

# Monetary Policy and Welfare in a Small Open Economy\*

Bianca De Paoli<sup>‡</sup>

## Abstract

This paper analyzes optimal monetary policy in a small open economy featuring monopolistic competition and nominal rigidities. It shows that the utility-based loss function for this economy can be written as a quadratic expression of domestic inflation, output gap and real exchange rate. The presence of an internal monopolistic distortion and a terms of trade externality drives optimal policy away from domestic inflation targeting and affects the optimal level of exchange rate volatility. When domestic and foreign goods are close substitutes for each other, the optimal policy rule implies lower real exchange rate volatility than a domestic inflation targeting regime. The reverse is true when the elasticity of substitution between goods is low.

*JEL Classification:* F41, E52, E58 and E61. *Keywords:* Welfare, Loss Function, Terms of Trade Externality, Optimal Monetary Policy, Small Open Economy.

First Draft: October 2003

Current Version: September 22, 2008

---

\*I would like to thank Gianluca Benigno, Anna Lipinska and Giancarlo Corsetti (the editor) for many helpful suggestions on the paper. I also thank two anonymous referees, Kosuke Aoki, Jordi Galí, Nobu Kiyotaki, and Jens Søndergaard for valuable comments. This work was largely carried out before the author joined the Bank of England, and the views expressed in this paper are those of the author and do not necessarily reflect the views of this institution.

<sup>†</sup>Bank of England, London School of Economics and CEP, London, United Kingdom.

<sup>‡</sup>Correspondence to: Bianca De Paoli, Bank of England, Threadneedle Street, London EC2R 8AH, United Kingdom. E-mail: bianca.depaoli@bankofengland.co.uk

# 1 Introduction

At the heart of the policy debate in open economies lays the question of whether central banks should react solely to fluctuations in domestic output and inflation, or whether there is a role for stabilizing international relative prices. While some academic studies show that policy objectives in an open economy can be isomorphic to the ones in a closed economy (e.g. Clarida, Gali and Gertler (2001) and Gali and Monacelli (2005)), other contributions have stressed that welfare in open economies may be affected by a terms of trade externality (e.g. Obstfeld and Rogoff (1998) and Corsetti and Pesenti (2001)). The current paper focuses on the implications of this externality for optimal monetary policy in a small open economy setting. We derive a welfare-based loss function for this economy, compute the optimal monetary policy plan, and rank the performance of simple policy rules. Our analysis illustrates how the presence of such externality and an internal monopolistic distortion drives optimal policy away from producer price inflation targeting and affects the optimal level of exchange rate volatility.

As documented in the trade theory literature (see, for example, Corden (1974)), a terms of trade externality arises in an open economy setting when the elasticity of demand for export (or supply of imports) is less than infinite. This fact implies that a social planner may wish to exploit domestic monopoly power by imposing an export tax (or an import tariff) and, thereby, improve its terms of trade.

Such externality has been extensively discussed in the international finance literature. Corsetti and Pesenti (2001) analyse the welfare implications of changes in money supply in a setting characterized by an internal distortion - related to monopolistic supply in the domestic market - and the aforementioned external distortion - related to the country's monopoly power in trade. In closed economies, the internal distortion implies that a monetary expansion can increase output toward its efficient level. But in open economies this expansion also reduces domestic consumers' purchasing power internationally. Because of the latter effect, expansionary policies may reduce welfare. As emphasized in Tille (2001), the overall impact of changes in the money supply depends on the relative size of these two distortions.<sup>1</sup> In a stochastic two-country environment, the terms of trade externality has also been shown to play a crucial role in the welfare and policy evaluation (see, for example, Corsetti and Pesenti (2005) and Benigno and Benigno (2003)).

Our analysis focuses on the implication of the terms of trade externality for monetary policy in a small open economy setting, which also features the internal monopolistic distortion and nominal rigidities. The small open economy model is derived as a limiting case of a two-country dynamic stochastic general equilibrium framework. The model assumes no trade or financial frictions, that is, the law of one price holds and asset markets are complete. In this setting, we follow the linear-quadratic approach developed by Sutherland (2002) and Benigno and Woodford (2006), and derive a utility-based loss function for the central bank. We show that this loss function can be approximated by a quadratic expression in domestic inflation, output gap and real exchange rate. Moreover, we obtain the optimal response to exogenous shocks in the form of a targeting rule. Our framework encompasses, as special cases, a closed economy setting with a general steady-state degree of monopolistic competition (as in Benigno and Woodford (2005)) and a small open

---

<sup>1</sup>Early contributions emphasizing international welfare spillovers of monetary policy shocks also include Obstfeld and Rogoff (1995) and (1998).

economy setting with unitary elasticity of intertemporal and intratemporal substitution (as in Galí and Monacelli (2005)).<sup>2</sup>

As a result of the external distortion, when domestic and foreign goods are close substitutes, an improvement in the terms of trade can increase welfare by inducing agents to consume more imported goods. These consumers are better off since they can reduce their labor effort without a corresponding fall in their consumption levels. Using a second order approximation of the model, we show that a less volatile real exchange rate is associated with a more appreciated exchange rate on average, or an improved terms of trade. Therefore, when domestic and foreign goods close substitutes in the utility and a terms of trade improvement can enhance welfare, the volatility of the real exchange rate is lower under the optimal rule than under a domestic inflation targeting regime. For sufficiently large values of the elasticity of intratemporal substitution, a policy that targets the exchange rate can outperform (that is, lead to higher welfare) one that stabilizes domestic prices. The conclusions are, nevertheless, reverted if the substitutability between domestic and foreign goods is low and the expenditure switching ability of terms of trade movements is reduced.

We should note that in our setting, monetary policy is affected by an external distortion because the small open economy retains some market power over its terms of trade. Other contributions to the literature, however, consider a small open economy model in which producers of tradable goods are price takers. In such a setting, and in the presence of a non-tradable sector in which prices are sticky, Lama and Medina (2007) find that the goal of monetary policy should be to replicate the flexible price equilibrium.<sup>3</sup>

Moreover, our analysis assumes a cashless economy and does not allow for any active fiscal instrument to operate on economic distortions. Models that consider different policy instruments, such as labor or consumption taxes, and monetary frictions, driven by transaction constraints, include Adao, Correia and Teles (2003), Correia, Nicolini and Teles (2003) and Hevia and Nicolini (2006).

Our results, nevertheless, complement the analysis of the policy implications of the terms of trade externality documented in the literature. Namely, Corsetti and Pesenti (2005), Devereux and Engel (2003), and Sutherland (2005) have shown that the presence of incomplete pass-through, arising from local currency pricing strategies of firms, can give rise to an international dimension of monetary policy. In particular, when prices of goods are sticky in local currencies, foreign firms' profits are a function of domestic monetary policy; and if the domestic central bank ignores this link, import prices would be too high relative to prices of domestically produced goods.<sup>4</sup> Under producer currency pricing, Benigno and Benigno (2003) identify gains from policy cooperation

---

<sup>2</sup>The framework considered in Galí and Monacelli (2005) only imposes unitary elasticity of intertemporal and intratemporal substitution when deriving the objective function.

<sup>3</sup>Similar conclusions are suggested in Aoki (2001), in which a two-sector closed economy model is applied to a small open economy that produces differentiated goods whose prices are sticky and imports goods whose prices are flexible. We should note that, given that the model considered in Lama and Medina (2007) also features segmented markets, the aforementioned result holds when abstracting from policy incentives to enhance risk sharing.

<sup>4</sup>Devereux and Engel (2003) suggest that optimal policy in this environment may consist of a fixed exchange rate regime. But, as Corsetti (2007) and Duarte and Obstfeld (2005) point out, the fact that the exchange rate does not move under the optimal policy can be a result of the (symmetric) structure of the model and not a product of efficient stabilization.

across countries, as policymakers acting independently have an incentive to affect the terms of trade.<sup>5</sup> Furthermore, Tille (2002) finds that in the presence of sector specific shocks, exchange rate fluctuations translate into misallocation of resources between different firms within a sector. As a result, the monetary authority has an incentive to restrict these fluctuations. In addition, as demonstrated in Corsetti, Dedola and Leduc (2007), strategic interactions between upstream and downstream firms may prevent perfect stabilization of the domestic price of final goods. These interactions can induce misalignment of prices both across firms and across sectors, even when shocks affect downstream firms only. Yet, these contributions assume a two-country setting. A specific advantage of studying a small open economy is that one can trace more directly the macro-economic implications of monetary and exchange rate policy disregarding cross-border strategic interactions in policy making.

Finally, while our approach is to consider a simple specification of an open economy and derive an analytical representation of the monetary policy problem, a large set of the literature have used numerical techniques to evaluate optimal policy in open economies (see, for example, the works of Kollman (2002), Søndergaard and Cova (2004), and Bergin et. al. (2007)). Although these studies have the disadvantage of not being able to illustrate analytically the economic mechanism behind their results, they can be applied to more complex, and possibly more realistic settings.

The remainder of the text is structured as follows. In Section 2, we describe the model features and present the small open economy's dynamics. Section 3 is dedicated to the derivation of welfare and the quadratic loss function. In Section 4, we derive the optimal policy plan, and the inefficiencies of the flexible price allocation are illustrated in Subsection 4.1. In Section 5, we examine the performance of standard policy rules in our small open economy. Finally, concluding remarks are presented in Section 6.

## 2 The Model

The baseline framework consists of a two-country dynamic general equilibrium model with complete asset markets, monopolistic competition in production and sticky prices. In particular, we assume that home price setting follows a Calvo-type contract. The model features complete pass-through, as prices are set in the producer's currency. Moreover, we abstract from monetary frictions by considering a cashless economy.<sup>6</sup>

In the setup considered, even though the law of one price holds, deviations from purchasing power parity arise from the existence of home bias in consumption. This bias depends on the degree of openness and the relative size of the economy. The specification allows us to characterize the small open economy by taking the limit of the home economy size to zero. Prior to applying the limit, we derive the optimal equilibrium conditions for the two-country model. After the limit is taken, the two countries, Home and Foreign, represent the small open economy and the rest of the world, respectively.

In this section we present a summary of the model's equilibrium conditions in log-linear form, while Appendix A contains the full derivation of the model. As illustrated in Table 1, the small

---

<sup>5</sup>A similar analysis is discussed in Pappa (2004).

<sup>6</sup>For details on this specification see Woodford (2003, chapter 2).

open economy's log-linearized equilibrium dynamics can be summarized by an aggregate supply (AS), an aggregate demand (AD) and a risk sharing equation (RS).

