed blue line – output gap weight $= 0.1$)
- In the next figure we repeat the exercise under the assumption of de-anchored expectations.

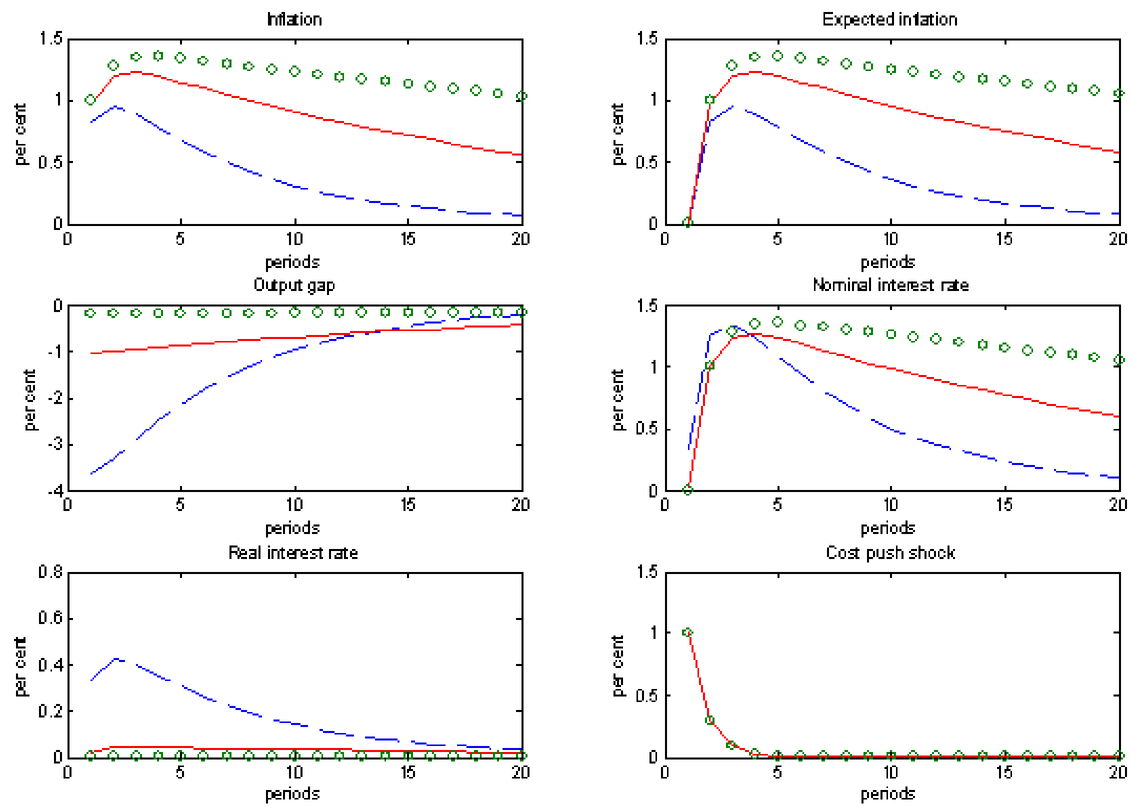
- When the central bank has a strong preference for output stabilization (the red solid lines and the green circles), expected and actual inflation remain significantly above target for a much longer period compared to the case of a low preference for output stabilization (the dashed blue line). This, of course, is done in order to minimize short run output volatility.

- A really committed 'inflation targeter' such as the one described by the dashed line increases real interest rates substantially more and, therefore, output falls dramatically.

- So does this mean that a central bank that does care about output volatility should avoid behaving like our 'inflation nutter' central bank? Maybe not.

- The central banks who care a lot about short term output volatility (the solid line and the green circles) may end up creating more output volatility in the long term by following a policy that risks de-anchoring inflation expectations.


Inflation
Expected inflation
Output gap
Nominal interest rate
Real interest rate
Cost push shock
per cent
periods



Inflation
Expected inflation
Output gap
Nominal interest rate
Real interest rate
Cost push shock
per cent
periods

- In contrast, the committed 'inflation targeter' (the dashed blue line), although creating a large recession in the short term, is more likely to keep inflation expectations under control, thus avoiding a loss of confidence by the public. Here is how this might work.

- Consider the viewpoint of a rational individual who behaves not like our automaton 'rational expectations' agents, but in a truly rational way, contemplating not just what inflation will be in the future, but what will be a good way to forecast it over the future.
- Suppose she believes that other consumers' expectations have been de-anchored, that they have given up on the inflation target.
- Suppose further that she believes that the central bank cares a lot about output volatility.
- She will then figure out that the economy's future adjustment path will be given by the green circles. Note how slowly inflation falls over time.
- This means that using the 'future inflation equals past inflation' rule-of-thumb is likely to work reasonably well. If the consumer has better things to do with her time than solve models like this one, she will probably fall in line with the others.

- This will then validate the 'de-anchored' expectations equilibrium, with the disastrous consequences for the output-inflation trade-off we demonstrated in the previous figure.

- Will our ‘committed inflation targeter’ avoid this unpleasant scenario of de-anchored expectations?

- Consider again the problem of a rational agent contemplating how to go about forecasting inflation when she believes that others’ expectations have been de-anchored.

- So if such an agent believes that others have lost faith in the target, should she switch to forecasting inflation using past inflation?

- The answer is probably ‘no’ if the central bank follows the policy prescriptions given by the dashed blue line. Such a policy-maker does not put much weight on stabilizing the real economy in the short run, and this naturally involves combating an inflationary shock by generating a big recession.

- Because of this, inflation falls relatively fast towards the target and therefore choosing the ‘inertial inflation’ ‘heuristic’ is likely to lead to large errors. Under this relatively harsh monetary policy, the forecasting behavior that leads to bad outcomes tends to get driven out, unlike in the case of the softer policy that worries more about the short term output losses from monetary policy actions.

- According to this analysis, a central bank might need to communicate very clearly to the public that any clear evidence of a de-anchoring of inflation expectations will lead to a significant tightening of policy
- And if this leads to short-term output losses, then this is a price he might want to pay to diminish future output/inflation trade-offs.