The Monetary Policy Transmission Mechanism in a Term-Structure Model with Unspanned Macro Risks*

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Abstract

In this paper I analyze how exogenous monetary policy impulses transmit jointly to the U.S. macroeconomy and the term structure of U.S. interest rates. I estimate a Macro-Affine Term Structure Model, which is similar to Joslin, Priebsch, and Singleton (2010), and use it to identify monetary policy shocks and term premia. My main finding is that monetary policy shocks trigger relevant movements in long-term bond premia, which in turn feed back into the macroeconomy. This gives rise to a "term-premium channel of monetary transmission". I show that it is particularly important in the pre-Volcker period; in the post-Volcker period, this channel turns out to be empirically irrelevant. I then estimate how shocks to future monetary policy expectations are transmitted to the economy. I find that, in the post-Volcker period, shocks to policy expectations produce more pronounced and more intuitive responses for macroeconomic variables than do standard shocks to the contemporaneous value of the monetary policy instrument. This suggests that communication between the Fed and private agents is a powerful instrument of monetary policy.

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1 Introduction

To a large extent, bond prices reflect the expectations that private agents form regarding the future path of the short-term interest rate. Not surprisingly, many recent papers in the macro-finance literature have tried to incorporate the information contained in these prices into the study of the monetary policy transmission mechanism\(^1\). There is also overwhelming evidence that, beyond short-rate expectations, bond prices reflect time-varying and potentially sizeable premia components\(^2\). Currently, the literature offers only scarce evidence regarding the role of these premia in the monetary transmission mechanism.

How exactly do bond premia, namely term premia\(^3\), respond (if at all) to monetary policy shocks? Is there any feedback from these movements to the macroeconomy? Do term premia responses change across different subsamples of the US data? And finally, what happens when, after isolating term premia, a shock to future monetary policy expectations occurs? Based on an empirical analysis of US data, I provide in this paper answers to all of these questions.

In order to succeed, I must overcome two crucial identification issues here. First, there is the issue of identifying the unobserved term premium component implicit in bond yields of different maturities. Then, I must devise a procedure for identifying unobserved monetary policy shocks. The framework I propose overcomes these two identification issues simultaneously.

To address the first issue, I model the joint reduced-form dynamics of the macroeconomy and the yield curve according to a Macro-Affine Term Structure Model (MTSM) similar to Ang and Piazzesi (2003). This family of models explores the discipline imposed by no-arbitrage in order to clarify the links between macroeconomic shocks and the entire yield curve. In this context, the identification of bond term premia naturally follows from the no-arbitrage conditions that form the core of the affine term-structure model.


\(^3\)Throughout this paper, I define term premia as the difference between the bond yield and the hypothetical yield that would arise if the Expectations Hypothesis of the term structure held. A formal definition can be found in Appendix D.
There are several variants of the Ang and Piazzesi (2003) framework available in the literature. In this paper I chose to follow the one proposed by Joslin, Priebsch, and Singleton (2010). This framework is attractive for at least two reasons. First, it is more general than Ang and Piazzesi (2003) in that it allows for two-way feedback effects between bond prices and macroeconomic variables. As a result, movements in term premia have the potential to affect not only bond prices but also the macroeconomic variables included in the model. Second, the Joslin, Priebsch, and Singleton (2010) framework does not have the property, shared by most models in the Ang and Piazzesi (2003) tradition, that the macroeconomic variables in the model are spanned by the yield curve\(^4\). Joslin, Priebsch, and Singleton (2010) show that this spanning property is at odds with the US data on bond yields and certain crucial macroeconomic variables, such as inflation and GDP.

Turning now to the identification of monetary policy shocks, the difficulty lies in finding a procedure for differentiating the truly exogenous movements in the monetary policy instrument from those movements that arise endogenously as the monetary authority responds to changes in the state of the economy. In particular, I design the MTSM such that identification through the \textit{recursiveness assumption} of Christiano, Eichenbaum, and Evans (1999) can be applied easily. More specifically, I include the monetary policy instrument, which I assume to be the short-term interest rate, in the state vector of the MTSM. Then I impose a particular ordering of the variables in this vector in order to give rise to a recursive causal relationship among these variables.

The model is fitted to quarterly US data on inflation, economic activity, commodity prices, and long-term yields. I split the data sample into two periods: 1959:1-1979:3 (pre-Volcker) and 1979:4-2007:4 (post-Volcker). My main finding is that monetary policy shocks trigger relevant movements in long-term bond premia in both subsamples. These movements feed back into the macroeconomy, giving rise to what I refer to as the \textit{term-premium channel of monetary transmission}.

More specifically, I find that an exogenous increase in the short rate temporarily raises term premia across different maturities by a statistically significant amount. This result holds for both samples. However, the responses of term premia are more pronounced and

\(^4\)That is, that a combination of yields explains all of the variation in the macro variables.
persistent in the pre-Volcker than in the post-Volcker subsample. I show that the responses of the macro variables included in the model after a monetary shock can be decomposed into one portion due to term premia movements and another due to movements in the term-structure that are consistent with the Expectations Hypothesis. That is, I quantify the term-premium channel of monetary transmission. Interestingly, my estimates show that the term-premium channel was particularly important in the pre-Volcker period, while this channel turns out to be empirically irrelevant in the latter period.

I then analyze how shocks to future monetary policy expectations affect the macroeconomy. This is motivated by the crucial role that modern central banks across the globe give to efficiently managing private agents’ expectations. My framework is convenient for this analysis because it allows for the isolation of policy expectations – i.e. it guarantees that the proposed shock affects the future policy expectations implied by the model, and not term premia. In the post-Volcker subsample, I find that a shock to monetary policy expectations up to one year ahead will lead to more pronounced and intuitive responses for the macroeconomic variables than would standard shocks to the contemporaneous value of the monetary policy instrument. In other words, while a contractionary shock to the current value of the short rate leads to counterintuitive rises in inflation and economic activity in the post-Volcker sample, a shock to policy expectations one-year ahead leads to declines in both macro variables.

I further show that if one directly shocks the long-term yield, which includes both policy expectations and term premia, then identification of the expectations shock becomes biased. In particular, the responses of the macro variables after a monetary policy shock become less pronounced than when I control for term premia.

There have been several earlier contributions to the macro-finance literature that, in some sense, tried to address one or more questions raised above. In particular, Evans and Marshall (1998) were among the first to study the monetary policy transmission in a system containing both macroeconomic and term structure variables. Their model consisted of a standard Vector Autoregression on macroeconomic variables and nominal yields that did not allow for feedback effects from bond yields to the macroeconomic variables. More recently,

\[5\text{See also Evans and Marshall (2007).}\]
in a context similar to mine, Diebold, Rudebusch, and Aruoba (2006) and Mumtaz and Surico (2009) used the recursiveness assumption to identify monetary policy shocks in a MTSM based on Nelson and Siegel (1987). My approach differs from these three papers in that my model rules out arbitrage opportunities across bonds of different maturities. In addition, none of them quantify the term premium channel of monetary transmission or study shocks to policy expectations.

This paper is also related to a literature that uses Fed funds futures’ data to identify monetary policy shocks. Kuttner (2001), for example, finds that long-term yields respond significantly to movements in the Fed funds rate that are not anticipated by Fed funds futures. On the other hand, anticipated changes in the Fed funds rate have only minimal effects on long-term yields. More recently, Piazzesi and Swanson (2008) show that accounting for premia in the Fed funds futures’ data is crucial in pursuing the identification scheme proposed by Kuttner (2001). Although this branch of the literature offers important insights into the identification of monetary policy shocks, it does not address the effects of these shocks on the rest of the economy. This paper focuses precisely on these effects.

This paper is organized as follows: Section 2 describes the data used to fit the model; Section 3 describes the Macro-Affine Term Structure Model in detail; Section 4 evaluates the responses of term premia to a standard monetary policy shock; Section 5.2 analyzes the effects of a shock to future monetary policy expectations; Section 6 concludes.

2 Data

I use quarterly data for the U.S. ranging from 1959:1 to 2007:4. The term-structure series that I include in my analysis are the nominal yields on 6-month and 1, 2, 3, 4, and 5-year zero-coupon bonds obtained from the CRSP database. For the short-term interest rate, I use the 3-month riskfree rate, also from CRSP. All of the yields are compounded continuously, and are observed on the last trading day of each quarter.

Three macro variables are included in the term structure model: the output gap \((GAP)\);
the rate of inflation (\(INF\)); and a measure of commodity prices (\(COMM\)). Although many term-structure models in the literature already incorporate measures of \(GAP\) and \(INF\), I add \(COMM\) as in an extensive branch of the macro literature that aims at identifying monetary policy shocks using structural VARs.

Inspired by Bernanke and Boivin (2003), Ang and Piazzesi (2003) and Moench (2008), I extract measures of \(GAP\) and \(INF\) from rich datasets that include several different output gap and inflation indicators. The motivation for this approach is that, in practice, central banks use many different economic indicators in order to form their views about the underlying levels of economic slack and inflation in the economy (in other words, central banks act in a “data-rich environment”). Therefore, I use the methodology proposed by Stock and Watson (1988), and measure \(GAP\) as the common factor extracted from a set of seven different economic slack indicators. The same methodology is applied to compute \(INF\) based on five different quarterly inflation indicators. Finally, \(COMM\) is a detrended and smoothed measure of commodity prices based on the CRB spot commodity prices index.