[Insert Table 1 about here]

[Insert Table 2 about here]

The variables  $\hat{C}_t$  and  $\hat{C}_t^*$  denote domestic and foreign consumption,  $\hat{Y}_t$  denotes domestic output,  $\hat{Q}_t$  denotes the real exchange rate and  $\hat{\pi}_t^H$  represents domestic (producer price) inflation. The stochastic environment is characterized by three structural shocks. Government spending,  $\hat{g}_t$ , is assumed to be completely financed by lump-sum taxes and is treated as an exogenous shock. Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production. We allow for fluctuations in this wedge by assuming a time-varying proportional tax. We refer to these fluctuations as mark-up shocks,  $\hat{\mu}_t$ . Productivity shocks are represented by  $\hat{\varepsilon}_t$ . The parameters of the model are described in Table 2.<sup>7</sup>

Equation (AS) represents the small open economy's Phillips curve. Note that the flexible price allocation is identical to the equilibrium allocation that would prevail were policymakers to target domestic inflation (that is, the case in which  $\alpha \rightarrow 0$  and, therefore,  $k \rightarrow \infty$ , is equivalent to the case in which  $\hat{\pi}_t^H = 0, \forall t$ ). Equation (AD) illustrates how the demand for the small open economy's products depends on foreign and domestic (private and public) consumption. Equation (RS) is derived from the complete market assumption, and represents the optimal risk sharing agreement between agents in the small economy and agents in the rest of the world.

Given domestic exogenous variables,  $\hat{\varepsilon}_t, \hat{g}_t, \hat{\mu}_t$ , and external conditions,  $\hat{C}_t^*$ , the small open economy system of equilibrium conditions is closed by specifying the monetary policy rule. In Sections 4 and 5, we examine different specifications for this rule. Apart from analyzing the optimal monetary policy regime in the form of a targeting rule, we evaluate the performance of alternative standard policy rules such as an exchange rate peg, and both consumer price index (CPI) and producer price index (PPI) inflation targeting.

Foreign dynamics are governed by foreign supply and demand conditions (AS\* and AD\*):

[Insert Table 3 about here]

The specification of the foreign policy rule completes the system of equilibrium conditions which determine the evolution of foreign inflation  $\hat{\pi}_t^*$ , foreign output  $\hat{Y}_t^*$  and foreign consumption  $\hat{C}_t^*$ . The economic dynamics of the rest of the world are independent of the dynamics of the real exchange rate or any variable in small open economy. Therefore, the policymaker of the small open economy can treat  $\hat{C}_t^*$  as exogenous. The policy choice of the rest of the world may modify the way in which foreign structural shocks affect  $\hat{C}_t^*$ , but it does not influence how the latter affects the small open economy.

### 3 Welfare

In a microfounded model, welfare and the policymaker's objective function can be precisely derived from households' utility. Following the linear-quadratic approach developed by Suther-

<sup>7</sup>Note that, in our framework, the parameter determining the degree of openness also governs the degree of home bias. In particular, the level of home bias is given by  $1 - \lambda$ . See Appendix A for details.

land (2002) and Benigno and Woodford (2006), we obtain the objective function from a second-order approximation of this utility (or, equivalently, a second-order Taylor expansion of Equation (A.1) in Appendix A). Moreover, we eliminate the linear terms in the Taylor expansion, using a second-order approximation of the model's equilibrium conditions. As a result, we obtain a purely quadratic approximation for the objective function.<sup>8</sup>

Alternative approaches to welfare evaluation include the computational methods described in Collard and Juillard (2001), Kim et al. (2003) and Schmitt-Grohé and Uribe (2004). Such techniques are based on perturbation methods and deliver a numerical evaluation of the optimal policy problem. In contrast, the methodology adopted here delivers an analytical representation of the policy problem, which is similar to the one used in the traditional literature on monetary policy evaluation.

A second-order Taylor expansion of the utility function implies the following approximation for welfare

$$W_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[ \begin{aligned} & \hat{C}_t - \frac{1}{\bar{\mu}} \hat{Y}_t + \frac{1}{2} (1 - \rho) \hat{C}_t^2 \\ & - \frac{1}{2} \frac{(\eta+1)}{\bar{\mu}} \left( \hat{Y}_t - \frac{\eta}{(\eta+1)} \hat{\varepsilon}_t \right)^2 - \frac{1}{2} \frac{\sigma}{k \bar{\mu}} \left( \hat{\pi}_t^H \right)^2 \end{aligned} \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (1)$$

where  $\bar{\mu}$  is the steady state level of markup,  $\mathcal{O}(\|\xi\|^3)$  represents terms of order higher than two, and the expression *t.i.p* stands for terms independent of policy.<sup>9</sup> Using the method described above, we can re-write the above expression as a quadratic function of output, real exchange rate and domestic inflation. Thus, as shown in Appendix B, the loss function for the small open economy can be expressed as

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[ \frac{1}{2} \Phi_y (\hat{y}_t)^2 + \frac{1}{2} \Phi_q (\hat{q}_t)^2 + \frac{1}{2} \Phi_\pi (\hat{\pi}_t^H)^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (2)$$

We define the welfare-relevant gaps as  $\hat{y}_t = (\hat{Y}_t - \hat{Y}_t^T)$  and  $\hat{q}_t = (\hat{Q}_t - \hat{Q}_t^T)$ , where  $\hat{Y}_t^T$  and  $\hat{Q}_t^T$  are policy targets that depend on the various shocks.<sup>10</sup> The weights of inflation, output and the real exchange rate gap in welfare losses,  $\Phi_\pi$ ,  $\Phi_y$ , and  $\Phi_q$ , are functions of the structural parameters of the model. Expressions for policy targets and weights are shown in Appendix B.

The welfare losses represented in Equation (2) are affected by two economic distortions which are common to the closed economy framework. These distortions are driven by the presence of price stickiness and monopolistic competition in domestic production. Price stickiness introduces an incentive to reduce inflation fluctuations, and monopolistic competition introduces an incentive to reduce steady-state production inefficiencies. These two factors justify the presence of  $(\hat{\pi}_t^H)^2$  and  $(\hat{y}_t)^2$  in Equation (2). In fact, if we characterize a closed economy (by setting  $\lambda = 0$ ), Equation

<sup>8</sup>As formalized in Judd (1996), and discussed in Benigno and Woodford (2006), in order to obtain an approximation of the objective function that is fully accurate to second-order, the effect of second moments on the mean of the variables should be taken into account. The linear-quadratic approach adopted in this paper incorporates these effects by obtaining a purely quadratic approximation for the policy objective.

<sup>9</sup>These terms depend solely on exogenous shocks and, therefore, are not affected by the policy choice.

<sup>10</sup>The terminology *policy target* is often used in works which consider that monetary policy follows targeting rules (see, for example, Svensson (2002)). It refers to variables that are composites of different shocks, and, thus, it does not refer to what one could consider as *observable policy targets*, such as actual inflation or exchange rates.

(2) becomes:

$$L_{to}^c = U_c \bar{C} E_{t_0} \sum \beta^t \left[ \frac{1}{2} \Phi_y^c (\hat{y}_t^c)^2 + \frac{1}{2} \Phi_\pi^c (\hat{\pi}_t^H)^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (3)$$

where the subscript  $c$  denotes a closed economy. The policymaker's problem in a closed economy can be illustrated by the relative weight of inflation with respect to output,  $\Phi_\pi/\Phi_y$ , and by the difference between  $\hat{Y}_t^T$  and  $\hat{Y}_t^{Flex}$  (where  $\hat{Y}_t^{Flex}$  represents the flexible price allocation for output). The solution to these terms is:

$$\frac{\Phi_\pi^c}{\Phi_y^c} = \frac{\sigma}{k(\eta + \rho)}, \quad (4)$$

$$\hat{Y}_t^{T,c} = \frac{\eta \hat{\varepsilon}_t + d \rho \hat{g}_t}{(\eta + \rho)} - \frac{d(\bar{\mu} - 1)(\eta + 1) \hat{\mu}_t}{(\eta + \rho)(\bar{\mu}\eta + \rho)}, \quad (5)$$

and

$$\hat{Y}_t^{Flex,c} = \frac{\eta \hat{\varepsilon}_t + \rho \hat{g}_t}{(\eta + \rho)} - \frac{\hat{\mu}_t}{(\eta + \rho)}, \quad (6)$$

where  $d = (\bar{\mu}\eta + \rho)(\bar{\mu}\eta + \rho + (\bar{\mu} - 1))^{-1}$ . As the above expressions show,  $\hat{Y}_t^{T,c} \neq \hat{Y}_t^{Flex,c}$ , so a policy of strict inflation targeting, which mimics the flexible price allocation, does not close the welfare-relevant output gap. Hence, there is a trade-off between stabilizing inflation and output. As shown in Benigno and Woodford (2003), this trade-off disappears only when the steady-state level of production is efficient (i.e.  $\bar{\mu} = 1$ ) and there are no mark-up fluctuations (i.e.  $\hat{\mu}_t = 0, \forall t$ ), resulting here in  $\hat{Y}_t^T = \hat{Y}_t^{Flex}$ .<sup>11</sup>

In our small open economy, apart from nominal rigidities and the internal monopolistic distortion, there is an external distortion that gives rise to a terms of trade externality (as described in Corsetti and Pesenti (2001)).<sup>12</sup> Such distortion comes from the fact that, because imported goods are not perfect substitutes to goods produced domestically, a social planner in an open economy may wish to exploit a certain degree of monopoly power.

We can illustrate some of the welfare implications of this external distortion by inspection of the linear terms in the welfare equation (1). As shown in section B.6 of the Appendix, these terms can be written as a function of  $E[(1 - \kappa_{\bar{\mu}, \rho\theta})\hat{Q}_t]$ , where  $\kappa_{\bar{\mu}, \rho\theta}$  is decreasing in  $\bar{\mu}$  and increasing in  $\rho\theta$ .<sup>13</sup> The larger the degree of internal monopolistic distortion - given by the mark-up  $\bar{\mu}$  - the larger are the gains from having a more depreciated real exchange rate on average, as this induces a higher demand for domestic goods and higher output. But the presence of the terms of trade externality implies that, when domestic and foreign goods are close substitutes, an appreciated

<sup>11</sup>Benigno and Woodford (2003) consider further specifications, such as a non-zero steady state government consumption, which are not addressed in this paper.

<sup>12</sup>Although the source of the external distortion present in the current framework is the same as the one in these references, the policy analysis is different. In Corsetti and Pesenti (2001), monetary policy is characterised by unanticipated movements in the money supply which can affect the level of production and the terms of trade (ex-post). In the current framework, monetary policy has an effect on the volatility of economic variables, which, as emphasised in Obstfeld and Rogoff (1998) and Henderson and Kim (1999), affects the expected (or ex-ante) value of variables.

