Incentives to Innovate and the Decision to Go Public or Private

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We model the impact of public and private ownership structures on firms' incentives to invest in innovative projects. We show that it is optimal to go public when exploiting existing ideas and optimal to go private when exploring new ideas. This result derives from the fact that private firms are less transparent to outside investors than are public firms. In private firms, insiders can time the market by choosing an early exit strategy if they receive bad news. This option makes insiders more tolerant of failures and thus more inclined to invest in innovative projects. In contrast, the prices of publicly traded securities react quickly to good news, providing insiders with incentives to choose conventional projects and cash in early. (*JEL* G24, G32, O32)

We introduce a model in which the form of equity financing—either public or private—affects managers' incentives to innovate. Our main contribution is to show that private ownership creates incentives for innovation, whereas public ownership disincentivizes innovation. As we allow for an endogenous choice of ownership structure, the model also provides, to the best of our knowledge, a novel explanation for the decision to go public or private. We find that this

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decision is affected by the relative profitability of innovative and conventional projects.

The logic of our model is as follows. A risk-neutral insider chooses between a conventional project and an innovative project. Following March (1991), we call the conventional project the exploitation of existing ideas and the innovative project the exploration of new ideas. Both projects generate cash flow in two consecutive periods. The insider has an option to liquidate his stake early by selling shares in the first period. Under private ownership, if the insider can time the market by choosing an early exit after receiving bad news, the insider becomes more tolerant of early failures and thus more inclined to invest in the innovative project. This tolerance-for-failure effect is the key determinant of innovation in private companies.

Under public ownership, cash flow is observable, and thus an early exit after receiving bad news is not profitable. Therefore, there is no tolerance for failures in public companies. Furthermore, the market prices of public securities react quickly to good news. This rapid incorporation of good news into market prices creates incentives for short-termist behavior. Thus, the insider may prefer the conventional project because it has a higher probability of early success. We show that the equilibrium under public ownership implies a positive probability of investment in the conventional project, even if innovation is ex ante efficient.

In sum, our model shows that the incentives in public firms are biased toward conventional projects, whereas the incentives in private firms are biased toward innovative projects. Consequently, holding all else constant, the optimal structure of ownership—public or private—changes with the firm's life cycle and depends on whether the exploitation of existing ideas or the exploration of new ideas is optimal.

We interpret our model as a theory of the evolution of ownership structures. Innovation is very important early in the life of a firm or industry. In an emerging industry, firms experiment with different varieties of products (Keppler 1996). Our model predicts that firms should start under private ownership to provide incentives for exploration and experimentation. Our model also predicts that firms should go private when they need to undertake risky restructurings. Whenever a firm needs to reinvent itself, it makes sense to do so out of the public eye. Major restructurings, involving radical changes in strategy, are more properly motivated under private ownership.[1]

There is evidence that private firms are more innovative than are public firms. Using patent citation data, Lerner, Sorensen, and Strömberg (2011) find that firms invest in more influential innovations after being acquired by private equity (PE) funds. Although most PE targets in their sample were already private, some of the most significant improvements in patent quality were associated with public-to-private transitions. For example, Seagate

[1] For an alternative incentive-based theory of the life cycle of speculative industries, see Biais, Rochet, and Woolley (2009).

Technologies, which is the largest patentee in their sample of PE targets, was initially a public company. Lerner, Sorensen, and Strömberg (2011) show that Seagate lagged behind its competitors in terms of the number of patents and citations in the years before they were bought by Silver Lake Partners. Seagate's innovative position improved significantly after the buyout.

Our model also has implications for the empirical literature dealing with the real effects of venture capital and buyout investments. Kaplan and Strömberg (2009) review this literature and conclude that private equity investment creates value because of tax benefits and the exploitation of mispricings in the debt and equity markets, and also by affecting corporate behavior, such as operations and investments. Our model suggests that PE funds can affect innovative investments via the decision to go public or private. Furthermore, our theory suggests that controlling for the type of transition (e.g., public-to-private vs. private-to-private) is at least as important in empirical work as is controlling for the type of investment (buyout vs. venture capital).[2]

The article is structured as follows. After discussing the related literature in Section 1, we present the model setup in Section 2 and separately discuss the private and public cases in Sections 3 and 4, respectively. We then bring these two cases together and discuss the choice between going public or private in Section 5. We discuss the case of illiquid private securities in Section 6 and conclude with a discussion of the empirical implications in Section 7, to which we add some final remarks in Section 8. All proofs are in the Appendix.

1. Related Literature

Our work fits with an emerging body of theoretical and empirical literature that deals with the roles of ownership structures and financing choices in corporate innovation. An early example is Aghion and Tirole (1994); more recent works include Aghion, Van Reenen, and Zingales (2009), Atanassov, Nanda, and Seru (2007), Belenzon, Berkovitz, and Bolton (2009), Bhattacharya and Guriev (2006, 2009), and Fulghieri and Sevilir (2009). These articles focus on related but different questions, such as the impact of capital structure, governance, organization, and ownership concentration on corporate innovation.

Our model is closely related to four different veins of theoretical literature:

(1) Interactions between stock prices and investment in firms. An extensive body of literature examines the role of stock prices in guiding

[2] Boucly, Sraer, and Thesmar (2011) provide evidence that the distinction between private-to-private and public-to-private transitions is relevant. They find that leveraged buyouts (LBOs) are followed by growth if the targets are financially constrained. LBOs are not followed by growth in public-to-private transitions (and in private-to-private LBOs of financially unconstrained targets). Though our model has no explicit implications for firm growth, if growth is related to periods of exploitation of existing technologies, then our model would predict that IPOs should be followed by growth. Moreover, in line with the evidence in Boucly, Sraer, and Thesmar (2011), public-to-private LBOs would be followed by restructuring or experimentation with an innovative process but no immediate growth.

corporate investment decisions and affecting insiders' incentives more generally. An incomplete list includes Holmström and Tirole (1993), Khanna, Slezak, and Bradley (1994), Dow and Gorton (1997), Faure-Grimaud and Gromb (2004), Almazan, Banerji, and Motta (2008), and Edmans (2009).

Our model is particularly related to models of managerial short-termism. Stein (1989) develops a model of rational short-termism driven by the stock market. In his model, in an attempt to mislead the market, firms take actions to boost current earnings at the cost of lower future earnings. In equilibrium, the market is not fooled and managers are stuck with an inefficient strategy. In a similar vein, Chemmanur and Jiao (2007) develop a model of the choice of security-voting structure, in which market-driven short-termism plays a key role. In their model, entrepreneurs may prefer to go public with a dual-class share structure to commit to pursuing long-term strategies. By selling equity without votes, the entrepreneur can insulate himself from short-term market pressure. This form of managerial entrenchment can be beneficial in situations in which agency costs are low.

Our model has similar implications. If the firm is public, a manager may choose the conventional project even if the innovative project has a higher net present value, because the former has a higher probability of generating high earnings in the short run. However, our model also depicts the alternative situation. If the firm is private, and thus free from pressure to boost current earnings, the manager puts too much emphasis on future cash flows. Without the stock market punishing short-term declines in earnings, managers become rationally biased toward innovative projects, which are risky but very profitable if successful. This bias gives rise to the phenomenon of *inefficient long-termism*. Innovation may be chosen even if it is inferior to conventional methods. Thus, our model provides a more balanced view of market incentives. Whereas managers of public firms may excessively focus on current earnings, managers of private firms may excessively focus on future earnings. The best structure thus depends on the nature of the projects available to the firm.

(2) Information disclosure and innovation. Bhattacharya and Ritter (1983) were the first to propose a model in which firms may compromise their ability to innovate if they disclose information to outside investors. In their model, an innovative firm in need of external finance faces a trade-off when choosing whether to disclose private information about its innovative capabilities. On the one hand, information disclosure allows the firm to obtain external funds with more advantageous terms. On the other hand, disclosure reveals crucial information to competitors and reduces the firm's initial advantage in a patent race. Maksimovic and Pichler (2001) develop a model that is based on a similar trade-off. In their model, firms choose between a new or existing technology and then decide whether to finance future rounds of investment with either public or private offerings. Public offerings are assumed to be cheaper, but they reveal information about industry profitability to potential competitors. Thus, firms may strategically delay financing or resort to private offerings to prevent entry. In a more recent article, Spiegel and Tookes (2009) develop and estimate a dynamic oligopoly model that incorporates some of the trade-offs originally highlighted in Bhattacharya and Ritter (1983) and Maksimovic and Pichler (2001), and they also analyze a number of new trade-offs. For example, their duopoly model generates predictions concerning the impact of the competitive environment on innovation and financing decisions. Large firms facing small rivals have more incentives to innovate because small firms find it too costly to compete by producing their own innovations. This effect changes the perceived costs and benefits of acquiring market share.

Our model differs from this body of literature because of our focus on the role of information asymmetry in incentives to innovate. In particular, our model is concerned with the effect of the way in which firms are financed on their internal incentives to choose between different technologies. Thus, our model allows us to address a different question: should the decision to go public or private depend on the relative profitability of innovative versus old technologies?

(3) Insider trading and incentives to innovate. In a seminal article, Hirshleifer (1971) shows that the option to trade on the basis of private information can provide additional incentives for engaging in innovative activities. He distinguishes between the technological benefits of innovations—the value created by the technological improvements made possible by an innovation—and their pecuniary benefits, which are the gains to the innovator from his ability to speculate in markets that will be affected by a particular innovation. If the pecuniary benefits are large, entrepreneurs may wish to pursue innovations even when the social value of those innovations is negative. A similar logic is present in our model. In opaque firms, insiders may choose to innovate mainly for the pecuniary benefits of innovation. Thus, private firms may innovate excessively.

Another article that is particularly related to ours is that of Bebchuk and Fershtman (1994). They show that the ability to trade on the basis of private information provides managers with incentives to undertake risky projects. The ability to sell shares before information about low profitability becomes public works as a put option that convexifies the payoffs enjoyed by insiders, which makes risky projects more attractive. The same effect is present in our model, but only in some cases. Our analysis is different in that we compare different levels of information asymmetry so that we can characterize the conditions under which the opposite result obtains, i.e., insider trading may also lead to the selection of safer projects.

(4) The decision to go public or private. Our article is also related to a large body of literature about the choice between public and private structures. Examples include Shah and Thakor (1988), Zingales (1995), Pagano and Roell (1998), Chemmanur and Fulghieri (1999), and Boot, Gopalan, and Thakor (2006), among many others. None of these articles consider the incentives for innovation as a determinant of ownership structures.

2. Model Setup

A risk-neutral insider initially holds all of the shares of a firm. The insider has no initial wealth, is protected by limited liability, and has outside utility normalized to zero. We view the insider as a manager-entrepreneur who founded the firm and initially owns it in full. Because the identity of the manager is not important in our model, we assume that the founder remains the manager regardless of the number of initial shares the founder sells to other investors. All results are unchanged if the founder is replaced by a newly hired professional manager.

2.1 Technology

The insider has to choose between two projects, projects 1 and 2, at two consecutive dates, dates 0 and 1. Each project has two possible outcomes: success or failure. Success yields payoff S, and failure yields payoff F, $S > F$. We call project 1 the exploitation of existing ideas and project 2 the exploration of new ideas. This setup is similar to that in Manso (2011).

If the insider chooses project 1, the conventional project, the probability of success is $p > 0$. The probability p is known to everyone. If the insider chooses project 2, the innovative project, the probability of success is $q > 0$, which is unknown. It is only possible to learn about q if the insider chooses project 2. We assume that $E[q|F] < E[q] < E[q|S]$. That is, the expectation of success increases if project 2 is successful at date 1 and decreases if project 2 fails at date 1.

The insider will only consider choosing the innovative project if the innovative project has a chance of delivering higher payoffs than does the conventional project. Thus, we also assume that $E[q|S] > p$ to eliminate the trivial case in which project 1 strictly dominates project 2. Conversely, the insider would always choose the innovative project if $E[q]$, the unconditional probability of success, is higher than p. We only consider the more interesting case in which $E[q] < p$. To economize on algebra and notation, we define δ and θ such that $\delta p = E[q]$ and $\theta p = E[q|S]$. Our assumptions imply that $0 < \delta < 1$ and $1 < \theta < 1/p$. To summarize,

$$\delta p = E[q] < p < E[q|S] = \theta p. \tag{1}$$

Equation (1) encapsulates all of the characteristics of project 2. Project 2 is exploratory because it is only possible to learn about the new method by trying it. Project 2 is promising because, conditional on being successful at date 1, its probability of success is higher than the probability of success associated with project 1. We can think of radical methods that seem unlikely to work but would greatly improve upon current methods if they did work. The interpretation of δ and θ is that a method is more radical the smaller the δ and the higher the θ.

The total profit (gross of any initial investment costs) is given by the undiscounted sum of payoffs, $\pi = x_1 + x_2$, where x_t is equal to F or S. We call x_t *earnings*. We assume that earnings are only liquid at date 2. That is, earnings

x_1 are realized at date 1, but dividends based on x_1 are paid at date 2. More generally, we wish to capture a situation in which it is possible to observe, at date 1, a signal x_1 about future profits. We call x_1 earnings at date 1 to simplify exposition, but it can also be understood as "a signal at date 1 about the profit at date 2."

The insider makes an initial investment, I, paid in cash, to produce positive earnings by investing in either project. Without this initial investment, all earnings are equal to zero, regardless of the project chosen.

The insider may switch from one project to the other after observing x_1. If the insider initially chooses to exploit the old method, the option to switch has zero value. However, if the initial choice is to explore the new method to maximize firm value, the insider switches to project 1 after observing $x_1 = F$. The option to switch is valuable under exploration. If the new method is tried but fails, the insider returns to the old method. Figure 1 provides a visual summary of the technology, taking into account the option to switch.