Figure 1 depicts the time series described above. The units associated with \(GAP\) and \(INF\) cannot be interpreted, because these factors were normalized to have zero mean and unit conditional variance. Nevertheless, the dynamics of \(GAP\) and \(INF\) capture the timing

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7 See, for example, Ang and Piazzesi (2003), Bikbov and Chernov (2010), and Joslin, Priebsch, and Singleton (2010).
8 Structural VARs estimated on post-war U.S. data in general give rise to a "pricing puzzle". That is, they have the counterintuitive property that inflation increases in response to a contractionary monetary policy shock. According to Christiano, Eichenbaum, and Evans (1996) and Sims and Zha (2006), this puzzle can be solved by including \(COMM\) as an additional endogenous in structural VARs. See also Christiano, Eichenbaum, and Evans (1999).
9 See Appendix A for details.
10 The series used to compute \(GAP\) are: (i) industrial production index, (ii) total nonfarm payrolls, (iii) real personal consumption expenditures, (iv) real GDP index, (v) the new orders component of the ISM manufacturing index, (vi) total housing starts, and (vii) civilian unemployment rate. (i)-(iv) were detrended using linear and quadratic deterministic trends, whereas (v)-(vii) were used directly in levels. All series were obtained from the St. Louis Fed.
11 The series used to compute \(INF\) are: (i) CPI fess food & energy, (ii) finished goods PPI less food & energy, (iii) personal consumption expenditures deflator less food and energy, (iv) GDP deflator, (v) average hourly earnings. All series were obtained from the St. Louis Fed, and were transformer into quarterly growth rates before applying the Kalman Filter.
12 The quarterly series used to construct \(GAP\) and \(INF\) represent figures observed in the first month of each quarter. The only two exception are the GDP and GDP deflator series, which are not observed on a monthly frequency and were therefore proxied by their one-quarter lag. This way the plausibility of the recursive identification scheme described in Section 3.2 is guaranteed.
13 More specifically, I detrend the CRB index (expressed in logs) by applying the standard Hodrick-Prescott filter. Then, in order to improve the fit of the model, I take a moving average of the detrended CRB index.
of the NBER recessions, and the peaks associated with the 1973 and 1979 oil shocks, very well.

In the remainder of this paper, the \( m \)-period bond yield is denoted by \( y_{m,t} \). The short-rate (i.e. the 3-month rate) is denoted by \( y_{1,t} \equiv r_t \). The yields used to evaluate the fit of the model are collected in the \( 7 \times 1 \) vector \( y_t \equiv [ r_t \ y_{2,t} \ y_{4,t} \ \cdots \ y_{20,t} ]' \). Finally, the macro variables are arranged in the \( 3 \times 1 \) vector \( M_t = [ GAP_t \ INF_t \ COMM_t ]' \).

## 3 The Macro-Affine Term Structure Model

I model the joint reduced-form dynamics of the macroeconomy and the yield curve according to a Macro-Affine Term Structure Model (MTSM) similar to Ang and Piazzesi (2003). This family of models explores the discipline imposed by no-arbitrage in order to clarify the links between macro shocks and the entire yield curve. The particular framework I adopt in this
paper follows Joslin, Priebsch, and Singleton (2010).

Section 3.1 describes the core equations of the model and applies no-arbitrage arguments to determine the prices of long-term bonds. Section 3.2 then shows how the recursiveness assumption is used to identify monetary policy shocks in the model. Finally, Section 3.3 describes the econometric methodology used to fit the model to the U.S. data.

3.1 Bond Prices and Macro Risks

Suppose that the state of the economy is summarized by the three macro variables described in Section 2 plus \( N \) additional yield-based factors. More specifically, at any time \( t \), the state-vector of the economy is given by \( Z_t \equiv [M_t' \; \mathcal{P}_t']' \in \mathbb{R}^{3+N} \), where the \( N \)-column vector \( \mathcal{P}_t \) contains the yield-based factors. Following Joslin, Singleton, and Zhu (2011), I assume that \( \mathcal{P}_t \) consists of returns on observed bond portfolios. More precisely, for a full-rank matrix of portfolio weights \( P \), I define \( \mathcal{P}_t \equiv Py_t \). My particular choice for \( P \) will be described in Section 3.2.

The Macro-Affine Term Structure Model is summarized by three equations:

\[
\begin{align*}
  r_t &= \rho_0 + \rho_1 \mathcal{P}_t \quad (1) \\
  \Delta \mathcal{P}_t &= \Theta_{0\mathcal{P}}^Q + \Theta_{1\mathcal{P}}^Q \mathcal{P}_{t-1} + \sqrt{\Sigma_{\mathcal{P}}} \epsilon_{\mathcal{P}}^Q \\
  \Delta Z_t &= \Theta_{0Z}^P + \Theta_{1Z}^P Z_{t-1} + \sqrt{\Sigma_Z} \epsilon_{Z}^P 
\end{align*}
\]

where \( \epsilon_{\mathcal{P}}^Q \sim N(0, I_N) \) and \( \epsilon_{Z}^P \sim N(0, I_{3+N}) \).\(^{14}\) According to equation (1), the short-term interest rate, \( r_t \), is assumed to be a linear function of \( \mathcal{P}_t \). In addition, the dynamics of the yield portfolios, \( \mathcal{P}_t \), under the risk-neutral probability measure (\( Q \)), follow the Gaussian process described in equation (2). The model is completed by assuming that the evolution of the complete state vector \( Z_t \) under the historical probability measure (\( \mathbb{P} \)) is given by the Gaussian process in equation (3).

\(^{14}\)The dimensions of the unknown coefficients present in the model are: \( \rho_0 \) is a scalar, \( \rho_1 \) and \( \Theta_{0\mathcal{P}}^Q \) are \( N \times 1 \), \( \Theta_{1\mathcal{P}}^Q \) and \( \Sigma_{\mathcal{P}} \) are \( N \times N \), \( \Theta_{0Z}^P \) is \( 3+N \times 1 \), and finally \( \Theta_{1Z}^P \) and \( \Sigma_Z \) are \( 3+N \times 3+N \). The matrix \( \sqrt{\Sigma_{\mathcal{P}}} \) is equal to the \( N \times N \) lower right corner of \( \sqrt{\Sigma_Z} \).
In the absence of arbitrage opportunities, bond prices are determined by equations (1) and (2). More specifically, letting $V_{m,t} \equiv \exp(-m \times y_{m,t})$ represent the time $t$ price of a bond that repays the investor at time $t+m$, it can be shown that the no-arbitrage bond price must respect $V_{m,t} = \mathbb{E}^Q_t [e^{-r_t} V_{m-1,t+1}]$. Appendix B shows that combining the bond-pricing condition to equations (1) and (2) yields the following solution for bond yields:

$$y_{m,t} = A_m + B_m P_t$$

where $A_m$ and $B_m$ are determined by the first-order difference equations described in Appendix B. Importantly, because of the assumed short-rate equation (1), the bond yields of different maturities are affine on $P_t$ and not on $M_t$. This means that only the risks associated with $P_t$ are priced explicitly by the model. Although macroeconomic risks are not priced explicitly, they may have important implications for bond prices, because under $\mathbb{P}$ the dynamics of $M_t$ interact with those of the yield portfolios (see equation (3)).

Standard models in the tradition of Ang and Piazzesi (2003) substitute equation (1) for an equation in which $r_t$ is a linear function of both $P_t$ and $M_t$. The bond prices implied by these more traditional models are then affine not only in $P_t$, but also in $M_t$. There are at least two reasons why having bond yields follow equation (4) is preferable to those implied by traditional models. First, having yields determined by equation (4) is consistent with the fact that a low-dimensional factor structure is sufficient to explain most of the variation in yields\(^{15}\). This avoids the problems that are likely to arise with estimating over-parameterized models, which will probably be the case when all variables in $Z_t$ are explicitly priced. Furthermore, Joslin, Priebsch, and Singleton (2010) show that in models where bond prices are affine on both $P_t$ and $M_t$, the macro factors are spanned by the term structure (i.e. a combination of yields explains all of the variation in $M_t$). In a setup similar to the one developed here, they show that this spanning property is empirically rejected in the U.S. data (more details follow in Section 4.1).

As in Diebold, Rudebusch, and Aruoba (2006), another important property of the model

\(^{15}\)Traditionally, a 3-factor structure consisting of Level, Slope and Curvature factors is sufficient to explain most of the cross-sectional variation in the term structure. See Litterman and Scheinkman (1992).
(1) - (3) is that it allows for two-way feedback effects between the macro variables and the yield curve. Mechanically, the interaction between \( M_t \) and \( P_t \) occurs because the conditional covariance matrix of \( Z_t, \Sigma_Z \), and the matrix of the slope coefficients \( \Theta^{P \Sigma Z} \) are potentially full. The model in which these feedback effects are not present is nested as a constrained version of equations (1) - (3).

3.2 Identifying Monetary Policy Shocks

Because the model described in the previous section consists of a reduced-form economic system, an identification scheme is needed in order to distinguish exogenous monetary impulses from those systematic responses of the Fed to changes in the state of the economy. In this paper, the identification of monetary policy shocks follows the *recursiveness assumption* of Christiano, Eichenbaum, and Evans (1999). According to this identification scheme, a particular ordering of the variables in \( Z_t \) is imposed in order to give rise to a recursive causal relationship among these variables.

Assume that the Fed’s monetary policy instrument is one of the endogenous variables included in \( Z_t \). Then the recursive identification scheme of Christiano, Eichenbaum, and Evans (1999) can be applied to equation (3) of the term-structure model, just as in any standard VAR. In particular, assume that \( \sqrt{\Sigma_Z} \) is the Cholesky factor associated with \( \Sigma_Z \). Then, the lower-triangular shape of \( \sqrt{\Sigma_Z} \) implies that the ordering of the variables in \( Z_t \) establishes a causal relation among the state variables. In particular, the variables that are ordered in \( Z_t \) above the monetary policy instrument do not move instantly when a monetary shock occurs. The values of these variables in a given period are assumed to be observed by the Fed before its monetary policy decision is taken. On the other hand, the variables ordered in \( Z_t \) below the policy instrument move instantly when a monetary shock occurs; thus, their values in a given period are assumed to be observed only after the Fed’s policy decision. As a result, this identification scheme implies that the policy shocks are orthogonal to the variables assumed to be included in the Fed’s information set\(^{16}\).\(^{17}\)

\(^{16}\)It can be shown that the monetary policy shocks identified through this recursive scheme do not depend on (i) the particular ordering of the variables above the policy instrument in \( Z_t \), and (ii) the particular ordering of the variables below the policy instrument in \( Z_t \). See Christiano, Eichenbaum, and Evans (1999).

