Problem Set 2
Macro II - November 2009
Bianca De Paoli
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Problem 1)
Consider the following model:

$$M_{t+1} = \beta \left(\frac{S_{t+1}C_{t+1}}{S_t C_t} \right)^{-\sigma} = \beta \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma}$$

where consumption follows an AR(1) process (i.e. we have an endowment economy):

$$c_{t+1} = (1-\phi)\overline{c} + \phi c_t + \varepsilon_{t+1}$$

a) Using second-order perthubation methods, compute the average equity risk premium (unconditional mean or stochastic steady state in Dynare) when $\beta = 0.99$, $\phi = 0.9$, $\sigma = 2$, $h = 0.8$ and a $\sigma_\varepsilon = 0.01$.

b) Compute the dynamics of the equity risk premium following a shock.

c) Analytically show that a second-order approximation to the risk premium can be written as

$$E_t(\hat{r}^y_{t+1} - \hat{r}_{t+1}) + \frac{1}{2} var_t(\hat{r}^y_{t+1})^2 = \frac{\sigma}{(1-h)} cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + o(\|\widehat{\xi}\|^3)$$

where $o(\|\widehat{\xi}\|^3)$ represents terms of order higher than 2.

c) If instead of using a second-order approximation we use a "log-normal approximation" would we obtain a constant risk premium?

Problem 2)
Phillips curve with indexation to steady-state inflation

Consider the Calvo model of staggered price setting in which, when firms cannot reset prices, their price is indexed to steady state inflation Π.

Firms maximize:

$$\max \sum_k E_t \theta^k Q_{t.t+k} \left[P_{t+k,t} Y_{t+k,t} - \Psi(Y_{t+k,t}) \right] = 0 \tag{1}$$

where $P_{t+k,t}$ denotes the price effective in period $t+k$ for a firm that last reset its price in period t.

Note that in periods in which firms cannot reset prices, we have

$$P_{t+k,t} = \Pi P_{t+k-1,t}$$

and only in period t this price is optimal

$$P_{t,t} = P_t^*$$

(a) Log-linearizing the price index $P = \left[\int_0^n p\left(z\right)^{1-\varepsilon} dz\right]^{\frac{1}{1-\varepsilon}}$ show that:

$$\hat{\pi}_t = (1-\theta)\left(\hat{p}_t^* - \hat{p}_{t-1}\right)$$

(b) Derive the first order condition of the firm that determines P_t^*.

(c) Log-linearize this condition and show that the Phillips curve can be written as

$$\hat{\pi}_t = \lambda \widehat{mc}_t + \beta E_t(\hat{\pi}_{t+1})$$

Problem 3)

Evaluating monetary policy rules

Assume that $U_t = E_t \sum\limits_{s=t}^{\infty} \beta^{s-t} \left[\frac{C_s^{1-\sigma}}{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi}\right]$, $Y_t = A_t N_t^{1-\alpha}$, and $\log(A_t) = a_t \sim AR(1)$

(a) Calculate the natural rate of output (as in Slides 5).

(b) Specify an interest rate rule that rule that mimics the optimal allocation.

(c) Assuming $\sigma = 1$, $\varphi = 1$, $\theta = 2/3$, $\rho_a = 0.9$, $\alpha = 1/3$, $\varepsilon = 6$ and $\sigma_\varepsilon = 0.01$, replicate (using Dynare or Reds and Solds) the output gap and inflation volatility in Table 4.1 for the cases in which the central bank is following a simple Taylor rule of the form

$$\hat{\imath}_t = \phi_y \hat{y}_t + \phi_\pi \pi_t$$

(d) For the case in which a Taylor rule is in terms of the output gap, i.e.

$$\hat{\imath}_t = \phi_y \tilde{y}_t + \phi_\pi \pi_t$$

and $\rho_a = 0$ calculate the the output gap and inflation volatility for the cases in which (i) $\phi_\pi = 1.5$ and $\phi_y = -0.1$ and (ii) $\phi_\pi = 0.5$ and $\phi_y = 0$. Explain the outcomes.