Technical Appendix of "Monetary Policy under Alternative Asset Market Structures: the Case of a Small Open Economy"

Bianca De Paoli
Bank of England and Centre for Economic Performance

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A Appendix: Complete Markets Specification

Our complete market setup closely follows the one of Chari *et al* (2002). In order to illustrate this specification we introduce some further notation, which explicitly accounts for the different states of nature s. We assume that agents meet and trade assets, which are contingent on the different states s, before monetary policy decisions are made. Given that agents do not know the policy decisions when trading financial assets, we implicitly consider the case in which the different policy choices are included in the state vector s.[1]

Denoting $s^t = (s_0, s_1, s_2, ..)$ the history of states up to period t, we assume that Home consumers choose consumption, labour and state-contingent bond holdings to maximize their utility

$$\sum_{t=0}^{\infty} \sum \beta^t \pi\left(s^t\right) U\left(s^t\right) \tag{A.1}$$

subject to a sequence of consumer budget constraints

$$P\left(s^t\right) C\left(s^t\right) + \sum_{s_{t+1}} Q\left(s^{t+1}/s^t\right) B\left(s^{t+1}\right) \leq B\left(s^t\right) + PI\left(s^t\right) \tag{A.2}$$

where we define private income as PI_t

$$PI_t \equiv \frac{(1-\tau_t)\int_0^n p_t(h) y_t(h) dh}{n} + P_{H,t} Tr_t \tag{A.3}$$

and $\pi\left(s^t\right)$ is the probability of state history s^t occurring. The variable $B\left(s^{t+1}\right)$ denotes home consumer's holdings of a one period nominal bond (denominated in domestic currency) purchased in period t and state s^t with payoffs contingent on state s_{t+1} at period $t+1$. Finally, the variable $Q\left(s^{t+1}/s^t\right)$ is the price of such a bond in period t and state s^t.

Domestic first order conditions with respect to bond holdings, in states s^{t-1} and s^t, imply

$$Q\left(s^t/s^{t-1}\right) = \beta^t \pi\left(s^t/s^{t-1}\right) \frac{U_C\left(s^t\right)}{U_C\left(s^{t-1}\right)} \frac{P\left(s^{t-1}\right)}{P\left(s^t\right)} \tag{A.4}$$

[1] That is, in this setup, one coordinate of the state vector effectively indexes the different policy choices, though we do not specify the mapping between policies and the state explicitly. For examples where this could be done, see Senay and Sutherland (2007), Hoffmann and Holtemoller (2008) and Berger (2008). In these works policy is written as a money supply rule governed by a single parameter. So this parameter could be included as a coordinate of the state vector and claims could be written contingent on it.

where $\pi\left(s^t/s^{t-1}\right) = \pi\left(s^t\right)/\pi\left(s^{t-1}\right)$. Similar optimality conditions in the rest of the world imply

$$Q\left(s^t/s^{t-1}\right) = \beta^t \pi\left(s^t/s^{t-1}\right) \frac{U_C^*\left(s^t\right)}{U_C^*\left(s^{t-1}\right)} \frac{S\left(s^{t-1}\right) P^*\left(s^{t-1}\right)}{S\left(s^t\right) P^*\left(s^t\right)}. \tag{A.5}$$

Given that domestic and foreign households face the same subjective probabilities, $\pi\left(s^t/s^{t-1}\right)$, and assuming no-arbitrage in asset markets, we can combine the two expressions above and iterate backward to get

$$\frac{U_C^*\left(s^t\right)}{U_C\left(s^t\right)} = k \frac{S\left(s^t\right) P^*\left(s^t\right)}{P\left(s^t\right)} \tag{A.6}$$

where

$$k = \frac{U_C^*\left(s^0\right)}{U_C\left(s^0\right)} \frac{P\left(s^0\right)}{S\left(s^0\right) P^*\left(s^0\right)}. \tag{A.7}$$

So the parameter k is pinned down by initial conditions in the market for state contingent bonds. In our analysis we assume that the monetary authorities chose policy taking k as given. That is, policy rules are determined after the state-contingent financial markets are closed. In addition, we assume a symmetric equilibrium in which $k = 1$, and given that the equation above holds in every state of the world, we can write

$$\frac{U_C(C_t^*)}{U_C(C_t)} = \frac{S_t P_t^*}{P_t}. \tag{A.8}$$

B Appendix: Approximating the Model

In this Appendix, we derive first and second order approximations to the equilibrium conditions of the model. Lowercase variables denote log deviations from steady state. We allow for an *asymmetric* steady state in the analysis of the incomplete market case. All variables in steady state are denoted with a bar. We assume that in steady state $1 + i_t = 1 + i_t^* = 1/\beta$ and $P_t^H/P_{t-1}^H = P_t^F/P_{t-1}^F = 1$. We normalize the price indexed such that $\overline{P}_H = \overline{P}_F$ and assume $\overline{G} = 0$. Moreover, we use the following isoelastic functional forms for the utility functions:

$$U(C_t) = \frac{C_t^{1-\rho}}{1-\rho} \tag{B.9}$$

$$V\left(y_t(h), \varepsilon_T\right) = \frac{\varepsilon_{y,t}^{-\eta} y_t(h)^{\eta+1}}{\eta+1} \tag{B.10}$$

B.1 Demand

The first order approximation to the small open economy demand is

$$y_H = -\theta p_H + d_b c + (1-d_b)c^* + \theta(1-d_b)q + g, \tag{B.11}$$

where $d_b = (1-\lambda)(1+a)$ and $a = \frac{\lambda(\overline{C}-\overline{C}^*)}{\overline{Y}}$. Moreover, Home relative prices are denoted by $p_H = P_H/P$ and the fiscal shock g_t is defined as $\frac{Gt-\overline{G}}{\overline{Y}}$, allowing for the analysis of this shock even when steady-state government consumption is non-zero. In the symmetric steady state, in which $d_b = 1-\lambda$, Equation (B.11) becomes

$$y_H = -\theta p_H + (1-\lambda)c + \lambda\hat{C}^* + \theta\lambda q + g. \tag{B.12}$$