\(^{17}\)The Cholesky factorization of \( \Sigma_Z \) actually implies a just-identification scheme. Therefore, it provides
To implement the recursive identification scheme in the model of Section 3.1, first one needs to choose the variable that will represent the monetary policy instrument of the Fed. A second decision must be made regarding the particular state variables that are assumed to be included in the information set available to the Fed before its policy decision (that is, one must choose whether each variable included in $Z_t$ should appear above or below the policy instrument).

I assume that the short-rate, $r_t$, represents Fed’s the monetary policy instrument. This choice is motivated by Bernanke and Mihov (1998) who find that, except for Volcker’s reserve targeting experiment (1979-1982), in practice the Fed actually has targeted the interest rate since the 1950s.\textsuperscript{18} Moreover, many recent empirical analyses of the term-structure – such as Ang, Dong, and Piazzesi (2007), Ang, Boivin, Dong, and Loo-Kung (2009), and Mumtaz and Surico (2009) – also view the short rate as the Fed’s monetary policy instrument. I introduce the policy instrument in the state vector $Z_t$ by assuming that one bond portfolio contained in $P_t$ simply replicates $r_t$. More specifically, I set one line of the matrix of portfolio weights $P$ to $[1 \ 0 \ \ldots \ 0]$.

In contrast to my approach, Diebold, Rudebusch, and Aruoba (2006) introduce $r_t$ in the state vector through $M_t$ and not through $P_t$. However, their approach does not rule out arbitrage opportunities in bond prices. By introducing the short-rate as a portfolio in $P_t$, I guarantee the absence of arbitrage opportunities across short-term and longer-term bonds.

With respect to the causal relations in the state vector, I choose the following ordering for the elements of $Z_t$:

$$Z_t = \begin{bmatrix} GAP_t & INF_t & COMM_t & r_t & P_{2,t} & \ldots & P_{N,t} \end{bmatrix}'$$

where $P_{i,t}$ represents the $i^{th}$ element of $P_t$ (with $P_{1,t} \equiv y_{1,t} \equiv r_t$). Note that the bond portfolio that replicates $r_t$ is ordered below $M_t$ and above all remaining $N-1$ bond portfolios. This implies that all elements of $M_t$ are included in the Fed’s time $t$ information set. As a simple recursive identification to $3 + N$ "structural" shocks in the model. In this paper I focus on monetary policy shocks, because in this case the recursive identification scheme is supported by many previous theoretical and empirical papers in the macro literature.

\textsuperscript{18}According to Cook (1988), movements in the fed funds rate followed judgemental actions of the Fed even during Volcker’s reserve targeting experiment.
result, $M_t$ responds with a lag to exogenous movements in the short-rate. On the other hand, the bond portfolios $\mathcal{P}_{\ell,t}, ..., \mathcal{P}_{N,t}$ are not in the Fed’s time $t$ information set, and therefore are allowed to adjust instantly to monetary shocks.

The motivation behind my ordering in $Z_t$ is as follows: because bond portfolios reflect asset prices that are purely forward-looking (see the bond pricing equation in Section 3), it is reasonable to assume that an exogenous change in $r_t$ triggers instant movements in $\mathcal{P}_{\ell,t}, ..., \mathcal{P}_{N,t}$. In other words, as soon as investors’ expectations are revised to incorporate the new level of $r_t$, the observed bond prices will be affected. In contrast, in case of $M_t$, the same policy shock in general will "affect economic conditions only after a lag that is both long and variable"\textsuperscript{19}. This lag could be rationalized in terms of the economic costs related, for example, to changes in production plans, revising goods’ prices, and etc. As a result, the policy shock will take longer to show up in the aggregate macroeconomic data.

Finally, note that the normalizations imposed to obtain econometric identification (see Appendix C) result in the coefficients of the short-rate equation (1) being $\rho_0 = 0$ and $\rho_1 = [ 1 \ 0 \ \cdots \ 0 ]'$. Therefore, in my identification scheme the short-rate dynamics are actually determined by the state equation (3) rather than equation (1). Following Christiano, Eichenbaum, and Evans (1999), the short-rate process implied by my framework therefore can be interpreted as an interest-rate feedback rule of the sort proposed by Taylor (1993) (expressed in reduced form). According to this view, endogenous short-rate movements would occur in response to changes in $M_t$, while all residual movements would be interpreted as monetary policy shocks.

### 3.3 Estimation Methodology

Because the first bond portfolio to enter $\mathcal{P}_t$ was already chosen in the previous section, it only remains to choose the other $N - 1$ yield-based factors (bond portfolios) in order to complete the model specification. The finance literature finds that most of the variation in bond yields is explained well by three unobserved factors usually referred to as level, slope and curvature. As Joslin, Singleton, and Zhu (2011) show, these estimated unobserved factors

\textsuperscript{19}Friedman (1961).
in general are similar to the first three principal components (PCs) of the term-structure data.

Accordingly, in this paper I allow for \( N = 3 \) yield-based pricing factors. As explained before, the first of these simply replicates the short-rate, \( r_t \). The two remaining factors are given by the second and third term-structure PCs, \( PC_2 \) and \( PC_3 \), which were extracted from my term-structure dataset. More specifically, the matrix of portfolio weights is given by:

\[
P = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-0.51 & -0.45 & -0.22 & 0.09 & 0.28 & 0.40 & 0.49 \\
0.60 & -0.01 & -0.61 & -0.34 & -0.11 & 0.17 & 0.33 \\
\end{bmatrix}
\]

Note that the loadings associated with the second bond portfolio roughly replicate the slope of the yield curve, while the third portfolio has the shape of a curvature factor with a trough on the 1-year maturity. I will therefore refer to \( PC_2 \) and \( PC_3 \) as the slope and curvature factors. In my dataset, the correlation between \( r_t \) and the first term-structure PC is above 0.97. Therefore, the fit of the model with my choice of \( P \) must be similar to that of a model where \( P_t \) contains the first three term-structure PCs.

The model is estimated by Maximum Likelihood (ML) after imposing the identifying normalizations proposed by Joslin, Singleton, and Zhu (2011). The bond portfolios \( P_t \) are assumed to be perfectly priced by the model. However, each observed yield \( y_{m,t}^{obs} \) (except for the short-rate) is assumed to be priced with a measurement error \( u_{m,t} = y_{m,t}^{obs} - y_{m,t} \) \( \sim N(0, \omega^2) \). The ML estimates of the \( P \)-dynamics of \( Z_t \) (except for \( \Sigma_Z \)) can be conveniently obtained by OLS. Conditional on these estimates, the likelihood function is optimized with respect to the parameters determining the \( Q \)-dynamics of \( P_t \) (\( \Sigma_Z \) included). For more details see Appendix C.

4 Model Analysis

This section analyzes the model estimation results. I show the results from using two different subsamples of my dataset, namely 1959:1-1979:3 and 1979:4-2007:4. This follows
from Boivin and Giannoni (2006); in the context of a VAR similar to the state equation (3) of my model, they find evidence of a structural break in the U.S. data in 1979:4. The model parameters’ estimates are reported in Appendix C.

Section 4.1 compares the model fit to alternative model specifications. Section 4.2 discusses the implications for the monetary policy transmission mechanism across the two samples used to fit the model.

4.1 Bond Pricing Errors

In order for the analysis carried out in the remainder of this paper to be meaningful, it is required that the long-term yields implied by the model track their observed counterparts reasonably well. Therefore, it is important to compare the fit of the MTSM described in Section 3 to other standard benchmark models in the literature. To form a fair comparison with the model from Section 3, all alternative models considered here have exactly three pricing factors.\(^{20}\)

The first alternative model is an Affine Term Structure Model with three yield-based pricing factors and no macro factors. As is standard in the finance literature, this yields-only model assumes that the pricing factors are the first three PCs of the term structure data. Based on their loadings on the term-structure data, these three PCs follow the usual Level, Slope and Curvature interpretation. Note that, unlike the model from Section 3, I use the first PC (i.e. the Level factor) in this case instead of \(r_t\) for the first yield-based factor.

The second model that I consider is an MTSM, as described in Section 3, with the exception that the macro factors in this case are assumed to be spanned by the term structure. More specifically, the model with spanned macro factors substitutes the short-rate equation (1) for another specification where \(r_t\) is a linear function of both \(P_t\) and \(M_t\) (i.e. both yield-based and macro factors are explicitly treated as pricing factors). In this case, it can be shown that the model-implied yields are linear in \(P_t\) and \(M_t\). Importantly, Joslin, Priebsch, and Singleton (2011) show that this model has the property that \(M_t\) can be replicated by appropriately chosen bond portfolios.\(^{21}\) – i.e. the macro variables are spanned by

\(^{20}\)In the model from Section 3 only the risks associated with the \(3 \times 1\) vector \(P_t\) were explicitly priced.

\(^{21}\)More precisely, the Spanned-Macro model implies that \(M_t = \zeta_0 + \zeta_1 \tilde{P}_t\), where \(\tilde{P}_t\) is a vector of bond
the information contained in the term structure. Models that have this spanning property include Ang and Piazzesi (2003), Ang, Dong, and Piazzesi (2007), and Bikbov and Chernov (2010). Because I am only focusing on three-factor models, the particular spanned MTSM that I estimate includes $GAP$, $INF$ and $COMM$ as pricing factors; no yield-based factor was included.

Panels (I) to (III) of Table 1 compare the fit of the three models discussed above for the 1959:1-1979:3 and 1979:4-2007:4 samples. Each model’s fit is evaluated according to the mean absolute bond pricing error, $\frac{1}{T} \sum_{t=1}^{T} |\tilde{u}_{m,t}|$, as well as the minimum and maximum estimated pricing error within each sample. All figures are expressed in annualized basis points. Table 1 focuses on 1-, 3-, and 5-year maturities.

The pricing errors associated with models "yields-only" and the unspanned MTSM are very similar in both samples. For these models, the pricing errors on average are small in absolute terms and they fluctuate inside a relatively narrow interval. Note that there is little deterioration in fit as we go from the yields-only model to the unspanned MTSM. This is because the former assumes that the pricing factors are the first three term-structure PCs, whereas the latter substitutes the first PC for the short-rate. Therefore, the cost of having the short-rate in the state vector to allow for monetary policy identification is very small.