To simplify the notation, we make $F=0$ and $S=1$, without loss of generality. Under exploitation (project 1), the ex ante value of the firm, gross of the initial investment cost, is $v_1 = p(1+p)+(1-p)p$. This expression implies

$$v_1 = 2p. \tag{2}$$

If the insider chooses exploration (project 2), the firm continues to use the innovative method in the case of a success at date 1. In the case of failure, the firm returns to the old method (project 1). The ex ante value of the firm under exploration is then $v_2 = \delta p(1+\theta p)+(1-\delta p)p$ or

$$v_2 = p\{1+\delta[1+p(\theta-1)]\}. \tag{3}$$

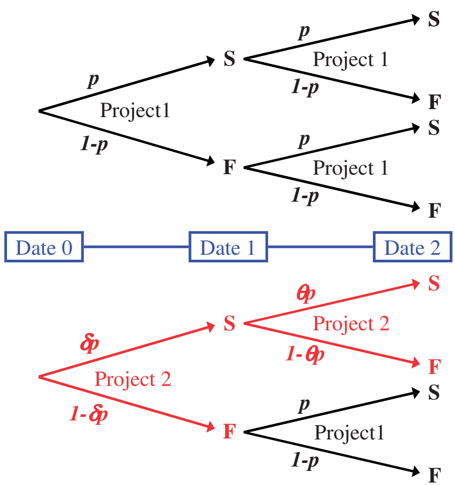

Figure 1
Earnings and probabilities associated with each initial project choice

The innovative project (project 2) is ex ante preferable to the conventional project (project 1) if and only if $v_2 - v_1 \geq 0$. We have

$$v_2 - v_1 > 0 \text{ if and only if } \delta[1 + p(\theta - 1)] > 1. \tag{4}$$

2.2 Liquidity and financial market frictions

The key financial market friction in our model is the existence of a demand for liquid assets caused by (unmodeled) borrowing constraints. The insider has a utility function, as in Diamond and Dybvig (1983), of

$$U(c_1, c_2) = \begin{cases} c_1 & \text{with probability } \mu, \\ c_2 & \text{with probability } 1 - \mu, \end{cases} \tag{5}$$

where c_t is consumption at date t. This reduced-form approach is common in microeconomic models of liquidity shocks (see, e.g., Freixas and Rochet 1997). With probability μ, a liquidity shock forces the insider to consume at date 1. With probability $1 - \mu$, there is no liquidity shock and dividends and consumption are synchronized at date 2. We can think of liquidity shocks as representing different types of consumers. Insiders that do not suffer a liquidity shock are called late consumers. Insiders that suffer a liquidity shock are early consumers.[3]

For liquidity shocks to have an impact on decisions, we need to assume that the insider faces borrowing constraints. The assumption of limited liability eliminates uncollateralized borrowing. The assumption of zero initial wealth implies that the insider has no initial collateral. We need to assume further that the insider cannot borrow by using his own shares as collateral.

Liquid securities, such as cash, can be stored from one period to the following period at no cost. There is no discounting or systematic risk in the economy.

2.3 Project financing

The insider must sell securities backed by future earnings to finance the initial investment, I, as the insider has no initial wealth. The insider may sell securities to either private or public investors. The initial investment, I, is observable to all and is contractible. Thus, the insider must pay I to undertake one of the projects if he sells securities to raise funds. The insider cannot run away with the money or invest in a third project.

We assume that share contracts are the only securities available. This assumption is for the simplicity of exposition. Capital structure choices are relevant in our model (i.e., the model does not exist in a Modigliani-Miller world), but they do not change the qualitative results regarding the choice between private and public ownership structures.[4]

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3 We interpret the liquidity shock as any reason, other than private information, for the insider to sell shares, including portfolio rebalancing, tax considerations, and behavioral biases. For evidence of such motives to trade, see, e.g., Kallunki, Nilsson, and Hellström (2009).

4 An extension of the model, in which the firm can also issue debt, can be found in some of the older Working Paper versions.

2.4 Investor types

There are two types of investors: sophisticated and unsophisticated. Both types of investors are fully rational. Unsophisticated investors only observe publicly available information. There are a large number of such investors in the economy. Thus, these investors behave competitively, and their trades are zero net present value transactions, conditional on all public information available at the time they occur. Sophisticated investors can observe inside information at the time they trade. That is, sophisticated investors always have the same information as the insider. Consistent with the idea that information and expertise are costly to acquire, we assume that sophisticated investors are in short supply.

We define the fundamental value of shares as the value that those shares will have if kept until the end of date 2. The fundamental value of shares may differ from the market value of shares, which is what unsophisticated investors will pay for the shares in equilibrium.

If the insider wishes to sell some of his shares, he can either sell them to some of the unsophisticated investors or search for a sophisticated investor who is willing to buy shares. Because sophisticated investors are in short supply (or, equivalently, they have shallow pockets), the insider can only find a sophisticated investor with some positive probability $e<1$. With probability $1-e$, the insider has no other option but to trade with unsophisticated investors. Once the insider meets a sophisticated investor, they bargain over the price of the shares to be sold. The surplus from trading with the sophisticated investor is $\Sigma \equiv v-V$, where v is the fundamental value of the shares being traded and V is the market value of those shares. The fraction of the surplus captured by the sophisticated investor is β, which measures the bargaining power of sophisticated investors. For simplicity, we assume that the market does not observe the negotiations between insiders and sophisticated investors.

Our assumptions about investor heterogeneity are standard. For example, Bolton, Santos, and Scheinkman (2011) similarly assume that informed investors are in short supply and uninformed investors are in infinite supply. We interpret informed, sophisticated investors as venture capitalists or private equity investors, who would only invest in businesses that they understand well. As it might not be possible for the insider to find an informed private buyer for his shares, sometimes the only option is to sell to small retail investors.

2.5 Differences between private and public ownership structures

The key results of our model depend on only one difference between private and public ownership. This difference is the ability of outsiders to observe the interim earnings, x_1, of a public firm but not of a private firm. Under public ownership, we assume that the interim earnings x_1 are observable by everyone. Under private ownership, in contrast, only the insider, current private investors, and future sophisticated investors observe x_1. These assumptions capture the fact that public companies are more transparent than are private companies.

Public companies are subject to tighter disclosure requirements, such as quarterly earnings reports and comprehensive annual reports, analyst coverage, and the aggregation of dispersed information into the stock price via trading.

For the sake of realism and to permit the analysis of different trade-offs, we also allow for other differences between the two structures, such as the cost of capital and liquidity costs. These enrich the model but are not necessary for any of the qualitative results linking innovation incentives and the choice between going public or private.

We assume that there are transaction costs associated with raising funds for investment through public offerings.[5] We capture the costs of issuing public equity by parameter $c_{pub} \in (0,1)$, such that each dollar sold in public offerings yields only c_{pub} to the firm. A large c_{pub} implies a small discount.

Raising capital through private sales also involves transaction costs. We denote the discount factor associated with private securities by $c_{priv} \in (0,1)$. This parameter is likely to change with changes in the institutional environment and the state of the economy. For example, when interest rates are relatively low, private equity funds can borrow cheaply, and thus going private becomes less costly for the firm. Private equity booms are thus associated with high levels of c_{priv}.[6]

We make no assumptions with respect to the relative cost of public equity capital $c_{priv} - c_{pub}$. Thus, our model allows for situations in which funds for investment are cheaper if financed by public securities ($c_{pub} > c_{priv}$) and cases in which being private reduces the cost of capital ($c_{pub} < c_{priv}$).

One justification for going public is to improve the liquidity of insider shareholdings (Chemmanur and Fulghieri 1999; Ritter and Welch 2002). For example, consider the case of a founder that suffers a liquidity shock and needs to sell shares quickly. If the firm is privately held, the founder may have to negotiate with a limited number of private investors. In contrast, under public ownership the founder may be able to sell his shares more easily through organized markets. To capture a potential liquidity advantage of public equity, we assume that each dollar in shares sold by the insider at date 1 (the liquidity shock period) yields only $k \leq 1$ if the company is private. No such discount happens if the firm is public. To focus on the main mechanism that explains our key results, we initially assume that there is no liquidity discount if the insider sells his own shares, $k = 1$. In Section 6, we analyze the case in which $k < 1$.

5 Lee et al. (1996) estimate that administrative and underwriting costs usually amount to approximately 11% of the IPO proceeds. IPO underpricing can create much higher costs, with total costs reaching the 20%–30% range (Ritter 1987). Seasoned equity offerings (SEOs) are less costly, but discounts are also common, with a typical negative stock price reaction after announcements of equity offerings of 3% of the pre-issue price (Asquith and Mullins 1986), to which direct costs of roughly 7% of the proceeds should be added (Lee et al. 1996).

6 Axelson et al. (2010) provide evidence that buyout activity increases in periods of low interest rates and that, in such periods, shareholders of target firms are able to sell their shares at higher premiums.

2.6 The structure of information and timing of events

At date 0, the insider decides to sell either a fraction $1-\alpha_{priv}$ of the shares to private investors or a fraction $1-\alpha_{pub}$ to public markets. We assume that public investors are unsophisticated. Private investors can be either sophisticated or unsophisticated. However, at date 0, this distinction is irrelevant because information is symmetric. The insider needs to raise at least I in cash to pay for the initial investment cost. After paying I, the insider chooses either project 1 or project 2. Outside investors cannot observe which project was chosen. Private investors, in contrast, have the same information as the insider.

At date 1, the insider observes the first realization of earnings $x_1 \in \{0, 1\}$ and then chooses project 1 or project 2. Again, this choice is unobservable to outsiders. The insider then learns about his liquidity needs. If the insider is an early consumer, he sells all of the shares that he owns. With probability e, the insider has the option to sell his shares to either a sophisticated private investor or the public market, where prices are determined by perfect competition among unsophisticated investors. Because sophisticated investors know everything that the insider knows, the insider may prefer to sell to public markets, even if private buyers are available. With probability $1-e$, the insider has no other option but to sell to public investors, regardless of the market valuation of the shares. If the insider is instead a late consumer, he may sell some of the shares or keep them until date 2. After observing whether the insider places orders to sell the shares, the market forms a price for the shares.

At date 2, the second-period earnings, $x_2 \in \{0, 1\}$, are realized, shareholders receive dividends, x_1+x_2, and the firm is liquidated. The liquidation value is normalized to zero. Figure 2 shows the time line.

2.7 Equilibrium

The game is played by one insider and infinitely many potential investors. Unsophisticated investors (also referred to as "the market") are in unlimited supply. Sophisticated investors are available with probability e. At date 0, before decisions are made, there is no meaningful difference between the two types of investors. All investors, regardless of type, become fully informed after buying shares in a private firm. At date 1, all sophisticated investors have the same information set as the insider. The market only observes public information.

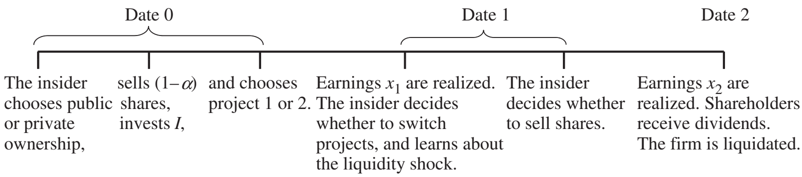

Figure 2
Time line

The insider takes actions at dates 0 and 1. At date 0, the insider first chooses between a private structure and a public structure, $\varphi \in \{priv, pub\}$. All of the actions that follow are conditional on the choice of φ. The insider also chooses the fraction of shares sold to investors, $\alpha_\varphi \in [0,1]$. Finally, the insider chooses project 2 with probability $\sigma_\varphi \in [0,1]$.

At date 1, the insider learns his type, $\tau \in \{\textit{early consumer}, \textit{late consumer}\}$, and whether sophisticated investors are available, $\varepsilon \in \{\textit{available, not available}\}$. The insider also learns $x_1 \in \{0,1\}$. The insider knows which project was chosen, $\pi \in \{1,2\}$. The insider sells shares to the market with probability $b_\varphi(x_1,\pi,\tau,\varepsilon) \in [0,1]$ and sells shares to sophisticated investors with probability $l_\varphi(x_1,\pi,\tau,\varepsilon) \in [0,1]$.

At date 0, the investors value the shares of the firm at u_φ. At date 1, the market observes whether the insider sells shares to the public, $n \in \{\textit{Sale}, \textit{No Sale}\}$. The market only observes the value of $x_1 \in \{0,1\}$ if the firm is public. To summarize, the market's information set at date 1 is $(n,\eta) \in \{\textit{Sale}, \textit{No Sale}\} \times \{x_1 = 0, x_1 = 1, (x_1 = 0) \cup (x_1 = 1)\}$. The market values the shares of the firm at date 1 at $V_\varphi(n,\eta)$. Because of perfect competition, $V_\varphi(n,\eta)$ is also the price that the market pays for each share. The sophisticated investors value the shares of the firm at date 1 at $\Lambda_\varphi(x_1,\pi,\tau,\varepsilon)$, i.e., they have the same information set as the insider. They are willing to pay $V_\varphi(n,\eta)+(1-\beta)(\Lambda_\varphi(x_1,\pi,\tau,\varepsilon)-V_\varphi(n,\eta))$ for each share, where $\beta \in [0,1]$ denotes the fraction of the surplus that is captured by the sophisticated investors, which is exogenously given.

The investors form beliefs about how the game is played in equilibrium. Without loss of generality, let all investors share the same beliefs $\rho \equiv (\sigma, b, l)$ about the unobservable choices made by the insider. Let υ denote the (stochastic) value of the company to shareholders.

Definition 1. For each set of parameters $\left(p,\delta,\theta,k,\mu,e,c_{priv},c_{pub},I,\beta\right)$, an equilibrium is a profile of strategies, valuations, and beliefs such that

1. at date 1, $b^*_\varphi(x_1,\pi,\tau,\varepsilon)$ and $l^*_\varphi(x_1,\pi,\tau,\varepsilon)$ maximize the insider's expected payoff, given $V^*_\varphi(n,\eta)$ and $\Lambda^*_\varphi(x_1,\pi,\tau,\varepsilon)$;
2. at date 0, φ^*, α^*_φ, and σ^*_φ maximize the insider's expected payoff, given u^*_φ, $b^*_\varphi(x_1,\pi,\tau,\varepsilon)$, $l^*_\varphi(x_1,\pi,\tau,\varepsilon)$, $V^*_\varphi(n,\eta)$, and $\Lambda^*_\varphi(x_1,\pi,\tau,\varepsilon)$;
3. new investors' valuations of shares are given by $V^*_\varphi(n,\eta) = E[\upsilon \mid n,\eta,\varphi,\rho^*]$; $\Lambda^*_\varphi(x_1,\pi,\tau,\varepsilon) = E[\upsilon \mid x_1,\pi,\tau,\varepsilon,\varphi,\rho^*]$ and $u^*_\varphi = E[\upsilon \mid \varphi,\rho^*]$;
4. beliefs are consistent with equilibrium play: $\rho^* = \left(\sigma^*_\varphi, b^*_\varphi(x_1,\pi,\tau,\varepsilon), l^*_\varphi(x_1,\pi,\tau,\varepsilon)\right)$;
5. probabilities are always updated in accordance with Bayes's rule.