<sup>13</sup>Note that Gali and Monacelli (2005) eliminate this term by assuming that  $\bar{\mu}^{-1} = 1 - \lambda$  and  $\rho = \theta = 1$ . But in general, using a first order approximation of demand equation and the risk sharing condition, we can write  $\hat{C}_t - \bar{\mu}^{-1}\hat{Y}_t$  as a function of  $E[(1 - \kappa_{\bar{\mu}, \rho\theta})\hat{Q}_t]$ . However, the full welfare implications of the linear term can only be accessed when this term is approximated to second order. This is derived in section B.5 of the Appendix.

real exchange rate (or an improvement in the terms of trade), on average, can enhance welfare by decreasing the disutility of producing at home without an equivalent reduction in the utility of consumption.<sup>14</sup> This is because the appreciation would be associated with lower consumption of domestic goods, while leading to larger consumption of foreign goods. In this case, the small open economy, as a monopolist over the goods it produces, can gain from restricting the supply and increasing the price of its goods. The larger is the degree of substitutability between goods (in particular, the larger  $\rho\theta$ ), the larger is the expenditure switching effect of changes in terms of trade, and, hence, the larger are the gains from an improvement in the terms of trade. An appreciation ceases to be welfare improving when these elasticities are small, and the terms of trade cannot divert consumption towards foreign goods. In this case, a more depreciated real exchange rate on average can be welfare improving. The reason is that, when domestic and foreign products are complements in the utility, the marginal utility from consuming domestic goods is increasing in the consumption of foreign goods, and vice versa. An average real depreciation is associated with a an increase in domestic output and a higher level of consumption of domestic goods, which, in turns leads to an increase in the marginal utility from imports (and vice versa).<sup>15</sup> As a result, the increase in overall utility of consumption outweighs the increase in the disutility of labour effort implied by the higher level of domestic production.

#### A note on the role of risk sharing

The implications of the aforementioned terms of trade externality crucially depend on the fact that agents in the small open economy have an optimal risk sharing arrangement with the rest of the world. Such arrangement prevents agents in the small economy from experiencing negative income effects if they were to reduce their production levels. Under financial autarky this would not be the case, as domestic agents' borrowing constraints imply a tight link between domestic consumption and income. As shown in the section B.6 of the Appendix, under this specification the linear terms in the welfare equation (1) can be written as a function of  $E[(1 - \kappa'_{\bar{\mu},\theta})\hat{Q}_t]$ , where  $\kappa'_{\bar{\mu},\theta}$  is a decreasing, rather than increasing, function of the elasticity of substitution between home and foreign goods ( $\theta$ ). That is, only when this elasticity is sufficiently low an appreciated exchange rate can improve welfare, as output would fall little relative the movement in real exchange rate, and the effect of the appreciation on agents' purchasing power would outweigh the reduction in output.<sup>16</sup>

## 4 Optimal Monetary Policy

In the previous section we illustrate how welfare is affected by the existence of a terms of trade externality. In the current section we analyze how this externality affects optimal policy. Following our linear-quadratic approach, the optimal monetary policy can be obtained by minimizing the

<sup>14</sup>We should note that in the present framework, given that all goods are traded, terms of trade and real exchange rates are directly related. In particular, from the definition of the real exchange rate and the price index (see Appendix for details), we can write  $Q_t^{\theta-1} = (1 - \lambda)(P_{F,t}/P_{H,t})^{\theta-1} + \lambda$ .

<sup>15</sup>I thank Giancarlo Corsetti for suggesting this explanation.

<sup>16</sup>This result is consistent with the findings of Sutherland (2004) who shows that gains from cooperation between two countries, which produce substitute goods, are larger when there is perfect risk sharing across such countries.

quadratic loss function (2) subject to the linear constraints specified in Table 1. Alternatively, we can depict these constraints in terms of the welfare-relevant output and exchange rate gaps,

[Insert Table 4 about here]

where  $u_t$  and  $\chi_t$  are linear combination of the structural shocks (these are shown in Appendix B).<sup>17</sup>

Equation (C1) shows that the welfare-relevant gaps,  $\hat{y}_t$  and  $\hat{q}_t$ , are not closed when prices are perfectly flexible. That is, the policy targets do not coincide with the allocations that would prevail if  $\alpha = 0$  (and consequently  $k \rightarrow \infty$ ). Moreover, Equation (C2) shows that closing the output gap fails to eliminate the real exchange rate gap.

We characterize the optimal plan assuming that policymakers can commit to maximizing the economy's welfare. The policy problem consists of choosing a path for  $\{\hat{\pi}_t^H, \hat{y}_t, \hat{q}_t\}$  to minimize (2), subject to the constraints shown in Table 3, and given the initial condition  $\hat{\pi}_{t_0}$ . In effect, the constraint on  $\hat{\pi}_{t_0}$  imposes that the first order conditions to the problem are time invariant. This method follows Woodford's (1999) timeless perspective approach, and thereby ensures that the policy prescription does not constitute a time-inconsistent problem. The multipliers associated with (C1) and (C2) are, respectively,  $\varphi_1$  and  $\varphi_2$ . Thus, the first order conditions with respect to  $\hat{\pi}_t^H, \hat{y}_t$  and  $\hat{q}_t$  are given by:

$$(\varphi_{1,t} - \varphi_{1,t-1}) = k\Phi_\pi \hat{\pi}_t^H, \quad (7)$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = \Phi_Y \hat{y}_t, \quad (8)$$

and

$$(1+l)\varphi_{2,t} + \rho\varphi_{1,t} = -\rho(1-\lambda)\Phi_q \hat{q}_t. \quad (9)$$

Combining equations (7), (8), and (9), we obtain the following expression:

$$\Phi'_y \Delta \hat{y}_t + \Phi'_q \Delta \hat{q}_t + \Phi'_\pi \hat{\pi}_t^H = 0, \quad (10)$$

where  $\Delta$  denotes the first difference operator,  $\Phi'_y = (1+l)\Phi_Y$ ,  $\Phi'_q = \rho(1-\lambda)\Phi_q$  and  $\Phi'_\pi = (\rho + \eta(1+l))k\Phi_\pi$ . The above expression characterizes the small open economy optimal targeting rule and stipulates how monetary policy should respond to different shocks (according to the composition of  $\hat{Y}_t^T$  and  $\hat{Q}_t^T$ ).<sup>18</sup>

So what are the implications of the terms of trade externality for optimal policy? The policy rule described in equation (10) prescribes responding to movements in inflation, output and the real exchange rate. Note that even if we express Equation (10) as a function of consumer price index (CPI) inflation instead of producer price index (PPI) inflation  $\hat{\pi}_t^H$ , the targeting rule still includes the term  $\Delta \hat{q}_t$ . So, when following this policy rule, the central bank may allow some variability in inflation in order to respond to costly movements in output and real exchange rates

<sup>17</sup>Note that the small open economy representation eliminates the need to formally characterize the open economy policy problem as a Nash equilibrium. For examples of how this is done in a two-country world, see Benigno and Benigno (2003) and Pappa (2004).

<sup>18</sup>Monetary policy is, therefore, modelled as a targeting rule and not as an instrument rule (or an interest rate rule). See Svensson (2002, 2003) and McCallum and Nelson (2004) for a discussion on these policy specifications.

(and, thus, terms of trade).<sup>19</sup> As a matter of fact, the optimal policy rule may imply a smaller or larger volatility in the real exchange rate relative to a domestic inflation targeting regime. As illustrated in Figure 1, under the parameter specification shown in Table 5, the volatility of real exchange rates under the optimal policy rule is lower (higher) than under a policy of domestic inflation target when domestic and foreign goods are substitutes (complements) to one another.

[Insert Figure 1 about here]

[Insert Table 5 about here]

In order to understand this result it is useful to examine how the mean and variance of the real exchange rate are related in our model. In general, by using a second order approximation of the equilibrium conditions shown in Appendix B, it is possible to express the unconditional mean of the real exchange rate as a function of second moments of the different macro variables.<sup>20</sup> Moreover, if, for illustration purposes, we consider the case in which prices are perfectly flexible (i.e.  $\alpha = 0$ ), the unconditional mean of the real exchange rate can actually be expressed solely as a function of the real exchange rate volatility.<sup>21</sup> That is, in this case we can write

$$E[\hat{Q}_t] = c_q var[\hat{Q}_t] \quad (11)$$

Figures 2 and 3 illustrate the relationship between the mean and variance of the real exchange rate (represented by  $c_q$ ) for different values of the elasticity of substitution ( $\theta$ ) and degree of openness ( $\lambda$ ). As the figures show, regardless of the value of  $\theta$  and  $\lambda$ , the higher the level of the exchange rate volatility, the more depreciated is the real exchange rate on average (i.e.  $c_q$  is always positive). Moreover, the effect of the real exchange rate volatility on its mean is increasing on the degree of openness ( $\lambda$ ) and the elasticity of substitution between home and foreign-produced goods ( $\theta$ ).

[Insert Figure 2 about here]

[Insert Figure 3 about here]

So, in our framework, a less volatile exchange rate is associated with a more appreciated exchange rate on average. Therefore, when a more appreciated exchange rate is welfare improving (and as shown in section 3 this is the when  $\rho\theta$  is large) optimal policy prescribes a lower volatility of the exchange rate relative to a domestic inflation targeting regime. In this case, the policy maker only partially stabilizes output relative to its flexible-price level – that is, output will be on average below the one that would prevail if prices were perfectly flexible. This translates into a fall in the variability of the real exchange rate relative to the case of producer price inflation targeting. The contrary is true when  $\rho\theta$  is low. Under this configuration, the policy maker finds it optimal to over-stabilize output relative to the flexible price allocation. That is, on average, the policy maker engineers a depreciation in real exchange rate which produces an external demand for domestic goods. In this case, the exchange rate is more volatile relative to the case of domestic price stability.

<sup>19</sup>Note that, as shown in footnote 15, the terms of trade and the real exchange rate are directly related in our framework. But, as emphasised in Corsetti, Dedola and Leduc (2007), in a different setting a policy that stabilizes the terms of trade, does not necessarily contains the volatility of the real exchange rate.

<sup>20</sup>For another study examining the effect of exchange rate uncertainty on the level of exchange rates, see Obstfeld and Rogoff (1998).

<sup>21</sup>In the case of sticky prices, the real exchange rate mean would be written as a function of inflation and real exchange rate volatility.