Now comparing now the unspanned and spanned MTSMs, observe that the latter displays pricing errors that are an order of magnitude larger in both samples. This is because most of the variation in yields can be explained by the first three term-structure PCs, and $GAP$, $INF$ and $COMM$ fail to replicate the variation on these PCs. Only at the cost of increasing the dimension of the vector of pricing factors (in particular, if extra yield-based factors are added) will the spanned MTSM fit the data as well as the model unspanned MTSM.

Therefore, in terms of fit to the term-structure data, the unspanned MTSM proposed in Section 3 is comparable to the standard yields-only model. The advantage of the unspanned portfolios with as many entries as the number of priced factors in the model. See Joslin, Priebsch, and Singleton (2011).

---

22 Keeping the three-factor specification, I also compared the unspanned MTSM from Section 3 to a spanned MTSM with the following pricing factors: $GAP$, $INF$ and $PC_1$ (instead of $GAP$, $INF$ and $COMM$). Still in this case, the unspanned MTSM fits the term-structure data significantly better than this specification of the spanned MTSM.

23 This point was first made by Joslin, Priebsch, and Singleton (2011).
Table 1: Comparing Price Errors Across Different Term-Structure Models

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<tbody>
<tr>
<td></td>
<td>$\frac{1}{T} \sum_{t=1}^{T}</td>
<td>u_{m,t}</td>
</tr>
<tr>
<td>(I) Unspanned MTSM:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = 4$</td>
<td>4.9</td>
<td>-15.3</td>
</tr>
<tr>
<td>$m = 12$</td>
<td>5.4</td>
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<tr>
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<td>6.6</td>
<td>-21.7</td>
</tr>
<tr>
<td>(II) Yields-only model:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = 4$</td>
<td>4.8</td>
<td>-17.4</td>
</tr>
<tr>
<td>$m = 12$</td>
<td>4.8</td>
<td>-16.5</td>
</tr>
<tr>
<td>$m = 20$</td>
<td>4.5</td>
<td>-15.8</td>
</tr>
<tr>
<td>(III) Spanned MTSM:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$m = 4$</td>
<td>62.7</td>
<td>-245.5</td>
</tr>
<tr>
<td>$m = 12$</td>
<td>50.5</td>
<td>-191.7</td>
</tr>
<tr>
<td>$m = 20$</td>
<td>46.7</td>
<td>-187.0</td>
</tr>
</tbody>
</table>

Notes: MTSM* is the model described in Section 3, with 3 yield-based pricing factors given by the portfolio weights in (5) and 3 unspanned macro factors; YTS is an Affine Term-Structure Model with 3 yield-based pricing factors (the first 3 PCs) and no macro factors; MTSM* is a 3-factor Macro-Affine Term Structure Model where the pricing factors are given by $M_t$. All figures are expressed in annualized basis points.
MTSM *vis-à-vis* the yields-only model is that the former allows for interactions between the term structure and the macroeconomy, crucial for the purposes of this paper. Additionally, in comparison to the three-factor spanned MTSM, the unspanned model delivers a much better fit to the term-structure data. I therefore conclude that the unspanned MTSM is an adequate tool for the study of monetary policy shocks carried out in the next sections.

### 4.2 The Monetary Transmission Mechanism in the Unspanned MTSM

In this subsection, I use the unspanned MTSM from Section 3 to study how shocks to the assumed monetary policy instrument, \( r_t \), transmit to the macroeconomy and to the term structure. I begin by computing the model-implied impulse-response functions (IRFs) to a monetary policy shock. In so doing, I pay particular attention to the differences that emerge between the two samples used to estimate the model, namely the 1959:1-1979:3 sample and the more recent 1979:4-2007:4 sample. In order to make the IRFs comparable across samples, I normalize the monetary policy shock to be an exogenous increase of 100 basis points in the (annualized) short-rate.

Figure 2 shows the movements of the state vector \( Z_t \equiv [ M_t \ P_t ]' \) following the monetary policy shock defined above. Model-implied IRFs based on the 1959:1-1979:3 sample are shown in the top panel, whereas the bottom panel shows the same for the 1979:4-2007:4 sample. In each chart the solid line corresponds to the mean response, whereas the shaded areas represent small-sample 95% confidence intervals.

I begin by analyzing the IRFs for the macro variables \( GAP \), \( INF \), and \( COMM \). Note that the IRFs for these variables differ substantially across the two samples. In the model estimated using the first sample, the policy shock leads to statistically significant and persistent movements in \( GAP \), \( INF \), and \( COMM \). For the second sample, the responses are much smaller – in fact, all three IRFs in the second sample are not significantly different

---

24The confidence intervals were computed using the following bootstrap method: (i) resample, with replacement, \( T \) residuals from the estimated model; (ii) given the initial state vector \( Z_0 \), use the resampled residuals and the estimated model parameters to construct new time series for the state-vector and the yield curve; (iii) re-estimate the model using the resampled data; (iv) repeat steps (i) - (iii) 500 times. The dashed lines report the 2.5 and 97.5 percentiles of the distribution of estimated impulse responses.
This result is in line with Boivin and Giannoni (2002), who find that the impacts of monetary policy shocks on output and inflation are much smaller since the beginning of the 1980s than in the period before. Based on estimated general equilibrium models, they associate these reduced monetary policy impacts to an increase in the Fed’s responsiveness to inflation expectations that began in the 1980s.

Note that for the model estimated over the 1959:1-1979:3 sample, the response of $INF$ stays positive for several quarters before turning negative. In the literature this is known as the "price puzzle"\textsuperscript{25}, and is usually treated by including a measure of commodity prices in the model. In fact, excluding $COMM$ from the unspanned MTSM amplifies this effect significantly.

Returning to Figure 2, the bottom row of charts in each panel shows the IRFs for the pricing factors $r$, $PC_2$, and $PC_3$. In both samples the monetary policy shock triggers a persistent reaction of the short rate. In the 1959:1-1979:3 sample, the short-rate reaction stays positive for two years, turns negative and then approaches zero from below. In contrast, for the 1979:4-2007:4 sample the reaction stays positive for the entire time until it disappears. The slope factor, $PC_2$, falls after the shock in both samples, implying that long-term yields react to the shock by less than the short-rate. Finally, the curvature factor, $PC_3$, does not react significantly to the shock in the 1959:1-1979:3 sample, whereas this factor decreases two periods after the shock hits in the 1979:4-2007:4 sample.

According to equation (4), as the three pricing factors respond to the shock, the model-implied yields are also likely to move. Moreover, because of the recursive ordering that I proposed in Section 3.2, all of the pricing factors, and consequently all individual bond yields, are allowed to move instantly with the shock. I now study how monetary policy shocks are transmitted to the term structure; in particular, I consider the model-implied decomposition of bond yields into premium and Expectations Hypothesis (EH) components.

To understand this decomposition, let us consider a risk neutral world. In this world, the risk-adjusted ($\mathbb{Q}$) and the historical ($\mathbb{P}$) probability measures coincide\textsuperscript{26}. Appendix D shows

\textsuperscript{25}See Christiano, Eichenbaum, and Evans (1996) and Sims and Zha (2006).

\textsuperscript{26}In other words, all prices of risk are zero.
Figure 2: A Contractionary Monetary Policy Shock: The State Vector

1959:1 - 1979:3

\[ \text{GAP} \]
\[ \text{INF} \]
\[ \text{COMM} \]

1979:4 - 2007:4

\[ \text{GAP} \]
\[ \text{INF} \]
\[ \text{COMM} \]

Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures time in quarters. \( \text{COMM} \) is expressed in % deviation from an HP trend, whereas \( r \) is expressed in percent per annum. The units of \( \text{GAP} \), \( \text{INF} \), \( \text{PC}_1 \) and \( \text{PC}_2 \) are not interpretable. The dashed lines are 95% bootstrapped confidence intervals.
that in this case bond yields will be given by 
\[ \hat{y}_{m,t}^{EH} = A_{m}^{EH} + B_{m}^{EH} Z_{t} \]
where \( A_{m}^{EH} \) and \( B_{m}^{EH} \) follow the recursions shown in the appendix. Interestingly, it can be shown that, up to a convexity term, the \( m \)-period bond yield in this hypothetical world, \( y_{m,t}^{EH} \), behaves according to the EH. In other words, the dynamics of \( y_{m,t}^{EH} \) are determined by the expected dynamics of the short-rate. Letting \( t_{m,t} \) capture the deviations of the \( m \)-period yield from the EH, I ultimately obtain the following yield decomposition:

\[ y_{m,t} = \hat{y}_{m,t}^{EH} + t_{m,t} \quad (6) \]

Following the macro and finance literatures, I will call \( t_{m,t} \) the "\( m \)-period term premium". A positive value for \( t_{m,t} \) indicates that investing in the long-term bond is riskier than investing in a sequence of short-term bonds for \( m \) periods.

Figures 3 and 4 show how the term-structure and its decomposition into premium and EH components react to the same monetary policy shock that was considered earlier. Each figure refers to a different sample over which the model is estimated. To make the comparison across samples easier, I also plot Figure 5, which juxtaposes the IRFs for the two samples. In all charts, the horizontal axes measure maturity going from one quarter to five years. The black lines represent mean responses across maturities and the shaded areas are the 95% bootstrapped confidence intervals (for presentational reasons, I ignored the confidence intervals in Figure 5). Each row of charts corresponds to a given number of periods after the shock hits: e.g. the first row displays IRFs for the quarter when the shock hits; the second row shows IRFs for two quarters later; and etc. Finally, the columns correspond to the yield decomposition of equation (6), i.e. \( y_{m,t} \), \( \hat{y}_{m,t}^{EH} \) and \( t_{m,t} \).

I begin by analyzing the IRFs for the model-implied yields, \( y_{m,t} \), as shown in the first columns of Figures 3 and 4. To help organize the discussion, I begin with my first result:

**Result I:** In both samples, an exogenous increase in the short rate moves long-term yields by a significantly positive amount, at the same time reducing the yield curve slope. The IRFs for the two samples are remarkably similar across maturities and over time.