This is a perfect Bayesian-Nash equilibrium. Parts 1 and 2 imply that an equilibrium must satisfy sequential rationality. Part 3 implies that the new investors' valuations must be rational. Part 4 implies that the investors must hold rational expectations, i.e., beliefs about the insider's behavior must be correct. Part 5 implies Bayesian rationality.

As will become clear when we characterize the equilibrium, all nodes of the game tree are reached with a strictly positive probability in equilibrium. There is no need to impose rules for updating beliefs at nodes off the equilibrium path, as there are no such nodes. Any deviation by the insider goes undetected, implying that the beliefs remain fixed at ρ^*, even if the insider chooses an off-the-equilibrium action.

3. Private Ownership

Characterizing the set of equilibria for this game requires many steps, as one sees in Definition 1. Because the choice of $\varphi \in \{priv,pub\}$ is effectively a choice between two quite distinct subgames, we first analyze each of these two cases separately. We then consider the decision to go public or private in Section 5.

First, consider the case of private ownership, i.e., at date 0, the insider sells $1-\alpha_{priv}$ shares to private investors. We take α_{priv} as exogenous for now and then work backwards to find the optimal α_{priv}.

After $1-\alpha_{priv}$ shares are sold, at the end of date 0, the insider chooses either project 1 or 2. Recall that the project choice is the private information of the insider. The intuition is that, although investments may be observable, the insider has unique information that allows him to assess the characteristics of the available projects. This is a natural assumption, which is consistent with the view that a manager's unique expertise may be essential for investment decisions.

Let $\sigma_{priv} \in [0,1]$ be the probability that the insider chooses project 2 (innovation). We allow for the possibility of equilibria involving mixed strategies. Intuitively, a strictly mixed strategy could also be interpreted as an intermediate project, which is more innovative than is project 1 but is not as radical as project 2. Our goal is to compute the equilibrium project choice, σ^*_{priv}, under private ownership.

3.1 Selling behavior at date 1

At the end of date 1, after observing x_1, the insider chooses whether to retain or sell the shares of the firm. We assume that the current private investors may also experience a liquidity shock and this shock is perfectly correlated with the insider's liquidity shock. Thus, the current private investors cannot buy out the insider after a liquidity shock. This assumption is stronger than necessary and is made only for simplicity.[7] As the insider and the current investors have

[7] Our results do not change qualitatively under the weaker assumption that there is a positive probability under which the private investors cannot offer liquidity insurance to the insider. There are many reasons that can make the private investors unable to offer liquidity insurance. One possibility is that all capital committed to a private equity fund has already been used. Even if there is still capital available, fund covenants may impose limits on the amount of fund capital invested in a single firm (Kaplan and Strömberg 2009). Fund covenants and restrictions on

identical preferences and share the same information set at date 1, they will exhibit the same optimal behavior. Thus, we need only to characterize the insider's behavior.

The insider sells either to new private buyers, who are sophisticated, or to public investors via an initial public offering (IPO). Private buyers are available with probability $e<1$. Trading with a private buyer is optimal only if the surplus $\Sigma \equiv v - V$ is positive, where v is the fundamental value of the firm and V is the value of the firm in an IPO. If the surplus is negative, the insider prefers an IPO to a private sale. To put it differently, a private sale is attractive only if the market undervalues the firm, i.e., if $v > V$. If $\Sigma > 0$ and a private buyer is available, the insider and the buyer find themselves in a bilateral monopoly situation. Let $\beta \in [0,1]$ denote the fraction of the surplus that is captured by the private buyer. The insider's payoff per share, conditional on selling to private investors, is given by $V + (1-\beta)(v - V)$. The insider strictly prefers a private sale if $\beta < 1$. To save on notation, we assume that β is zero so that the insider always captures the full surplus when trading with a private buyer. This assumption is not necessary; the analysis that follows is well defined for any value of β (although it is trivial if $\beta = 1$).[8]

The insider receives the fundamental value of the shares v if he sells to private buyers. Private buyers thus offer liquidity insurance to the insider. We say that the insider has liquidity needs if the insider suffers a liquidity shock and there are no private buyers available. Insiders with liquidity needs must sell shares in public markets. Insiders without liquidity needs may behave strategically and go public to exploit potential mispricings.

We now consider how the market updates its beliefs if there is an IPO at date 1. Let m be the posterior probability that the insider has liquidity needs, conditional on a public sale (IPO), at date 1. A small m means that the market assigns a high probability to the case in which the insider sells for strategic reasons.

An insider with liquidity needs (i.e., an early-consumer insider who cannot find a private investor) has no other option but to sell shares to the market (i.e., to make an IPO). An insider without liquidity needs chooses whether or not to sell to the market. The following lemma describes the insider's behavior when earnings are $x_1 = 1$.

Lemma 1. In the private ownership case, an insider without liquidity needs never sells shares to the market at date 1 after observing a success ($x_1 = 1$).

raising additional capital can be rationalized as potential solutions to agency conflicts between general partners (fund managers) and limited partners (Axelson, Strömberg, and Weisbach 2009). Finally, it could also be the case that funds need to exit early to produce evidence of good performance and raise more capital (Gompers 1996).

8 It may seem strange to assume that private buyers are in short supply but have no bargaining power. This is only for simplicity; there is no loss of generality. All we need is that insiders have some bargaining power. If insiders had no bargaining power ($\beta = 1$), then the existence of private buyers would not improve the insider's situation, and thus sophisticated investors would play no role in the model.

An insider without liquidity needs who sees $x_1 = 1$ would only sell shares in public markets if he believes that the shares are overvalued. After a success, the fundamental value of one share is either $1+p$ or $1+\theta p$. The proof of Lemma 1 shows that the market price at date 1 is lower than $1+p$. Intuitively, the market expects the insider to be more likely to sell after a failure than after a success. Market rationality then rules out those share prices that are not compatible with the insider's selling behavior. Consequently, prices at date 1 are never high enough to entice an insider to sell shares after receiving good news. In short, as the market does not observe earnings at date 1, the market always assigns a strictly positive probability to failure, which encourages the insider to keep the shares in the case of success.

Let $b \in [0,1]$ be the probability that an insider without liquidity needs sells shares to the market after observing a failure, $x_1 = 0$.[9] For a given pair of equilibrium values (σ_{priv}, b), we define $m(\sigma_{priv}, b) \equiv \Pr(\mathit{Liquidity\ needs} \mid \mathit{Sale})$. By Bayes's rule, rational market beliefs imply that

$$m(\sigma_{priv}, b) = \frac{\Pr(\mathit{Sale} \mid \mathit{Liquidity\ needs}) \Pr(\mathit{Liquidity\ needs})}{\Pr(\mathit{Sale})}. \tag{6}$$

The inputs for this formula are as follows. In an equilibrium in which the probability of choosing project 2 is σ_{priv}, the unconditional probability of selling shares to the market at date 1 is

$$\Pr(\mathit{Sale}) = \mu(1-e) + b(1-\mu+\mu e)\left[\sigma_{priv}(1-\delta p) + (1-\sigma_{priv})(1-p)\right]. \tag{7}$$

The first term on the right-hand side is the probability that the insider has liquidity needs, in which case the insider sells with probability 1. The second term is given by the probability of no liquidity needs $(1-\mu+\mu e)$, times the probability of failure, times b, which is the probability of a sale conditional on a failure and no liquidity needs.

Conditional on having liquidity needs, the insider sells to the market with probability 1. As the probability of the insider experiencing liquidity needs is $\mu(1-e)$, we have

$$m(\sigma_{priv}, b) = \frac{\mu(1-e)}{\mu(1-e) + b(1-\mu+\mu e)\left[\sigma_{priv}(1-\delta p) + (1-\sigma_{priv})(1-p)\right]}. \tag{8}$$

The equilibrium value of shares if the market holds rational beliefs is

$$V_{priv}(\sigma_{priv}, b) = m(\sigma_{priv}, b)\left[\sigma_{priv} v_2 + (1-\sigma_{priv}) v_1\right] + \left(1 - m(\sigma_{priv}, b)\right) p. \tag{9}$$

If a public offering is caused by liquidity needs, which happens with probability $m(\sigma_{priv}, b)$, the market value per share is given by a weighted average of the

9 Because b can only be nonzero if $x_1 = 0$, b does not need to be conditional on the project choice. For brevity, we omit the proof of this claim.

fundamental values of the innovative and the conventional projects, $\sigma_{priv}v_2+\left(1-\sigma_{priv}\right)v_1$. If the public offering is not caused by liquidity needs, then by Lemma 1 the market knows that the insider does not sell shares after a success. As the optimal action after a failure is to switch to the conventional project, the value of the firm after a failure is p.

A necessary condition for the insider to sell shares to the market after a failure is $V_{priv}\left(\sigma_{priv},b\right)\geq p$. The next lemma shows that the insider always sells to the market after a failure.

Lemma 2. In the private ownership case, an insider without liquidity needs sells shares to the market at date 1 with probability $b=1$ after observing a failure ($x_1=0$).

The insider always sells after a failure because the market assigns a strictly positive probability to $x_1=1$. This belief is rational because an insider with liquidity needs always sells.

Lemma 2 shows that a key aspect of the private ownership case is the insider's ability to sell shares at date 1 after observing a failure. A late-consumer insider only sells shares at date 1 if they are overvalued. Overvaluation may occur in equilibrium because the market does not observe x_1 and thus cannot distinguish between a liquidity-motivated sale and an opportunistic sale. This information asymmetry creates a valuable option for a late-consumer insider.

Let $T\left(\sigma_{priv}\right)\equiv V_{priv}\left(\sigma_{priv},1\right)-p$ denote the intrinsic value of the option to exit early for a late-consumer insider conditional on $x_1=0$. Selling shares is a real option to the insider. The value of the underlying asset is the market value of shares in equilibrium V_{priv}, whereas the exercise price of the option is p. Lemma 2 implies that $T\left(\sigma_{priv}\right)>0$.

3.2 Project choice at date 0

Now we return to date 0 and analyze the choice between projects 1 and 2. Suppose that the market expects project 2 to be chosen with probability σ_{priv}. At date 0, the expected value of each share held by the insider if the insider chooses project 1 is given by

$$u_{priv,1}\equiv\mu(1-e)V_{priv}\left(\sigma_{priv},1\right)+(1-\mu+\mu e)\left[(1-p)V_{priv}\left(\sigma_{priv},1\right)+p(1+p)\right]. \tag{10}$$

This expression accounts for the fact that at date 0 the insider does not yet know his type. With probability $\mu(1-e)$, the insider has liquidity needs and will be forced to sell at date 1. With probability $1-\mu+\mu e$, the insider has no liquidity needs but may sell voluntarily. Lemmas 1 and 2 imply that the insider sells after a failure and does not sell after a success.

Similarly, if the insider chooses project 2, whereas the market expects project 2 to be chosen with probability σ_{priv}, the expected value of each share at date 0 is

$$u_{priv,2} \equiv \mu(1-e)V_{priv}\left(\sigma_{priv},1\right) + (1-\mu+\mu e)\left[(1-\delta p)V_{priv}\left(\sigma_{priv},1\right)+\delta p(1+\theta p)\right]. \tag{11}$$

An equilibrium with a positive probability of exploration, $\sigma_{priv}>0$, exists only if $u_{priv,2} \geq u_{priv,1}$. That is, choosing project 2 at date 0 must be incentive compatible for the insider. Using $u_{priv,2}$ and $u_{priv,1}$, and substituting $V_{priv}\left(\sigma_{priv},1\right)=T\left(\sigma_{priv}\right)+p$, we obtain

$$u_{priv,2} \geq u_{priv,1} \Leftrightarrow v_2 - v_1 + p(1-\delta)T\left(\sigma_{priv}\right) \geq 0. \tag{12}$$

An equilibrium in which the insider chooses project 2 with probability $\sigma_{priv}>0$ exists only if the incentive compatibility condition (12) holds. Similarly, an equilibrium with a positive probability of choosing project 1, $\sigma_{priv}<1$, exists only if $v_2-v_1+p(1-\delta)T\left(\sigma_{priv}\right)\leq 0$. A strictly mixed strategy equilibrium, $0<\sigma_{priv}<1$, exists only if condition (12) holds with equality.

The intuition for the incentive effects of private ownership on innovation can be obtained from the incentive compatibility condition (12). Using Hirshleifer's (1971) terminology, we call v_2-v_1 the technological benefit of innovation. It is the expected fundamental value of innovation, v_2, minus its opportunity cost, v_1. The technological benefit can be positive or negative. $p(1-\delta)T\left(\sigma_{priv}\right)$ is the pecuniary benefit of innovation. It represents the net expected gain to the insider from the option to trade on the basis of private information. Unlike the technological benefit, the pecuniary benefit is always positive:

$$p(1-\delta)T\left(\sigma_{priv}\right)=(1-\delta p)T\left(\sigma_{priv}\right)-(1-p)T\left(\sigma_{priv}\right)>0. \tag{13}$$

Because the innovative project has a higher probability of failure than does the conventional project, the expected value of the option to exit early is higher under innovation, $(1-\delta p)T\left(\sigma_{priv}\right)>(1-p)T\left(\sigma_{priv}\right)$.

The value of the option to exit early $T\left(\sigma_{priv}\right)$ reflects the fact that the private ownership structure displays a high degree of tolerance for failure. Tolerance for failure has been shown to be a key feature of optimal incentive schemes for innovation (Manso 2011). Here, in contrast, the incentive to innovate is given by the ownership structure itself. The key insight of our model is that tolerance for failure is more valuable for innovation because the option to exit early is exercised more often if exploration is chosen. To emphasize the underlying mechanism, we refer to the pecuniary benefit, $p(1-\delta)T\left(\sigma_{priv}\right)$, as the tolerance-for-failure effect.