## 4.1 Price stability in a small open economy

When is domestic price stability optimal in a small open economy? As illustrated by the targeting rule expression (10), in our general specification, optimal monetary policy deviates from pure domestic inflation targeting, or, equivalently, the flexible price allocation is inefficient. In the current section, we examine the special case in which this ceases to be the case. And, to understand the inefficiency of the flexible price allocation in the general case, we step back from the linear-quadratic analysis and compare the social planner's non-linear equilibrium conditions with those under flexible prices.<sup>22</sup> This analysis allows us to evaluate how the optimality of domestic inflation targeting depends on structural parameters and shocks.

The planner's objective is to maximize agents' utility subject to the small open economy's demand, risk sharing and relative price equations (Equations (A.11), (A.17) and (A.6) in the appendix).<sup>23</sup> Moreover, the planner in a small open economy takes external conditions as given. Hence, the first order condition can be written as

$$p_{H,t}^e U_c(C_t^e) = s_t^e V_y(Y_t^e, \varepsilon_t), \quad (12)$$

where the relative price of domestic goods is denoted by  $p_{H,t} = P_{H,t}/P_t$ ,  $s_t^e = [1 + \lambda(\rho\theta - 1)(Q_t^e)^{1-\theta} + \frac{\lambda}{1-\lambda}\rho\theta(Q_t^e)^{\frac{\rho\theta-1}{\rho}}]$ , and the superscript  $e$  denotes the efficient allocation. A full characterization of the efficient allocation can be obtained by combining the above equation with the constraints of the policy problem.

On the other hand, in the decentralized problem, the equilibrium condition implied by monopolistic competition and price stickiness is given by the price setting condition (Equation (A.15) in the Appendix). If we assume, however, that prices are flexible, this condition becomes

$$p_{H,t}^{Flex} U_c(C_t^{Flex}) = \mu_t V_y(Y_t^{Flex}, \varepsilon_t), \quad (13)$$

Comparing conditions (12) and (13), it is clear that even with perfectly flexible prices, mark-up shocks and movements in the real exchange rate generate inefficient fluctuations in the ratio of marginal disutility of production and marginal utility of consumption. This comparison presents an alternative demonstration of the fact that, in general, a policy of domestic price stabilization that mimics the flexible price allocation does not implement the efficient allocation. This is only the case under certain assumptions concerning parameter values, but it also depends on the sources of shocks hitting the economy. We now illustrate these special cases.

If we impose that  $\rho\theta = 1$ , the efficiency condition (12) and the decentralized flexible price allocation (13) can be written as

$$(1 - \lambda)(Y_t^e - G_t)^{-\rho} = \varepsilon_t^{-\eta} (Y_t^e)^\eta, \quad (14)$$

and

$$\frac{1}{\mu_t} (Y_t^{Flex} - G_t)^{-\rho} = \varepsilon_t^{-\eta} (Y_t^{Flex})^\eta. \quad (15)$$

<sup>22</sup>In a subsequent work, Faia and Monacelli (2008) follow a similar approach and characterize the optimal Ramsey allocation. Their analysis focuses the role of home bias.

<sup>23</sup>Note that here the social planner is not constrained by the pricing decisions of firms. When computing optimal policy this further constraint has to be included, as done in the Appendix of Benigno and Benigno (2003).

The above expressions illustrate that the steady-state flexible price allocation is inefficient (i.e.  $\bar{Y}^{flex} \neq \bar{Y}^e$ ) unless  $\bar{\mu} = 1/(1 - \lambda)$ . It also shows that, the flexible price allocation and the efficient allocation move proportionally to one another following productivity shocks. Therefore, if the small open economy is subject solely to productivity shocks, a policy that mimics the flexible price allocation (that is, a producer price inflation targeting regime) is optimal when  $\rho\theta = 1$ . The same result holds for foreign shocks.<sup>24</sup> In the case of exogenous fluctuations in government expenditure,  $Y^{flex}$  and  $Y^e$  do not move proportionally, unless the steady-state level of output is efficient. Finally, mark-up shocks generate disproportional movements in  $Y^{flex}$  and  $Y^e$  regardless of the steady-state characterization. This result is consistent with the findings of Benigno and Woodford (2004) in a closed economy setting.

Therefore, the assumptions needed in order to have a producer price inflation targeting as the optimal plan are:

$$\begin{aligned}\rho\theta &= 1, \\ \hat{\mu}_t &= 0, \forall t,\end{aligned}$$

and, in the case of fiscal shocks,

$$\bar{\mu} = 1/(1 - \lambda).$$

Under this specification, the weights on the loss function are:

$$\frac{\Phi_y}{(1 - \lambda)} = (\eta + \rho), \quad (16)$$

$$\Phi_q = 0, \quad (17)$$

and

$$\frac{\Phi_\pi}{(1 - \lambda)} = \frac{\sigma}{k}, \quad (18)$$

and the target for output is

$$\hat{Y}_t^T = \hat{Y}_t^{Flex} = (\eta + \rho)^{-1} \{ \eta \varepsilon_{Y,t} + \rho g_t \}. \quad (19)$$

The relative weights specified in equations (16) and (18) are analogous to those in the closed economy, and the policy target coincides with the flexible price allocation. When  $\rho\theta = 1$ , international relative price movements have income and substitution effects that offset each other, and thus, do not have expenditure switching implications. Under this assumption, and provided that the steady-state is efficient (which is guaranteed by assuming that  $\bar{\mu} = 1/(1 - \lambda)$ ), welfare is no longer affected by the terms of trade externality or the internal monopolistic distortion. Thus, since nominal rigidity is the only relevant economic distortion, domestic price stability is optimal. Moreover, the optimal plan does not respond to external shocks. Under this specification, optimal monetary policy in a small open economy is isomorphic to optimal policy in a closed economy.

This result is consistent with the findings of Galí and Monacelli (2005), who characterized the loss function for the case in which  $\rho = \theta = 1$  and the steady-state level of output is efficient (the authors also assume that  $\bar{\mu} = 1/(1 - \lambda)$ ). But we should note that their framework only includes productivity shocks, and as the above result suggests, considering other sources of shocks can have implications for the efficiency of the flexible price allocation.

---

<sup>24</sup>Note that, when  $\rho\theta = 1$ , the flexible price allocation and the efficient allocation of output are insular to foreign shocks.

## 5 Ranking standard policy rules

In section 4, we present the optimal monetary policy in the form of a targeting rule. But the implementation of such a rule may not be straightforward, either because its targets are difficult to monitor ( $\hat{Y}_t^T$  and  $\hat{Q}_t^T$  depend on unobservable shocks) or because its weights are complex functions of structural parameters. For this reason, it is useful to compare the performances of simple policy rules and analyze how these are affected by the terms of trade externality.

We compute a ranking, based on our welfare measure, of policies that target domestic prices (or the producer price index - PPI), consumer prices (or CPI) and a fixed exchange rate regime (or PEG). This is done for different values of  $\rho$  and  $\theta$ . Table 6 shows the policy rule that leads to the highest level of welfare when the economy is subject to all shocks.<sup>25</sup> The exercise shows that while PPI targeting leads to higher welfare when the intratemporal and intertemporal elasticities are low, for sufficiently high values of  $\rho$  and  $\theta$ , a fixed exchange rate regime is the one associated with the highest level of welfare.<sup>26</sup>

In line with the discussion in section 4, the ranking of policy rules illustrated in Table 6 is, by and large, driven by the amount of stabilization of output relative to the flexible price allocation implied by the different policy regimes. When domestic and foreign products are good substitutes to one another (corresponding to the lower right corner of Table 6), the welfare gains from a terms of trade improvement (or a real exchange rate appreciation) outweigh the gains from stabilizing domestic prices. As a result, a policy of fixed exchange rates, which ties policymakers' hands, only partially stabilizes the output relative to the flexible price allocation, and is associated with a more appreciated real exchange rate on average, leads to a higher welfare than a policy of domestic inflation targeting. On the other hand, when the intratemporal and intertemporal elasticities are low (corresponding to the upper left corner of the table), a policy that overstabilize output and (i.e. leads to a higher level of output and a more depreciated real exchange rate relative to the flexible price allocation) is welfare improving. So, in this case a PPI inflation targeting regime outperforms the exchange rate peg regime. For an intermediate value of  $\rho$  and  $\theta$ , a policy of CPI targeting is the preferred regime.

[Insert Table 6 about here]

To assess the relevance of the above results, we evaluate the welfare costs associated with adopting different rules. Table 7 illustrates the welfare gains or losses of adopting PPI inflation targeting rather than an exchange rate peg, when the economy is faced by all the different shocks. The numbers represent the percentage difference in steady-state consumption between a PPI and a PEG.<sup>27</sup> The results show that these gains (or losses) are between 0.032% to -0.279%. We have also conducted some sensitivity analysis and computed the welfare losses following each shock individually. We found that the losses were significantly larger following external shocks.

<sup>25</sup>The stochastic environment is parameterized to characterize a small industrialized economy. See Table 5 for a full description of the parameter values used in this exercise.

<sup>26</sup>We should, however, qualify that our setting presents a stylized specification for the small open economy, which does not allow for liquidity runs which could compromise the performance of exchange rate pegs.

<sup>27</sup>In particular, we compute  $W_d^{PPI,PEG} = \frac{W^{PPI} - W^{PEG}}{U_c(\bar{C})} = \frac{2(1-\beta)(U_0^{PPI} - U_0^{PEG})}{U_c(\bar{C})}$ , where  $U_0$  is the expected life-time utility of the representative agent.

[Insert Table 7 about here]

## 6 Conclusion

In this paper we formalize a small open economy model as a limiting case of the two-country general equilibrium framework; characterize its utility-based loss function; and derive the optimal monetary plan. The model developed in this work encompasses, as special cases, a small open economy with efficient levels of steady-state output and a closed economy framework.

The utility-based loss function for a small open economy can be written as a quadratic expression of domestic inflation, output gap and real exchange rate. In our framework, when the economy experiences productivity and foreign shocks exclusively, domestic inflation targeting is optimal only under a particular specification for preferences. If fiscal disturbances are also present, optimal price stability additionally requires a production subsidy. We have also shown that a policy of domestic inflation targeting leads to higher welfare than CPI or exchange rate targeting if domestic and foreign goods are not close substitutes for each other. Conversely, when this substitutability is high, the optimal rule implies lower real exchange rate volatility than a PPI regime.

The tools developed in this paper can be applied to different economic environments. The model presented here assumes that asset markets are complete. Relaxing this assumption can lead to a more realistic framework, and, as hinted in our analysis, can change the results obtained. Also, this paper characterizes a setting in which the monetary authority has one policy instrument but faces three policy incentives (driven by monopolistic competition, price rigidities and the terms of trade externality). Previous works have demonstrated that steady-state inefficiencies generated by monopolistic competition can sometimes be resolved with a production subsidy.<sup>28</sup> An interesting extension of the present paper could assess whether the introduction of further policy instruments would implement the first best.<sup>29</sup> Other interesting avenue for future research may include the introduction of different sectors.<sup>30</sup>

---

<sup>28</sup>See Woodford (2003) and Benigno and Woodford (2005).