The second order approximation to the demand function is

$$\sum \beta^t \left[d_y' y_t + \frac{1}{2} y_t' D_y y_t + y_t' D_e e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{B.13}$$

where

$$y_t = \left[\begin{array}{cccc} y_t & c_t & p_{Ht} & q_t \end{array} \right],$$

$$e_t = \left[\begin{array}{cccc} \varepsilon_t & \mu_t & g_t & c_t^* \end{array} \right],$$

$$d_y' = \left[\begin{array}{cccc} -1 & d_b & -\theta & \theta(1-d_b) \end{array} \right],$$

$$D_y' = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & (1-d_b)d_b & 0 & -\theta(1-d_b-d_g)d_b \\ 0 & 0 & 0 & 0 \\ 0 & -\theta(1-d_b)d_b & 0 & \theta^2(1-d_b)d_b \end{array} \right],$$

and

$$D_e' = \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -d_b & -(1-d_b)d_b \\ 0 & 0 & 0 & \theta & 0 \\ 0 & 0 & 0 & -\theta(1-d_b) & \theta(1-d_b)d_b \end{array} \right].$$

B.2 The Real Exchange Rate

Given that, in the rest of the world, $P_F = SP^*$, Equation (6) in the main text can be expressed as:

$$\left(\frac{P_t}{P_{H,t}} \right)^{1-\theta} = (1-\lambda) + \lambda \left(Q_t \frac{P_t}{P_{H,t}} \right)^{1-\theta}. \tag{B.14}$$

The first order approximation to the above expression is:

$$p_{H,t} = -\frac{\lambda q_t}{1-\lambda}. \tag{B.15}$$

The second order approximation to Equation (B.14) is:

$$\sum E_t \beta^t \left[f_y' y_t + \frac{1}{2} y_t' F_y y_t + y_t' F_e e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0, \tag{B.16}$$

where

$$f_y' = \left[\begin{array}{cccc} 0 & 0 & -(1-\lambda) & -\lambda \end{array} \right],$$

$$f_e' = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \end{array} \right],$$

$$F_y' = \lambda(\theta-1) \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & (1-\lambda/(1-\lambda)) \end{array} \right],$$

and

$$F_e' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

B.3 Price Setting

The first and second-order approximations to the price setting equation follow Benigno and Benigno (2003). These conditions are derived from the following first order condition of sellers that can reset their prices:

$$E_t \left\{ \sum (\alpha\beta)^{T-t} U_c(C_T) \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_T \left[\frac{\tilde{p}_t(h)}{P_{H,T}} \frac{P_{H,T}}{P_T} - \mu_T \frac{V_y\left(\tilde{y}_{t,T}(h), \varepsilon_t\right)}{U_c(C_T)} \right] \right\} = 0, \qquad \text{(B.17)}$$

where

$$\tilde{y}_t(h) = \left(\frac{\tilde{p}_t(h)}{P_{H,t}} \right)^{-\sigma} Y_t, \qquad \text{(B.18)}$$

and

$$(P_{H,t})^{1-\sigma} = \alpha P_{H,t-1}^{1-\sigma} + (1-\alpha)\left(\tilde{p}_t(h)\right)^{1-\sigma}. \qquad \text{(B.19)}$$

With markup shocks, μ_t, defined as $\frac{\sigma}{(\sigma-1)(1-\tau_t^H)}$, the first order approximation to the price setting equation can be written in the following way:

$$\pi_t = k\left(\rho c_t + \eta y_t - p_{H,t} + \mu_t - \eta \varepsilon_t\right) + \beta E_t \pi_{t+1}, \qquad \text{(B.20)}$$

where $k = (1-\alpha\beta)(1-\alpha)/\alpha(1+\sigma\eta)$.

The second order approximation to Equation (B.17) can be written as follows:

$$\begin{aligned} Q_{to} = \phi \sum E_t \beta^t & \left[a_y' y_t + \frac{1}{2} y_t' A_y y_t + y_t' A_e e_t + \frac{1}{2} a_\pi \pi_t^2 \right] \\ & + t.i.p + \mathcal{O}(||\xi||^3), \end{aligned} \qquad \text{(B.21)}$$

$$a_y' = \begin{bmatrix} \eta & \rho & -1 & 0 \end{bmatrix},$$

$$a_e' = \begin{bmatrix} -\eta & 1 & 0 & 0 \end{bmatrix},$$

$$A_y' = \begin{bmatrix} \eta(2+\eta) & \rho & -1 & 0 \\ \rho & -\rho^2 & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_e' = \begin{bmatrix} -\eta(1+\eta) & 1+\eta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$a_\pi = (\eta+1)\frac{\sigma}{k}.$$

B.4 Incomplete Markets: Approximating the Current Account equation

We assume that home-currency denominated bonds are in zero net supply. The net foreign asset position is therefore fully denominated in foreign currency. Aggregating private and public budget constraints, the law of motion for b_t can be written as

$$P_t C_t + \frac{S_t B_{F,t}}{(1+i_t^*)\,\psi\left(\frac{S_t B_{F,t}}{P_t}\right)} = S_t B_{F,t-1} + P_{H,t}(Y_t - G_t) \tag{B.22}$$

Defining $\frac{B_{F,t}S_t}{P_t} \equiv B_t$ we can rewrite the government budget constraint as

$$B_t = B_{t-1}\frac{S_t P_{t-1}}{S_{t-1}P_t}(1+i_t^*)\,\psi(B_t) + \frac{P_{H,t}}{P_t}(Y_t - G_t)(1+i_t^*)\,\psi(B_t) - C_t(1+i_t^*)\,\psi(B_t)\,. \tag{B.23}$$

From agents' intertemporal choice,

$$U_C(C_t) = (1+i_t^*)\psi(B_t)\,\beta E_t\left[U_C(C_{t+1})\frac{S_{t+1}P_t}{S_t P_{t+1}}\right]. \tag{B.24}$$

We can therefore write (B.23) as

$$B_t\beta E_t\left[U_C(C_{t+1})\frac{S_{t+1}P_t}{S_t P_{t+1}}\right] = B_{t-1}\frac{S_t P_{t-1}}{S_{t-1}P_t}U_C(C_t) + \frac{P_{H,t}}{P_t}(Y_t - G_t)U_C(C_t) - C_t U_C(C_t)\,. \tag{B.25}$$

And the log linear representation of the above equation, defining $a_\beta = \frac{a}{1-\beta}$, is

$$\begin{aligned} &-\rho a_\beta c_t + b_{t-1} + a_\beta(\Delta Q_t - \pi_t^*) \\ &= -y_t + (1+a)c_t - \rho a c_t - p_{H,t} + g_t \\ &+ \beta E_t\left[-\rho a_\beta c_{t+1} + b_t + a_\beta(\Delta Q_{t+1} - \pi_{t+1}^*)\right]. \end{aligned} \tag{B.26}$$

