Problem 1) Consider a different version of the basic RBC model (with fixed labour supply and no real rigidities) presented in the lecture. In particular, assume that:

\[ u(C_t) = C_t - \kappa(C_t + v_t)^2 \]

where \( v_t \) is a white noise preference shock. Moreover assume that:

- \( Z_t = \bar{Z} = \frac{1-\beta}{\delta} \)
- \( \alpha = 1 \)
- \( \delta = 0 \)

(a) Derive the first order conditions.
(b) Using the method of undetermined coefficient derive the expression for consumption and capital as a function of the states.
(c) Draw (without the use of numerical methods) an impulse response to a one-time shock to \( v_t \).

Problem 2) Consider a different version of the basic RBC model (without real rigidities but with endogenous labour supply decision) presented in the lecture. In particular, assume that:

\[ U(C_t+i, L_t+i) = \theta (C_t+i L_t+i)^{1-\sigma} \]

where \( L_t \) denotes leisure (and, thus, \( L_t = 1 - N_t \)).

(a) Calculate the equilibrium conditions and the steady state (which value of \( \theta \) is consistent with \( \bar{N} = 0.257 \)).
(b) Simulate (numerically - using either the reds/solds algorithm or Dynare freeware) impulse responses of the endogenous variables in the model to a productivity shock for different values of \( \sigma \) and \( \rho \).
(c) How does the responses compare with the case of separable labour.

Problem 3) Going back to the separable utility case, assume that the per-period utility function is separable in consumption and leisure and logarithmic in consumption:

\[ U(C_{t+i}, L_{t+i}) = \log C_{t+i} + \theta \frac{L_{t+i}^{1-\varphi}}{1-\varphi} \]
add government spending to the model. In particular, assume that the resource constraint is now

\[ K_t = (1 - \delta)K_{t-1} + Y_t - C_t - X_t \]

where \( X_t \) is government spending.\(^1\) Assume that in the long run government spending is a constant fraction of GDP, say 0.2, but that government spending follows the stochastic process

\[ x_t = \rho x_{t-1} + \varepsilon_t. \]

Finally, assume that there are no shocks to technology.

(a) After deriving the deterministic steady-state, derive the log-linear equilibrium conditions of the model.

(b) Using the method of undetermined coefficients, derive the state-space representation of the model.

(c) Plot the impulse-response functions of the endogenous variables in the model to a one-time shock in \( x_t \) using the following combinations of \( \varphi \) and \( \rho \).\(^2\) What do your results suggest about government spending as a plausible source of business cycle?

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\(^1\) Note from the way we have written the budget constraint that we are implicitly assuming that government spending is financed by lump-sum taxation. Right? Are we also assuming that the government budget is balanced?

\(^2\) Note that, given that you have already calculated the state space presentation of the model, you can code the impulse responses manually, without the need of solution algorithms.