The option to exit early pushes the insider toward choosing the innovative project. If innovation is efficient from a technological perspective ($v_2-v_1\geq 0$), this extra incentive for innovation is not necessary; the incentive compatibility condition is not binding. The case of negative technological benefits ($v_2-v_1<0$) is more surprising. In this case, innovation is inefficient. We would then have $\sigma^*_{priv}=0$ without the tolerance-for-failure effect. However, because of the tolerance-for-failure effect, we can have $\sigma^*_{priv} > 0$ or even $\sigma^*_{priv} = 1$. Innovation may be chosen with certainty, despite being inefficient. If the tolerance-for-failure effect is larger than the technological benefit of innovation, the private ownership structure inefficiently encourages innovation.

The next proposition characterizes the equilibrium value of σ_{priv} under all possible pure strategy and mixed strategy equilibria. In particular, we show that there is a unique σ^*_{priv} for a given set of parameters (p,δ,θ,μ,e). The proposition follows from the incentive compatibility condition (12) and the properties of $T\left(\sigma_{priv}\right)$.

Proposition 1. For each set of parameters (p,δ,θ,μ,e), there exists a unique equilibrium probability of exploration for the private ownership case, $\sigma^*_{priv} \in [0,1]$, such that

1. if $v_2 \geq v_1$, then $\sigma^*_{priv} = 1$ (exploration is certain if innovation is efficient);
2. if $v_2 < v_1$, then

$$\sigma^*_{priv} = \begin{cases} 1, & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1), \\ \sigma \text{ is such that } T(\sigma) = \frac{v_1 - v_2}{p(1-\delta)}, & \text{if } T(1) < \frac{v_1 - v_2}{p(1-\delta)} < T(0), \\ 0, & \text{if } T(0) \leq \frac{v_1 - v_2}{p(1-\delta)}, \end{cases} \tag{14}$$

where $T\left(\sigma_{priv}\right) \equiv V_{priv}\left(\sigma_{priv},1\right) - p$.

Figure 3 shows the three possible cases if $v_2 < v_1$. The horizontal dashed lines represent different values for $\frac{v_1 - v_2}{p(1-\delta)}$. Consider, e.g., decreasing $v_1 - v_2$ and at the same time keeping $p(1-\delta)$ fixed (this can be achieved by increasing θ). The R_1 line represents a case in which the difference $v_1 - v_2$ is large. In such a case, the (negative) technological benefit of innovation is large and dominates the tolerance-for-failure effect, which implies that the first-best action, $\sigma^*_{priv} = 0$, is chosen in equilibrium. The R_2 line represents an intermediate value of $v_1 - v_2$. In this case, there is a probability of innovation, $\sigma^*_{priv} \in (0,1)$, that makes the insider indifferent between projects 1 and 2. The technological benefit is exactly offset by the tolerance-for-failure effect. Thus, the equilibrium involves some inefficient amount of innovation. Figure 3 also shows that σ^*_{priv} increases if the probability of the shock, μ, increases. This is so because $T\left(\sigma_{priv}\right)$ increases with μ (Proposition 2 below proves this result). The R_3 line is a case in which $v_1 - v_2$ is positive but small so that the option to exit early is so valuable that the insider chooses the least profitable project in equilibrium, $\sigma^*_{priv} = 1$.

In sum, our model shows that the private ownership structure is biased toward innovation. This bias is welcome if $v_2 \geq v_1$ but may lead to inefficiencies if $v_1 > v_2$.

The effects of θ, δ, e, and μ on the intensity of innovation σ^*_{priv} are described in Proposition 2. If $v_2 - v_1 \geq 0$, then $\sigma^*_{priv} = 1$. In this case, small changes in the

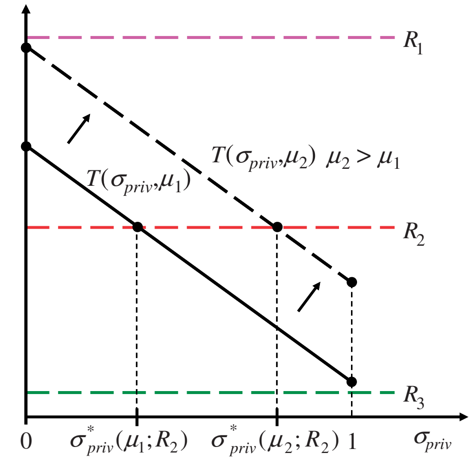

Figure 3
Equilibrium probability of innovation if exploitation is efficient

parameters do not affect the equilibrium. Therefore, Proposition 2 focuses on the case $\sigma^*_{priv} \in (0,1)$, for which $v_2 - v_1 < 0$. This is case R_2 in Figure 3.

Proposition 2. If $\sigma^*_{priv} \in (0,1)$, then $\frac{\partial \sigma^*_{priv}}{\partial \theta} > 0$, $\frac{\partial \sigma^*_{priv}}{\partial \delta} > 0$, $\frac{\partial \sigma^*_{priv}}{\partial \mu} > 0$, and $\frac{\partial \sigma^*_{priv}}{\partial e} < 0$.

Increases in θ and δ increase the net present value of innovation. Thus, the equilibrium intensity of innovation increases. This proposition also shows that the radicalism of an innovation has ambiguous effects on the likelihood of its adoption. If an innovative project becomes more radical because it is less likely to pay off, i.e., if δ decreases, then the firm is less likely to innovate. If an innovative project becomes more radical because its payoffs increase more dramatically in the case of success, i.e., if θ increases, then the firm is more likely to innovate.

An increase in μ helps the insider disguise a trade after $x_1 = 0$ as a sale motivated by a liquidity shock. As a result, innovation becomes more attractive, and in equilibrium there is more innovation. An increase in e, however, means that the insider can more easily find a private buyer in the case of a liquidity shock. A public sale then becomes less likely to be caused by a liquidity shock. Thus, the IPO share price falls after an increase in e. Such an effect attenuates the tolerance-for-failure effect, which then reduces the intensity of innovation.

3.3 The value of being private

We now calculate the expected value of the firm to the insider at $t = 0$, immediately after raising capital from private investors to pay for the initial investment cost I. Let α_{priv} be the fraction of shares that the insider retains after raising capital. Let $u_{priv} \equiv \sigma^*_{priv} u_{priv,2} + (1-\sigma^*_{priv}) u_{priv,1}$ denote the expected value of each share retained by the insider. We have the following lemma.

Lemma 3. For any equilibrium value of σ^*_{priv}, we have that $u_{priv} = \sigma^*_{priv} v_2 + (1-\sigma^*_{priv}) v_1$.

The intuition behind this lemma is as follows. Although the insider sells strategically at date 1 to exploit his informational advantage, share prices at date 1 must adjust until investors make zero profits on average. Whatever the insider gains by trading strategically is perfectly compensated in expectation by the loss that occurs when he is forced to liquidate his shares after a success. Thus, at date 0, he expects, on average, zero profits from future trading.

Because we assumed that private investors may suffer a liquidity shock that is perfectly correlated with that of the insider, private investors also value shares at u_{priv}. Assuming as before that the insider has full bargaining power with respect to investors, the insider can sell each share for u_{priv}. To cover the investment cost, the revenue from selling shares must satisfy $\left(1-\alpha_{priv}\right) c_{priv} u_{priv} \geq I$. Because of the trading costs implied by $c_{priv} < 1$, the insider will sell the minimum number of shares necessary for the investment. That is, α_{priv} is such that

$$\left(1-\alpha_{priv}\right) c_{priv} u_{priv} = I. \tag{15}$$

To avoid uninteresting cases in which the investment can never be financed, let $I \in (0,\ c_{priv} \min\{v_1,\ v_2\})$. That is, the firm's cost of capital is sufficiently low, and funds for investment can always be raised. Using Lemma 3, the insider's stake in equilibrium is

$$\alpha^*_{priv} = 1 - \frac{I}{c_{priv}\left[\sigma^*_{priv} v_2 + (1-\sigma^*_{priv}) v_1\right]}. \tag{16}$$

We can thus express the value of the firm to the insider under private ownership as

$$W_{priv} \equiv \alpha^*_{priv} u_{priv} = \sigma^*_{priv} v_2 + (1-\sigma^*_{priv}) v_1 - \frac{I}{c_{priv}}. \tag{17}$$

The first two terms on the right-hand side represent the expected outcome from the project decision, and the third term is the initial investment cost, adjusted for the cost of raising private capital. One reason that W_{priv} differs from its first-best counterpart—the value of the firm in a frictionless economy—is because raising funds for investing is costly, $c_{priv} < 1$. Moreover, a surprising result is that W_{priv} may also differ from its first-best counterpart because the equilibrium level of innovation, σ^*_{priv}, may be excessive compared with the first best. That is, we can have $\sigma^*_{priv} > 0$ even though $v_1 > v_2$. The intuition here is the same as in Hirshleifer (1971). That is, an agent may innovate too much to create opportunities for trading. The opposite problem never occurs; under private ownership, there is never too little innovation in equilibrium.

4. Public Ownership

Now consider the case of public ownership. In this case, the insider pays for the investment cost, I, by selling a fraction $1-\alpha_{pub}$ of the shares to the public market. As in the case of private ownership, the insider sells the remaining shares at date 1 if there is a liquidity shock. As before, if sophisticated private buyers are available, which occurs with probability e, the insider may prefer to sell shares to them. The difference between the public and private cases is the transparency of earnings. In the case of public ownership, the earnings, x_1, can be observed by all investors.

4.1 Selling behavior at date 1

The steps to analyze the equilibrium are similar to those in the case of private ownership. In what follows, we denote the probability that the insider chooses project 2 by $\sigma_{pub} \in [0,1]$.

Earnings transparency means that the market always knows whether the firm has experienced a failure, $x_1=0$. The market also knows that project 1 is always chosen after $x_1=0$. Therefore, although the market does not know which project was chosen at date 0, this lack of knowledge is not relevant for computing the value of the firm conditional on $x_1=0$. Regardless of the project chosen, the expected market value of the firm after $x_1=0$ is p because there is no information asymmetry between the insider and the market. Thus, shares are always fairly valued if $x_1=0$ and the insider gains nothing by selling shares. We can assume that the insider either sells or retains his shares if $x_1=0$. The equilibrium payoffs are not affected by this choice.

The insider may, however, choose to sell shares to the market after a success, $x_1=1$. Although the market knows that $x_1=1$, the market does not know which project was chosen at date 0. If project 1 was chosen, the expected value of the firm is $1+p$. If project 2 was chosen, the expected value of the firm is $1+\theta p$. Thus, the insider is better off if the market believes that project 2 was initially chosen. This creates a value-relevant information asymmetry.

The next lemma characterizes the behavior of an insider without liquidity needs after $x_1=1$.

Lemma 4. In the public ownership case, after observing a success, $x_1=1$, an insider without liquidity needs

1. never sells shares to the market if the innovative project has been chosen;
2. weakly prefers to sell shares to the market if the conventional project has been chosen.

According to part 1 of Lemma 4, the insider never sells to the market voluntarily at date 1 after exploration. The intuition is that, if project 2 is chosen, the firm is sold with a discount after $x_1=1$ because the market can never be certain that project 2 was chosen.

According to part 2 of Lemma 4, the insider sells to the market with probability 1 if the insider chooses the conventional project and is successful (to simplify the exposition, we assume that the insider sells in the case of indifference). Selling after $x_1 = 1$ if the insider chooses project 1 is always profitable as long as the market assigns a strictly positive probability to project 2.

It is instructive to compare this case to the private ownership case. Under private ownership, the insider never voluntarily sells to the market after a success. The reason for the difference in behavior is that outsiders can observe successes in the case of a public firm but not in the case of a private firm. In the private case, a firm may have had a success, but the market always assigns a positive probability to failure. As a result, selling to the market after a success is never optimal. In the public case, the market can observe successes but still cannot observe which project was chosen. Thus, under public ownership, it is optimal to sell after a success if the conventional project was chosen.

Lemma 4 implies that, if there was no liquidity shock, trading after $x_1 = 1$ would reveal the choice of project. Liquidity shocks allow insiders who choose project 1 to trade after $x_1 = 1$ without revealing the choice of project. In equilibrium, late-consumer insiders who have chosen project 1 pool with early-consumer insiders.

In equilibrium, the market must have correct beliefs and thus must assign probability σ_{pub} to the likelihood of project 2 being chosen. If the market observes a success and the insider sells shares, the market assigns probability s to project 2 being chosen. The difference between σ_{pub} and s is that σ_{pub} is the unconditional probability of choosing project 2, whereas s is the probability of project 2 being chosen given that the insider sells shares and the market observes $x_1 = 1$,

$$s \equiv \Pr(\mathit{Project}\ 2 \mid \mathit{Sale},\ x_1 = 1) = \frac{\Pr(\mathit{Sale}, x_1 = 1 \mid \mathit{Project}\ 2)\Pr(\mathit{Project}\ 2)}{\Pr(\mathit{Sale}, x_1 = 1)}. \quad (18)$$

The values of the probabilities are as follows. From Lemma 4, the probability of selling and $x_1 = 1$ is

$$\Pr(\mathit{Sale}, x_1 = 1) = \left(1 - \sigma_{pub}\right) p + \sigma_{pub}\mu(1-e)\delta p, \quad (19)$$

and the probability of selling and $x_1 = 1$ conditional on project 2 is

$$\Pr(\mathit{Sale}, x_1 = 1 \mid \mathit{Project}\ 2) = \mu(1-e)\delta p. \quad (20)$$

Finally, the unconditional probability of project 2 is σ_{pub}. Therefore, equilibrium beliefs must be

$$s(\sigma_{pub}) = \frac{\sigma_{pub}\mu(1-e)\delta}{\left(1-\sigma_{pub}\right) + \sigma_{pub}\mu(1-e)\delta}. \quad (21)$$

Given such beliefs, the market value of shares sold in public markets at $t = 1$ after a success is

$$V_{pub}\left(\sigma_{pub}\right) = 1 + s\left(\sigma_{pub}\right)\theta p + \left[1 - s\left(\sigma_{pub}\right)\right] p. \quad (22)$$

4.2 Project choice at date 0

We determine which project is chosen at date zero by first calculating the expected payoffs of projects 1 and 2 for the insider. The expected value of one share if the insider chooses project 1 is

$$u_{pub,1} = pV_{pub}\left(\sigma_{pub}\right) + (1-p)\,p. \tag{23}$$

If the insider chooses project 1, the probability of success is p. In the case of a success, the insider sells to the market and obtains $V_{pub}\left(\sigma_{pub}\right)$. If there is a failure, the market value of the firm becomes p, because the best project to choose at date 1 is project 1, again with probability p of success.