<sup>29</sup>In particular, this extension could follow the approach proposed by Adao, Correia and Teles and Correia, Nicolini, and Teles (2003).

<sup>30</sup>Some of this analysis can already be found in Lipińska (2008), who extends the current work to includes a non-tradable sector.

## A The Model

### Preferences

We consider two countries,  $H$  (Home) and  $F$  (Foreign). The world economy is populated with a continuum of agents of unit mass, where the population in the segment  $[0, n)$  belongs to country  $H$  and the population in the segment  $(n, 1]$  belongs to country  $F$ . The utility function of a consumer  $j$  in country  $H$  is given by:<sup>31</sup>

$$U_t^j = E_t \sum_{s=t}^{\infty} \beta^{s-t} [U(C_s^j) - V(y_s(j), \varepsilon_s)]. \quad (\text{A.1})$$

Households obtain utility from consumption  $U(C^j)$  and contribute to the production of a differentiated good  $y(j)$  attaining disutility  $V(y(j), \varepsilon)$ . Productivity shocks are denoted by  $\varepsilon_s$ .  $C$  is a Dixit-Stiglitz aggregator of home and foreign goods, defined by

$$C = \left[ v^{\frac{1}{\theta}} C_H^{\frac{\theta-1}{\theta}} + (1-v)^{\frac{1}{\theta}} C_F^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (\text{A.2})$$

where  $\theta > 0$  is the intratemporal elasticity of substitution and  $C_H$  and  $C_F$  are consumption sub-indices that refer to the consumption of home-produced and foreign-produced goods, respectively. The parameter determining home consumers' preferences for foreign goods,  $(1-v)$ , is a function of the relative size of the foreign economy,  $1-n$ , and of the degree of openness,  $\lambda$ ; more specifically,  $(1-v) = (1-n)\lambda$ .

Similar preferences are specified for the rest of the world,

$$C = \left[ v^*{}^{\frac{1}{\theta}} C_H^*{}^{\frac{\theta-1}{\theta}} + (1-v^*)^{\frac{1}{\theta}} C_F^*{}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \quad (\text{A.3})$$

with  $v^* = n\lambda$ . That is, foreign consumers' preferences for home goods depend on the relative size of the home economy and the degree of openness. Note that, for  $\lambda < 1$ , the specification of  $v$  and  $v^*$  generates a home bias in consumption, as in Sutherland (2005).

The sub-indices  $C_H$  ( $C_H^*$ ) and  $C_F$  ( $C_F^*$ ) are Home (Foreign) consumption of the differentiated products produced in countries  $H$  and  $F$ . These are defined as follows:

$$C_H = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad C_F = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.4})$$

$$C_H^* = \left[ \left( \frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad C_F^* = \left[ \left( \frac{1}{1-n} \right)^{\frac{1}{\sigma}} \int_n^1 c^*(z)^{\frac{\sigma-1}{\sigma}} dz \right]^{\frac{\sigma}{\sigma-1}}, \quad (\text{A.5})$$

where  $\sigma > 1$  is the elasticity of substitution across the differentiated products. The consumption-based price indices that correspond to the above specifications of preferences are given by

$$P = \left[ v P_H^{1-\theta} + (1-v) (P_F)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (\text{A.6})$$

<sup>31</sup>In the subsequent sections, we assume the following isoelastic functional forms:  $U(C_t) = \frac{C_t^{1-\rho}}{1-\rho}$  and  $V(y_t, \varepsilon_{Y,t}) = \frac{\varepsilon_{Y,t}^{-\eta} y_t^{1+\eta}}{1+\eta}$ , where  $\rho$  is the coefficient of relative risk aversion and  $\eta$  is equivalent to the inverse of the Frisch elasticity of labor supply.

and

$$P^* = \left[ v^* P_H^{*1-\theta} + (1-v^*) (P_F^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad (\text{A.7})$$

where  $P_H$  ( $P_H^*$ ) is the price sub-index for home-produced goods expressed in the domestic (foreign) currency and  $P_F$  ( $P_F^*$ ) is the price sub-index for foreign produced goods expressed in the domestic (foreign) currency:

$$P_H = \left[ \left( \frac{1}{n} \right) \int_0^n p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, \quad (\text{A.8})$$

$$P_H^* = \left[ \left( \frac{1}{n} \right) \int_0^n p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}, P_F^* = \left[ \left( \frac{1}{1-n} \right) \int_n^1 p^*(z)^{1-\sigma} dz \right]^{\frac{1}{1-\sigma}}. \quad (\text{A.9})$$

We assume that the law of one price holds, so

$$p(h) = S p^*(h) \text{ and } p(f) = S p^*(f), \quad (\text{A.10})$$

where the nominal exchange rate,  $S_t$ , denotes the price of foreign currency in terms of domestic currency. Equations (A.6) and (A.7), together with condition (A.10), imply that  $P_H = S P_H^*$  and  $P_F = S P_F^*$ . However, as Equations (A.8) and (A.9) illustrate, the home bias specification leads to deviations from purchasing power parity; that is,  $P \neq S P^*$ . For this reason, we define the real exchange rate as  $Q = \frac{S P^*}{P}$ .

From consumers' preferences, we can derive the total demand for a generic good  $h$ , produced in country H, and the demand for a good  $f$ , produced in country F:

$$y_t^d(h) = \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\sigma} \left\{ \left[ \frac{P_{H,t}}{P_t} \right]^{-\theta} \left[ v C_t + \frac{v^*(1-n)}{n} \left( \frac{1}{Q_t} \right)^{-\theta} C_t^* \right] + G_t \right\}, \quad (\text{A.11})$$

$$y_t^d(f) = \left[ \frac{p_t(f)}{P_{F,t}} \right]^{-\sigma} \left\{ \left[ \frac{P_{F,t}}{P_t} \right]^{-\theta} \left[ \frac{(1-v)n}{1-n} C_t + (1-v^*) \left( \frac{1}{Q_t} \right)^{-\theta} C_t^* \right] + G_t^* \right\}, \quad (\text{A.12})$$

where  $G$  and  $G^*$  are country-specific government shocks. We assume that the public sector in the Home (Foreign) economy only consumes Home (Foreign) goods and has preferences for differentiated goods analogous to the ones of the private sector (given by Equations (A.4) and (A.5)). Moreover, we consider the case in which fluctuations in government spending,  $G_t$  ( $G_t^*$ ), or proportional taxes,  $\tau_t$  ( $\tau_t^*$ ), are exogenous and completely financed by lump-sum transfers,  $Tr_t$  ( $Tr_t^*$ ), made in the form of domestic (foreign) goods.

Finally, to portray our small open economy, we use the definition of  $v$  and  $v^*$  and take the limit for  $n \rightarrow 0$ . Consequently, conditions (A.11) and (A.12) can be rewritten as

$$y^d(h) = \left[ \frac{p_t(h)}{P_{H,t}} \right]^{-\sigma} \left\{ \left[ \frac{P_{H,t}}{P_t} \right]^{-\theta} \left[ (1-\lambda) C_t + \lambda \left( \frac{1}{Q_t} \right)^{-\theta} C_t^* \right] + G_t \right\}, \quad (\text{A.13})$$

$$y^d(f) = \left[ \frac{p_t^*(f)}{P_{F,t}^*} \right]^{-\sigma} \left\{ \left[ \frac{P_{F,t}^*}{P_t^*} \right]^{-\theta} C_t^* + G_t^* \right\}. \quad (\text{A.14})$$

Equations (A.13) and (A.14) show that external changes in consumption affect the small open economy, but the opposite is not true. Moreover, movements in the real exchange rate do not affect the total demand for goods produced in the rest of the world.

### Price-setting mechanism

Prices follow a partial adjustment rule à la Calvo (1983). Producers of differentiated goods know the form of their individual demand functions (given by Equations (A.13) and (A.14)), and maximize profits taking overall market prices and products as given. In each period a fraction,  $\alpha \in [0, 1)$ , of randomly chosen producers is not allowed to change the nominal price of the goods they produce. The remaining fraction of firms, given by  $(1 - \alpha)$ , chooses prices optimally by maximizing the expected discounted value of profits. The optimal choice of producers that can set their price  $\tilde{p}_t(j)$  at time  $T$  is, therefore:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left( \frac{\tilde{p}_t(j)}{P_{H,T}} \right)^{-\sigma} Y_{H,T} \left[ \frac{\tilde{p}_t(j) P_{H,T}}{P_{H,T} P_T} - \frac{\sigma V_y(\tilde{y}_{t,T}(j), \varepsilon_{Y,T})}{(1 - \tau_T)(\sigma - 1) U_c(C_T)} \right] \right\} = 0. \quad (\text{A.15})$$

Monopolistic competition in production leads to a wedge between marginal utility of consumption and marginal disutility of production, represented by  $\frac{\sigma}{(1 - \tau_t)(\sigma - 1)}$ . We allow for fluctuations in this wedge by assuming a time-varying proportional tax  $\tau_t$ . Hereafter, we refer to these fluctuations as mark-up shocks  $\mu_t$ , where  $\mu_t = \frac{\sigma}{(1 - \tau_t)(\sigma - 1)}$ . In the present model,  $\tau_t$  only affects the pricing mark-up and has no further fiscal consequences. So we name these disturbances mark-up shocks, but they are essentially driven by labor tax fluctuations.

Given the Calvo-type setup, the price index evolves according to the following law of motion,

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1 - \alpha) (\tilde{p}_t(h))^{1-\sigma}. \quad (\text{A.16})$$

The rest of the world has an analogous price setting mechanism.

### The asset market structure

We assume that, as in Chari et al. (2002), markets are complete both domestically and internationally. In this setting, agents have access to a complete set of state contingent securities, and the intertemporal marginal rate of substitution is equalized across countries,

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} = \frac{U_C(C_{t+1})}{U_C(C_t)} \frac{P_t}{P_{t+1}}. \quad (\text{A.17})$$

Using the definition of the real exchange rate and assuming symmetric initial conditions across countries, it follows that

$$U_C(C_t^*) = U_C(C_t) Q_t. \quad (\text{A.18})$$

The presence of home bias leads to different shadow prices of consumption across countries, or, equivalently, different consumption based price indexes. As intuitively explained in Corsetti, Dedola and Leduc (2008), efficient risk sharing requires that "the marginal benefit of an extra unit of foreign consumption should be equal to its marginal cost, given by the domestic marginal utility of consumption times  $Q_t$ , i.e. the relative price of  $C_t^*$  in terms of  $C_t$ ". That is, efficient risk sharing does not imply real risk sharing, as real marginal utilities are not equalized across countries.

