Slides 1: The RBC Model
Analytical and Numerical solutions

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1 Theory of business cycles

Business Cycle Facts:

- Macroeconomic fluctuations vary in size and persistence
- Modern theory of business cycles assumes economy is perturbed by shocks which propagate into the economy
- Different output components have different properties in terms of economic fluctuations: Inventories, consumption of durables, resident investment are very volatile, while non-durable consumption, government expenditure and net exports are relatively stable.
- Size of upturns and downturns somewhat similar, but the former are more persistent (different this time around?)

Explaining the business cycles: need to modify neoclassical model of Macro I as follows

- Include shocks (to technology, government expenditure, preferences, monetary conditions, etc)
- Include fluctuations in employment (endogenising labour supply)
- Result is a "frictionless" real business cycle model based on microeconomic foundation
- Extensions to introduce real and nominal rigidities to explain empirical facts in asset prices and nominal variables (future lectures)

Solving the model:

- Solution to non-linear dynamic forward looking rational expectations stochastic models
- Traditional linearization approach
- Evaluation through impulse response analysis

Foundations: the basic Real Business Cycle model

The social planner's maximization problem

Basic problem: maximize lifetime utility, given resource constraint

$$\max E_t \sum_{i=0}^{\infty} \beta^i \left\{ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} \right\}$$

s.t.

$$Y_{t+i} = C_{t+i} + K_{t+i} - (1-\delta) K_{t+i-1} \tag{1}$$

$$Y_{t+i} = Z_{t+i} K_{t+i-1}^{\alpha} N_{t+i}^{1-\alpha} \tag{2}$$

$$N_{t+i} = 1 \tag{3}$$

$$\hat{z}_{t+i} = \rho \hat{z}_{t+i-1} + \varepsilon_{t+i}, \quad \varepsilon_t \sim i.i.d.N\left(0, \sigma^2\right) \tag{4}$$

- Y_t, C_t, K_t, Z_t are time t levels of output, consumption, capital and productivity, respectively. (**NB** Upper case are levels, lower case variables are logs and variables with hats are log-deviations from steady-state - more later)
- **NB** K_t represents the amount of capital available for production in period $t+1$ - it is an end of period stock.*
- N_t is a measure of labour input. This will be fixed at 1 for now, but we will analyse models with variable labour supply later.
- β is the subjective discount factor, α is a measure of the returns to scale of capital, σ is the coefficient of relative risk aversion or the inverse the elasticity of intertemporal substitution and δ is the rate of depreciation.

*Some frameworks adopt a start of period stock notation, that is, in this case, K_t represents the amount of capital available for production in period t. The reseource contrain is then $Y_t = C_t + K_{t+1} - (1-\delta) K_t$

- C_t and K_t are the planner's *choice* variables.
- Z_t and K_{t-1} are so called *state* variables, i.e. they are *predetermined.*
- Λ_t is the Lagrange multiplier associated with the resource constraint

First-order conditions:

$$C_t^{-\sigma} = \Lambda_t \tag{5}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_t} + 1 - \delta \right) \right] \tag{6}$$

$$Y_t = Z_t K_{t-1}^{\alpha} \tag{7}$$

$$Y_t = C_t + K_t - (1 - \delta_t) K_{t-1} \tag{8}$$

Alternatively, one can assume that agents can trade bonds:

$$Y_{t+i} + B_{t+i-1} R_{t+i} = C_{t+i} + B_{t+i} + K_{t+i} - (1-\delta) K_{t+i-1} \tag{9}$$

First-order conditions:

$$C_t^{-\sigma} = \Lambda_t \tag{10}$$

$$\Lambda_t = \beta E_t \left[\Lambda_{t+1} R_{t+1} \right] \tag{11}$$

$$\frac{\beta E_t[\Lambda_{t+1} R_{t+1}]}{\Lambda_t} = \frac{\beta E_t[\Lambda_{t+1} R^k_{t+1}]}{\Lambda_t} \tag{12}$$

where $R^k_{t+1} \equiv \alpha \frac{Y_{t+1}}{K_t} + 1 - \delta$

But does the introduction of bonds change the dynamics of the model?

All agents need is one asset to store savings from one period to the other.

Representative agents: bonds will be in zero net supply ($B_{t+i-1} = B_{t+i-1} = 0$) and all saving will be stored in the form of capital

Solving the deterministic steady state (DSS)

All expectations are realized and uncertainty is absent. How do we find the DSS?