Furthermore, if we allow $B_{W,t} = B_{F,t-1}\frac{P_{t-1}}{P_t}\frac{S_t}{S_{t-1}}U_C(C_t)$ and $s_t = -\frac{P_{H,t}}{P_t}(Y_t - G_t) + C_t$, the intertemporal government solvency condition can be written as

$$b_{W,t} = U_C(C_t)\,s_t + E_t\beta b_{W,t+1} = E_t\sum_{T=t}^{\infty}\beta^{T-t}U_C(C_T)\,s_t, \tag{B.27}$$

and the term $U_C(C_T, \xi_{C,T})\,s_t$ can be approximated, up to the second order, by

$$U_C\overline{Y}\left\{\begin{array}{c} a - \widehat{Y}_t + (1+a(1-\rho))c_t - p_{H,t} - \frac{1}{2}y_t^2 + \rho y_t c_t - y_t p_{H,t} \\ +\frac{1}{2}(a\rho^2 + (1+a)(1-2\rho))c_t^2 + \rho c_t p_{H,t} - \frac{1}{2}p_{H,t}^2 \end{array}\right\}.$$

Thus, defining $b_{W,t} = \frac{B_{W,t} - \overline{B}_W}{\overline{B}_W'}$ and $\overline{B}_W' = \frac{U_c\overline{Y}}{(1-\beta)}$, we have,

$$\begin{aligned} b_{W,t} = {} & (1-\beta)\left[b_y^{i\prime}y_t + \frac{1}{2}y_t'B_y^i y_t + y_t'B_e^i e_t\right] + \beta E_t b_{W,t+1} \\ & + t.i.p + \mathcal{O}(||\xi||^3) \end{aligned} \tag{B.28}$$

where

$$b_y^{i\prime} = \left[\begin{array}{cccc} -1 & 1+a(1-\rho) & -1 & 0 \end{array} \right],$$

$$B_y^{i\prime} = \left[\begin{array}{cccc} -1 & \rho & -1 & 0 \\ \rho & a(1-\rho)^2+(1-2\rho) & \rho & 0 \\ -1 & \rho & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right],$$

and

$$B_e^{i\prime} = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Special Case: Note that if $\rho = \theta = 1$, $a = 0$ and $b_{-1} = 0$, the second order current account approximation becomes

$$c_t = y_t + p_{H,t} - g_t - g_t p_{H,t} + \widehat{g_t C}_t, \tag{B.29}$$

which combining with the demand equation implies

$$c_t = q_t + c_t^*. \tag{B.30}$$

This is identical to the perfect risk sharing condition.

B.5 Financial Autarky: the Extreme Case of Market Incompleteness

In this case we assume that there is no risk sharing between countries. The inability to trade bonds across borders impose that the value of imports equates the value of exports:

$$(1-n)SP_{H,t}^*C_{H,t}^* = nP_{F,t}C_{F,t}, \tag{B.31}$$

given the preference specification, we can write:

$$C_{H,t} = v\left[\frac{P_{H,t}}{P_t}\right]^{-\theta} C_t,\ C_{F,t} = (1-v)\left[\frac{P_{F,t}}{P_t}\right]^{-\theta} C_t, \tag{B.32}$$

$$C_{H,t}^* = v^*\left[\frac{P_{H,t}^*}{P_t^*}\right]^{-\theta} C_t^*, C_{F,t}^* = (1-v^*)\left[\frac{P_{F,t}^*}{P_t^*}\right]^{-\theta} C_t^*. \tag{B.33}$$

Substituting in Equation (B.31):

$$C_t = \left[\frac{p_{H,t}}{Q_t}\right]^{1-\theta} [Q_t]^{\theta} C_t^*. \tag{B.34}$$

And using the definition of the consumption indexes and market clearing, condition (B.34) implies

$$P_{H,t}(Y_t - G_t) = P_t C_t. \tag{B.35}$$

Assuming $\overline{C} = \overline{C}^*$, the second order approximation is

$$p_{H,t} + y_t - g_t + y_t g_t = c_t, \tag{B.36}$$

and can be represented as follows:

$$\sum E_t \beta^t \left[b_y^{fa\prime} y_t + \frac{1}{2} y_t' B_y^{fa} y_t + y_t' B_e^{fa} e_t \right] + t.i.p + \mathcal{O}(||\xi||^3) = 0,$$

$$b_y^{fa\prime} = \left[\begin{array}{cccc} -1 & 1 & -1 & 0 \end{array} \right],$$

$$B_y^{fa\prime} = 0,$$

and

$$B_e^{fa\prime} = \left[\begin{array}{cccc} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right].$$

Special Case: when $\theta = 1$, Equation (B.36) combined with the demand equation becomes

$$c_t^* = c_t + q_t. \tag{B.37}$$

B.6 Complete markets: the Risk Sharing Equation

Assuming a symmetric steady-state equilibrium, the log linear approximation to the risk sharing Equation (29) in the main text is

$$c_t^* = c_t + \frac{1}{\rho} q_t. \tag{B.38}$$

Given our utility function specification, Equation (29) gives rise to an exact log linear expression and therefore the first and second order approximations are identical. In matrix notation, we have

$$\sum E_t \beta^t \left[b_y^{c\prime} y_t + \frac{1}{2} y_t' B_y^{c} y_t + y_t' B_e^{c} e_t \right] = 0,$$

where

$$b_y^{c\prime} = \left[\begin{array}{cccc} 0 & -1 & 0 & \frac{1}{\rho} \end{array} \right],$$

$$b_y^{c\prime} = \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array} \right],$$

$$B_y^{c\prime} = 0,$$

and

$$B_e^{c\prime} = 0.$$

C Appendix: Welfare and Optimal Monetary Policy

C.1 Welfare with Incomplete Asset Markets:

Following Benigno and Benigno (2003), the second order approximation to the utility function, U_t, can be written as:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} \left[U(C_s) - \frac{1}{n} \int_0^n V(y_s^j, \varepsilon_{Y,s}) dj \right], \tag{C.39}$$

$$W_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[w_y' y_t - \frac{1}{2} y_t' W_y y_t - y_t' W_e e_t - \frac{1}{2} w_\pi \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.40}$$

where

$$w_\pi' = \frac{\sigma}{\mu k},$$

$$w_y' = \begin{bmatrix} -1/\mu(1+a) & 1 & 0 & 0 \end{bmatrix},$$

$$W_y' = \begin{bmatrix} \frac{(1+\eta)}{(1+a)\mu} & 0 & 0 & 0 \\ 0 & -(1-\rho) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and

$$W_e' = \begin{bmatrix} -\frac{\eta}{\mu(1+a)} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Using the second order approximation to the equilibrium condition, we can eliminate the term $w_y' y_t$. We derive the vector Lx, such that