## B Welfare Approximations

In this Appendix, we derive a second order approximation of the equilibrium conditions of the model and the utility function under the assumptions that  $\bar{Q} = 1$ ,  $\bar{C} = \bar{C}^*$  and  $\bar{G}^* = \bar{G} = 0$ . Nevertheless, we allow for an inefficient level of steady state output (and the degree of this inefficiency is given by  $\bar{\mu}$ ). Our derivation follows closely the approach presented in Benigno and Benigno (2003, 2006).

### B.1 Demand

The second order approximation of the demand condition in the small open economy (Equation (A.13)) can be written as:

$$\sum \beta^t \left[ d'_y y_t + \frac{1}{2} y'_t D_y y_t + y'_t D_e e_t \right] + t.i.p + \mathcal{O}(\|\xi\|^3) = 0, \quad (\text{B.19})$$

where

$$\begin{aligned} y_t &= \begin{bmatrix} \hat{Y}_t & \hat{C}_t & \hat{p}_{H,t} & \hat{Q}_t \end{bmatrix}, \quad e_t = \begin{bmatrix} \hat{\varepsilon}_t & \hat{\mu}_t & \hat{g}_t & \hat{C}_t^* \end{bmatrix} \\ d'_y &= \begin{bmatrix} -1 & 1 - \lambda & -\theta & \theta\lambda \end{bmatrix}, \quad d'_e = \begin{bmatrix} 0 & 0 & 1 & \lambda \end{bmatrix}, \\ D'_y &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \lambda(1 - \lambda) & 0 & -\theta\lambda(1 - \lambda) \\ 0 & 0 & 0 & 0 \\ 0 & -\theta\lambda(1 - \lambda) & 0 & \theta^2\lambda(1 - \lambda) \end{bmatrix}, \quad D'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -(1 - \lambda) & -\lambda(1 - \lambda) \\ 0 & 0 & \theta & 0 \\ 0 & 0 & -\theta\lambda & \theta\lambda(1 - \lambda) \end{bmatrix}. \end{aligned}$$

Note that  $\hat{g}_t$  is defined as  $\log(G_t/\bar{Y})$  in order to allow for a zero steady state level of government expenditure.

### B.2 Risk Sharing Equation

Given our utility function specification, Equation (A.18) gives rise to a exact log linear expression, and the first and second-order approximations are therefore identical. In matrix notation, we have:

$$\sum E_t \beta^t \left[ c'_y y_t + \frac{1}{2} y'_t C_y y_t + y'_t C_e e_t \right] = 0, \quad (\text{B.20})$$

$$\begin{aligned} c'_y &= \begin{bmatrix} 0 & -1 & 0 & \frac{1}{\rho} \end{bmatrix}, \quad c'_e = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \\ C'_y &= 0, \quad C'_e = 0. \end{aligned}$$

### B.3 The Real Exchange Rate

Using the definition of the real exchange rate, we can write the second-order approximation of Equation (A.6) as:

$$\sum E_t \beta^t \left[ f'_y y_t + \frac{1}{2} y'_t F_y y_t + y'_t F_e e_t \right] + t.i.p + \mathcal{O}(\|\xi\|^3) = 0, \quad (\text{B.21})$$

where

$$f'_y = \begin{bmatrix} 0 & 0 & -(1-\lambda) & -\lambda \end{bmatrix}, f'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix},$$

$$F'_y = \lambda(\theta-1) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1-\lambda)/(1-\lambda) \end{bmatrix}, F'_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

#### B.4 Price Setting

The second order approximation of the price setting equation (A.15), given (A.16), can be written in the following way:

$$Q_{to} = \phi \sum E_t \beta^t \left[ a'_y y_t + \frac{1}{2} y'_t A_y y_t + y'_t A_e e_t + \frac{1}{2} a_{\hat{\pi}} \hat{\pi}_t^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (\text{B.22})$$

where

$$a'_y = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix}, a'_e = \begin{bmatrix} -\eta & 1 & 0 & 0 \end{bmatrix}, a_{\hat{\pi}} = (\eta+1) \frac{\sigma}{k},$$

$$A'_y = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & 0 \\ \rho & -\rho^2 & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, A'_e = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

#### B.5 Welfare

The second-order approximation of the utility function,  $U_t$ , can be written as:

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[ w'_y y_t - \frac{1}{2} y'_t W_y y_t - y'_t W_e e_t - \frac{1}{2} w_{\hat{\pi}} \hat{\pi}_t^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (\text{B.23})$$

where

$$w'_{\hat{\pi}} = \frac{\sigma}{\bar{\mu}k}, w'_y = \begin{bmatrix} -1/\bar{\mu} & 1 & 0 & 0 \end{bmatrix},$$

$$W'_y = \begin{bmatrix} \frac{(1+\eta)}{\bar{\mu}} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, W'_e = \begin{bmatrix} -\frac{\eta}{\bar{\mu}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

In addition, using the second-order approximation of the equilibrium condition derived in Sections B.1 to B.4, we can eliminate the term  $w'_y y_t$  from Equation (B.23). In order to do so, we derive the vector  $Lx$ , such that

$$\begin{bmatrix} a_y & d_y & f_y & c_y \end{bmatrix} Lx = w_y,$$

where  $a_y, d_y, f_y, c_y$  were previously defined in this Appendix. We have:

$$Lx_1 = \frac{1}{(\rho + \eta) + l\eta} [-l\bar{\mu}^{-1} + (1 - \lambda) - \bar{\mu}^{-1}], \quad (\text{B.24})$$

$$Lx_2 = \frac{1}{(\rho + \eta) + l\eta} [\rho(\bar{\mu}^{-1} - (1 - \lambda)) + (1 - \lambda)(\eta + \rho)], \quad (\text{B.25})$$

$$Lx_3 = \frac{1}{(\rho + \eta) + l\eta} [-(\rho\theta - 1)(1 - \lambda)\bar{\mu}^{-1} - (\eta\theta + 1)], \quad (\text{B.26})$$

where  $l = (\rho\theta - 1)\lambda(2 - \lambda)$ . The loss function  $L_{t_0}$  will have the following form:

$$L_{t_0} = U_c \bar{C} E_{t_0} \sum \beta^t \left[ \frac{1}{2} y_t' L_y y_t + y_t' L_e e_t + \frac{1}{2} l_{\hat{\pi}} \hat{\pi}_t^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3), \quad (\text{B.27})$$

where:

$$L_y = W_y + Lx_1 A_y + Lx_2 D_y + Lx_3 F_y,$$

$$L_e = W_e + Lx_1 A_e + Lx_2 D_e,$$

$$L_{\hat{\pi}} = w_{\hat{\pi}} + Lx_1 a_{\hat{\pi}}.$$

We can also use the model's equilibrium conditions in order to express the loss function in terms of the output, the real exchange rate and inflation. Moreover, these variables can be expressed as deviations from their targets, as follows

$$L_{t_0}^i = U_c \bar{C} E_{t_0} \sum \beta^t \left[ \frac{1}{2} \Phi_Y (\hat{Y}_t - \hat{Y}_t^T)^2 + \frac{1}{2} \Phi_q (\hat{Q}_t - \hat{Q}_t^T)^2 + \frac{1}{2} \Phi_\pi (\hat{\pi}_t^H)^2 \right] + t.i.p + \mathcal{O}(\|\xi\|^3) \quad (\text{B.28})$$

where, using  $(1 - \phi) = 1/\bar{\mu}$ , we have

$$\begin{aligned} \Phi_Y &= (\eta + \rho)(1 - \phi) + \frac{(\rho - 1)[-l(1 - \phi) - (\lambda - \phi)]}{(1 + l)} \\ &\quad + Lx_1 \left[ (\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l)} \right] - \frac{Lx_2(1 - \lambda)^2 \lambda(\rho\theta - 1)}{(1 + l)}, \\ \Phi_q &= -\frac{(\lambda + l)(\rho - 1)}{(1 - \lambda)\rho^2} + \frac{Lx_1 l(\rho - 1 - l)}{(1 - \lambda)^2 \rho} \\ &\quad + \frac{Lx_2 \lambda(\rho\theta - 1)[\rho\theta(1 - \lambda) + \lambda + l]}{\rho^2} + \frac{Lx_3 \lambda(\theta - 1)}{1 - \lambda}, \\ \Phi_\pi &= \frac{\sigma}{\bar{\mu}k} + (1 + \eta) \frac{\sigma}{k} Lx_1, \end{aligned}$$

and

$$\hat{Y}_t^T = q_y^e e_t, \text{ and } \hat{Q}_t^T = q_q^e e_t, \text{ }^{32}$$

where

$$\begin{aligned} q_y^e &= \frac{1}{\Phi_Y} \left[ \frac{\eta}{\bar{\mu}} + Lx_1(1 + \eta)\eta \quad -Lx_1(1 + \eta) \quad \frac{(\rho-1)(1-\lambda)+Lx_2-\rho(\rho-1)Lx_1}{1+l} \quad 0 \right], \\ q_q^e &= \frac{1}{\Phi_q} \left[ 0 \quad 0 \quad \frac{lLx_1}{(1-\lambda)} + \frac{Lx_2\lambda(\lambda(1-\lambda)+1)(\rho\theta-1)}{\rho(1-\lambda)} \quad \frac{-Lx_2\lambda(1-\lambda)(\rho\theta-1)}{\rho} \right]. \end{aligned}$$

<sup>32</sup>Note that the targets were specified in order to allow for any value of  $\Phi_q$ .