The case of no growth: set $C_{t+i} = \bar{C}\ \forall i$, then combine and reduce equations such that we obtain variables as a function of only the deep parameters (α, β, δ). E.g. combining Euler equation and expression for R_t gives

$$\bar{R} = 1/\beta$$

("upperbar" denotes the steady-state value of the variables)

Using this in R_t and assuming $\bar{Z} = 1$, we obtain

$$\bar{K} = \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha} \right)^{\frac{1}{\alpha - 1}}.$$

Similarly we find

$$
\begin{aligned}
\bar{Y} &= \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} \\
\bar{I} &= \delta \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}} \\
\bar{C} &= \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{\alpha}{\alpha-1}} - \delta \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)^{\frac{1}{\alpha-1}} .
\end{aligned}
$$

- At this point we can turn to the data to find out e.g. long run level of $\frac{C}{Y}, \frac{I}{Y}$ and from these we can judge the value of deep parameters.

Solving the RBC model

Linearizing the model

- For some special case we can find reduce form solutions for the non-linear equilibrium conditions
- We could also simulate the model using numerical methods
- But solving the model explicitly can deliver better economic insights
- A log linear approximation to equilibrium allows the model to be solved analytically

Linearization:

A first order Taylor expansion:

- Taylor expansions approximate analytical functions around a fixed point, assuming that all their derivatives exist.

- In particular, we consider the case in which the fixed point constitutes the steady-state value of the variables.

- A first-order Taylor expansion of a function $F(X)$ is given by:

$$F(X) = F(\overline{X}) + F_X(\overline{X})(X - \overline{X}) + o(\|\xi - \overline{\xi}\|^2), \tag{13}$$

- where the term $o(\|\xi - \overline{\xi}\|)^2$ stands for terms of order higher than one

- and $F_X(\overline{X})$ is the first derivative of $F(.)$ evaluated at the steady-state value of X.

The log approximation:

- We have seen how to write the non-linear function $F(X)$ as a linear expression of $(X - \overline{X})$
- But macroeconomic models are often presents the system of equilibrium conditions in *log deviations* from steady-state.
- That is, the models are expressed in terms of $\hat{x} = \log\left(X/\overline{X}\right)$ or $\hat{x} = x - \overline{x}$, where $x = \log(X)$
- We should note that the logarithmic function must also be approximated to first-order.

- In order to do so we use the following identity:

$$X = \exp\{x\},$$

- The first-order expansion to the above equation can be written as:

$$X = \exp(\overline{x}) + \exp(\overline{x})(x - \overline{x}) + o(\|\xi - \overline{\xi}\|^2), \tag{14}$$

or, alternatively,

$$\begin{aligned} \frac{X - \overline{X}}{\overline{X}} &= \frac{X - \exp(\overline{x})}{\exp(\overline{x})} = (x - \overline{x}) + o(\|\xi - \overline{\xi}\|^2) \\ &= \hat{x} + o(\|\xi - \overline{\xi}\|^2) \end{aligned} \tag{15}$$

- We can rewrite equation 13 as:

$$F(X) = F(\overline{X}) + F_X(\overline{X})\overline{X}\hat{x} + o(\|\widehat{\xi}\|^2) \tag{16}$$

- We have seen how to write the non-linear function $F(X)$ as a linear expression of $\hat{x}$

The model's linearized conditions:

$$-\sigma\hat{c}_t = \hat{\lambda}_t \tag{17}$$

$$\hat{\lambda}_t = E_t\left[\hat{\lambda}_{t+1} + \hat{r}^k_{t+1}\right] \tag{18}$$

$$\bar{R}\hat{r}^k_{t+1} = \alpha\frac{\bar{Y}}{\bar{K}}\left(\hat{y}_{t+1} - \hat{k}_t\right) \tag{19}$$

$$\hat{y}_t = \hat{z}_t + \alpha\hat{k}_{t-1} \tag{20}$$

$$\frac{\bar{Y}}{\bar{K}}\hat{y}_t = \frac{\bar{C}}{\bar{K}}\hat{c}_t + \hat{k}_t - (1-\delta)\,\hat{k}_{t-1} \tag{21}$$

And here we can use the steady state conditions derived above, and summarize the dynamics as:

$$\sigma E_t \left[\hat{c}_{t+1} - \hat{c}_t\right] = E_t \left[\hat{r}^k_{t+1}\right] \tag{22}$$

$$\hat{r}^k_t = \alpha\beta y_k \left(\hat{z}_t - (1-\alpha)\hat{k}_{t-1}\right) \tag{23}$$

$$y_k(\hat{z}_t + \alpha\hat{k}_{t-1}) = (y_k - \delta)\hat{c}_t + \hat{k}_t - (1-\delta)\,\hat{k}_{t-1} \tag{24}$$

where $y_k = \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)$

The rational expectation solution

Solving linear difference equations:

We can write the model in the general form

$$AE_t\mathbf{y}_{t+1} = B\mathbf{y}_t + C\mathbf{x}_t$$

where:

- $\mathbf{x}_t$ is the vector of exogenous shocks
- $\mathbf{y}_t$ is the vector of endogenous variables
- A, B and C are general matrices of structural parameters

Methods to solve linear rational expectations systems:

- Normally relies on numerical methods
  - Summary in McCallum, B. (1998)
  - Approaches of Klein (1997), King and Watson (1995), Sims
- McCallum, B. (1983), Uhlig (1997) and Blinder and Peseran (1995) use methods of undetermined coefficients (guess and verify)
- Existence and the uniqueness of a solution - Blanchard and Kahn (1980)

Analytical solution: the state space representation:

Solutions for all variables in terms of state variables

$$\hat{k}_t = c_{kk}\hat{k}_{t-1} + c_{kz}\hat{z}_t \tag{25}$$

$$\hat{c}_t = c_{ck}\hat{k}_{t-1} + c_{cz}\hat{z}_t \tag{26}$$

$$\hat{r}^k_t = c_{rk}\hat{k}_{t-1} + c_{rz}\hat{z}_t \tag{27}$$

Example - method of undetermined coefficients:

Assume:

- $\delta = 0$ (no depreciation) and
- $\sigma = 1$(log utility)

The system of equilibrium conditions becomes

$$E_t\left[\hat{c}_{t+1}-\hat{c}_t\right]=E_t\left[\hat{r}^k_{t+1}\right] \tag{28}$$

$$\hat{r}^k_t=(1-\beta)(\hat{z}_t-(1-\alpha)\hat{k}_{t-1}) \tag{29}$$

$$(1-\beta)(\hat{z}_t+\alpha\hat{k}_{t-1})=(1-\beta)\hat{c}_t+\alpha\beta(\hat{k}_t-\hat{k}_{t-1}) \tag{30}$$

Or, given that $E_t\left[\hat{z}_{t+1}\right]=\rho\hat{z}_t$, the system can be written as

$$E_t\left[\hat{c}_{t+1}-\hat{c}_t\right]=(1-\beta)\left(\rho\hat{z}_t-(1-\alpha)\hat{k}_t\right) \tag{31}$$

$$(1-\beta)(\hat{z}_t+\alpha\hat{k}_{t-1})=(1-\beta)\hat{c}_t+\alpha\beta(\hat{k}_t-\hat{k}_{t-1}) \tag{32}$$

So, we can guess a formulation for c_t as a function of the states, such as

$$\hat{c}_t = c_{cz}\hat{z}_t + c_{ck}\hat{k}_{t-1} \tag{33}$$

and find the coefficients c_{cz} and c_{ck} by plugging this expression into the the above system.

From equation 32

$$\hat{k}_t = \frac{1-\beta}{\alpha\beta}(1-c_{cz})\hat{z}_t + \frac{\alpha - c_{ck}(1-\beta)}{\alpha\beta}\hat{k}_{t-1} \tag{34}$$

And eliminating $\hat{k}_t$ from 31

$$\begin{aligned} & c_{cz}(\rho-1)\hat{z}_t + c_{ck}\frac{1-\beta}{\alpha\beta}(1-c_{cz})\hat{z}_t + \frac{\alpha c_{ck} - c_{ck}^2(1-\beta)}{\alpha\beta}\hat{k}_{t-1} - c_{ck}\hat{k}_{t-1} \\ = \; & (1-\beta)\left(\rho\hat{z}_t - (1-\alpha)\frac{1-\beta}{\alpha\beta}(1-c_{cz})\hat{z}_t - \frac{\alpha - c_{ck}(1-\beta)}{\alpha\beta}(1-\alpha)\hat{k}_{t-1}\right) \end{aligned}$$

Equalizing the RHS and LHS coefficients in $\hat{k}_{t-1}$

$$-c_{ck}^2 + [\alpha - (1-\beta)(1-\alpha)]\, c_{ck} + \alpha(1-\alpha) = 0$$

- Pick the solution that guarantees $c_{kk} < 1$. As shown Campbell (1994), this is given by the positive root of the above equation.

- After finding c_{ck}, we can then follow the same approach to find c_{cz}

- As shown in Campbell (1994), this solution leads to a very weak propagation mechanism of shocks (no persistence) (*See discussion O&R*).