$$\begin{bmatrix} a_y & d_y & f_y & b_y^i \end{bmatrix} Lx = w_y.$$

Thus, the loss function can be written as

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} y_t' L_y^i y_t + y_t' L_e^i e_t + \frac{1}{2} l_\pi^i \pi_t^2 \right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.41}$$

where

$$L_y^i = W_y + Lx_1^i A_y + Lx_2^i D_y + Lx_3^i F_y + Lx_4^i B_y^i,$$

$$L_e^i = W_e + Lx_1^i A_e + Lx_2^i D_e + Lx_4^i B_e,$$

and

$$l_\pi^i = w_\pi + Lx_1^i a_\pi.$$

Given the values of a_y, d_y, b_y^i, f_y, defined in this Appendix, we have:

$$Lx_1^i = \frac{(1+a)\lambda(l_1(2-\lambda)+(\phi-\lambda))-a(l_1+\phi)}{(1+a)\lambda(l_2(2-\lambda)-(\rho+\eta)(1-\lambda)-\lambda(\rho-1))-al_2},$$

$$Lx_2^i = \frac{-\lambda((\rho+\eta)-\phi(\rho-1))}{(1+a)\lambda(l_2(2-\lambda)-(\rho+\eta)(1-\lambda)-\lambda(\rho-1))-al_2},$$

$$Lx_3^i = \frac{-(\lambda+(1-\lambda)a)\theta((\rho+\eta)-(\rho-1)(\phi-(\phi-1)a))}{(1+a)\lambda(l_2(2-\lambda)-(\rho+\eta)(1-\lambda)-\lambda(\rho-1))-al_2},$$

and

$$Lx_4^i = \frac{(1+a)\lambda(l_3(2-\lambda)-(\phi-\lambda))-a(l_3-\phi)}{(1+a)\lambda(l_2(2-\lambda)-(\rho+\eta)(1-\lambda)-\lambda(\rho-1))-al_2},$$

where $l_1 = \theta a(\rho-\phi(\rho-1))+\phi(\theta-1)$,$l_2 = \theta((\rho+\eta)-\eta(\rho-1)a)$, $l_3 = \theta((\rho+\eta(1+a)-(\rho-1)\phi)$ and $(1-\phi) = \frac{1}{\mu}$.

We write the model just in terms of the output, real exchange rate and inflation, using the matrixes N and N_e, as follows:

$$y_t' = N\,[Y_t, T_t] + N_e e_t,$$

$$N = \begin{bmatrix} 1 & 0 \\ \frac{1}{d_b} & -\frac{d_q}{d_b} \\ 0 & -\frac{\lambda}{(1-\lambda)} \\ 0 & 1 \end{bmatrix},$$

and

$$N_e = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{d_b} & -\frac{(1-d_b)}{d_b} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

where $d_q = \frac{\theta(\lambda+(1-d_b)(1-\lambda))}{(1-\lambda)}$.

Equation (C.41) can therefore be expressed as

$$L_{to} = U_c\bar{C}E_{t_0}\sum\beta^t\left[\begin{array}{c} \frac{1}{2}\,[y_t,q_t]'\,L_y^{i'}\,[y_t,q_t] \\ +\,[y_t,q_t]'\,L_e^{i'}e_t + \frac{1}{2}l_\pi^i\pi_t^2 \end{array}\right] + t.i.p + \mathcal{O}(||\xi||^3), \tag{C.42}$$

where:

$$L_y^{i\prime} = N' L_y^i N,$$

$$L_e^{i\prime} = N' L_y^i N_e + N' L_e,$$

$$[y_t,q_t]'\,L_y^{i'}\,[y_t,q_t] = [y_t,q_t]'\left[\begin{array}{cc} l_{yy}^i & l_{yq}^i \\ l_{yq}^i & l_{qq}^i \end{array}\right][y_t,q_t]$$

$$[y_t,q_t]'\,L_e^{i'}e_t = [y_t,q_t]'\left[\begin{array}{c} l_{ye}^i \\ l_{qe}^i \end{array}\right]L_e'e_t, \tag{C.43}$$

$$l^i_{ye} = \left[\begin{array}{cccc} l^i_{y\varepsilon} & l^i_{y\mu} & l^i_{yg} & l^i_{yc^*} \end{array} \right],$$

$$l^i_{qe} = \left[\begin{array}{cccc} l^i_{q\varepsilon} & l^i_{q\mu} & l^i_{qg} & l^i_{qc^*} \end{array} \right],$$

$$\begin{aligned} l^i_{yy} &= \frac{(\eta+1)(1-\phi)}{(1+a)} + \frac{\rho-1}{d_b^2} + \left(\frac{\rho\,(\rho-2d_b)}{d_b^2} + \eta(2+\eta)\right) Lx_1 \\ &+ \frac{(1-d_b)}{d_b^2} Lx_2 + \frac{\rho\left(a\rho + 2(a(1-\lambda)-\lambda) + (1+a)^{-1}\rho^{-1}\right)}{d_b^2} Lx_4, \end{aligned}$$

$$\begin{aligned} l^i_{yq} = l^i_{qy} &= \frac{\rho(1-\lambda)^{-1} + (1+a)}{d_b^2}\left((r_1-\lambda)(1+a) + a\rho\theta\right) Lx_1 + \frac{\theta(a-\lambda\,(1-\lambda)^{-1})}{d_b^2} Lx_2 \\ &+ \left((1-\lambda)^{-1}\lambda + \frac{\rho(1+a)\lambda}{d_b^2}\right) Lx_4 + \left(\left(\rho d_b^{-1} + r_3\right) Lx_4 - 1\right)\frac{r_2}{d_b}, \end{aligned}$$

$$\begin{aligned} l^i_{qq} &= r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2 Lx_1 + \theta d_b(1-d_b)(1+r_2) Lx_2 \\ &+ (1-\lambda)^{-1}\lambda(\theta-1) Lx_3 \\ &+ \left((\lambda(1-\lambda)^{-1} + r_2 r_3) r_2 + \lambda(1-\lambda)^{-1}(-\lambda(1-\lambda)^{-1} + r_2\rho)\right) Lx_4, \end{aligned}$$

$$l^i_{\pi} = \frac{\sigma}{k}\left(\frac{(1-\phi)}{(1+a)} + (\eta+1) Lx_1\right),$$

$$l^i_{y\varepsilon} = \frac{-\eta(1-\phi)}{(1+a)} - \eta(\eta+1) Lx_1,$$

$$l^i_{y\mu} = (\eta+1) Lx_1,$$

$$l^i_{yg} = \frac{-(\rho-1)}{d_b^2} + \frac{\rho\,(d_b-\rho)}{d_b^2} Lx_1 - \frac{Lx_2}{d_b} - \frac{(r_3+\rho) Lx_4}{d_b},$$