The expected gain per share from choosing project 2 is

$$u_{pub,2} = \delta p\left[\mu(1-e)V_{pub}\left(\sigma_{pub}\right) + (1-\mu+\mu e)(1+\theta p)\right] + (1-\delta p)\,p. \tag{24}$$

At date 1, the probability of success is δp. In the case of a success, the insider only sells to the market if he has liquidity needs, which occurs with probability $\mu(1-e)$. Without liquidity needs, the insider retains the shares until date 2 and continues with project 2, now with a probability of success equal to θp. If $x_1 = 0$, which occurs with probability $(1-\delta p)$, the insider obtains p, regardless of whether the insider retains the shares or sells them.

The next proposition fully characterizes the equilibrium σ^*_{pub} for all mixed strategy and pure strategy equilibria. For a given set of parameters, the equilibrium σ^*_{pub} is unique.

Proposition 3. For each set of parameters $(p, \delta, \theta, \mu, e)$, there exists a unique equilibrium probability of exploration for the public ownership case, $\sigma^*_{pub} \in [0,1)$, given by

$$\sigma^*_{pub} = \frac{s^*}{\mu(1-e)\delta + s^*[1-\delta\mu(1-e)]}, \tag{25}$$

where

$$s^* = \max\left\{\frac{v_2 - v_1 - \delta\mu(1-e)p^2(\theta-1)}{p^2(\theta-1)[1-\delta\mu(1-e)]}, 0\right\}. \tag{26}$$

Moreover, σ^*_{pub} is such that

1. if $v_1 \geq v_2 - \delta\mu(1-e)p^2(\theta-1)$, then $\sigma^*_{pub} = 0$ (in particular, exploitation is certain if $v_1 > v_2$);
2. if $v_1 < v_2 - \delta\mu(1-e)p^2(\theta-1)$, then $\sigma^*_{pub} \in (0,1)$.

Proposition 3 shows that an equilibrium with full innovation, $\sigma^*_{pub} = 1$, is never possible. If the market expects exploration with probability 1, then choosing exploitation becomes a dominant strategy. By choosing project 1, the insider increases the probability of success and, if successful, makes a profit by selling shares at date 1.

The proposition also shows that under public ownership the insider chooses project 1 if $v_1 > v_2$. This contrasts with the case of private ownership, in which the insider may choose the innovative project even if the conventional project has a higher expected return. However, if $v_2 > v_1$, the insider never chooses to explore with probability 1 under public ownership. In fact, the insider may choose project 1 with probability 1 even though $v_2 > v_1$. These results show that public ownership creates a bias against innovation. However, public ownership always induces the efficient project choice if $v_1 > v_2$.

Proposition 4 shows the effects of δ, θ, μ, and e on σ^*_{pub}. If $v_1 \geq v_2$, then $\sigma^*_{pub} = 0$. Therefore, the proposition focuses on the case $\sigma^*_{pub} \in (0,1)$, for which $v_2 > v_1$.

Proposition 4. If $\sigma^*_{pub} \in (0,1)$, then $\frac{\partial \sigma^*_{pub}}{\partial \delta} > 0$, $\frac{\partial \sigma^*_{pub}}{\partial \theta} > 0$, $\frac{\partial \sigma^*_{pub}}{\partial \mu} < 0$, and $\frac{\partial \sigma^*_{pub}}{\partial e} > 0$.

Parameter δ increases the probability of success at $t = 1$, and θ increases the probability of success at $t = 2$, given that the project was successful at $t = 1$. Because the innovative project becomes more valuable as θ or δ increases, an increase in one of these parameters makes innovation more likely, i.e., σ^*_{pub} increases. As in the case of private ownership, the radicalism of an innovation has ambiguous effects on project choice.

Innovation becomes less likely after an increase in the probability of a liquidity shock, $\partial \sigma^*_{pub} / \partial \mu < 0$. Recall that Proposition 4 only considers the case in which $v_2 > v_1$, which implies $\sigma^*_{pub} \in (0,1)$. Therefore, $\sigma^*_{pub} < 1$ means that the insider chooses the conventional project with positive probability, although the conventional project is inefficient. The insider behaves in this way because the probability of success at $t = 1$ under the conventional project is higher than the probability of success under the innovative project, $p > \delta p$. If liquidity shocks occur frequently, the insider can more easily hide the choice of project 1. Frequent liquidity shocks make the market more likely to believe that the insider is selling because of a liquidity shock and not because of a success under exploitation. Thus, as it becomes easier to hide the choice of project 1, the incentives to choose innovation are reduced.[10]

Unlike the case of private ownership, under public ownership innovation becomes more likely as finding informed private buyers becomes easier ($\partial \sigma^*_{pub} / \partial e > 0$). This result suggests that a well-developed buyout market is beneficial for innovation in public firms. The intuition is as follows. Insiders with liquidity needs at date 1 may have to sell undervalued shares if they innovate and are successful. This possibility makes the innovative project

[10] We can also interpret an increase in μ to be an improvement in stock liquidity, because a higher μ reduces the price impact of insider sales. Under this interpretation, improvements in stock liquidity reduce the probability of innovation.

less attractive. If sophisticated private buyers are willing to buy the insider out after a success, then the incentives for innovation are restored.

4.3 The value of being public

We now compute the expected value of the firm to the insider at $t=0$, immediately after raising capital from public investors to pay for the initial investment cost I. Let α_{pub} be the fraction of shares that the insider retains after raising capital. Let $u_{pub} \equiv \sigma^*_{pub} u_{pub,2} + (1-\sigma^*_{pub}) u_{pub,1}$ denote the expected value of each share retained by the insider. We have the following lemma.

Lemma 5. For any equilibrium value of σ^*_{pub}, we have that $u_{pub} = \sigma^*_{pub} v_2 + (1-\sigma^*_{pub}) v_1$.

As in the private case, share prices at date 1 adjust until investors make zero profits on average.

To cover the investment cost, the revenue from selling shares must satisfy $\left(1-\alpha_{pub}\right) c_{pub} u_{pub} \geq I$. Because of the trading costs implied by $c_{pub} < 1$, the insider will sell the minimum number of shares necessary for the investment. That is, α_{pub} is such that

$$\left(1-\alpha_{pub}\right) c_{pub} u_{pub} = I. \tag{27}$$

Substituting u_{pub} from Lemma 5, the insider's stake in equilibrium is

$$\alpha^*_{pub} = 1 - \frac{I}{c_{pub}\left[\sigma^*_{pub} v_2 + (1-\sigma^*_{pub}) v_1\right]}. \tag{28}$$

We can thus express the value of the firm to the insider under public ownership as

$$W_{pub} \equiv \alpha^*_{pub} u_{pub} = \sigma^*_{pub} v_2 + (1-\sigma^*_{pub}) v_1 - \frac{I}{c_{pub}}. \tag{29}$$

The ex ante value of the public firm differs from the value of the private firm for two reasons. First, the costs of public and private capital may differ ($c_{priv} \neq c_{pub}$). Second, the intensities of innovation under public and private ownership may differ ($\sigma^*_{priv} \neq \sigma^*_{pub}$).

5. The Decision to Go Public or Private

We now complete the characterization of the equilibrium by considering the decision $\varphi \in \{priv, pub\}$. The decision to go private or public at date 0 depends only on the values of W_{priv} and W_{pub}. If $W_{priv} > W_{pub}$, the insider chooses to go private. If $W_{pub} > W_{priv}$, the insider chooses to go public.

To simplify the notation, we define the relative cost advantage of public offerings compared to private offerings as

$$a \equiv \frac{1}{c_{priv}} - \frac{1}{c_{pub}} = \frac{c_{pub} - c_{priv}}{c_{priv} c_{pub}}. \tag{30}$$

If public offerings are cheaper than private offerings ($c_{pub} > c_{priv}$), then $a > 0$.

Using (17) and (29), we obtain

$$W_{priv} - W_{pub} = \left(\sigma^*_{priv} - \sigma^*_{pub}\right) v_2 + (\sigma^*_{pub} - \sigma^*_{priv}) v_1 - \frac{I}{c_{priv}} + \frac{I}{c_{pub}}, \tag{31}$$

which proves Proposition 5.

Proposition 5. For a given set of parameters $\left(p, \delta, \theta, \mu, e, c_{priv}, c_{pub}, I\right)$, the private ownership structure is (weakly) preferable to the public ownership structure if and only if

$$\left(\sigma^*_{priv} - \sigma^*_{pub}\right)(v_2 - v_1) \geq aI, \tag{32}$$

where σ^*_{priv} and σ^*_{pub} are given by Propositions 1 and 3.

From Proposition 5, we see that the choice between public and private structures is driven by three key forces: (1) the difference in innovation intensity between private and public structures, $\sigma^*_{priv} - \sigma^*_{pub}$; (2) the relative efficiency of innovative projects, $v_2 - v_1$; and (3) the relative capital cost advantage of public offerings, aI.

We use the results from the two previous sections to prove the following proposition.

Proposition 6. For any $(p, \delta, \theta, \mu, e)$, $\sigma^*_{priv} - \sigma^*_{pub} \geq 0$. That is, the intensity of innovation under private ownership is at least as large as the intensity of innovation under public ownership.

This result follows from the fact that the private structure sometimes creates a bias toward innovation (tolerance for failure), whereas the public structure sometimes creates a bias against innovation (short-termism). These biases distort innovation away from its first-best level but in different directions. For a given set of parameters, one of the two following cases must hold: either there are no biases or at least one structure has a bias that distorts innovation. If biases are not present, then both structures lead to the same intensity of innovation. If at least one of these biases is operational, then there is either too much innovation under the private structure or too little innovation under the public structure. In either case, we have $\sigma^*_{priv} \geq \sigma^*_{pub}$.

This result has important empirical consequences. It formally shows that private firms are more innovative than are public firms, holding all else constant.

This result is also important because it implies that, apart from differences in the cost of capital, going private is more attractive than going public if innovation is efficient ($v_2 - v_1 > 0$). In fact, if we shut down the effect of the cost of capital by setting $a=0$, innovation efficiency is the only consideration in the choice of ownership structure, as shown in the next corollary.

Corollary 1. Let $a=0$ so that the private ownership structure is preferable to the public ownership structure if and only if $\left(\sigma_{priv} - \sigma_{pub}\right)(v_2 - v_1) \geq 0$ for a given set of parameters $(p, \delta, \theta, \mu, e)$. Then,

1. if innovation is efficient ($v_2 > v_1$), the insider chooses to go private;
2. if the conventional project is efficient ($v_1 > v_2$), the insider strictly prefers to go public if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ and is indifferent between going public or private if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$;
3. if both projects are equivalent ($v_2 = v_1$), the insider is indifferent between going public or private.

If $v_2 > v_1$, then Propositions 1 and 3 imply $\sigma_{priv}^* = 1$ and $\sigma_{pub}^* < 1$. Therefore, the condition for going private is satisfied. If $v_1 > v_2$, then Proposition 3 implies $\sigma_{pub}^* = 0$. The corollary above implies that the insider will either choose to go public or may choose to go private if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$. In the latter case, Proposition 1 implies that $\sigma_{priv}^* = 0$. Thus, if the insider optimally chooses the ownership structure, the first-best outcome is always achieved. The innovative project is chosen with probability 1 if $v_2 > v_1$, and the conventional project is chosen with probability 1 if $v_1 > v_2$.

6. Illiquid Private Securities

As discussed in Subsection 2.5, private securities are probably more difficult to sell than public securities. To capture the relative illiquidity of private securities, we now assume that, if the firm is private, the insider only pockets $k < 1$ for each dollar of shares sold at date 1. Because the algebra is substantially more complex in this case, without loss of generality, we set $e=0$.

The analysis of the public case is unchanged. Most of the analysis of the private case also remains unchanged. In particular, Lemma 1 still holds. Therefore, an insider without liquidity needs never sells shares to the market after $x_1 = 1$. However, with $k < 1$, the necessary condition for selling shares to the market after a failure changes to

$$k V_{priv}\left(\sigma_{priv}, b\right) \geq p. \tag{33}$$

Because $V_{priv}\left(\sigma_{priv}, b\right) > p$, we have $k V_{priv}\left(\sigma_{priv}, b\right) > p$ for k sufficiently close to 1. As a result, the insider sells shares with probability 1 after a failure if the market for private securities is liquid enough. As k approaches 1, we eventually get $b=1$. On the other hand, if the market at date 1 is very illiquid (k close to zero), then a late-consumer insider never sells, $b=0$. For intermediate values of k, the equilibrium is in strictly mixed strategies, with $b\in(0,1)$ and b increasing in k. The next lemma formalizes these results.

Lemma 6. In the private ownership case with $k\in(0,1]$ and $e=0$, a late-consumer insider sells shares with equilibrium probability $b(\sigma_{priv})$ at date 1 after observing $x_1=0$, where

$$b(\sigma_{priv})=\begin{cases} 1, & \text{if } k\geq k_1,\\ \mu\frac{k[v_1+\sigma_{priv}(v_2-v_1)]-p}{(1-k)(1-\mu)[1-p+\sigma_{priv}p(1-\delta)]p}, & \text{if } k_2<k<k_1,\\ 0, & \text{if } k\leq k_2,\end{cases} \tag{34}$$

$$k_1\equiv\frac{\mu p+(1-\mu)[1-p+\sigma_{priv}p(1-\delta)]p}{\mu[v_1+\sigma_{priv}(v_2-v_1)]+(1-\mu)[1-p+\sigma_{priv}p(1-\delta)]p},$$

$$k_2\equiv\frac{p}{v_1+\sigma_{priv}(v_2-v_1)}.$$

The threshold values k_1 and k_2 define three regions for the behavior of the insider, as shown in Figure 4. In Region 3, the insider never sells shares. In Region 2, the insider plays a strictly mixed strategy. If the market for private securities is liquid enough, $k\geq k_1$, as shown in Region 1, then the insider sells after a failure with probability 1.

Figure 4 also illustrates the effect of the liquidity shock on the insider's selling behavior. If μ increases, k_1 decreases. So, a late-consumer insider sells shares with probability 1 for a larger set of values of k. Intuitively, if μ increases, it becomes easier for the insider to disguise a failure behind a liquidity shock.