Impulse response analysis

With the state space representation for $\hat{c}_t$, we can use the other equations to obtain a full state space representation for the model - of the form:

$$\hat{k}_t = c_{kk}\hat{k}_{t-1} + c_{kz}\hat{z}_t \tag{35}$$

$$\hat{c}_t = c_{ck}\hat{k}_{t-1} + c_{cz}\hat{z}_t \tag{36}$$

$$\hat{r}_t^k = c_{rk}\hat{k}_{t-1} + c_{rz}\hat{z}_t \tag{37}$$

and given the evolution of the exogenous variable

$$\hat{z}_t = \rho\hat{z}_{t-1} + \varepsilon_{t+i}, \quad \varepsilon_t \sim i.i.d.N\left(0, \sigma^2\right) \tag{38}$$

Can then express solutions to all variables in terms of ARMA representation:

$$\hat{z}_t = \frac{1}{1-\rho L}\varepsilon_t \qquad \text{AR(1)} \tag{39}$$

$$\hat{k}_t = \frac{c_{kz}}{(1-c_{kk}L)(1-\rho L)}\varepsilon_t \qquad \text{AR(2)} \tag{40}$$

$$\hat{c}_t = \frac{c_{cz}+(c_{ck}c_{kz}-c_{cz}c_{kk})L}{(1-c_{kk}L)(1-\rho L)}\varepsilon_t \qquad \text{ARMA(2,1)} \tag{41}$$

$$\hat{r}_t^k = \frac{c_{rz}+(c_{rk}c_{kz}-c_{rz}c_{kk})L}{(1-c_{kk}L)(1-\rho L)}\varepsilon_t \qquad \text{ARMA(2,1)} \tag{42}$$

to see how each variable react to shock in each period

Details for the ARMA representation

Eg $\hat{k}_t$

$$\begin{aligned}
\hat{k}_t &= c_{kk}\hat{k}_{t-1} + c_{kz}\hat{z}_t \\
&= (1 - Lc_{kk})^{-1} c_{kz}\hat{z}_t = (1 - Lc_{kk})^{-1}(1 - Lc_{kk})^{-1} c_{kz}\varepsilon_t
\end{aligned}$$

why is this an AR(2)? Mechanically - it contains L^2.

Illustration:

$$\begin{aligned}
\hat{k}_t &= c_{kk}\hat{k}_{t-1} + c_{kz}\rho\hat{z}_{t-1} + c_{kz}\varepsilon_t \\
\hat{k}_{t-1} &= c_{kk}\hat{k}_{t-2} + c_{kz}\hat{z}_t \\
\hat{k}_t &= (\rho + c_{kk})\hat{k}_{t-1} - c_{kk}\rho\hat{k}_{t-2} + c_{kz}\varepsilon_t \text{ - AR(2)}
\end{aligned}$$

Numerical methods:

King and Watson algorithm - MATLAB REDS-SOLDS code

REDS-SOLDS is a package of Matlab codes written to solve rational expectations models numerically. It takes as input a model written in the form:

$$AE_t\mathbf{y}_{t+1} = B\mathbf{y}_t + C\mathbf{x}_t$$

where:

- $\mathbf{x}_t$ is the vector of exogenous shocks
- $\mathbf{y}_t$ is the vector of endogenous variables

Moreover:

- $\mathbf{y}_t$ is ordered so that variables that are predetermined appear last in a subvector $\mathbf{k}_t$[†]
- we denote $NY = dim(\mathbf{y}_t)$, $NX = dim(\mathbf{x}_t)$, and $NK = dim(\mathbf{k}_t)$
- in the program, we input A, B, C, NY, NX, NK

[†]Eg: lagged variables are preditermined. Or if you define K_t represents the amount of capital available for production in period t, then k_t is also a predetermined variable.

- The program REDS.M reduces the system, i.e., transforms it so that it contains a non-singular subsystem that can be solved and turned into a solution of the whole model.

- This whole solution operation is performed by SOLDS.M, whose output are the matrices D, F, G, and H in:

$$
\begin{aligned}
\mathbf{y}_t &= D\mathbf{k}_t + F\mathbf{x}_t \\
\mathbf{k}_{t+1} &= G\mathbf{k}_t + H\mathbf{x}_t
\end{aligned}
$$

- So the solution delivers a state space representation of the model

Together, REDS.M and SOLDS.M are a simplified version of a package of codes written by Robert King and Mark Watson, implementing the algorithms described in their paper "System Reduction and Solution Algorithms for Singular Linear Difference Systems Under Rational Expectations" (mimeo, 1995).

We can use D, F, G, and H as inputs to compute impulse responses using the m-function:

$$IRF(SHOCK, NIR, D, F, G, H)$$

- the first entry specifies the impulse (i.e., the component of $\mathbf{x}_t$ to which the system is responding)
- NIR is the number of periods for impulse-response computation
- The output of $IRF.M$ is a NY x NIR matrix in which each row corresponds to the path of the corresponding component of $\mathbf{y}_t$ along the NIR periods.