$$l^i_{yc^*} = \frac{-(\rho-1)(1-d_b)}{d_b^2} + - \frac{\rho\,(d_b-\rho)}{d_b^2}\frac{(1-d_b)}{d_b} Lx_1 - \frac{(\lambda(1+a)-a) Lx_2}{d_b} - \frac{(1-d_b)}{d_b}\left(\rho + \frac{r_3}{d_b}\right) Lx_4,$$

$$l^i_{q\varepsilon} = 0,$$

$$l^i_{q\mu} = 0,$$

$$\begin{aligned} l^i_{qg} &= \frac{\theta\,(\rho-1)\,(a-\lambda(2-\lambda)(1+a))}{(1-\lambda)d_b^2} + \frac{((r_1+\lambda)(1+a) - a\rho\theta)\,\rho}{(1-\lambda)d_b^2}(Lx_1 + Lx_4) \\ &+ \frac{(\lambda(1+a)-a) Lx_2}{(1-\lambda)d_b} - \frac{((r_3+\rho)(2-\lambda)+\lambda)(1+a) + a\theta r_3)\,\rho}{(1-\lambda)d_b^2}\frac{Lx_4}{d_b}, \end{aligned}$$

and

$$l^i_{qc^*} = \frac{\theta\,(\rho-1)\,(\lambda(1+a)-a)((1-\lambda)^{-1}+d_b)}{d_b^2} - \frac{((r_1+\lambda)(1+a)-a\rho\theta)\,(\lambda(1+a)-a)Lx_1}{(1-\lambda)d_b^2}$$
$$+\frac{\theta(\lambda(1+a)-a)Lx_2}{(1-\lambda)d_b} + \frac{(\lambda(1+a)-a)}{d_b}\left(\rho\lambda(1-\lambda)^{-1}+r_2r_3\right)Lx_4,$$

where $r_1 = \lambda(2-\lambda)(\rho\theta-1)$, $r_2 = \frac{\theta((1-d_b)+(1-\lambda)^{-1}\lambda)}{d_b}$ and $r_3 = (1+\rho^2)a + 1 - 2\rho$

C.2 Optimal Plan with Incomplete Asset Markets:

The optimal plan consists of minimizing (31) subject to equations in Table 4. Therefore, the first order conditions with respect to π_t, y_t, q_t, c_t, and b_t are:

$$l^i_\pi \pi_t + \Delta\varphi_{1,t} = 0, \tag{C.44}$$

$$0 = l^i_{yy}y_t + l^i_{yq}q_t + l^i_{ye}\widehat{e}_t - k\eta\varphi_{1,t} + \varphi_{2,t} - \varphi_{4,t},$$

$$0 = l^i_{yq}y_t + l^i_{qq}q_t + l^i_{qe}\widehat{e}_t - k\frac{\lambda}{(1-\lambda)}\varphi_{1,t} - d_q\varphi_{2,t} + \varphi_{3,t} - \beta^{-1}\varphi_{3,t-1}$$
$$+\frac{\lambda}{(1-\lambda)}\varphi_{4,t} - a_\beta\Delta\varphi_{4,t} + a_\beta\beta\Delta E_t\varphi_{4,t+1}, \tag{C.45}$$

$$0 = -\rho k(1+a)\varphi_{1,t} - (1+a)(1-\lambda)\varphi_{2,t} - \rho\varphi_{3,t} + \rho\beta^{-1}\varphi_{3,t-1} + (1+a)\varphi_{4,t} + \rho a_\beta\Delta\varphi_{4,t}, \tag{C.46}$$

and

$$E_t\Delta\varphi_{4,t+1} = \beta^{-1}\delta\varphi_{3,t}. \tag{C.47}$$

The case of no intermediation costs:
When $\delta = 0$, the first order conditions can be written as

$$Q^i_y E_t\Delta(y_{t+1} - y^{T,i}_{t+1}) + Q^i_q E_t\Delta(q_{t+1} - q^{T,i}_{t+1}) + Q^i_\pi E_t\pi_{t+1} = 0, \tag{C.48}$$

with

$$Q^i_y = l^i_{yq} + (d_q + (1+a)(1-\lambda)\rho^{-1})l^i_{yy},$$

$$Q^i_q = l^i_{qq} + (d_q + (1+a)(1-\lambda)\rho^{-1})l^i_{yq},$$

$$Q^i_\pi = k\left[(1+a)(\rho + \eta(1-\lambda)\rho^{-1}) + \eta d_q + \lambda(1-\lambda)^{-1}\right]l^i_\pi,$$

$$q^{T,i}_t = \frac{-l_{qe}}{Q^i_q}\widehat{e}_t,$$

and

$$y^{T,i}_t = \frac{-(d_q + (1+a)(1-\lambda)\rho^{-1})l^i_{ye}}{Q^i_y}\widehat{e}_t.$$

Special Case: Incomplete markets, symmetric steady state, no trade imbalances and specific level of steady-state output
In the case we have

1. $\mu = 1/(1-\lambda)$
2. $\rho = \theta = 1$
3. $a = 0$

In this case, the first order conditions can be written as:

$$0 = (l^i_{yy} + l^i_{yq}(1-\lambda))\Delta y_t + ((1-\lambda)l^i_{qq} + l^i_{yq})\Delta q_t + (l^i_{ye} + l^i_{qe}(1-\lambda))\Delta \widehat{e}_t + k(\eta+1)l^i_\pi \pi_t \quad \text{(C.49)}$$

Moreover:

$$Lx_1 = 0;\ Lx_2 = -1;\ Lx_3 = -1;\ \text{and } Lx_4 = 2-\lambda.$$

And therefore:

$l_{yy} + l_{yq}(1-\lambda) = (\eta+1)(1-\lambda)$
$(1-\lambda)l_{qq} + l_{yq} = 0$
$l_\pi = (1-\lambda)\sigma/k$
$(l_{ye} + l_{qe}(1-\lambda)) = \begin{bmatrix} -\eta(1-\lambda) & 0 & -(1-\lambda) & 0 \end{bmatrix}$

Hence, the targeting rule can be written as

$$0 = \Delta\left(y_t - \frac{\eta}{(\eta+1)}\varepsilon_t - \frac{1}{(\eta+1)}g_t\right) + \sigma\pi^H_t. \quad \text{(C.50)}$$

In addition, using Equation (B.29), we can write the Phillips Curve as follows:

$$\pi_t = k\left((\eta+1)y_t - \eta\varepsilon_{Yt} - g_t + \mu_t\right) + \beta E_t \pi_{t+1}. \quad \text{(C.51)}$$

By inspection of Equation (C.50) and (C.51), we can see that domestic inflation target is the optimal plan if there are no markup shocks, μ_t.

