Problem 1)
Consider the following model:

\[ M_{t+1} = \beta \left( \frac{S_{t+1}C_{t+1}}{S_tC_t} \right)^{-\sigma} = \beta \left( \frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \]

where consumption follows an AR(1) process (i.e. we have an endowment economy):

\[ c_{t+1} = (1 - \phi)\bar{c} + \phi c_t + \varepsilon_{t+1} \]

a) Using second-order perthubation methods, compute the average equity risk premium (unconditional mean or stochastic steady state in Dynare) when \( \beta = 0.99, \phi = 0.9, \sigma = 2, h = 0.8 \) and a \( \sigma_e = 0.01 \).

b) Compute the dynamics of the equity risk premium following a shock.

c) Analytically show that a second-order approximation to the risk premium can be written as

\[ E_t(\hat{r}_{t+1}) + \frac{1}{2} var_t(\hat{r}_{t+1})^2 = \frac{\sigma}{(1-h)} \text{cov}_t(\hat{c}_{t+1}, \hat{r}_{t+1}) + o(\|\xi\|^3) \]

where \( o(\|\xi\|^3) \) represents terms of order higher than 2.

c) If instead of using a second-order approximation we use a "log-normal approximation" would we obtain a constant risk premium?

Problem 2)
Phillips curve with indexation to steady-state inflation
Consider the Calvo model of staggered price setting in which, when firms cannot reset prices, their price is indexed to steady state inflation \( \Pi \).

Firms maximize:

\[ \max_k \sum_k E_t \theta^k Q_{t,t+k} [P_{t+k,t} Y_{t+k,t} - \Psi(Y_{t+k,t})] = 0 \quad (1) \]

where \( P_{t+k,t} \) denotes the price effective in period \( t+k \) for a firm that last reset its price in period \( t \).

Note that in periods in which firms cannot reset prices, we have

\[ P_{t+k,t} = \Pi P_{t+k-1,t} \]

and only in period \( t \) this price is optimal

\[ P_{t,t} = P^*_t \]
(a) Log-linearizing the price index $P = \left[ \int_0^\infty p(z)^{1-\varepsilon}\,dz \right]^{1/\varepsilon}$ show that:

$$\hat{\pi}_t = (1 - \theta) (\hat{p}_t^* - \hat{p}_{t-1})$$

(b) Derive the first order condition of the firm that determines $P_t^*$.

(c) Log-linearize this condition and show that the Phillips curve can be written as

$$\hat{\pi}_t = \lambda \hat{\pi}_{t+1} + \beta \hat{E}_t(\hat{\pi}_{t+1})$$

**Problem 3)**

**Evaluating monetary policy rules**

Assume that $U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[ C_s^{1-\sigma} - \frac{N_s^{1+\varphi}}{1+\varphi} \right]$, $Y_t = A_t N_t^{1-\alpha}$, and $\log(A_t) = a_t \sim AR(1)$

(a) Calculate the natural rate of output (as in Slides 5).

(b) Specify an interest rate rule that mimics the optimal allocation.

(c) Assuming $\sigma = 1$, $\varphi = 1$, $\theta = 2/3$, $\rho_a = 0.9$, $\alpha = 1/3$, $\varepsilon = 6$ and $\sigma_s = 0.01$, replicate (using Dynare or Reds and Solds) the output gap and inflation volatility in Table 4.1 for the cases in which the central bank is following a simple Taylor rule of the form

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_y \pi_t$$

(d) For the case in which a Taylor rule is in terms of the output gap, i.e.

$$\hat{i}_t = \phi_y \hat{y}_t + \phi_y \pi_t$$

and $\rho_a = 0$ calculate the the output gap and inflation volatility for the cases in which (i) $\phi_x = 1.5$ and $\phi_y = -0.1$ and (ii) $\phi_x = 0.5$ and $\phi_y = 0$. Explain the outcomes.