Userguide and files can be found in Woodford's webpage[‡]

[‡]http://www.columbia.edu/~mw2230/Tools/
Ulhlig also has a toolkit for solving RE models in his web: http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

Example:

Summary of model in log linear terms

$$\sigma E_t \left[\hat{c}_{t+1} - \hat{c}_t\right] = E_t \left[\hat{r}^k_{t+1}\right] \tag{43}$$

$$\hat{r}^k_t = \alpha\beta y_k \left(\hat{z}_t - (1-\alpha)\hat{k}_{t-1}\right) \tag{44}$$

$$y_k(\hat{z}_t + \alpha\hat{k}_{t-1}) = (y_k - \delta)\hat{c}_t + \hat{k}_t - (1-\delta)\,\hat{k}_{t-1} \tag{45}$$

where $y_k = \left(\frac{\frac{1}{\beta} - 1 + \delta}{\alpha}\right)$

MATLAB file (instructions and example):

```
% Specify parameter values:

alpha = 0.3;

sigma = 1.0;

rho = 0.95;

beta = 1/1.01;

delta = 0.025;

% Constructed parameters

y_k=((1/beta)-1+delta)/alpha;
```

```
% Specify model - matrices A, B and C
% Dimensions
NY=6;
NK=2;
NX=1;
A = zeros(NY,NY);
B = zeros(NY,NY);
C = zeros(NY,NX);
% Can enter matrix directly in the code, or organize them by indexing
  the variables
```

```
% index variables - with pre-determined variables last
% 1st) index endogenous variables:
ic=1;
ir=2;
ik=3;
iz=4;
% 2nd) index pre-determined variables:
iklag=5;
izlag=6;
% 3rd) index exogenous variables (shocks):
eps=1;
```

```
% Model Equations
% Euler Equation
A(1,ic)=1;
B(1,ic)=1;
A(1,ir)=-sigma^-1;
% Marginal product of capital
B(2,ir)=-1;
B(2,iz)=alpha*beta*y_k;
B(2,ik)=-alpha*beta*y_k*(1-alpha);
```

```
 % Capital accumulation equation
B(3,ik)=-1;
B(3,iz)=y_k;
B(3,iklag)=y_k*alpha+(1-delta);
B(3,ic)=-y_k+delta;
 % Identity for productivity process
B(4,iz)=-1;
C(4,eps)=1;
B(4,izlag)=rho;
```

```
 % Lag identity for k
A(5,iklag)=1;
B(5,ik)=1;
 % Lag identity for z
A(6,izlag)=1;
B(6,iz)=1;
```

% Load solution program

% program for REDUCTION OF DYNAMIC SYSTEMS

reds;

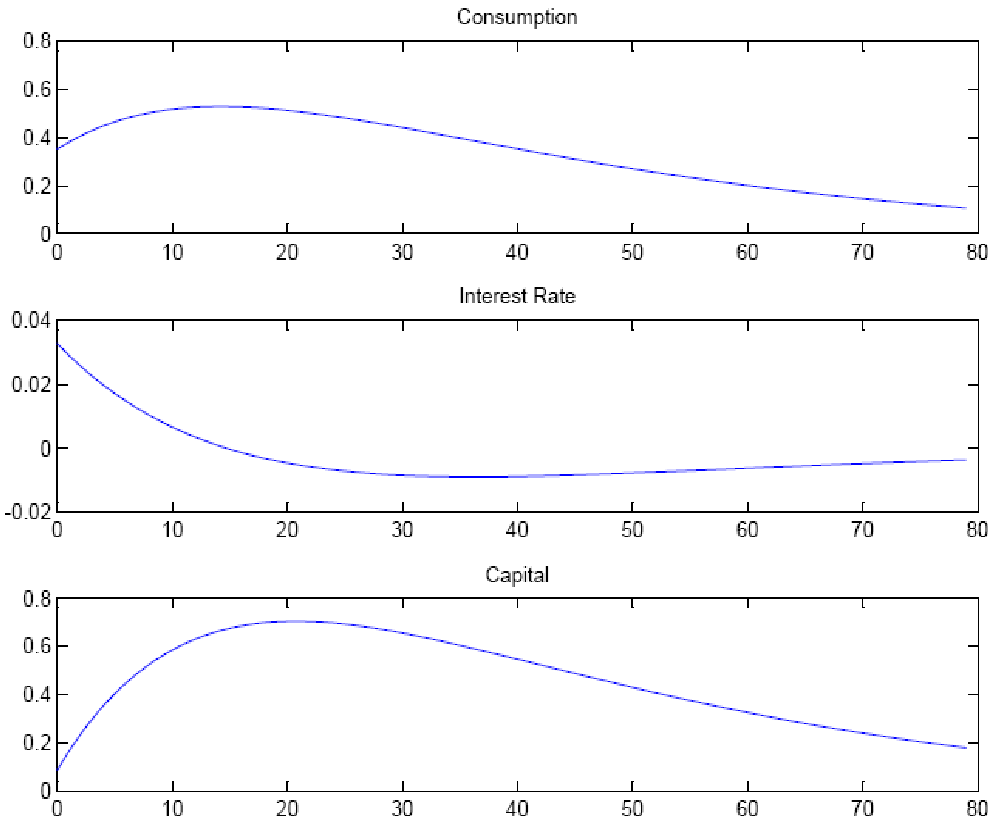
% (the program checks for solvability- that is, checks if |Az-B| is identically null)