C.3 Welfare under Financial Autarky

Using an analogous derivation for welfare, but substituting the matrices b^i_y, B^i_y and B^i_e for b^{fa}_y, B^{fa}_y and B^{fa}_e, the loss function under financial autarky has the following weights[2]:

$$\begin{aligned} l^{fa}_{yy} &= (\eta+1)(1-\phi) + \frac{\rho-1}{d_b^2} \\ &+ \left(\frac{\rho\,(\rho-2d_b)}{d_b^2} + \eta(2+\eta)\right) Lx^{fa}_1 + \frac{(1-d_b)}{d_b^2} Lx^{fa}_2, \end{aligned}$$

$$\begin{aligned} l^{fa}_{yq} = l^{fa}_{qy} &= \frac{\rho(1-\lambda)^{-1}}{d_b^2}\left((r_1-\lambda)(1+a) + a\rho\theta\right) Lx^{fa}_1 \\ &+ \frac{\theta(a-\lambda\,(1-\lambda)^{-1})}{d_b^2} Lx^{fa}_2 - \frac{r_2}{d_b}, \end{aligned}$$

[2] Note that for the derivation of wefare under Complete Market and Financial autarky, we assume $a = 0$.

$$l_{qq}^{fa} = r_2^2(\rho-1) + (\lambda(1-\lambda)^{-1} - r_2\rho)^2 Lx_1^{fa} + \theta d_b(1-d_b)(1+r_2)Lx_2^{fa}$$
$$+ (1-\lambda)^{-1}\lambda(\theta-1)Lx_3^{fa},$$

$$l_{\pi}^{fa} = \frac{\sigma}{k}\left((1-\phi) + (\eta+1)Lx_1^{fa}\right),$$

$$l_{y\varepsilon}^{fa} = -\eta(1-\phi) - \eta(\eta+1)Lx_1^{fa},$$

$$l_{y\mu}^{fa} = (\eta+1)Lx_1^{fa},$$

$$l_{yg}^{fa} = -\frac{(Lx_2(1-\lambda) + \rho Lx_4^{fa})}{d_b},$$

$$l_{yc^*}^{fa} = \frac{-(\rho-1)(1-d_b)}{d_b^2} - \frac{\rho(d_b-\rho)}{d_b^2}\frac{(1-d_b)}{d_b}Lx_1^{fa} - \frac{(\lambda(1+a)-a)Lx_2^{fa}}{d_b},$$

$$l_{q\varepsilon}^{fa} = 0,$$

$$l_{q\mu}^{fa} = 0,$$

$$l_{qg}^{fa} = \frac{(r_2+\lambda)Lx_4^{fa}}{d_b^2},$$

$$l_{yc^*}^{fa} = \frac{\theta(\rho-1)(\lambda(1+a)-a)((1-\lambda)^{-1}+d_b)}{d_b^2} - \frac{((r_1+\lambda)(1+a)-a\rho\theta)(\lambda(1+a)-a)Lx_1^{fa}}{(1-\lambda)d_b^2}$$
$$+ \frac{\theta(\lambda(1+a)-a)Lx_2^{fa}}{(1-\lambda)d_b},$$

and

$$l_{yc^*}^{fa} = -\frac{(Lx_2^{fa}(1-\lambda) + (-r_2-\lambda+\rho)Lx_4^{fa})}{d_b},$$

with

$$Lx_1^{fa} = \frac{\lambda(l_1^{fa}(2-\lambda) + (\phi-\lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$
$$Lx_2^{fa} = \frac{-\lambda((\rho+\eta) - \phi(\rho-1))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$
$$Lx_3^{fa} = \frac{-\lambda\theta((\rho+\eta) - (\rho-1)\phi)}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

and

$$Lx_4^{fa} = \frac{\lambda(l_3^{fa}(2-\lambda) - (\phi-\lambda))}{\lambda(l_2^{fa}(2-\lambda) - (\rho+\eta)(1-\lambda) - \lambda(\rho-1))},$$

where $l_1^{fa} = \phi(\theta-1)$,$l_2^{fa} = \theta((\rho+\eta)$ and $l_3^{fa} = \theta((\rho+\eta-(\rho-1)\phi)$.

C.4 Optimal Plan under Financial Autarky

We can write the system of equations given in Table 2 in terms of y_t and q_t as follows:

$$\pi_t = \phi\left((\eta+\rho)y_t - (\rho-1)\lambda(1-\lambda)^{-1}q_t + \mu_t - \eta\varepsilon_t\right) + \beta E_t\pi_{t+1}, \tag{C.52}$$

and

$$y_t = q_t\frac{(1+l_i)}{(1-\lambda)} + c_t^* + \lambda^{-1}g_t. \tag{C.53}$$

The policymaker minimizes the loss function subject to the problem (C.52) and (C.53). Given that the multipliers associated with (C.52) and (C.53) are, respectively, φ_1 and φ_2, the first order conditions with respect to π_t, y_t and q_t are given by:

$$(\varphi_{1,t} - \varphi_{1,t-1}) = kl_\pi^{fa}\pi_t, \tag{C.54}$$

$$\varphi_{2,t} - (\eta+\rho)\varphi_{1,t} = l_{yy}^{fa}y_t + l_{yq}^{fa}q_t + l_{ye}^{fa}\widehat{e}_t, \tag{C.55}$$

and

$$-(1+l_i)(1-\lambda)^{-1}\varphi_{2,t} + (\rho-1)\lambda(1-\lambda)^{-1}\varphi_{1,t} = l_{qy}^{fa}y_t + l_{qq}^{fa}q_t + l_{qe}^{fa}\widehat{e}_t. \tag{C.56}$$

The last 3 equations can be combined, giving rise to the following targeting rule

$$Q_y^{fa}\Delta(y_t - y_t^{T,fa}) + Q_q^c\Delta(q_t - q_t^{T,fa}) + Q_\pi^{fa}\pi_t = 0, \tag{C.57}$$

where

$$Q_y^{fa} = (l_{yy}^{fa} + l_{qy}^{fa}(1-\lambda)(1+l_i)^{-1}),$$

$$Q_q^{fa} = ((1-\lambda)(1+l_i)^{-1}l_{qq}^{fa} + l_{qy}^{fa}),$$

$$Q_\pi^{fa} = k((\eta+\rho) - (\rho-1)\lambda(1-\lambda)(1+l_i)^{-1})l_\pi^{fa},$$

$$y_t^{T,fa} = (Q_y^{fa})^{-1}l_{ye}^{fa}\widehat{e}_t,$$

and

$$q_t^{T,fa} = (Q_q^{fa})^{-1}l_{qe}^{fa}(1-\lambda)(1+l_i)^{-1}\widehat{e}_t.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (C.50). Also, in the less restrictive case that only $\theta = 1$, the targeting rule would be given by

$$0 = \Delta\left(y_t - y_t^{Flex}\right) + \sigma\pi_t^H, \tag{C.58}$$

where $y_t^{Flex} = \frac{\eta}{(\eta+\rho)}\varepsilon_t + \frac{\rho}{(\eta+\rho)}g_t$. In other words, producer price stability consists the optimal plan under the assumptions of $\mu = 1/(1-\lambda)$ and $\theta = 1$, regardless of the value of ρ.