\(^{27}\)Impulse response functions and variance decompositions of \( y_{m,t} \), \( \hat{y}_{m,t}^{EH} \) and \( t_{m,t} \) are trivial to compute in the unspanned MTSM because these variables are all linear functions of the state vector \( Z_{t} \).
Figure 3: 1959:1-1979:3; Yield Curve Decomposition after a Monetary Policy Shock

Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum. The gray areas are 95% bootstrapped confidence intervals.
Figure 4: 1979:4-2007:4; Yield Curve Decomposition after a Monetary Policy Shock

Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum. The gray areas are 95% bootstrapped confidence intervals.
Figure 5: Yield Curve Decomposition after a Monetary Policy Shock; Comparing Samples

- Bond Yields ($y_{m,t}$)
- EH Consistent Yields ($y_{m,t}^{EH}$)
- Term Premium ($tp_{m,t}$)

On Impact ($t_0$)
Two quarters later ($t_0 + 2$)
Four Quarters Later ($t_0 + 4$)
Six Quarters Later ($t_0 + 6$)

Notes: Impulse responses to an increase of 100 basis points in the short-rate. The horizontal axes measures bond maturities from one quarter to five years. All responses are expressed in percent per annum.
As soon as the monetary policy shock hits the economy, all long-term yields reported increase by a significantly positive amount in both samples. The impact of the shock on yields declines as maturity increases. In fact, whereas the short rate increases by 100 basis points, the 5-year yield increases by only about 50-to-60 basis points in both samples. This means that the monetary shock considered here leads to a drop of about 40-to-50 basis points in the 5-year slope of the yield curve. Going back to Figure 2, this is in line with the fact that the slope factor $PC_2$ drops in response to the shock.

Over the quarters following the shock, the IRFs of bond yields die out in both samples. Note that the shape of the responses over time and at different maturities is remarkably similar across the two samples (see the first column of Figure 5). If yields move by similar magnitudes across samples, and we know from Figure 2 that the short-rate IRFs differ quite significantly across samples, then term premia must necessarily move differently following a monetary policy shock depending on the sample analyzed. This leads me to formalize my second result:

**Result II:** *In both samples an exogenous increase in the short rate leads to statistically significant increases in term premia across maturities. These responses are more pronounced and persistent in the 1959:1-1979:3 than in the 1979:4-2007:4 sample.*

In fact, the two last columns of Figures 3 and 4 show that term premia of all maturities increase by a significant amount following the shock. However, this increase is larger in general for the model estimated over the 1959:1-1979:3 samples than when I use data from the later sample (see the third column of Figure 5). For example, while in the 1979:4-2007:4 sample the 5-year term premium instantly increases by 26 basis points on average, in the earlier sample it increases by more than double that amount. Moreover, in the first sample the term premia responses of all maturities persistently stay above zero for more than six quarters after the shock. For the second sample, on the other hand, the term premia responses statistically become zero after about four quarters following the shock.

The fact that term premia move significantly following the shocks may have crucial implications for the monetary policy transmission mechanism. Remember from Section 3
that the term-structure model I analyze allows for two-way feedback effects between the macro variables and the yield curve. As a result, there is a *term-premium channel* through which monetary policy affects output and inflation. More specifically, movements in term premia following a monetary shock in general will have an impact on the pricing factors contained in $P_t$, which then will affect the dynamics of the macro variables contained in $M_t$. Interestingly, the fact that term premia respond more to the shock in the 1959:1-1979:3 than in the 1979:4-2007:4 sample suggests that the term-premium channel may be particularly important in the earlier sample.

I now try to quantify the term-premium channel of monetary transmission. Let $\Psi_{Z,h}$ be the impulse response of the state vector $h$ periods after a monetary policy shock occurs. Appendix E shows that $\Psi_{Z,h}$ can be decomposed into two components: $\Psi_{Z,h}^{EH}$ and $\Psi_{Z,h}^{TP}$. The first component, $\Psi_{Z,h}^{EH}$, measures what the response of $Z_t$ would be to a monetary policy shock if the term premium were held constant. In this case, the response of $Z_t$ will be affected only by the EH component of the term structure. I refer to this as the EH channel of monetary transmission. The second component, $\Psi_{Z,h}^{TP}$, holds the EH component of the term structure fixed and accounts only for the change in $Z_t$ that is caused by movements in term premia; this is the term-premium channel mentioned above.

Figure 6 shows the decomposition of the impulse responses of $GAP$, $INF$ and $COMM$ into their EH and term-premium components. The top panel shows IRFs for the 1959:1-1979:3 sample; the bottom panel refers to the 1979:4-2007:4 sample. Based the decomposition of the IRFs shown in this figure, I now formalize my third result:

**Result III:** *In the 1959:1-1979:3 sample, the term-premium channel is responsible for a large fraction of the movements in the macro variables following a monetary policy shock. In this case, the term premium channel acts to diminish the effectiveness of monetary policy shocks. The opposite is true for the 1979:4-2007:4 sample, for which the term-premium channel is close to being non-existent.*

The top panel of Figure 6 shows that a large portion of the responses of $GAP$ and $INF$ in the 1959:1-1979:3 sample are due to the term premium channel. The exact mechanism
Figure 6: Decomposing the Impulse Responses of the Macro Variables

1959:1 - 1979:3

$GAP$

$INF$

$PC_2$

1979:4 - 2007:4

$GAP$

$INF$

$PC_2$

--- Total  + Expectations Hypothesis  − Term Premium

Notes: Impulse responses to an exogenous increase of 100 basis points in the short-rate. The horizontal axes measures time in quarters. The units of $GAP$ and $INF$ are not interpretable.
behind this result is as follows: movements in term premia caused by the monetary impulse are reflected directly into the pricing (yield-based) factors included in $Z_t$. These in turn feed back into the macro variables through equation (3) which, in the 1959:1-1979:3 sample, causes large movements in $GAP$ and $INF$.

Going one step further, note that in the first sample the term-premium and EH channels of $GAP$ and $INF$ move in opposite directions. For example, the drop in $GAP$ in response to the shock is the result of a sharp drop in activity due to the EH channel, which is significantly moderated by the term-premium channel. For both $GAP$ and $INF$, the term premium channel in the first sample acts to diminish the effectiveness of monetary policy shocks.

To understand this last point, let me focus for a moment on the IRF of $PC_2$ shown in the top right corner of Figure 6. This chart shows that the movements in term premia following a monetary impulse pressure the slope factor to move up (not down, as the EH channel would predict). In fact, going back to Figure 3, note that in this sample the IRFs of term-premia are increasing in bond maturity – thus they exert upward pressure on the yield curve slope. In turn, this positive pressure on the slope factor feeds back into the macroeconomy through equation (3), causing the term premium channel to pressure $GAP$ and $INF$ in the direction opposite of the EH channel.

In terms of the 1979:4-2007:4 sample, note that the term-premium channel barely moves in the responses of both $GAP$ and $INF$. In fact, for these variables the lines corresponding to the total IRF and the EH channel are almost exactly on top of each other, as shown in the bottom panel of Figure 6. This striking difference across samples occurs mostly because the term-premium channel pressures the slope factor by much less in the second than in the first sample.

To conclude this section, I study the variance decomposition of the yield curve components. Table 2 shows the portion of the variance of $h$-period ahead forecasts of $y_{m,t}$, $y_{m,t}^{EH}$ and $tp_{m,t}$ that is due to identified monetary policy shocks. For purposes of presentation, I only report here the cases where $m$ is equal to 20 quarter and $h$ is set to 4, 12, and 20 quarters. The left panel corresponds to the 1959:1-1979:3 sample; the right panel refers to
Table 2: Forecast Error Variance Due to Monetary Policy Shocks

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<tr>
<td></td>
<td>( h = 4 )</td>
<td>( h = 12 )</td>
<td>( h = 20 )</td>
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<tr>
<td>( y_{20} )</td>
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<td>(1.9, 50.1)</td>
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<tr>
<td>( t_{p20} )</td>
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<td>60.9</td>
<td>54.9</td>
<td>21.3</td>
</tr>
<tr>
<td></td>
<td>(32.3, 76.8)</td>
<td>(22.7, 68.1)</td>
<td>(20.4, 63.6)</td>
<td>(5.2, 39.8)</td>
</tr>
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Notes:

the 1979:4-2007:4 sample. The numbers in parentheses are bootstrapped 95% confidence intervals.

Even though the bootstrapped confidence intervals are quite wide, at all horizons considered in Table 2 monetary policy shocks explain a substantial fraction of the forecast error variance of the 5-year yield and its premium and EH components. In the 1959:1-1979:3 sample, in particular, monetary policy shocks explain about 60% of the 5-year term-premium forecast error variance. As we look at the more recent sample, though, this fraction drops quite substantially, to around 15 to 20%. This leads me to my fourth result:

**Result IV:** A substantial portion of the term-premia forecast error variance is due to monetary policy shocks. In the 1959:1-1979:3 sample, these shocks explain around 60% of the forecast error variance of the 5-year term-premium. In the second sample, this fraction drops to around 15 to 20%.

This result is surprising in light of some recent theoretical results obtained from general equilibrium models. Rudebusch and Swanson (2008) study the nominal term premium in the context of a dynamic stochastic general equilibrium model (DSGE) with long-run risks, following Bansal and Yaron (2004). They find that the variability of term premia is dictated by technological shocks. Monetary policy shocks are responsible for only a small
portion of the variability in term premia. In contrast, the results in Table 2 show that
monetary policy shocks, at least in the context of the model developed here, are crucial to
understanding the variability in term-premia.

5 Shocks to Future Monetary Policy

Up until now I have considered only shocks that directly affect the short-term interest rate $r_t$. However, one of the major challenges of central banking is to coordinate the public’s expectations, allowing the monetary authority to reach its objectives with minimum costs.\(^{28}\)

In particular, modern macroeconomic theory shows that expectations about the future directions of key macroeconomic variables such as GDP, inflation and the short-rate are crucial to determining the success of current monetary policy actions. In fact, the introduction to Michael Woodford’s classic book states that:

"For successful monetary policy is not so much a matter of effective control of overnight interest rates as it is shaping market expectations of the way in which interest rates, inflation, and income are likely to evolve over the coming year and later". [Woodford (2003)]

It follows that central banks’ communication with the private sector is now considered to be one of the most important instruments for the conduct of monetary policy. In the words of Woodford (2003):

"... insofar as it is possible for the central bank to affect expectation, this should be an important tool of policy stabilization... Not only do expectations about policy matter, but, at least under current conditions, very little else matters"

(author’s italic).