% program for SOLUTION OF DYNAMIC SYSTEMS

solds;

% (the program obtain the final expressions for C, D, E and F)

```
% Plot impulse responses under both policy rules

NIR=80;

lead = 0:(NIR-1);

IMP = irf (eps, NIR, D, F, G, H);

figure ('Name', 'Responses to productivity shock')

subplot (311), plot (lead, IMP (ic, :)), title ('Consumption')

subplot (312), plot (lead, IMP (ir, :)), title ('Interest Rate')

subplot (313), plot (lead, IMP (ik, :)), title ('Capital')
```


Consumption
Interest Rate
Capital

Capital: Productivity shock => higher marginal product of capital => capital cannot jump (predetermined variable - see capital accumulation equation) => capital slowly goes up and then down

Return on capital: higher marginal product of capital implies higher return on capital => but as capital increases interest, marginal product of capital and its rental return falls => hump in the response of capital implies return on capital undershooting => Similarly for interest rates

Consumption: higher productivity and output leads to an increase in consumption => and as long as interest rates are falling, and so is the marginal cost of consuming today relative to tomorrow, consumption is increasing => only when interest rates start increasing, consumption starts coming back to steady state => hump in the response of capital implies an interest rate undershooting, which in turn implies a hump in consumption

Perturbation methods

- Userguide and files can be found in Dynare's webpage:

http://www.cepremap.cnrs.fr/dynare/

- Can write the model in non-linear form
- Program derives a first (or second) order approximation of the model
- For details on Perturbation methods, see:
- Kenneth Judd. 1996. "Approximation, Perturbation, and Projection Methods in Economic Analysis". 511-585. Hans Amman, David Kendrick, and John Rust. Handbook of Computational Economics. 1996. North Holland Press.

Simulating using Dynare

The following code solves the above model in Dynare.

// Variable declaration

// endogenous variables listed by ‘var’, exogenous variables listed by ‘varexo’ commands

var C, K, Z, R, Y, K_Y, I_Y, C_Y, I, MU;

varexo e;