Moreover, we can write the constraints of the maximization problem as:

$$\hat{\pi}_t^H = k \left( \eta(\hat{Y}_t - \hat{Y}_t^T) + (1 - \lambda)^{-1}(\hat{Q}_t - \hat{Q}_t^T) + \hat{u}_t \right) + \beta E_t \hat{\pi}_{t+1}^H, \quad (\text{B.29})$$

$$(\hat{Y}_t - \hat{Y}_t^T) = (\hat{Q}_t - \hat{Q}_t^T) \frac{(1 + l)}{\rho(1 - \lambda)} + \hat{\chi}_t, \quad (\text{B.30})$$

where

$$\begin{aligned} \hat{u}_t &= \left[ \eta, \frac{1}{1 - \lambda} \right] \left[ (\hat{Y}_t^T - \hat{Y}_t^{Flex}), (\hat{Q}_t^T - \hat{Q}_t^{Flex}) \right]', \\ \hat{\chi}_t &= \left[ -1, \frac{(1 + l)}{\rho(1 - \lambda)} \right] \left[ (\hat{Y}_t^T - \hat{Y}_t^{Flex}), (\hat{Q}_t^T - \hat{Q}_t^{Flex}) \right]'. \end{aligned}$$

We can also define  $\hat{Y}_t^{Flex}$  and  $\hat{Q}_t^{Flex}$  as the flexible price allocation for output and the real exchange rate:

$$\hat{Y}_t^{Flex} = [(\eta + \rho) + \eta l]^{-1} \left\{ \eta(1 + l)\hat{\varepsilon}_t - (1 + l)\hat{\mu}_t + \rho\hat{g}_t - \rho l\hat{C}_t^* \right\}, \quad (\text{B.31})$$

$$\frac{\hat{Q}_t^{Flex}}{(1 - \lambda)} = [(\eta + \rho) + \eta l]^{-1} \rho \left\{ \eta\hat{\varepsilon}_{y,t} - \hat{\mu}_t - \eta\hat{g}_t - (\eta + \rho)\hat{C}_t^* \right\}. \quad (\text{B.32})$$

## B.6 Re-writing the linear terms in the Taylor expansion

Using a first-order approximation of demand equation (RS) and the constraint of the policy problem (C1), we can write the linear terms  $w'_y y_t$  as follows

$$\hat{C}_t - \frac{1}{\bar{\mu}} \hat{Y}_t = \frac{1}{\rho} (1 - \kappa_{\bar{\mu}, \rho\theta}) \hat{Q}_t + t.i.p. + \mathcal{O}(\|\xi\|^2)$$

where

$$\kappa_{\bar{\mu}, \rho\theta} = \frac{1 + (2 - \lambda)\lambda(\rho\theta - 1)}{\bar{\mu}(1 - \lambda)}$$

We should note, however, that the full welfare implications of the linear term can only be accessed when this term is approximated to second order. This was derived in the section B.5 in this Appendix.

Under **Financial Autarky**, equation (RS) no longer holds. Instead, in this case, domestic budget constraint implies  $P_{H,t}Y_t = P_tC_t$ . In this case can write

$$\hat{C}_t - \frac{1}{\bar{\mu}} \hat{Y}_t = \frac{1}{(1 - \lambda)} (1 - \kappa'_{\bar{\mu}, \theta}) \hat{Q}_t + t.i.p. + \mathcal{O}(\|\xi\|^2)$$

where

$$\kappa'_{\bar{\mu}, \theta} = 1 + (1 - \lambda) - \bar{\mu}^{-1} - (1 - \bar{\mu}^{-1})(2 - \lambda)(\theta - 1)$$

## References

- [1] Adao, B., Correia, I. and Teles, P., 2003. Gaps and Triangles. *Review of Economic Studies* 70(4), 699-713.
- [2] Adolfson, M., Laséen, S. Linde, J. and Villani, M., 2007. Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through. *Journal of International Economics* 72(2), 481-511.
- [3] Aoki, K., 2001. Optimal Monetary Policy Response to Relative Price Changes. *Journal of Monetary Economics* 48(1), 55-80.
- [4] Benigno, G. and Benigno, P., 2003. Price Stability in Open Economies. *Review of Economic Studies* 70(4), 743-764.
- [5] Benigno, G. and Benigno, P. 2006. Designing Targeting rules for International Monetary Policy Coordination. *Journal of Monetary Economics* 53(3), 473-506.
- [6] Benigno, P. and Woodford, M. 2005. Inflation Stabilization And Welfare: The Case Of A Distorted Steady State. *Journal of the European Economic Association* 3(6), 1185-1236.
- [7] Benigno, P. and Woodford, M., 2006. Linear-Quadratic Approximation of Optimal Policy Problems. NBER Working Paper 12672.
- [8] Bergin, P., Shin, H. and Tchakarov, I., 2007. Does exchange rate variability matter for welfare? A quantitative investigation of stabilization policies. *European Economic Review* 51(4), 1041-1058.
- [9] Calvo, G., 1983. Staggered Prices in a Utility-Maximizing Framework. *Journal of Monetary Economics* 12(3), 383-398.
- [10] Chari, V., Kehoe, P. and McGrattan, E., 2002. Can Sticky Price Models Generate volatile and Persistent Real Exchange Rates? *Review of Economic Studies* 69(3), 533-63.
- [11] Clarida, R., Gali, J., Gertler, M., 2001. Optimal Monetary Policy in Closed versus Open Economies: An Integrated Approach. NBER Working Paper 8604.
- [12] Collard, F. and Juillard, M., 2001. Accuracy of stochastic perturbation methods: The case of asset pricing models. *Journal of Economic Dynamics and Control* 25(6-7), 979-999.
- [13] Corden, M., 1974. *Trade policy and economic welfare*, Clarendon Press, Oxford.
- [14] Corsetti, G., Dedola, L. and Leduc, S., 2008. International Risk-sharing and the Transmission of Productivity Shocks. forthcoming *Review of Economic Studies*.
- [15] Corsetti, G., Dedola, L. and Leduc, S., 2007. Optimal Monetary Policy and the Sources of Local-Currency Price Stability. NBER Working Paper 13544.
- [16] Corsetti, G. and Pesenti, P., 2001. Welfare and Macroeconomic Interdependence. *Quarterly Journal of Economics* 116(2), 421-445.

- [17] Corsetti, G. and Pesenti, P., 2005. International dimensions of optimal monetary policy. *Journal of Monetary Economics* 52(2), 281-305.
- [18] Corsetti, G., 2007. Openness and the case for flexible exchange rates. *Research in Economics* 60 (1), 1-21.
- [19] Correia, M., Nicolini, J. and Teles, P., 2003. Optimal Fiscal and Monetary Policy: Equivalence Results. CEPR Discussion Papers 3730.
- [20] Cova, P. and Søndergaard, J., 2004. When Should Monetary Policy Target The Exchange Rate? Royal Economic Society Annual Conference, paper number 51.
- [21] Devereux, M., and Engel, C., 2003. Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility. *Review of Economic Studies* 70(4), 765-84.
- [22] Duarte, M. and Obstfeld, M., 2008. Monetary Policy in the Open Economy Revisited: The Case for Exchange-Rate Flexibility Restored. Forthcoming *Journal of International Money and Finance* .
- [23] Faia, E. and Monacelli, T., 2008. Optimal Monetary Policy in a Small Open Economy with Home Bias. Forthcoming *Journal of Money Credit and Banking*.
- [24] Gali, J. and Monacelli, T., 2005. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. *Review of Economic Studies* 72(3), 707-734.
- [25] Henderson, D. W. and Kim J., 1999. Exact Utilities under Alternative Monetary Rules in a Simple Macro Model with Optimizing Agents. *International Tax and Public Finance* 6, 507–535.
- [26] Hevia, C. and Nicolini, J., 2004. Optimal Devaluations. *Econometric Society Latin American Meetings* 337, Econometric Society.
- [27] Judd, K., 1996. Approximation, Perturbation and Projection Methods in Economic Analysis. In: Amman, H. M., Rust, J. and Kendrick, D. A. (Ed.), *Handbook of Computational Economics*, vol. I.
- [28] Kehoe, P. and Perri, F., 2002. International Business Cycles with Endogenous Incomplete Markets. *Econometrica* 70(3), 907-928
- [29] Kim, J., Kim, S. and Levin, A., 2003. Patience, persistence, and welfare costs of incomplete markets in open economies. *Journal of International Economics* 61(2), 385-396.
- [30] Kollmann, R., 2002. Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics* 49(5), 989-1015.
- [31] Lama, R. and Medina, J., 2007. Optimal Monetary Policy in a Small Open Economy Under Segmented Asset Markets and Sticky Prices. IMF Working Papers 217.
- [32] Lipínska, A., 2007. The Maastricht Convergence Criteria and Optimal Monetary Policy for the EMU Accession Countries. CEP Discussion Papers 0808.

- [33] Lubik, T. and Schorfheide, F., 2005. A Bayesian Look at New Open Economy Macroeconomics. Economics Working Paper Archive 521, The Johns Hopkins University, Department of Economics.
- [34] Lubik, T. and Schorfheide, F., 2007. Do Central Banks Respond to Exchange Rate Movements? A Structural Investigation. *Journal of Monetary Economics* 54(4), pages 1069-1087.
- [35] McCallum, B. and Nelson, E., 2005. Targeting vs. Instrument Rules for Monetary Policy. *Federal Reserve Bank of St. Louis Review* 87(5), 597-611.
- [36] Obstfeld, M. and Rogoff, R., 1995. Exchange Rate Dynamics Redux. *Journal of Political Economy* 103(3), 624-660.
- [37] Obstfeld, M. and Rogoff, R., 1998. Risk and Exchange Rates. NBER Working Papers 6694.
- [38] Pappa, E., 2004. Do the ECB and the fed really need to cooperate? Optimal monetary policy in a two-country world. *Journal of Monetary Economics* 51(4), 753-779
- [39] Rotemberg, J. and Woodford, M., 1997. An Optimization-Based Framework for the Evaluation of Monetary Policy. In: Bernanke, B. S. and Rotemberg, J. J. (Ed.), *NBER Macroeconomics Annual 1997*, Cambridge, MIT Press.
- [40] Schmitt-Grohé, S. and Uribe, M., 2004. Solving dynamic general equilibrium models using a second-order approximation to the policy function. *Journal of Economic Dynamics and Control* 28(4), 755-775.
- [41] Smets, F. and Wouters, R., 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. *Journal of the European Economic Association* 1(5), 1123-1175.
- [42] Sutherland, A., 2002. A Simple Second-Order Solution Method For Dynamic General Equilibrium Models. CEPR Discussion Papers 3554.
- [43] Sutherland, A., 2004. International Monetary Policy Coordination and Financial Market Integration. Mimeo, St Andrews University.
- [44] Sutherland, A., 2005. Incomplete pass-through and the welfare effects of exchange rate variability. *Journal of International Economics* 65(2), 375-399.
- [45] Svensson, L., 2002. Inflation Targeting: Should It Be Modeled as an Instrument Rule or a Targeting Rule?. *European Economic Review* 46(4-5), 771-780.
- [46] Svensson, L., 2003. What Is Wrong with Taylor Rules? Using Judgment in Monetary Policy through Targeting Rules. *Journal of Economic Literature* 41(2), 426-477.
- [47] Tille, C., 2001. The Role of Consumption Substitutability in the International Transmission of Monetary Shocks. *Journal of International Economics* 53(2), 421-444
- [48] Tille, C., 2002. How valuable is exchange rate flexibility? Optimal Monetary policy under sectorial shocks. Staff Reports 147, Federal Reserve Bank of NY.