C.5 Welfare with Complete Markets

Following the derivation in De Paoli (2009), the loss function with complete markets can be written as

$$\begin{aligned} L_{to}^{i} &= U_c \bar{C} E_{t_0} \sum \beta^t \left[\frac{1}{2} l_{yy}^c (y_t - y_t^{T,c})^2 + \frac{1}{2} l_{qq}^c (q_t - q_t^{T,c})^2 + \frac{1}{2} l_\pi^c (\pi_t)^2 \right] \\ &\quad + t.i.p + \mathcal{O}(||\xi||^3) \end{aligned} \tag{C.59}$$

where:

$$\begin{aligned} l_{yy}^c &= (\eta + \rho)(1 - \phi) + \frac{(\rho - 1)\left[-l^c(1 - \phi) - (\lambda - \phi)\right]}{(1 + l^c)} \\ &\quad + Lx_1^c \left[(\eta + \rho) + \eta(\eta + 1) - \frac{\rho(\rho - 1)}{(1 + l^c)} \right] \\ &\quad - \frac{Lx_2^c (1 - \lambda)^2 \lambda(\rho\theta - 1)}{(1 + l^c)}, \end{aligned}$$

$$\begin{aligned} l_{qq}^c &= -\frac{(\lambda + l^c)(\rho - 1)}{(1 - \lambda)\rho^2} \\ &\quad + \frac{Lx_1^c l^c (\rho - 1 - l^c)}{(1 - \lambda)^2 \rho} \\ &\quad + \frac{Lx_2^c \lambda(\rho\theta - 1)\left[\rho\theta(1 - \lambda) + \lambda + l^c\right]}{\rho^2} \\ &\quad + \frac{Lx_3^c \left[1 + \lambda^2(2 - \lambda)\right] \lambda(\theta - 1)}{1 - \lambda}, \end{aligned}$$

$$l_\pi^c = \frac{\sigma}{\mu k} + (1 + \eta)\frac{\sigma}{k} Lx_1^c,$$

$$y_t^{T,c} = q_y^e e_t,$$

and

$$q_t^{T,c} = q_q^e e_t,$$

where

$$q_y^e = \frac{1}{\Phi_Y} \left[\begin{array}{cccc} \frac{\eta}{\mu} + Lx_1^c (1 + \eta)\eta & -Lx_1^c (1 + \eta) & \frac{(\rho - 1)(1 - \lambda) + Lx_2^c}{1 + l^c} & 0 \end{array} \right],$$

$$q_Q^e = \frac{1}{\Phi_Q} \left[\begin{array}{cccc} 0 & 0 & \frac{(\rho - 1 - l^c) Lx_1^c}{(1 - \lambda)} + \frac{Lx_2^c \lambda(\lambda(1 - \lambda) + 1)(\rho\theta - 1)}{\rho(1 - \lambda)} & \frac{-Lx_2^c \lambda(1 - \lambda)(\rho\theta - 1)}{\rho} \end{array} \right],$$

$$\begin{aligned} Lx_1^c &= \frac{1}{(\rho + \eta) + l^c \eta} \left[l\mu^{-1} + (1 - \lambda) - \mu^{-1} \right], \\ Lx_2^c &= \frac{1}{(\rho + \eta) + l^c \eta} \left[\rho(\mu^{-1} - (1 - \lambda)) + (1 - \lambda)(\eta + \rho) \right], \\ Lx_3^c &= \frac{1}{(\rho + \eta) + l^c \eta} \left[(\rho\theta - 1)(1 - \lambda)\mu^{-1} - (\eta\theta + 1) \right], \end{aligned} \tag{C.60}$$

and $l^c = (\rho\theta - 1)\lambda(2 - \lambda)$.

C.6 Optimal Plan with Complete Markets

The optimal plan consists of minimizing the loss function subject to

$$\pi_t = k\left(\eta\hat{Y}_t + (1-\lambda)^{-1}q_t + \mu_t - \eta\varepsilon_t + \rho c_t^*\right) + \beta E_t\pi_{t+1}, \tag{C.61}$$

and

$$y_t = q_t\frac{(1+l^c)}{\rho(1-\lambda)} + g_t + c_t^*. \tag{C.62}$$

The multipliers associated with (C.61) and (C.62) are, respectively, φ_1 and φ_2. The first order conditions with respect to π_t, y_t and q_t are, therefore, given by

$$(\varphi_{1,t} - \varphi_{1,t-1}) = kl_\pi^c\pi_t, \tag{C.63}$$

$$\varphi_{2,t} - \eta\varphi_{1,t} = l_{yy}^c(y_t - y_t^T), \tag{C.64}$$

and

$$-\varphi_{2,t} - \frac{\rho}{(1+l)}\varphi_{1,t} = \frac{\rho(1-\lambda)}{(1+l)}l_{qq}^c(q_t - q_t^T). \tag{C.65}$$

To obtain a targeting rule for the small open economy, we combine equations (C.63), (C.64), and (C.65):

$$Q_y^c\Delta(y_t - y_t^{T,c}) + Q_q^c\Delta(q_t - q_t^{T,c}) + Q_\pi^c\pi_t = 0, \tag{C.66}$$

where

$$Q_y^c = (1+l^c)l_{yy}^c,$$

$$Q_q^c = \rho(1-\lambda)l_{qq}^c,$$

and

$$Q_\pi^c = (\rho + \eta(1+l))kl_\pi^c.$$

Special Case: when $\mu = 1/(1-\lambda)$ and $\rho = \theta = 1$, the targeting rule is identical to (C.50). This confirms that, under these circumstances, the asset market structure is irrelevant for monetary policy.