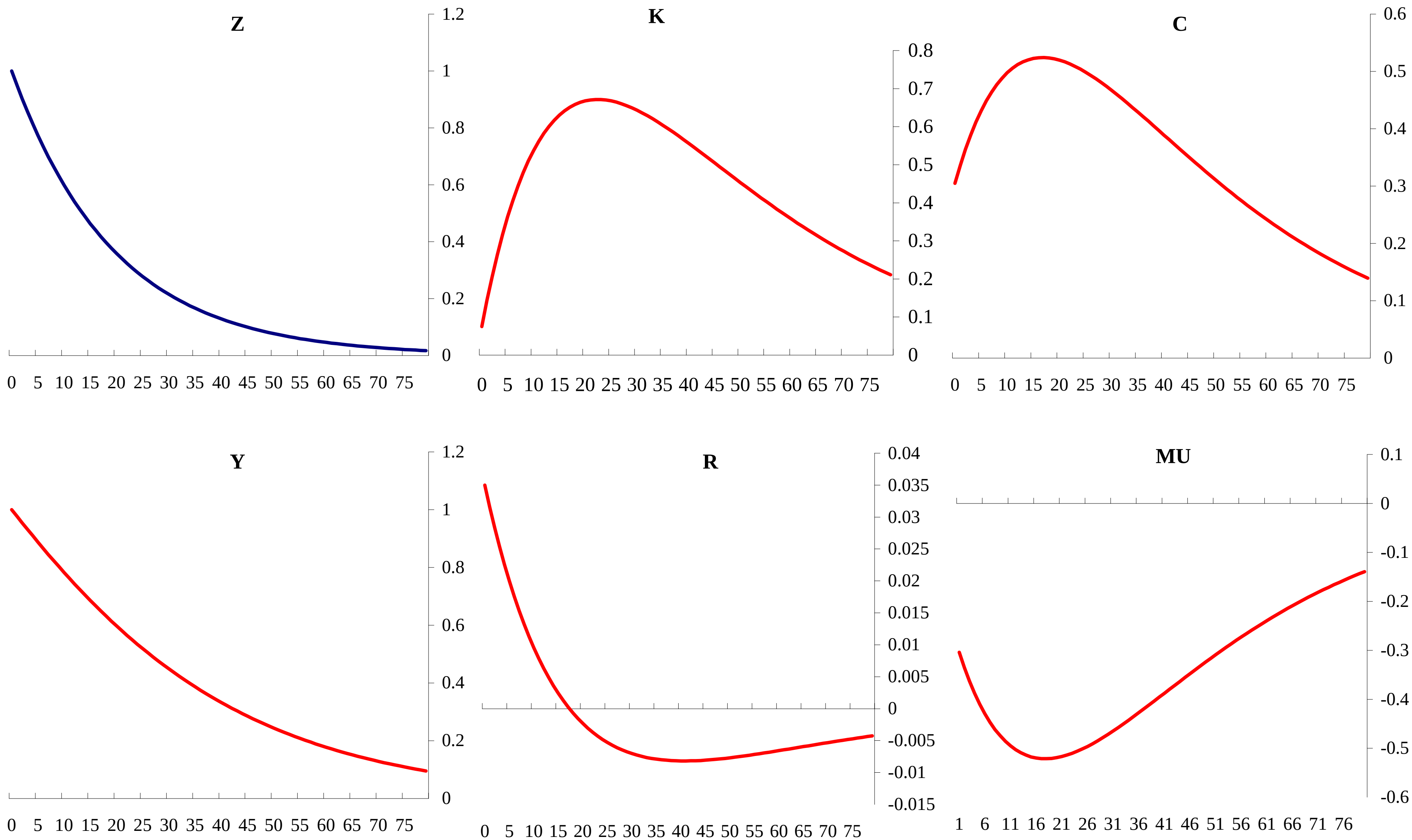
```
// Parameter declaration and calibration

// List parameters

parameters beta, sigma, rho, delta, alpha, Zbar;

// Calibration of Parameters (in quarterly units)

alpha = 0.3;

sigma = 1.0;

rho = 0.95;

beta = 1/1.01;

delta = 0.025;

Zbar = 1;
```

// Model declaration

```
model;

K = Z*K(-1)^alpha - C + (1-delta)*K(-1);

C^(-sigma) = (beta*C(+1)^(-sigma))*(1 + alpha*Z(+1)*K^(alpha-1) - delta);

Z = Zbar^(1-rho)*Z(-1)^rho*exp(e);

R = 1 + alpha*Z*K(-1)^(alpha-1) - delta;

Y = Z*K(-1)^alpha;

MU = C^(-sigma);

I =K-(1-delta)*K(-1);

K_Y = K/Y;

C_Y = C/Y;
```

```
I_Y = I/Y;
end;
// Steady-state values
initval;
Z = 1;
K = ((1/beta - (1-delta))/(Z*alpha))^(1/alpha-1);
C = Z*K^alpha - delta*K;
R = 1/beta;
MU = C^(sigma);
Y = Z*K^alpha;
I = delta*K;
```

```
K_Y = K/Y;

C_Y = C/Y;

I_Y = I/Y;

e = 0;

end;

steady;

// Shock declaration

shocks;

var e = 0.01^2;

end;

stoch_simul(order=1, irf=80);
```


Z
1.2 1 0.8 0.6 0.4 0.2 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
K
0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
C
0.6 0.5 0.4 0.3 0.2 0.1 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
Y
1.2 1 0.8 0.6 0.4 0.2 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
R
0.04 0.035 0.03 0.025 0.02 0.015 0.01 0.005 0 -0.005 -0.01 -0.015
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
MU
0.1 0 -0.1 -0.2 -0.3 -0.4 -0.5 -0.6
1 6 11 16 21 26 31 36 41 46 51 56 61 66 71 76