\(^{28}\) The importance of expectations’ coordination became clear with the advent of micro-founded models for the analysis of monetary policy. These models highlight that future expectations play a crucial role in the decision making of private agents, which will in turn affect the macroeconomic equilibrium. See Woodford (2003).
In this section I study the macroeconomic implications of shocks to expectations about the future path of the monetary policy instrument \( r_t \). The MTSM from Section 3 is a particularly convenient tool for this study because it allows for the decomposition of long-term yields into policy expectations (i.e. the EH component of \( y_{m,t} \)) and term premia.

Section 5.1 shows how the unspanned MTSM can be used to identify the future policy expectations shock discussed above. Section ?? studies the transmission of shocks to future policy expectations to the macroeconomy. Since the consensus that expectations are crucial for monetary policy started to form in the mid 1980’s, I will only show results for the more recent sample, 1979:4-2007:4\(^{29}\), in this section.

### 5.1 Identifying Shocks to Future Monetary Policy

I define a shock to policy expectations \( m \) periods ahead as an exogenous impulse in \( y_{m,t}^{EH} \) that is neither accompanied by an instantaneous movement in the short-rate \( r_t \) nor by an instantaneous change in the macro variables in \( M_t \). Because the EH states that (up to a Jensen’s inequality term) the \( m \)-period yield equals the average short-rate from \( t \) until maturity, the expectations shock that I consider directly affects the future path of the traditional monetary policy instrument, \( r_t \), from the current period until maturity. The requirement that \( r_t \) does not move on impact guarantees that the shock will not be confused with the standard monetary policy shock considered in Sections 3 and 4. Also, the assumption that \( M_t \) does not move on impact maintains a coherence with the monetary policy transmission mechanism discussed in the previous sections.

Two points about my shock definition are worth mentioning. Firstly, although the short rate is assumed to remain constant on impact, it is allowed to move endogenously in the periods after the shock hits. Second, I allow the term premia to move instantly when the shock hits.

To make the discussion more concrete, I now show how to identify shocks to monetary policy expectations in the MTSM from Section 3. \(^{30}\) The main idea is to use a recursive identification scheme as described in Section 3.2, except that instead of using the state

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\(^{29}\)Results for the pre-Volcker sample are available upon request.

\(^{30}\)See Appendix F for more details of the identification methodology.
equation (3) to identify the shocks, I now use a rotated version of this equation. In the rotated model, the quantity I want to shock, $y_{m,t}^{EH}$, appears explicitly as an endogenous variable in the rotated state vector.

More precisely, let the rotated state vector be given by $\tilde{Z}_t \equiv \left[ M'_t \ r_t \ y_{m,t}^{EH} \ tp_{m,t} \right]'$. Appendix F shows that there exists a $6 \times 1$ vector $W_0$ and an invertible $6 \times 6$ matrix $W_1$ such that

$$
\begin{pmatrix}
M_t \\
r_t \\
y_{m,t}^{EH} \\
\dot{tp}_{m,t}
\end{pmatrix}
\equiv W_0 + W_1
\begin{pmatrix}
M_t \\
r_t \\
PC_{1,t} \\
PC_{2,t}
\end{pmatrix}
\equiv \tilde{Z}_t
$$

(7)

The choices of $W_0$ and $W_1$ rely on the fact that in the unspanned MTSM $y_{m,t}^{EH}$ and $tp_{m,t}$ are linear functions of $Z_t$, namely $y_{m,t}^{EH} = A_{m}^{EH} + B_{m}^{EH} Z_t$ and $tp_{m,t} = (A_{m} - A_{m}^{EH}) + ([0 \ B_{m}] - B_{m}^{EH}) Z_t$. Note that the decomposition of $y_{m,t}$ into policy expectations, $y_{m,t}^{EH}$, and term premium, $tp_{m,t}$, appears explicitly in the last two entries of the rotated state vector. Appendix F shows that $\tilde{Z}_t$ evolves according to

$$
\tilde{Z}_t = \tilde{\Theta}_{0Z}^p + \tilde{\Theta}_{1Z}^p \tilde{Z}_{t-1} + \sqrt{W_1 \Sigma Z W_1'} \epsilon_{zt}
$$

where $\tilde{\Theta}_{0Z}^p$ and $\tilde{\Theta}_{1Z}^p$ are known functions of $W_0$, $W_1$, $\Theta_{0Z}^p$ and $\Theta_{1Z}^p$.

As before, I identify shocks using the recursiveness assumption and thus assume that $\sqrt{W_1 \Sigma Z W_1'}$ is the Cholesky factor associated with matrix $W_1 \Sigma Z W_1'$. The ordering of the variables in $\tilde{Z}_t$ is again crucial. In the particular way I defined $\tilde{Z}_t$ in equation (7), an exogenous shock to $y_{m,t}^{EH}$ agrees with my definition of a policy expectations shock. That is, on impact all three variables in $M_t$ and the short rate shows no response to the shock; on the other hand, the term premium is allowed to move instantly. Only one period after the shock hits, all variables in $\tilde{Z}_t$ will be allowed to respond to the shock.
5.2 The Transmission of Policy Expectations Shocks

Figure 7 plots the IRFs of GAP and INF to a +100-basis-point shock to future monetary policy expectations. In each chart, the lines correspond to alternative choices for $m$. In particular, the dotted lines refer to IRFs when $m$ equals 1, in which case the shock collapses to the standard monetary policy shock studied in Section 4 (note that the scales of the vertical axes in this figure are different from the ones in Figure 2). The marked lines, on the other hand, refer to IRFs to policy expectations shocks for the cases where $m$ is equal to 2, 3 and 4.

Consider first the IRFs to a standard monetary policy shock, i.e. $m = 1$. In this model, this shock generates counter-intuitive responses for the macro variables shown in Figure 4. That is, both GAP and INF increase persistently following the standard monetary policy shock (even though we showed in Section 4 that these movements were not statistically significant), contrary to what is suggested by monetary theory. On the other hand, a shock of the same magnitude to policy expectations 2, 3 or 4 periods ahead causes both GAP and INF to drop persistently. Intuitively, once private agents learn that monetary policy on average will be tighter from now until $m$, both activity and inflation drop for several quarters. Interestingly, these drops occur quite quickly, with the trough of the responses occurring just one quarter after the shock hits. Also, increasing the shock horizon, $m$, from 2 to 4 reduces the initial impact of the shock on both GAP and INF. I summarize these results as follows:

**Result V:** In the 1979:4-2007:4 sample, an exogenous shock to monetary policy expectations leads to drops in activity and inflation. The troughs of these responses occur only one quarter after the shock hits. Increasing the horizon of the expected future policy shock from 2 to 4 quarters attenuates the IRFs.

As mentioned before, an important advantage of using the MTSM to study policy expectations shocks lies in the fact that this model allows for the separation of premium and EH component of yields\(^{31}\). Therefore, I now ask the following question: what would be the

\(^{31}\)The importance of separating policy expectations and premia in the context of monetary policy shocks’ identification was also highlighted by Piazzesi and Swanson (2008).
difference in the IRFs shown in Figure 7 had I ignored the term premium component of long-term yields and directly shocked $y_{m,t}$ instead of $y_{m,t}^{EH}$?

To answer this question, I compare the IRFs to the policy expectations shocks shown in Figure 7 to the new IRFs following a shock to $y_{m,t}$. Figure 8 plots the results. The IRFs to $y_{m,t}^{EH}$ shocks correspond to the red lines; the black lines are responses to shocks to $y_{m,t}$. For each shock, I continue to show responses for the case where $m$ is set to 2, 3 and 4.

Note that for all policy horizons ($m$) shown in Figure 8, the IRFs following a shock to $y_{m,t}$ are less pronounced than those following a shock to $y_{m,t}^{EH}$. In general, the responses to a $y_{m,t}^{EH}$ shock are about two times stronger on impact than the corresponding IRFs to a $y_{m,t}$ shock. Therefore, if I had ignored the term-premium component of long-term yields when identifying policy expectations shocks, I would have gotten substantially weaker IRFs than in the case where this component is accounted for. This result is summarized below:

**Result VI:** If the objective is to identify shocks to future policy expectations, then shocking $y_{m,t}$ instead of the true model-implied policy expectations $y_{m,t}^{EH}$ leads to biased IRFs of GAP and INF. These IRFs following a shock to $y_{m,t}$ are substantially weaker than the corresponding ones for the case where $y_{m,t}^{EH}$ is shocked. This is because the shock to $y_{m,t}^{EH}$ controls for the term premium component of the long-term yield.
6 Conclusions

This paper has shown that, in the context of a Macro-Affine Term Structure Model, movements in bond premia may play an important role in the monetary transmission mechanism. I found that, during the pre-Volcker sample, a large portion of the movements in inflation and economic activity following a monetary policy shock are due to movements in term premia that feed back into the economy. In the post-Volcker period, in contrast, this channel of monetary transmission is empirically irrelevant. I also have shown that in the post-Volcker period, shocks to policy expectations produce more pronounced and more plausible responses for the macroeconomic variables than do standard shocks to the contemporaneous value of the monetary policy instrument.