C.7 The welfare cost of inflation

Under some simplifying assumptions, the weight of inflation in the loss function for, l_π, can be expressed as:[3]

$$l_\pi^i = l_\pi^{fa} = \frac{\sigma(1-\lambda)}{k}\left(1 + \frac{l_i\lambda(1-\lambda)^{-1}(\eta+1)}{l_i(\rho+\eta) + \rho(1-\lambda) + \eta + \lambda}\right)$$

and

$$l_\pi^c = \frac{\sigma(1-\lambda)}{k}\left(1 - \frac{l_c(\eta+1)}{(\rho+\eta) + \eta l}\right)$$

[3]Here we assume that the level of output is efficient in the steady state (for the small open economy) and that the net foreign asset position is zero. In particular, we set $\overline{\mu} = 1/(1-\lambda)$ and $\overline{B} = 0$.

with $l_i = (\theta - 1)(2 - \lambda)$ and $l_c = (\rho\theta - 1)\lambda(2 - \lambda)$.

These expressions demonstrate that when domestic and foreign goods are substitutes in the utility function, inflation variability is less costly when asset markets are complete. More specifically, when $\rho\theta > 1$ and $\theta > 1$ then $l_\pi^i = l_\pi^{fa} > q_\pi$ and $l_\pi^c < q_\pi$. When welfare is expressed as a purely quadratic expression, the weight on inflation under incomplete markets and financial autarky increases, while in the case of complete markets it decreases. It follows that, with complete markets, the linear term $c_t - \frac{1}{\bar{\mu}(1+a)} y_t$ in Equation (C.40) can be written as an increasing function of inflation variability. On the other hand, with imperfect risk sharing, either in the case of financial autarky or market incompleteness, this term is a decreasing function of $(\pi_t)^2$. This conclusion if reversed if domestic and foreign goods are complements in the utility. Now, if $\rho\theta < 1$ and $\theta < 1$, the conclusion is reversed: $l_\pi^i = l_\pi^{fa} < q_\pi$ and $l_\pi^c > q_\pi$.

C.8 Randomization Problem - the Financial Autarky Case

To ensure that the policy obtained from the minimization of the loss function is indeed the best available policy, we should confirm that no other random policy plan can be welfare improving. De Paoli (2009) analyses the case of complete markets. We present the conditions under which no random policy can enhance welfare. As shown in Woodford and Benigno (2003), these conditions coincide with the second order condition for the linear quadratic optimization problem. In the present Section, we study the case of financial autarky.

Following the same steps as De Paoli (2009), we characterize the relationship between inflation and output and inflation and the real exchange rate. Equation (AS) combined with Equation (FA) leads to the following expression:

$$\pi_t = k\left(\frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{d_1} y_t + \mu_t + \eta\varepsilon_t\right) + \beta E_t \pi_{t+1}, \tag{C.67}$$

where $d_1 = (\theta - 1)(1 - \lambda) - \lambda\theta$. Alternatively,

$$\pi_t = k\left(\frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{(1-\lambda)} q_t + \mu_t + \eta\varepsilon_t\right) + \beta E_t \pi_{t+1}. \tag{C.68}$$

A random sunspot realization that adds $\varphi_j v_j$ to π_{t+j}, will, therefore, add a contribution of $\alpha_y k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ to y_t and $\alpha_q k^{-1}(\varphi_j - \beta\varphi_{j+1})v_j$ to q_t, where

$$\alpha_q^{fa} = \frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{(1-\lambda)}, \tag{C.69}$$

and

$$\alpha_y^{fa} = \frac{(\eta+\rho)d_1 - (\rho-1)\lambda}{d_1}. \tag{C.70}$$

To obtain what is the contribution to the loss function of the realization $\varphi_j v_j$ to π_{t+j}, we rewrite the loss function as follows. Noticing that $(d_q\lambda^{-1} - 1)q_t = y_t + t.i.p$, the loss function under financial autarky can be written as

$$L_{to} = U_c \bar{C} E_{t_0} \sum \beta^t \left[\begin{array}{c} \frac{1}{2}(l_{yy}^{fa} + (d_q\lambda^{-1} - 1)^{-1} l_{yq}^{fa})(y_t - y_t^T)^2 \\ +\frac{1}{2}(l_{qq}^{fa} + (d_q\lambda^{-1} - 1) l_{yq}^{fa})(q_t - q_t^T)^2 + \frac{1}{2} l_\pi^{fa} \pi_t^2 \end{array} \right] + t.i.p, \tag{C.71}$$

where

$$y_t^T = l_y^e e_t,$$

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_q\lambda^{-1} - 1)^{-1} l_{yq}^{fa})} \begin{bmatrix} l_{ye}^{fa} & l_{y\mu}^{fa} & l_{yg}^{fa} & l_{yc^*}^{fa} \end{bmatrix},$$

$$q_t^T = l_q^e e_t,$$

and

$$l_y^e = \frac{1}{(l_{yy}^{fa} + (d_q\lambda^{-1} - 1) l_{yq}^{fa})} \begin{bmatrix} l_{qe}^{fa} & l_{q\mu}^{fa} & l_{qg}^{fa} & l_{qc^*}^{fa} \end{bmatrix}.$$

Consequently, the contribution to the loss function of a random realization in $\varphi_j v_j$ is

$$U_c \bar{C} \beta^t \sigma_v^2 E_{t_0} \sum \beta^t \left[\Phi^{fa} k^{-1} (\varphi_j - \beta \varphi_{j+1})^2 + l_\pi^{fa} (\varphi_j)^2 \right], \tag{C.72}$$

where

$$\Phi^{fa} = \Phi_Y^{fa} \alpha_y^2 + \Phi_Q^{fa} \alpha_q^2,$$

$$\Phi_Y^{fa} = (l_{yy}^{fa} + (d_q\lambda^{-1} - 1)^{-1} l_{yq}^{fa}),$$

and

$$\Phi_Q^{fa} = (l_{qq}^{fa} + (d_q\lambda^{-1} - 1) l_{yq}^{fa}).$$

It follows that policy randomization cannot improve welfare if the expression given by Equation (C.72) is positive definite. Hence, the first order conditions to the minimization problem are indeed a policy optimal if Φ^{fa} and l_π^{fa} are not both equal to zero and either: (a) $\Phi^{fa} \geq 0$ and $\Phi^{fa} + (1 - \beta^{1/2})^2 k^{-2} l_\pi^{fa} \geq 0$, or (b) $\Phi^{fa} \leq 0$ and $\Phi^{fa} + (1 - \beta^{1/2})^2 k^2 l_\pi^{fa} \geq 0$ holds.