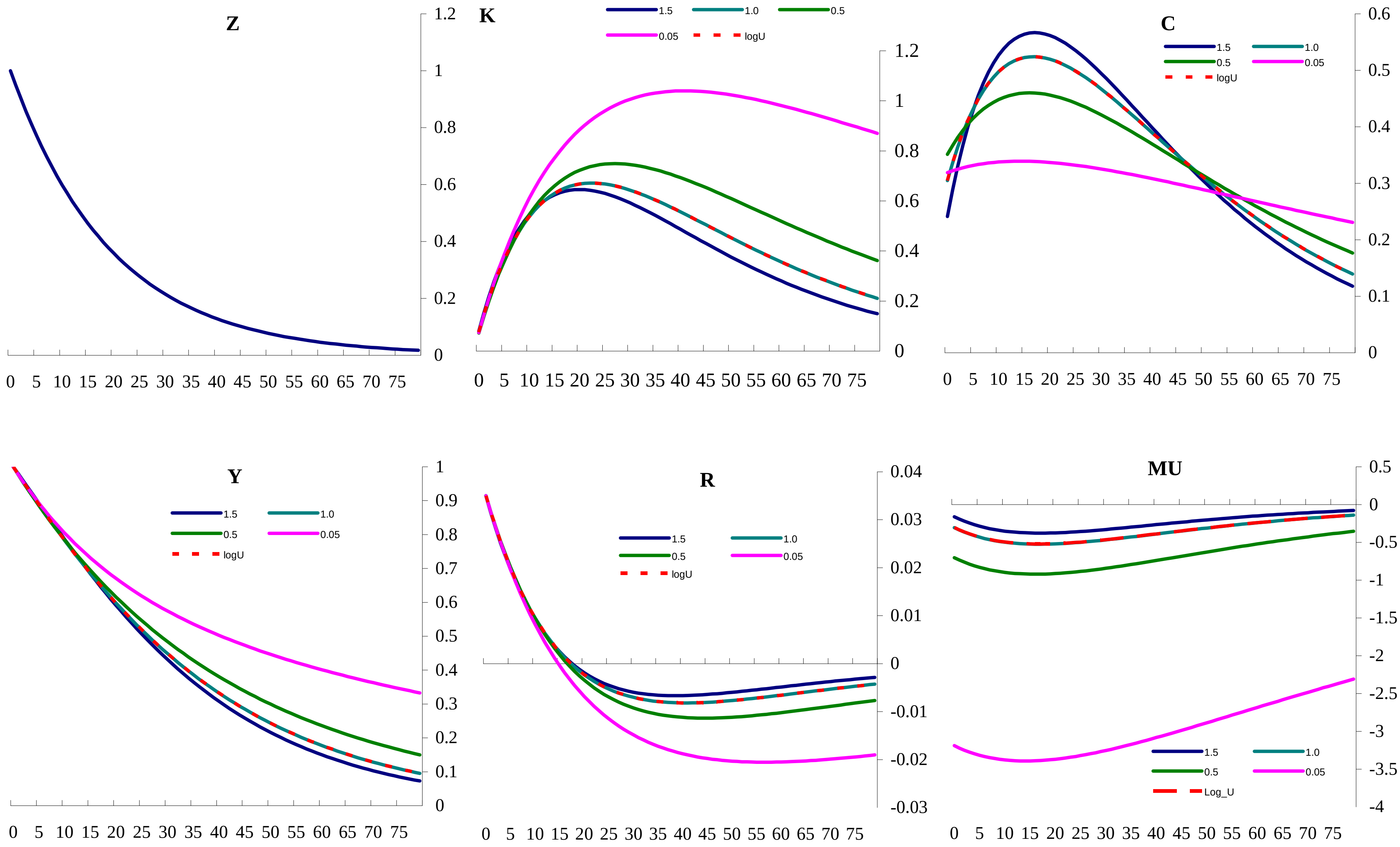
Varying the elasticity of intertemporal substitution

Calibration:

$$\alpha = 0.3; \;\; \delta = 0.025$$

But now we vary σ to see what happens to the responses.

$$\sigma^{-1} = [0.05, 0.5, 1.0, 1.5]$$


Z
1.2 1 0.8 0.6 0.4 0.2 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
K
1.5 1.0 0.5 0.05 logU
1.2 1 0.8 0.6 0.4 0.2 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
C
1.5 1.0 0.5 0.05 logU
0.6 0.5 0.4 0.3 0.2 0.1 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
Y
1.5 1.0 0.5 0.05 logU
1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
R
1.5 1.0 0.5 0.05 logU
0.04 0.03 0.02 0.01 0 -0.01 -0.02 -0.03
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75
MU
1.5 1.0 0.5 0.05 Log_U
0.5 0 -0.5 -1 -1.5 -2 -2.5 -3 -3.5 -4
0 5 10 15 20 25 30 35 40 45 50 55 60 65 70 75

Possible homework:

- Replicate the linearization and the method of undetermined coefficients

- Arrive at the ARMA representation and illustrate the autoregressive process

- Simulate the model with preference shocks - ie utility $\frac{\varepsilon_{t+i} C_{t+i}^{1-\sigma}}{1-\sigma}$