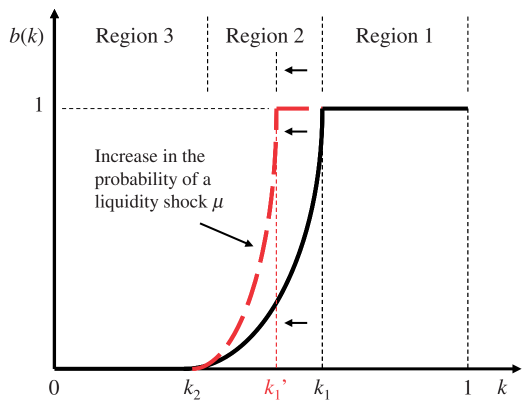

Figure 4
$b(k)$: probability of a late-consumer insider selling shares after $x_1=0$

We redefine T (the intrinsic value of the option to exit early for a late-consumer insider) as

$$T\left(\sigma_{priv}\right) \equiv \max\left\{k V_{priv}\left(\sigma_{priv}, b\left(\sigma_{priv}\right)\right) - p, 0\right\}. \tag{35}$$

This option has zero value if the underlying value $k V_{priv}\left(\sigma_{priv}, b\left(\sigma_{priv}\right)\right)$ is low, which may happen either because the market for private securities is very illiquid (low k) or because the market is "cold," i.e., the market believes that $x_1 = 0$ is very likely if an insider sells shares (that is, μ is low). In Figure 4, $T\left(\sigma_{priv}\right)$ is strictly positive in Region 1, and zero in Regions 2 and 3.

The next proposition generalizes our results in Proposition 1 to the case in which $k \leq 1$.

Proposition 7. For each set of parameters $(p, \delta, \theta, \mu, k)$ and $e = 0$, there exists an equilibrium probability of exploration for the private ownership case, $\sigma^*_{priv} \in [0, 1]$, given by

1. if $v_2 > v_1$, then $\sigma^*_{priv} = 1$ (exploration is certain if innovation is efficient);
2. if $v_2 < v_1$, then

$$\sigma^*_{priv} = \begin{cases} 1, & \text{if } \frac{v_1 - v_2}{p(1-\delta)} \leq T(1), \\ \sigma \text{ such that } T(\sigma) = \frac{v_1 - v_2}{p(1-\delta)}, & \text{if } T(1) < \frac{v_1 - v_2}{p(1-\delta)} < T(0), \\ 0, & \text{if } T(0) \leq \frac{v_1 - v_2}{p(1-\delta)}, \end{cases} \tag{36}$$

   where $T\left(\sigma_{priv}\right) \equiv \max\left\{k V_{priv}\left(\sigma_{priv}, b\left(\sigma_{priv}\right)\right) - p, 0\right\}$;
3. if $v_2 = v_1$, then $\sigma^*_{priv} \in \arg\min_{\sigma \in [0,1]} T(\sigma)$.

The private ownership innovation bias is still present in this case. We can have $\sigma^*_{priv} = 1$ with $v_1 > v_2$ and $k < 1$. That is, the insider may choose the innovative project with certainty even though the conventional project is the efficient choice *and* the market for private securities is illiquid.

Starting from an equilibrium with $\sigma^*_{priv} = 1$ and $v_1 > v_2$, as k falls the insider eventually chooses a mixed strategy between the innovative and the conventional project ($0 < \sigma^*_{priv} < 1$). As k continues to decrease, the insider eventually selects the conventional project with certainty ($\sigma^*_{priv} = 0$). If $v_1 = v_2$, the insider may be indifferent among several strategies and we can have multiple probabilities σ^*_{priv} in equilibrium. If $v_2 > v_1$, the insider always selects the innovative project with certainty ($\sigma^*_{priv} = 1$) for any k.

As before, the insider chooses to go private or public to maximize the ex ante value of the firm. The value of W_{pub} is unchanged, $W_{pub} = u_{pub} - \frac{I}{c_{pub}}$. In the private case, on the other hand, the value of the firm must now take into account the discount implied by $k < 1$.

Lemma 7. For each set of parameters $(p, \delta, \theta, \mu, k)$ and $e = 0$, the ex ante value of each share to the insider under private ownership is given by

$$W_{priv} = \sigma^*_{priv} v_2 + \left(1 - \sigma^*_{priv}\right) v_1 - \frac{I}{c_{priv}} - L(k), \tag{37}$$

where

$$L(k)=\begin{cases}(1-k)\left\{\mu\left[\sigma^*_{priv}v_2+\left(1-\sigma^*_{priv}\right)v_1\right]+(1-\mu)\left[1-p+\sigma^*_{priv}p(1-\delta)\right]p\right\} & \text{if } k\geq k_1\\ \mu\left[\sigma^*_{priv}v_2+\left(1-\sigma^*_{priv}\right)v_1-p\right] & \text{if } k\in(k_2,k_1)\\ (1-k)\mu\left[\sigma^*_{priv}v_2+\left(1-\sigma^*_{priv}\right)v_1\right] & \text{if } k\leq k_2\end{cases}.$$

The new term $L(k)$ represents the expected cost of illiquidity associated with the sale of shares at date 1. This cost is another source of inefficiency associated with private ownership. Selling shares is costly because private securities are illiquid. Simple inspection reveals that $L(k)>0$ (recall that $\mu>0$), unless $k=1$, in which case $L(1)=0$.

The illiquidity cost $L(k)$ affects the choice between public and private ownership structures. Now the private ownership structure is preferable to the public ownership structure if and only if

$$\left(\sigma^*_{priv}-\sigma^*_{pub}\right)(v_2-v_1)\geq aI+L(k), \tag{38}$$

where a is as defined in Equation (30). If $c_{priv}=c_{pub}$ $(a=0)$, we have the following proposition.

Proposition 8. Let $k\leq 1$. If $c_{priv}=c_{pub}$, then

1. if $v_1\geq v_2$, the insider chooses the public structure;
2. if $v_2>v_1$, there is a unique $k^*\in(0,1)$ such that the insider chooses the public structure if $k<k^*$ and chooses the private structure if $k\geq k^*$.

If private securities are less liquid than public securities ($k<1$), the insider faces a trade-off if $v_2>v_1$. The trade-off shows up because the private structure provides appropriate incentives to innovate but imposes illiquidity costs. If the illiquidity costs are large (k small), the insider prefers the public structure even though it leads to less innovation. If we think of k as representing the costs of selling some shares of an originally private company, such as the IPO costs, our model suggests that innovation is fostered by the development of IPO markets (i.e., an increase in k).

7. Model Implications

Our model has a number of new empirical implications. Here, we briefly discuss some of the key predictions and the existing empirical evidence. This section also serves as a summary of the main results in the article.

Prediction 1. Firms undertake more innovative projects after going private.

Prediction 2. Firms undertake fewer innovative projects after going public.

Both predictions follow from Proposition 6. As discussed in the introduction, the evidence in Lerner, Sorensen, and Strömberg (2011) is consistent with (but not a direct test of) Prediction 1. Recent work by Bernstein (2011) aims at explicitly testing Prediction 2. In a data set of innovative firms that filed for an IPO, he compares the innovation performance of firms that successfully completed their IPOs with those that decided to withdraw the IPO for exogenous reasons. Consistent with Prediction 2, he finds that firms that proceed with their IPOs experience a decline in patent citations and other innovation measures.

Prediction 3. Firms should go or stay private if innovative projects have higher net present values than do conventional projects. Similarly, firms should go or stay public if conventional projects have higher net present values than do innovative projects.

This is a direct consequence of Corollary 1 (i.e., it is also a corollary of Predictions 1 and 2). Holding all else constant, the relative profitability of innovative versus conventional projects should affect the decision to go public or private. We are unaware of empirical work directly testing this prediction.

Prediction 4. A reduction in the costs of an IPO fosters innovation.

This prediction follows from Proposition 8. An IPO becomes less costly as k increases. If $k \geq k^*$ and innovation is efficient, firms optimally choose the private structure, which leads to more innovation. An empirical consequence of this prediction is that countries with more developed IPO markets (high k) should have more innovative firms.

Prediction 5. An active buyout market fosters innovation in public firms but harms innovation in private firms.

This prediction follows from Propositions 2 and 4. In a more developed buyout market, sophisticated private equity investors (buyout and VC) are more easily available to provide liquidity to managers and entrepreneurs. In our model, this corresponds to an increase in e. From Proposition 2, an increase in e harms innovation in private firms. From Proposition 4, an increase in e fosters innovation in public firms.

Prediction 6. An increase in the degree of information asymmetry in IPOs fosters innovation in private firms.

This prediction follows from Proposition 2. In the case of a private firm, parameter μ can be seen as a proxy for an information asymmetry between insiders and outside investors. If $\mu = 0$, insiders cannot benefit from IPO timing, as IPO prices become fully informative about date 1 earnings.

If $\mu = 1$, IPO prices contain no information about earnings. Proposition 2 shows that innovation increases with μ. Intuitively, more asymmetric information makes the option to sell after a failure more valuable, which strengthens the tolerance-for-failure effect, thus fostering innovation in private firms.

Prediction 7. A decrease in stock liquidity fosters innovation in public firms.

This prediction follows from Proposition 4. In the case of a public firm, there is no asymmetry of information concerning x_1 at date 1. Parameter μ is proportional to the price impact of an insider trade. A small μ implies a large price decline if the insider sells. Thus, larger values of μ are associated with smaller price declines due to insider trading, which is equivalent to a more liquid market for the stock. Proposition 4 shows that an increase in liquidity (larger μ) hurts innovation in public firms. The evidence in Fang, Tian, and Tice (2010) supports this prediction. They find that exogenous increases in stock liquidity adversely affect innovation. Such an effect is stronger for firms in which managers are more likely to yield to pressure to maximize short-term stock prices, which is consistent with the mechanism behind Prediction 7.

Although most of the direct predictions of the model still need to be tested, there is some additional evidence in support of the forces underlying our model. Asker, Farre-Mensa, and Ljungqvist (2011) investigate the effects of public and private ownership on corporate investment. They find that public firms invest less than similar private firms and that firms reduce their investment levels after going public. They argue that their evidence is best explained by managerial short-termism, as in Stein (1989). In particular, they show that there are no significant differences in investment behavior when comparing private firms with public firms in which prices are less sensitive to accounting earnings. This evidence is consistent with our assumption that the key difference between private and public companies is the information contained in earnings. In our model, a public company with uninformative earnings would invest in the same way as a private company.

Evidence consistent with the tolerance-for-failure effect is provided by Acharya and Subramanian (2009), who empirically demonstrate that innovation is more prevalent in countries with debtor-friendly bankruptcy codes, and Acharya, Baghai, and Subramanian (2009), who show that more stringent labor laws lead to more innovation inside firms. Similarly, Chemmanur, Loutskina, and Tian (2011) show that firms generate more and better patents after the adoption of antitakeover provisions. They argue that antitakeover provisions make firms more tolerant of short-run failures and allow them to focus on long-run projects. Tian and Wang (2012) develop a measure of failure-tolerance for venture capitalists and show that IPO firms that are backed by failure-tolerant VCs are more innovative. Chemmanur, Loutskina, and Tian (2011) provide related evidence that VCs create value for their portfolio firms partly because they exhibit tolerance for failures, which spurs innovation.

8. Final Remarks

Our results suggest that public and private firms invest in fundamentally different ways. Private firms take more risks, invest more in new products and technologies, and pursue more radical innovations. Private firms are more likely to choose projects that are complex, difficult to describe, and untested. Organizational change is also more likely under private ownership. Mergers and acquisitions, divestitures, and changes in organizational structure and management practices are more easily motivated under private ownership.

Conversely, public firms choose more conventional projects. Their managers appear short-sighted; they care too much about current earnings. They find it difficult to pursue complex projects that the market does not appear to understand well. Public firms go private after adverse shocks, when it is clear that their business models are no longer working and there is a need for restructuring.

There are still many untested implications of our model. Our model predicts that cash-flow volatility should be higher in private firms. Private firms should be more profitable during technological revolutions, whereas public firms should be more valuable in mature but growing industries. Our model also has implications for the decision to go public or private. Firms are likely to go public after a technological breakthrough, i.e., when it makes sense to exploit a newly discovered technology. Firms are likely to go private after suffering permanent negative productivity shocks, i.e., when their existing technologies or business models become permanently unprofitable. Chemmanur, He, and Nandy (2010) find that firms go public at the peak of their productivity, and then performance declines after going public. This is consistent with firms going public only after perfecting a new technology; they become public in the "harvesting" period. Our model also explains why companies go private when performance is particularly poor.

Finally, we note that there are many directions in which the model can be extended. Our model emphasizes two important effects—short-termism and the lack of tolerance for failures—that make public firms ill-suited to pursue innovations. However, one could also argue, along the lines of Burkart, Gromb, and Panunzi (1997), that the "hands-off" approach of public shareholders is necessary to foster managerial initiative and may counteract the effects we emphasize here. This is a promising avenue for future theoretical and empirical explorations.

Appendix: Proofs

Lemma 1.

Proof. Let $b^F \equiv \Pr(Sale \mid x_1 = 0)$ denote the probability of selling shares to the market after a failure and $b^S \equiv \Pr(Sale \mid x_1 = 1)$ denote the probability of selling shares to the market after a success, both for the case of no liquidity needs. Let h denote the probability that the project failed, given that the insider sells shares to the market, $h \equiv \Pr(x_1 = 0 \mid Sale)$. To prove that $b^S = 0$, we first need to prove two preliminary results.

Result 1: $b^F \geq b^S$. Proof: Let V be the market value of shares at date 1. The insider sells shares at date 1 after a success with project 1 only if $V \geq 1+p$. Similarly, the insider sells shares at date 1 after a success with project 2 only if $V \geq 1+\theta p$. After a failure, the insider sells only if $V \geq p$, regardless of the project chosen. Thus, in any equilibrium such that $b^S > 0$, it must be that $V \geq \min\{1+p, 1+\theta p\} = 1+p$, which implies that $V > p$. In such a case, the insider must sell with probability 1 after a failure, i.e., $b^F = 1$. Therefore, the probability of selling to the market after a failure must be at least as large as the probability of selling after a success, $b^F \geq b^S$.