My findings show that accounting for premia is important not only to understanding movements in bond prices, as highlighted by Dai and Singleton (2002), but also to understanding the dynamics of key macroeconomic variables, such as inflation and output. This reaffirms, in the context of a monetary policy transmission analysis, the crucial role that financial assets play in the behavior of the macroeconomy.
References


A Extraction of GAP and INF from a Rich Dataset

This appendix describes the details behind the estimates of GAP and INF shown in Section 2. Suppose that one wants to extract a common factor from a set of $K$ different observed economic indicators $\{x_{1,t}, \ldots, x_{K,t}\}$. Following Stock and Watson (1988), one way to estimate this common factor is to consider the following state-space model:

**Signal Equation:**

$$
\begin{bmatrix}
    x_{1,t} \\
    x_{2,t} \\
    \vdots \\
    x_{K,t}
\end{bmatrix} = 
\begin{bmatrix}
    \mu_1 \\
    \mu_2 \\
    \vdots \\
    \mu_K
\end{bmatrix} + 
\begin{bmatrix}
    \gamma_1 & 1 & 0 & \cdots & 0 \\
    \gamma_2 & 0 & 1 & \cdots & 0 \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
    \gamma_K & 0 & 0 & \cdots & 1
\end{bmatrix} 
\begin{bmatrix}
    C_t \\
    \lambda_{1,t} \\
    \vdots \\
    \lambda_{K,t}
\end{bmatrix}
$$

**Transition Equation:**

$$
\begin{bmatrix}
    C_t \\
    \lambda_{1,t} \\
    \vdots \\
    \lambda_{K,t}
\end{bmatrix} = 
\begin{bmatrix}
    \phi_C & 0 & \cdots & 0 \\
    0 & \phi_1 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & \phi_K
\end{bmatrix} 
\begin{bmatrix}
    C_{t-1} \\
    \lambda_{1,t-1} \\
    \vdots \\
    \lambda_{K,t-1}
\end{bmatrix} + 
\begin{bmatrix}
    \nu_{C,t} \\
    \nu_{1,t} \\
    \vdots \\
    \nu_{K,t}
\end{bmatrix}
$$

where $C_t$ is a common factor, $\lambda_{i,t}$ for $i = 1, \ldots, K$ are idiosyncratic factors (one for each economic indicator). Also, the iid innovations follow $\nu_{C,t} \sim N(0,1)$ and $\nu_{i,t} \sim N(0, \sigma_i^2)$ for $i = 1, \ldots, K$, and are assumed to be uncorrelated with each other at all lags. Therefore, each observed economic indicator loads both on the common and on an idiosyncratic factors.

For simplicity, I assume that both the common and the idiosyncratic factors follow AR(1) processes, even though more complex lag structures could also be considered. The common-factor model is estimated via Maximum Likelihood using standard Kalman filtering techniques. Details about the economic indicators used to extract GAP and INF are shown in Table 3. Note that the Kalman Filter is particularly convenient in handling missing observations in the economic indicators’ time series, a feature that according to Table 3 occurs often in my dataset.

B No-Arbitrage Bond Pricing

This appendix shows how to derive the first-order difference equations that determine $A_m$ and $B_m$ in equation (4). No-arbitrage pricing implies that the price of an $m$-period bond, $V_{m,t}$, is determined by $V_{m,t} = E_t^Q [e^{-r_t V_{m-1,t+1}}]$. Start by guessing that the model-implied $V_{m,t}$ follows an affine function of the pricing factors, that is $V_{m,t} = \exp(A_t + B_t X_t)$. Using equations (1) and (2) this guess is verified as follows:

$$
V_{m,t} = E_t^Q \left[ \exp \left( -r_t + \tilde{A}_{m-1} + \tilde{B}_{m-1} \tilde{P}_{t+1} \right) \right]
= E_t^Q \left[ \exp \left( -\rho_0 - \rho'_1 \tilde{P}_t + \tilde{A}_{m-1} + \tilde{B}_{m-1} \Theta_0^{Q_P} + \tilde{B}_{m-1} \left( \Theta_1^{Q_P} + I_N \right) \tilde{P}_t + \tilde{B}_{m-1} \Sigma_P \sigma_{Pt+1} \right) \right]
= \exp \left( \tilde{A}_{m-1} + \tilde{B}_{m-1} \Theta_0^{Q_P} + \frac{1}{2} \tilde{B}_{m-1} \Sigma_P \sigma_{P} \tilde{B}_{m-1} - \rho_0 + \tilde{B}_{m-1} \left( \Theta_1^{Q_P} + I_N \right) - \rho'_1 \right) X_t
$$

where the third equality uses the properties of the log-normal distribution. It follows that:

$$
\tilde{A}_{m} - \tilde{A}_{m-1} = \tilde{B}_{m-1} \Theta_0^{Q_P} + \frac{1}{2} \tilde{B}_{m-1} \Sigma_P \sigma_{P} \tilde{B}_{m-1} - \rho_0
$$
$$
\tilde{B}_{m} - \tilde{B}_{m-1} = \tilde{B}_{m-1} \Theta_1^{Q_P} - \rho'_1
$$

where $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0$. The $m$-period continuously compounded yield, $y_{m,t} \equiv -\frac{P_{m,t}}{m}$, is then given by:

$$
y_{m,t} = A_m + B_m X_t
$$

where $A_m \equiv -\frac{\tilde{A}_{m}}{m}$ and $B_m \equiv -\frac{\tilde{B}_{m}}{m}$. 

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Table 3: Economic Indicators Used to Construct the Common Macro Factors

<table>
<thead>
<tr>
<th>(1) SERIES USED IN GAP:</th>
<th>long name:</th>
<th>short name:</th>
<th>series ID*:</th>
<th>sample:</th>
<th>transf.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Industrial Production Index</td>
<td>INPROD</td>
<td>INDPRO</td>
<td>58:4 - 07:4</td>
<td>DT</td>
<td></td>
</tr>
<tr>
<td>(ii) Total Nonfarm Payrolls</td>
<td>PAYROLL</td>
<td>PAYEMS</td>
<td>58:4 - 07:4</td>
<td>DT</td>
<td></td>
</tr>
<tr>
<td>(iii) Real Personal Consumption Expenditures</td>
<td>PCE</td>
<td>PCEC96</td>
<td>59:1 - 07:4</td>
<td>DT</td>
<td></td>
</tr>
<tr>
<td>(iv) Real GDP Index</td>
<td>GDP</td>
<td>GDPC96</td>
<td>58:4 - 07:4</td>
<td>DT</td>
<td></td>
</tr>
<tr>
<td>(v) ISM manufacturing index (New Orders)</td>
<td>ISM</td>
<td>NAPMNOI</td>
<td>58:4 - 07:4</td>
<td>LEV</td>
<td></td>
</tr>
<tr>
<td>(vi) Total Housing Starts</td>
<td>HOUST</td>
<td>HOUST</td>
<td>59:1 - 07:4</td>
<td>LEV</td>
<td></td>
</tr>
<tr>
<td>(vii) Civilian Unemp. Rate</td>
<td>URATE</td>
<td>UNRATE</td>
<td>58:4 - 07:4</td>
<td>LEV</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(2) SERIES USED IN INF:</th>
<th>long name:</th>
<th>short name:</th>
<th>series ID*:</th>
<th>sample:</th>
<th>transf.:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) CPI less food &amp; energy</td>
<td>CPI</td>
<td>CPILFESL</td>
<td>58:4 - 07:4</td>
<td>QG</td>
<td></td>
</tr>
<tr>
<td>(ii) Finished Goods PPI less food &amp; energy</td>
<td>PPI</td>
<td>PPILFE</td>
<td>74:2 - 07:4</td>
<td>QG</td>
<td></td>
</tr>
<tr>
<td>(iii) PCE Deflator less food and energy</td>
<td>PCEDEF</td>
<td>PCEPILFE</td>
<td>59:2 - 07:4</td>
<td>QG</td>
<td></td>
</tr>
<tr>
<td>(iv) GDP deflator</td>
<td>GDPDEF</td>
<td>GDPDEF</td>
<td>58:4 - 07:4</td>
<td>QG</td>
<td></td>
</tr>
<tr>
<td>(v) Average Hourly Earnings</td>
<td>EARN</td>
<td>AHEMAN</td>
<td>58:4 - 07:4</td>
<td>QG</td>
<td></td>
</tr>
</tbody>
</table>

Notes: The transformations applied to the series are the following: DT = quadratic deterministic trend removed, QG = series transformed into quarterly growth rates, and N = no transformation applied. All series were obtained from the St. Louis Fed database. * corresponds to ID codes for the St. Louis Fed database.
C Details of the Econometric Methodology

Joslin, Singleton, and Zhu (2011) propose a set of normalizations that not only allow for identification of the model described in Section 3.1, but also simplifies the task of finding a global maximum of the likelihood function. More specifically, the model with observable pricing factors has the property that the parameters governing bond pricing, \( \Theta_{0P}, \Theta_{1P}, \rho_0, \rho_1 \), are uniquely mapped into a smaller set of parameters \( \left( r^Q_\infty, \lambda^Q, \Sigma_{PP} \right) \), where \( r^Q_\infty \) represents the long-run mean of the short rate under \( Q \), and \( \lambda^Q \) is the \( N \)-vector of ordered eigenvalues of \( \Theta_{1P}^Q \). Also, the bond portfolios contained in \( P_t \) are assumed to be priced perfectly by the model. The observed yields, except the short-rate (which is included in \( P_t \)), are allowed to differ from their model-implied counterpart through a \( (J - 1) \)-vector of measurement errors \( u_t \sim N \left( 0, \omega^2 I_{J-1} \right) \). Note that it is assumed for simplicity that the variance of the measurement errors is the same across all long-term yields used to fit the model.

The likelihood function of the model is then given by

\[
L \left( \hat{y}^{obs}_t, Z_t | Z_{t-1}; \Phi \right) = L \left( \hat{y}^{obs}_t | Z_t, Z_{t-1}; r^Q_\infty, \lambda^Q, \Sigma_Z, \omega \right) \times L \left( Z_t | Z_{t-1}; \Theta_{0Z}, \Theta_{1Z}, \Sigma_Z \right)
\]

where \( \hat{y}^{obs}_t \) contains the yields observed with measured errors. A convenient feature of the normalization proposed by Joslin, Singleton, and Zhu (2011) is that the ML estimate of \( \Theta_{0Z}^P \) and \( \Theta_{1Z}^P \), that is \( \hat{\Theta}_{0Z}^P \) and \( \hat{\Theta}_{1Z}^P \), are obtained by OLS. Conditional on \( \hat{\Theta}_{1Z}^P \), an optimization algorithm searches for the values of \( r^Q_\infty, \lambda^Q, \Sigma_Z \), and \( \omega \) in order to find the global maximum of the likelihood function\(^{32}\).