- [49] Woodford, M., 1999. Optimal Monetary Policy Inertia. Proceedings, Federal Reserve Bank of San Francisco.
- [50] Woodford, M., 2003. Interest Rates and Prices: Foundation of a Theory of Monetary Policy. Princeton University Press.

Figures

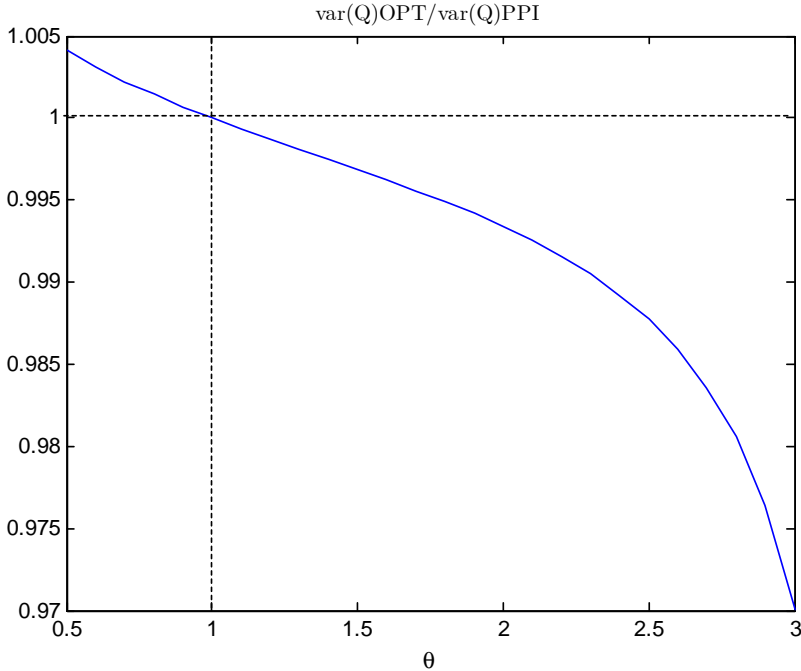


Figure 1: Variance of real exchange rate under the optimal plan relative to variance of real exchange rate under PPI inflation targeting - for different values of  $\theta$ . The value of the remaining structural parameters is shown in Table 5.

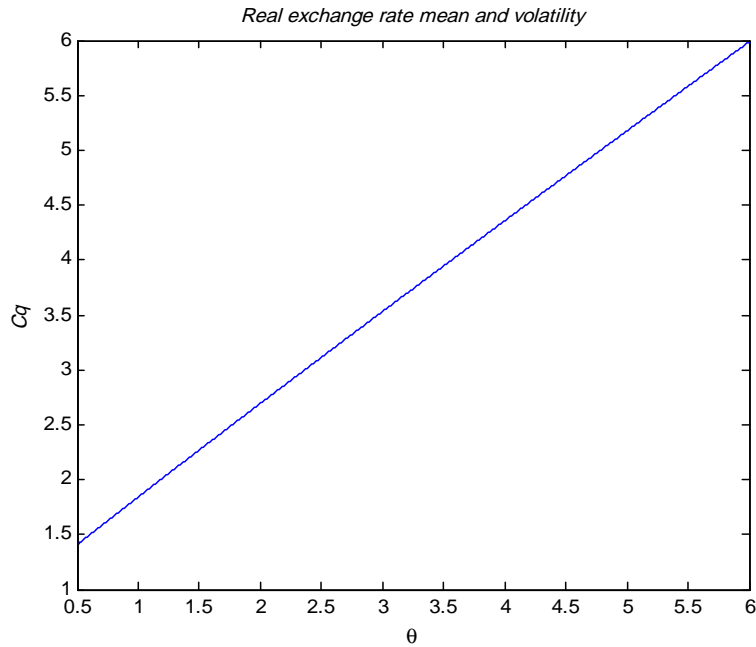


Figure 2: Relationship between the mean and variance of the real exchange rate - given by  $E[Q_t] = c_q var[Q_t]$  - for different values of  $\theta$ , given  $\alpha = 0$ . The value of the remaining structural parameters is shown in Table 5.

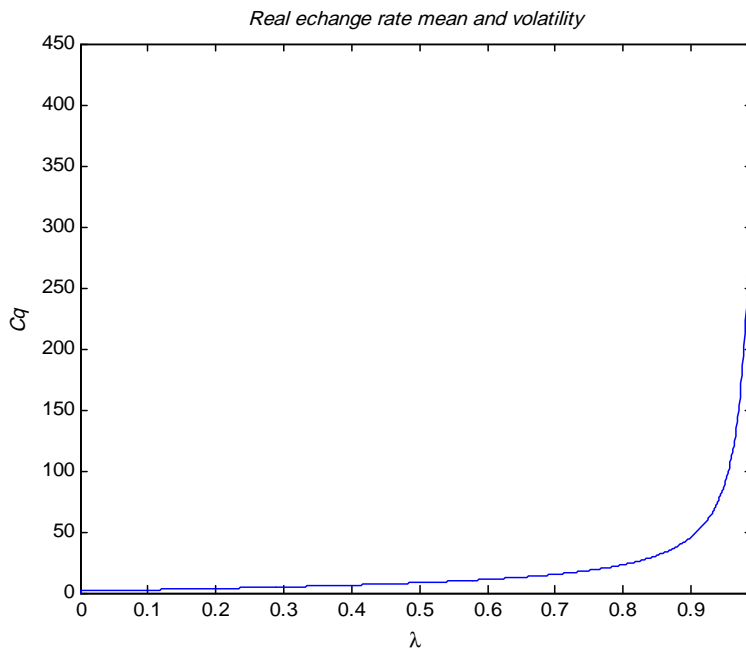


Figure 3: Relationship between the mean and variance of the real exchange rate - given by  $E[Q_t] = c_q var[Q_t]$  - for different values of  $\lambda$ , given  $\alpha = 0$ . The value of the remaining structural parameters is shown in Table 5.

## Tables

Table 1: Home equilibrium conditions

$$\begin{aligned}\hat{\pi}_t^H &= k(\rho\hat{C}_t + \eta\hat{Y}_t + \lambda(1-\lambda)^{-1}\hat{Q}_t + \hat{\mu}_t - \eta\hat{\varepsilon}_t) + \beta E_t \hat{\pi}_{t+1}^H & \text{(AS)} \\ \hat{Y}_t &= (1-\lambda)\hat{C}_t + \lambda\hat{C}_t^* + \gamma\hat{Q}_t + \hat{g}_t & \text{(AD)} \\ \hat{C}_t &= \hat{C}_t^* + \frac{1}{\rho}\hat{Q}_t & \text{(RS)}\end{aligned}$$

where variables are expressed in log deviations from steady-state, i.e.  $\hat{X} \equiv \log(X/\bar{X})$

Table 2: Model parameters

$\rho^{-1}$	Intertemporal elasticity of substitution
$\theta$	Intratemporal elasticity of substitution
$\eta^{-1}$	Frisch elasticity of labor supply
$\lambda$	Degree of openness
$\beta$	Subjective discount factor
$\sigma$	Elasticity of substitution across the differentiated products
$k$	$(1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$
$\gamma$	$\theta\lambda(2-\lambda)/(1-\lambda)$

Table 3: Foreign equilibrium conditions

$$\begin{aligned}\hat{\pi}_t^* &= k(\rho\hat{C}_t^* + \eta\hat{Y}_t^* + \hat{\mu}_t^* - \eta\hat{\varepsilon}_t^*) + \beta E_t \hat{\pi}_{t+1}^* & \text{(AS}^*) \\ \hat{Y}_t^* &= \hat{C}_t^* + \hat{g}_t^* & \text{(AD}^*)\end{aligned}$$

Table 4: Constraints of the policy problem

$$\begin{aligned}\hat{\pi}_t^H &= k\left(\eta\hat{y}_t + \frac{1}{(1-\lambda)}\hat{q}_t + \hat{u}_t\right) + \beta E_t \hat{\pi}_{t+1}^H & \text{C1} \\ \hat{g}_t &= \frac{(1+l)}{\rho(1-\lambda)}\hat{q}_t + \hat{\chi}_t & \text{C2}\end{aligned}$$

where  $l = (\rho\theta - 1)\lambda(2 - \lambda)$

Table 5: Parameter values used in the quantitative analysis

Parameter	Value	Notes:
$\beta$	0.99	Specifying a quarterly model with 4% steady-state real interest rate
$\eta$	0.47	Following Rotemberg and Woodford (1997)
$\lambda$	0.4	This implies a 40% import share of the GDP
$\alpha$	0.66	Characterizing an average length of price contract of 3 quarters
$\bar{\mu}$	$(1 - \lambda)^{-1}$	Characterizing an efficient steady-state
$\sigma$	10	Following Benigno and Woodford (2005)
$\rho$	1	(unless specified otherwise)
$\theta$	3	(unless specified otherwise)
$sdv(\hat{\varepsilon})$	0.0071	Consistent with Galí and Monacelli (2005) and Kehoe and Perri (2002)
$sdv(\hat{g})$	0.0062	Following Lubik and Schorfheide (2005)
$sdv(\hat{\mu})$	0.0013	Consistent with Adolfson et al (2007) and Smets and Wouters (2003)
$sdv(\hat{C}^*)$	0.0129	Following Lubik and Schorfheide (2007)
$\kappa(\varepsilon), \kappa(C^*)$	0.66	Following Galí and Monacelli (2005)
$\kappa(g)$	0.94	Following Lubik and Schorfheide (2005)
$\kappa(\mu)$	0.99	Following Adolfson et al (2007)

Shocks are assumed to be uncorrelated and follow an AR(1) process with persistence  $\kappa^{(\cdot)}$  and standard deviation  $sdv(\cdot)$ .

Table 6: Policy rule associated with highest welfare

$\rho \backslash \theta$	0.5	2	3.5	5
0.5	PPI	PPI	PPI	PEG
2	PPI	PPI	PEG	PEG
3.5	PPI	CPI	PEG	PEG
5	PPI	PEG	PEG	PEG

Table 7:  $W_d^{PPI, PEG}$  following all shocks

$\rho \backslash \theta$	0.5	2	3.5	5
0.5	0.032%	0.014%	0.004%	-0.002%
2	0.065%	0.008%	-0.028%	-0.051%
3.5	0.128%	-0.003%	-0.088%	-0.142%
5	0.215%	-0.023%	-0.179%	-0.279%