Intuitively, this result follows from the fact that the condition to sell in case of success is more stringent than the condition to sell in case of failure.

Result 2: $h \geq 1-p$. Proof: By definition, $\Pr(Sale) = b^F \Pr(x_1 = 0) + b^S (1 - \Pr(x_1 = 0))$. Result 1 implies that $\Pr(Sale) \leq b^F \Pr(x_1 = 0) + b^F (1 - \Pr(x_1 = 0)) = b^F$.

By Bayes's rule,

$$h = \frac{b^F \Pr(x_1 = 0)}{\Pr(Sale)}. \tag{A1}$$

Because $b^F \geq \Pr(Sale)$, $h \geq \Pr(x_1 = 0)$. The lowest possible value for $\Pr(x_1 = 0)$ occurs if the insider chooses project 1 with probability 1, in which case $\Pr(x_1 = 0) = 1-p$, proving that $h \geq 1-p$.

Now, to prove that $b^S = 0$, it suffices to show that $V < 1+p$ always (because $1+p < 1+\theta p$). Let s denote the probability that the insider has chosen the innovative project given that shares are sold to the market at date 1, $s \equiv \Pr(\textit{Project } 2 \mid \textit{Sale})$. In any equilibrium in which the market has rational beliefs, each share sold at date 1 must be valued at $V(s,h) \equiv hp + (1-h)[s(1+\theta p) + (1-s)(1+p)]$. Notice that $V(s,h)$ is increasing in s and decreasing in h. Result 2 implies that h cannot be lower than $1-p$; therefore, the upper bound for $V(s,h)$ is given by $\overline{V} \equiv V(1, 1-p) = (1-p)p + p(1+\theta p)$.

A necessary (but not sufficient) condition for the insider to sell shares to the market after a success is that the maximum possible value for V must be at least as large as the minimum possible value for the fundamental value of shares: $\overline{V} \geq \min\{1+\theta p, 1+p\} = 1+p$. Because $\overline{V}$ is increasing in θ, setting $\theta = 1/p$ (the highest possible value of θ) implies that this condition can expressed as $(1-p)p + 2p \geq 1+p$. It is straightforward to check that this condition never holds for any $p < 1$ (the case in which $p = 1$ is ruled out by assumption, as there would be no uncertainty). Thus, there is no combination of parameters and rational market beliefs h and s such that $V \geq 1+p$, which proves that $b^S = 0$. ■

Lemma 2.

Proof. For any given pair of market beliefs (σ_{priv}, b), an insider without liquidity needs sells with probability 1 after a failure if $V_{priv}(\sigma_{priv}, b) > p \Leftrightarrow m(\sigma_{priv}, b)[\sigma_{priv} v_2 + (1-\sigma_{priv}) v_1] + (1 - m(\sigma_{priv}, b)) p > p$. Because $\Pr(\textit{Liquidity needs}) = \mu(1-e) > 0$, from Equation (6) we have that $m(\sigma_{priv}, b) > 0$. Thus, $V_{priv}(\sigma_{priv}, b) > p$ holds for any (σ_{priv}, b) because $v_1 > p$ and $v_2 > p$. ■

Proposition 1.

Proof. The equilibrium value of σ_{priv} must satisfy the incentive compatibility (IC) constraints

$$\begin{array}{ll} v_2 - v_1 + p(1-\delta)T(1) \geq 0 & \text{if } \sigma_{priv} = 1 \text{ (project 2)}, \\ v_2 - v_1 + p(1-\delta)T(\sigma_{priv}) = 0 & \text{if } \sigma_{priv} \in (0,1) \text{ (mixed strategies)}, \\ v_2 - v_1 + p(1-\delta)T(0) \leq 0 & \text{if } \sigma_{priv} = 0 \text{ (project 1)}, \end{array} \tag{A2}$$

where $T(\sigma_{priv}) \equiv V_{priv}(\sigma_{priv}, 1) - p$. We have $T(\sigma_{priv}) > 0$ (see the proof of Lemma 2).

Case 1. If $v_2 - v_1 \geq 0$, then the IC condition for project 2, $v_2 - v_1 + p(1-\delta)T(1) \geq 0$, is trivially satisfied as $T(\sigma_{priv}) > 0$ for any σ_{priv}. On the other hand, there is no σ_{priv} such that the IC conditions for project 1 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 1$ is the only equilibrium.

Case 2. If $v_1 - v_2 > 0$, then

$$\frac{\partial T(\sigma_{priv})}{\partial \sigma_{priv}} = \frac{\partial m(\sigma_{priv}, 1)}{\partial \sigma_{priv}} \left[\sigma_{priv} v_2 + (1-\sigma_{priv}) v_1 - p\right] - (v_1 - v_2) m(\sigma_{priv}, 1) < 0, \tag{A3}$$

because $\frac{\partial m(\sigma_{priv}, 1)}{\partial \sigma_{priv}} < 0$ and $\sigma_{priv} v_2 + (1-\sigma_{priv}) v_1 - p > 0$. Therefore, the value for the option to exit is minimized at $\sigma_{priv} = 1$ and maximized at $\sigma_{priv} = 0$.

If $\frac{v_1 - v_2}{p(1-\delta)} \leq T(1)$, then the IC condition for project 2 is satisfied for any σ_{priv}, whereas there is no $\sigma_{priv} < 1$ such that the IC conditions for project 1 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 1$ is the only equilibrium.

If $T(1) < \frac{v_1 - v_2}{p(1-\delta)} < T(0)$, as $T(\sigma_{priv})$ is continuous and decreasing in $\sigma_{priv} \in (0,1)$, there exists a unique $\sigma^*_{priv} \in (0,1)$ such that $T(\sigma^*_{priv}) = \frac{v_1 - v_2}{p(1-\delta)}$. In this case, the IC condition for mixed strategies holds exactly at σ^*_{priv} and $u_{priv,1} = u_{priv,2}$.

If $T(0) \leq \frac{v_1 - v_2}{p(1-\delta)}$, then the IC condition for project 1 is satisfied for any σ_{priv}, whereas there is no $\sigma_{priv} > 0$ such that the IC conditions for project 2 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 0$ is the only equilibrium. ■

Proposition 2.

Proof. Suppose that $\sigma^*_{priv} \in (0,1)$. In this case, the IC condition implies that σ^*_{priv} is defined implicitly by $v_2 - v_1 + p(1-\delta)T(\sigma^*_{priv}) = 0$. Define $G(\sigma, \mu, \theta, \delta, e) = v_2 - v_1 + p(1-\delta)T(\sigma)$. Substituting $T(\sigma) = V_{priv}(\sigma, 1) - p$ implies $G(\sigma, \mu, \theta, \delta, e) = v_2 - v_1 + p(1-\delta)\{m(\sigma, 1)[\sigma v_2 + (1-\sigma) v_1 - p]\}$. Using the implicit function theorem, $\frac{\partial \sigma^*_{priv}}{\partial x} = -\left.\frac{\partial G/\partial x}{\partial G/\partial \sigma}\right|_{\sigma = \sigma^*_{priv}}$, where x is the parameter of interest and $\frac{\partial G}{\partial \sigma} < 0$, as $\frac{\partial G(\sigma,\mu,\theta,\delta,e)}{\partial \sigma} = p(1-\delta)\{\frac{\partial m}{\partial \sigma}[\sigma v_2 + (1-\sigma) v_1 - p] + (v_2 - v_1) m(\sigma, 1)\}$, $\frac{\partial m}{\partial \sigma} < 0$, and $v_2 - v_1 < 0$. We have $\frac{\partial G(\sigma^*_{priv},\mu,\theta,\delta,e)}{\partial \theta} = p(1-\delta)[p^2\delta + p(1-\delta) m(\sigma^*_{priv}, 1) p^2 \delta \sigma^*_{priv}] > 0$, which implies $\frac{\partial \sigma^*_{priv}}{\partial \theta} > 0$. Moreover, after some algebra, it can be shown that

$$\begin{aligned}
&\frac{\partial G(\sigma^*_{priv}, \mu, \theta, \delta, e)}{\partial \delta} \\
&\quad = p[1 + p(\theta - 1)] - pm(\sigma^*_{priv}, 1)[p + \sigma^*_{priv}(v_2 - v_1)] + \\
&\quad p(1-\delta)\left\{\frac{\partial m(\sigma^*_{priv}, 1)}{\partial \delta}[p + \sigma^*_{priv}(v_2 - v_1)] + \sigma^*_{priv} p[1 + p(\theta - 1)]\right\} > 0,
\end{aligned} \tag{A4}$$

which implies $\frac{\partial \sigma^*_{priv}}{\partial \delta} > 0$. For the effect of e on σ^*_{priv}, we have $\frac{\partial G(\sigma^*_{priv},\mu,\theta,\delta,e)}{\partial e} = p(1-\delta)\frac{\partial m}{\partial e}[p + \sigma^*_{priv}(v_2 - v_1)] < 0$, because $\frac{\partial m}{\partial e} < 0$, which implies $\frac{\partial \sigma^*_{priv}}{\partial e} < 0$. Similarly, for the effect of μ on σ^*_{priv}, we have $\frac{\partial G(\sigma^*_{priv},\mu,\theta,\delta,e)}{\partial \mu} = p(1-\delta)\frac{\partial m}{\partial \mu}[p + \sigma^*_{priv}(v_2 - v_1)] > 0$, because $\frac{\partial m}{\partial \mu} > 0$, and thus $\frac{\partial \sigma^*_{priv}}{\partial \mu} > 0$. ■

Lemma 3.

Proof. The proof follows by algebra. To save on notation, we use σ instead of σ^*_{priv}. We have $u_{priv} \equiv \sigma u_{priv,2} + (1-\sigma)u_{priv,1}$; thus, using the expressions for $u_{priv,1}$ and $u_{priv,2}$, we get

$$u_{priv} = \mu(1-e)V_{priv}(\sigma,1) + (1-\mu+\mu e)\{[\sigma(1-\delta p)+(1-\sigma)(1-p)]$$
$$\times V_{priv}(\sigma,1) + \sigma\delta p(1+\theta p) + (1-\sigma)p(1+p)\}.$$

Substituting (8) into (9), we get

$$V_{priv}(\sigma,1)$$
$$= \frac{\mu(1-e)[\sigma v_2 + (1-\sigma)v_1] + (1-\mu+\mu e)[\sigma(1-\delta p)+(1-\sigma)(1-p)]p}{\mu(1-e) + (1-\mu+\mu e)[\sigma(1-\delta p)+(1-\sigma)(1-p)]}.$$

Substituting $V_{priv}(\sigma,1)$ in the expression for u_{priv} yields (after algebra)

$$u_{priv} = \mu(1-e)[\sigma v_2 + (1-\sigma)v_1] + (1-\mu+\mu e)[\sigma(1-\delta p)+(1-\sigma)(1-p)]p$$
$$+ (1-\mu+\mu e)\{\sigma\delta p(1+\theta p) + (1-\sigma)[p(1+p)]\}$$
$$= \mu(1-e)[\sigma v_2 + (1-\sigma)v_1] + (1-\mu+\mu e)[\sigma v_2 + (1-\sigma)v_1]$$
$$= \sigma v_2 + (1-\sigma)v_1,$$

which completes the proof. ■

Lemma 4.

Proof. *Part 1*. Rational market beliefs imply that shares sold after $x_1 = 1$ can be valued at most at $1+\theta p$. Therefore, an insider without liquidity needs strictly prefers to keep his shares, unless the market believes that $\sigma_{pub} = 1$. However, $\sigma_{pub} = 1$ cannot be an equilibrium. If the market believes that $\sigma_{pub} = 1$, then the insider would instead exploit (i.e., choose project 1 with probability 1), sell at date 1 in case of a success, and obtain an expected payoff $p(1+\theta p) + (1-p)p > \delta p(1+\theta p) + (1-\delta p)p$. (Recall that the market observes $x_1 = 1$ but cannot observe the project.) Therefore, $\sigma_{pub} = 1$ cannot be an equilibrium. Thus, if an equilibrium exists, it must be that $\sigma_{pub} < 1$. As $\sigma_{pub} < 1$, the insider never sells after a success.

Part 2. Rational market beliefs imply that shares sold after $x_1 = 1$ must be valued at least at $1+p$. An insider without liquidity needs then strictly prefers to sell his shares, unless the market believes that $\sigma_{pub} = 0$, in which case he is indifferent between selling or not selling. ■

Proposition 3.

Proof. For the insider to be willing to randomize between projects 1 and 2, we must have equal expected gains from both projects, i.e.,

$$pV_{pub}(\sigma_{pub}) + (1-p)p = \delta p\left[\mu(1-e)V_{pub}(\sigma_{pub})\right.$$
$$\left. + (1-\mu+\mu e)(1+\theta p)\right] + (1-\delta p)p. \quad \text{(A5)}$$

The term on the left-hand side is the expected value of choosing project 1. This expression uses the fact that the insider always sells to the market after $x_1 = 1$ (Lemma 4, part 2). The term on the right-hand side is the expected value of choosing project 2. This expression uses the fact that an insider without liquidity needs never sells to the market after $x_1 = 1$ (Lemma 4, part 1).

Replacing $V_{pub}\left(\sigma_{pub}\right)$ with (22) and solving for $s\left(\sigma_{pub}\right)$ yields (after algebra)

$$s\left(\sigma_{pub}\right)=\frac{\delta[1+p(\theta-1)]-\delta\mu(1-e)p(\theta-1)-1}{p(\theta-1)[1-\delta\mu(1-e)]}$$

$$=\frac{v_2-v_1-\delta\mu(1-e)p^2(\theta-1)}{p^2(\theta-1)[1-\delta\mu(1-e)]}, \tag{A6}$$

as long as the numerator is positive. If negative, the equilibrium $s\left(\sigma_{pub}\right)$ is zero, because in that case project 1 gives higher payoffs than project 2. In any case, by (A6), $s\left(\sigma_{pub}\right)<1$. Thus, the equilibrium $s\left(\sigma^*_{pub}\right)$ is given by

$$s^*=\max\left\{\frac{v_2-v_1-\delta\mu(1-e)p^2(\theta-1)}{p^2(\theta-1)[1-\delta\mu(1-e)]},0\right\}. \tag{A7}$$

Using (21), $\sigma^*_{pub}=s^*/[\mu(1-e)\delta+s(1-\mu\delta+\mu e)]$ if $s^*>0$, and $\sigma^*_{pub}=0$ if $s^*=0$; thus, there is a one-to-one mapping between σ^*_{pub} and s^*.