Note that the search algorithm in the second stage of the estimation procedure usually converges very quickly because good starting values are easy to obtain. First, good starting values for the parameters in \( \Sigma_Z \) can be obtained by running an OLS regression of \( Z_t \) on \( Z_{t-1} \). Also, good starting values for \( r^Q_\infty \) and \( \lambda^Q \) are not difficult to obtain because these parameters are rotation-invariant and therefore carry economic interpretation.

D Risk Premium Accounting in the MTSM

The most general form of the MTSM described in Section 3 is given by\(^{33}\)

\[
\begin{align*}
Z_t &= \Theta_{0Z}^Q + \Theta_{1Z}^Q Z_{t-1} + \sqrt{\Sigma_Z} \varepsilon_t^Z \\
Z_t &= \Theta_{0Z}^P + \Theta_{1Z}^P Z_{t-1} + \sqrt{\Sigma_Z} \varepsilon_t^P \\
r_t &= \rho_0 + \rho_1 Z_t
\end{align*}
\]

where \( Z_t \equiv \left[ M'_t \quad P'_t \right]' \). Note that, differently from the model described in Section 3, here I let all factors, macro and yield-based, be treated as pricing factors (i.e. I do not impose that the macro factors are unspanned by the yield curve). The case of unspanned macro factors is easily recovered by setting \( \rho_1 = \left[ 0 \quad \rho_1 \right]' \).

In a risk neutral world, the risk-adjusted (\( Q \)) and the actual (\( P \)) probability measures coincide, which implies that bond prices would be given by

\[
V^{EH}_{m,t} = E^P_t \left[ e^{-r_t V^{EH}_{m-1,t+1}} \right]
\]

Note that this would imply that, up to a convexity term, the EH holds. I follow the derivations in Appendix B and, also in the risk neutral world, I guess and verify that the log of bond price is an affine function of

\(^{32}\)The search algorithm in the second stage of the estimation procedure usually converges very quickly because: (1) the OLS estimate of \( \Sigma \) provides a good starting value for \( \Sigma_{PP} \), and (2) since \( r^\infty \) and \( \lambda^Q \) are rotation-invariant (i.e. they carry economic interpretation), good starting values are not difficult to obtain. Estimating the model using my dataset takes about 30 seconds.

\(^{33}\)In this paper I am restricting attention to models where the factors, \( Z_t \), are markovian. More general models can be obtained by relaxing this assumption.
where $y_{m,t}^E = \exp(\tilde{A}_m^E + \tilde{B}_m^E Z_t)$. I therefore obtain

\[
\begin{align*}
\gamma_{m,t}^E &= A_m^E + B_m^E Z_t \\
\tilde{A}_m^E &= \tilde{A}_{m-1}^E + \tilde{B}_{m-1}^E \Theta_{\pi Z} + \frac{1}{2} \tilde{B}_{m-1}^E \sqrt{\Sigma Z} \left( \sqrt{\Sigma Z} \right)^\prime \left( \tilde{B}_{m-1}^E \right)^\prime - \rho_0 \\
\tilde{B}_m^E &= \tilde{B}_{m-1}^E \Theta_{\pi Z} - \rho_1 Z
\end{align*}
\]

where $A_m^E \equiv -\frac{\tilde{A}_m^E}{m}$ and $B_m^E \equiv -\frac{\tilde{B}_m^E}{m}$. Finally, defining the term premium at maturity $m$, $t_{m,t}$, as the deviation of $y_{m,t}$ from $m$-period yield consistent with the EH, it is easy to see that

\[
t_{m,t} \equiv y_{m,t} - y_{m,t}^E = (A_m - A_m^E) + \left( \begin{array}{c} 0 \\ B_m \end{array} \right) - B_m^E Z_t
\]

## E Decomposing the Impulse Response Functions

Let the column of $\sqrt{\Sigma Z}$ that corresponds to the short-rate be given by $\left( \begin{array}{c} \omega' \\ \omega' \end{array} \right)$. This follows from the assumption that the short-rate is ordered as the first bond portfolio and is below the macro variables in $M_t$. The impulse response of $Z_t$ after $h$ periods since a one standard deviation monetary policy shock hits, $\Psi_{Z,h}$, is given by:

\[
\Psi_{Z,h} = \left( \Theta_{\pi Z} \right)^h \left( \begin{array}{c} 0 \\ \omega \end{array} \right)
\]

From Section 3, it is easy to write the model-implied yields as an affine function of the state-vector $Z_t$:

\[
y_t = A_{7 \times 1} + \left( \begin{array}{c} 0 \\ \tilde{B}_{7 \times 3} \end{array} \right) Z_t = A_{7 \times 1} + B Z_t
\]

where $A = (A_1 \ A_2 \ A_4 \ \cdots \ A_{20})'$, $\tilde{B} = (B_1' \ B_2' \ B_4' \ \cdots \ B_{20}')'$, and $B = (0 \ \tilde{B})$. Setting the prices of risk to zero, the version of $y_t$ that is consistent with the EH is an affine function of $Z_t$, which implies that the residual term premium is also affine in $Z_t$:

\[
y_{t}^E = A_{7 \times 1}^E + B Z_t
\]

\[
t_{t} = A_{7 \times 1}^{tp} + B_{7 \times 6}^{tp} Z_t
\]

where $y_{t}^E \equiv (y_{t,1}^E \ y_{t,2}^E \ y_{t,3}^E \ \cdots \ y_{t,20}^E)'$ and $t_{t} \equiv (t_{1,t} \ t_{2,t} \ t_{4,t} \ \cdots \ t_{20,t})'$. Note that $A_{7 \times 1}^{tp} = A - A_{7 \times 1}^E$ and $B_{7 \times 6}^{tp} = B - B_{7 \times 6}^E$.

Since $\mathcal{P}_t = P y_t$, the impulse response of $Z_t$ when $h = 0$ is given by:

\[
\Psi_{Z,0} = \left( \begin{array}{c} 0 \\ \omega \end{array} \right) = \left( \begin{array}{c} 0 \\ P \Psi_{y,0} \end{array} \right)
\]

where $\Psi_{y,0}$ is the impulse response of $y_t$ on impact. Moreover, $\Psi_{y,0}$ can be decomposed into an EH and a term premium component using equations (8):

\[
\Psi_{y,0} = B \left( \begin{array}{c} 0 \\ \omega \end{array} \right) = \Psi_{y,0}^{EH} + \Psi_{y,0}^{tp}
\]

\[
= B_{7 \times 6}^{EH} \left( \begin{array}{c} 0 \\ \omega \end{array} \right) + B_{7 \times 6}^{tp} \left( \begin{array}{c} 0 \\ \omega \end{array} \right)
\]
Therefore, $\Psi_{Z,0}$ can be decomposed into

$$\Psi_{Z,0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} (P\Psi_{yEH,0}) + \begin{pmatrix} 0 \\ 0 \end{pmatrix} (P\Psi_{yP,0})$$

and it follows that the impulse response of $Z_t$ to a monetary policy shock $h$ periods after the shock occurs is given by

$$\Psi_{Z,h} = \begin{pmatrix} \Theta^p_{1Z} \\ \Theta^w_{1Z} \end{pmatrix}^h \begin{pmatrix} 0 \\ 0 \end{pmatrix} (P\Psi_{yEH,0}) + \begin{pmatrix} 0 \\ 0 \end{pmatrix} (P\Psi_{yP,0})$$

where $\Psi_{Z,h}^{EH}$ and $\Psi_{Z,h}^{IP}$ are respectively the EH and the term premium components of the impulse response to a monetary policy shock.

F Identifying Shocks to $y_{m,t}^{EH}$

To be able to shock monetary policy expectations and term premia, I will rotate $Z_t \equiv [ M_t' \ P_t' ]'$ so that the quantities shocked appears explicitly in the rotated state vector $\tilde{Z}_t$. Let $W_0$ be a $3 + N \times 1$ vector and $W_1$ be an invertible $3 + N \times 3 + N$ matrix such that

$$\tilde{Z}_t \equiv W_0 + W_1 Z_t$$

i.e. $W_0$ and $W_1$ rotate $Z_t$ linearly. If, for example one wants to rotate $Z_t \equiv [ M_t' \ P_t' ]'$ in order to obtain $\tilde{Z}_t = [ GAP_t \ INF_t \ COMM_t \ r_t \ y_{m,t}^{EH} \ \theta_{P,t,0,1} ]'$, the choices of $W_0$ and $W_1$ would be

$$W_0 \equiv \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ A_{mEH} \end{pmatrix} \quad W_1 \equiv \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $B_m^{(i)}$ is the $i^{th}$ entry in the vector $B_m$ and $B_m^{IP} \equiv [ 0 \ B_m ] - B_m^{EH}$ with $B_m^{IP(i)}$ being the $i^{th}$ entry in $B_m^{IP}$.

Using the $\tilde{Z}_t$ definition, equation (3) can be written in terms of the rotated state vector $\tilde{Z}_t$:

$$W_1^{-1} (\tilde{Z}_t - W_0) = \Theta^p_{0Z} + [ \Theta^p_{1Z} + I ] W_1^{-1} (\tilde{Z}_{t-1} - W_0) + \sqrt{\Sigma_Z} \epsilon^{p}_{zt}$$

$$\tilde{Z}_t = \tilde{\Theta}^p_{0Z} + \tilde{\Theta}^p_{1Z} \tilde{Z}_{t-1} + \left( W_1 \sqrt{\Sigma_Z} \right) \epsilon^{p}_{zt}$$

where $\tilde{\Theta}_{0Z}^p = W_1 (\Theta_{0Z}^p - \Theta_{1Z}^p W_1^{-1} W_0)$ and $\tilde{\Theta}_{1Z}^p = W_1 (\Theta_{1Z}^p + I) W_1^{-1}$. A recursive identification of the shocks that hit the rotated state vector can therefore be obtained by taking the Cholesky decomposition of the variance of the rotated residuals $W_1 \Sigma_Z W_1'$. Moreover, the dynamics following the shock can be obtained through the rotated matrices of VAR coefficients, namely $\tilde{\Theta}_{0Z}^p$ and $\tilde{\Theta}_{1Z}^p$. 

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