If $v_1\geq v_2-\delta\mu(1-e)p^2(\theta-1)$, then from (A7) $s^*=0\Rightarrow\sigma^*_{pub}=0$. If $v_1<v_2-\delta\mu(1-e)p^2(\theta-1)$, then we must have $\sigma^*_{pub}\in(0,1)$. ■

Proposition 4.

Proof. From Proposition 3, σ^*_{pub} is strictly increasing in s^* if $\sigma^*_{pub}\in(0,1)$. Therefore, we can obtain the effect of each parameter on σ^*_{pub} by its effect on s^* using $\frac{\partial\sigma^*_{pub}}{\partial x}=\frac{\partial\sigma^*_{pub}}{\partial s^*}\frac{\partial s^*}{\partial x}$, where x is the parameter of interest and $\frac{\partial\sigma^*_{pub}}{\partial s^*}>0$. From (A6), we have $\frac{\partial s^*}{\partial\theta}=\frac{1-\delta}{p[1-\delta\mu(1-e)](\theta-1)^2}>0$, $\frac{\partial s^*}{\partial\delta}=\frac{(1-\mu+\mu e)[p(\theta-1)+1]}{p(\theta-1)[1-\delta\mu(1-e)]^2}>0$, $\frac{\partial s^*}{\partial e}=\delta(1-\delta)\mu\frac{1+p(\theta-1)}{p(\theta-1)[1-\delta\mu(1-e)]^2}>0$, and $\frac{\partial s^*}{\partial\mu}=-\delta(1-\delta)(1-e)\frac{1+p(\theta-1)}{p(\theta-1)[1-\delta\mu(1-e)]^2}<0$. ■

Lemma 5.

Proof. The proof follows by algebra. To save on notation, we use σ instead of σ^*_{pub}. We have $u_{pub}\equiv\sigma u_{pub,2}+(1-\sigma)u_{pub,1}$; thus, using the expressions for $u_{pub,1}$ and $u_{pub,2}$, we get

$$u_{pub}=[(1-\sigma)p+\sigma\mu(1-e)\delta p]V_{pub}(\sigma) \tag{A8}$$

$$+(1-\sigma)(1-p)p+\sigma[\delta p(1-\mu+\mu e)(1+\theta p)+(1-\delta p)p].$$

Substituting (21) into (22), we get

$$V_{pub}(\sigma)=1+p+s(\sigma)(\theta-1)p \tag{A9}$$

$$=\frac{(1-\sigma)p(1+p)+\sigma\mu(1-e)\delta p(1+\theta p)}{(1-\sigma)p+\sigma\mu(1-e)\delta p}.$$

Thus,

$$u_{pub}=[(1-\sigma)p+\sigma\mu(1-e)\delta p]\frac{(1-\sigma)p(1+p)+\sigma\mu(1-e)\delta p(1+\theta p)}{(1-\sigma)p+\sigma\mu(1-e)\delta p}$$

$$+(1-\sigma)(1-p)p+\sigma[\delta p(1-\mu+\mu e)(1+\theta p)+(1-\delta p)p] \tag{A10}$$

$$=(1-\sigma)2p+\sigma[\delta p(1+\theta p)+(1-\delta p)p]$$

$$=(1-\sigma)v_1+\sigma v_2,$$

which completes the proof. ■

Proposition 5.

Proof. From the expressions of W_{priv} and W_{pub} in (17) and (29), we obtain $W_{priv} \geq W_{pub} \Leftrightarrow \left(\sigma^*_{priv} - \sigma^*_{pub}\right)(v_2 - v_1) \geq aI$. ■

Proposition 6.

Proof. Suppose $v_2 \geq v_1$. Then, from Proposition 1, we have $\sigma^*_{priv} = 1$, and from Proposition 3, we know that $\sigma^*_{pub} < 1$ always. Thus, $\sigma^*_{priv} > \sigma^*_{pub}$ if $v_2 \geq v_1$. Suppose instead that $v_2 < v_1$. Proposition 3 implies that $\sigma^*_{pub} = 0$; thus, $\sigma^*_{priv} \geq \sigma^*_{pub}$. ■

Corollary 1.

Proof. By Proposition 5, if $a = 0$, the insider prefers the private ownership structure if $\left(\sigma^*_{priv} - \sigma^*_{pub}\right)(v_2 - v_1) \geq 0$. We need to consider three cases. (1) If $v_2 > v_1$, the condition reduces to $\sigma^*_{priv} \geq \sigma^*_{pub}$. From Propositions 1 and 3, $\sigma^*_{priv} = 1$ and $\sigma^*_{pub} < 1$. Therefore, $\sigma^*_{priv} > \sigma^*_{pub}$ and the insider goes private. (2) If $v_1 > v_2$, then the condition to go private reduces to $\sigma^*_{priv} \leq \sigma^*_{pub}$. By Proposition 1, $\sigma^*_{priv} > 0$ if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ and $\sigma^*_{priv} = 0$ if $\frac{v_1 - v_2}{p(1-\delta)} \geq T(0)$. By Proposition 3, $\sigma^*_{pub} = 0$. Thus, we have $\sigma^*_{priv} > \sigma^*_{pub}$ if $\frac{v_1 - v_2}{p(1-\delta)} < T(0)$ (the insider then goes public), and $\sigma^*_{priv} = \sigma^*_{pub}$ otherwise (the insider is then indifferent between going public or private). (3) If $v_1 = v_2$, then $\left(\sigma^*_{priv} - \sigma^*_{pub}\right)(v_2 - v_1) = 0$ and the insider is indifferent between going public or private. ■

Lemma 6.

Proof. From $V_{priv}\left(\sigma_{priv}, b\right) = m\left(\sigma_{priv}, b\right)\left[\sigma_{priv} v_2 + \left(1 - \sigma_{priv}\right) v_1\right] + (1-m)p$ and

$$m\left(\sigma_{priv}, b\right) = \frac{\mu}{\mu + (1-\mu)\left[\sigma_{priv}(1-\delta p) + \left(1-\sigma_{priv}\right)(1-p)\right]b}, \tag{A11}$$

we obtain

$$V_{priv}\left(\sigma_{priv}, b\right) = \frac{\mu\left[v_1 + \sigma_{priv}(v_2 - v_1)\right] + (1-\mu)\left[1 - p + \sigma_{priv} p(1-\delta)\right]pb}{\mu + (1-\mu)\left[1 - p + \sigma_{priv} p(1-\delta)\right]b}. \tag{A12}$$

We split the proof into three parts, for $b = 1$, $b = 0$, and $0 < b < 1$.

(1) For $b = 1$ to be part of an optimal strategy for the insider, we need $kV_{priv}\left(\sigma_{priv}, 1\right) \geq p$. Substituting the expression of $V_{priv}\left(\sigma_{priv}, 1\right)$, the condition for selling is

$$k \geq \frac{\mu p + (1-\mu)\left[1 - p + \sigma_{priv} p(1-\delta)\right]p}{\mu\left[v_1 + \sigma_{priv}(v_2 - v_1)\right] + (1-\mu)\left[1 - p + \sigma_{priv} p(1-\delta)\right]p} \equiv k_1. \tag{A13}$$

Because $v_1 + \sigma_{priv}(v_2 - v_1) > p$, $k_1 < 1$. Thus, there exist values for k such that $k > k_1$, in which case $b = 1$ is the optimal action for the insider.

(2) For $b = 0$ to be part of an equilibrium strategy for the insider, we need $kV_{priv}\left(\sigma_{priv}, 0\right) \leq p$. Similar algebra shows that this condition is equivalent to

$$k \leq \frac{p}{v_1 + \sigma_{priv}(v_2 - v_1)} \equiv k_2, \tag{A14}$$

where $0 < k_2 < k_1$.

(3) If $k \in (k_2, k_1)$, any equilibrium must be in strictly mixed strategies. Imposing the condition $kV\left(\sigma_{priv}, b\right) = p$ leads to

$$b = \mu \frac{k\left[v_1 + \sigma_{priv}(v_2 - v_1)\right] - p}{(1-k)(1-\mu)\left[1 - p + \sigma_{priv} p(1-\delta)\right]p}. \tag{A15}$$

Substituting in (A15) shows that $b = 0$ if $k = k_2$, and that $b = 1$ if $k = k_1$. Furthermore, b is strictly increasing in k, as $\frac{\partial b}{\partial k} = \mu \frac{v_1 + \sigma(v_2 - v_2) - p}{p(1-\mu)(1-k)^2(1 - p + p\sigma(1-\delta))} > 0$. Therefore, $b \in (0,1)$ for $k \in (k_2, k_1)$. ■

Proposition 7.

Proof. The equilibrium value of σ_{priv} must satisfy the incentive compatibility (IC) constraints

$$\begin{array}{ll} v_2 - v_1 + p(1-\delta)T(1) \geq 0 & \text{if } \sigma_{priv} = 1 \text{ (project 2)}, \\ v_2 - v_1 + p(1-\delta)T\left(\sigma_{priv}\right) = 0 & \text{if } \sigma_{priv} \in (0,1) \text{ (mixed strategies)}, \\ v_2 - v_1 + p(1-\delta)T(0) \leq 0 & \text{if } \sigma_{priv} = 0 \text{ (project 1)}, \end{array} \tag{A16}$$

where $T\left(\sigma_{priv}\right) \equiv \max\left\{kV_{priv}\left(\sigma_{priv}, b\left(\sigma_{priv}\right)\right) - p, 0\right\} \geq 0$.

Case 1. If $v_2 - v_1 > 0$, then the IC condition for project 2, $v_2 - v_1 + p(1-\delta)T(1) \geq 0$, is trivially satisfied as $T\left(\sigma_{priv}\right) \geq 0$ for any σ_{priv}. On the other hand, there is no σ_{priv} such that the IC conditions for project 1 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 1$ is the only equilibrium.

Case 2. If $v_1 - v_2 > 0$. Suppose first that we have an equilibrium where $\sigma_{priv} > 0$. From the IC constraints, we know that we need $T\left(\sigma_{priv}\right) > 0$, which implies $b\left(\sigma_{priv}\right) = 1$. Thus, conditional on $\sigma_{priv} > 0$ and $v_1 - v_2 > 0$, we have

$$\frac{\partial V\left(\sigma_{priv}\right)}{\partial \sigma_{priv}} = \frac{\partial m\left(\sigma_{priv}, 1\right)}{\partial \sigma_{priv}}\left[\sigma_{priv} v_2 + \left(1-\sigma_{priv}\right)v_1 - p\right] - (v_1 - v_2)m\left(\sigma_{priv}, 1\right) < 0 \tag{A17}$$

because $\frac{\partial m\left(\sigma_{priv}, 1\right)}{\partial \sigma_{priv}} < 0$ and $\sigma_{priv} v_2 + \left(1-\sigma_{priv}\right)v_1 - p > 0$. Therefore, the value for the option to exit is minimized at $\sigma_{priv} = 1$. Notice that, unlike the case of $k = 0$, there might be a set of values $\sigma \in [0,1]$ that minimize $T(\sigma)$, because $T(\sigma)$ is no longer strictly positive.

If $\frac{v_1 - v_2}{p(1-\delta)} \leq T(1)$, then the IC condition for project 2 is satisfied for any σ_{priv}, whereas there is no $\sigma_{priv} < 1$ such that the IC conditions for project 1 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 1$ is the only equilibrium.

If $T(1) < \frac{v_1 - v_2}{p(1-\delta)} < T(0)$, as $T\left(\sigma_{priv}\right)$ is continuous and (weakly) decreasing in $\sigma_{priv} \in (0,1)$, there exists a unique $\sigma^*_{priv} \in (0,1)$ such that $T(\sigma^*_{priv}) = \frac{v_1 - v_2}{p(1-\delta)}$. In this case, the IC condition for mixed strategies holds exactly at σ^*_{priv} and $u_{priv,1} = u_{priv,2}$.

If $T(0) \leq \frac{v_1 - v_2}{p(1-\delta)}$, then the IC condition for project 1 is satisfied for any σ_{priv}, while there is no $\sigma_{priv} > 0$ such that the IC conditions for project 2 or for mixed strategies can be satisfied. Therefore, $\sigma^*_{priv} = 0$ is the only equilibrium.

Case 3. $v_1 = v_2$. Define the interval $\phi \equiv [\sigma_L, 1] = \arg\min_\sigma T(\sigma)$. If $T(1) > 0$, then the IC constraint for project 2 is satisfied with strict inequality, implying a unique equilibrium $\sigma^*_{priv} = 1$. In this case, $\sigma_L = 1$ and ϕ is a singleton.

If $T(1) = 0$, then $T\left(\sigma_{priv}\right) = \max\{kV_{priv}\left(\sigma_{priv}, b\left(\sigma_{priv}\right)\right) - p, 0\} = 0$ for any $\sigma^*_{priv} \in [\sigma_L, 1]$, which implies that the insider is indifferent between any $\sigma^*_{priv} \in [\sigma_L, 1]$, proving the result. ■

Lemma 7.

Proof. The ex ante share values for the insider under projects 1 and 2 are

$$u_{priv,1} \equiv \mu k V_{priv}(\sigma, b) + (1-\mu)\left\{(1-p)[bkV_{priv}(\sigma, b) + (1-b)p] + p(1+p)\right\} \tag{A18}$$

$$\begin{aligned} u_{priv,2} \equiv \mu k V_{priv}(\sigma, b) + (1-\mu)\left\{(1-\delta p)\left[bkV_{priv}(\sigma, b) + (1-b)p)\right]\right. \\ \left. + \delta p(1+\theta p)\right\}, \end{aligned} \tag{A19}$$

which are also the valuations for the investors at date 0, given our assumption that investors share the same liquidity shock. Because investors do not know which project will be chosen at the time they invest, in equilibrium they are willing to buy each share by $\sigma u_{priv,2}+(1-\sigma)u_{priv,1}$, which implies

$$\alpha^* = 1 - \frac{I}{c_{priv}[\sigma u_{priv,2}+(1-\sigma)u_{priv,1}]}. \tag{A20}$$

We thus have

$$\begin{aligned} W_{priv} &= \left\{1 - \frac{I}{c_{priv}[\sigma u_{priv,2}+(1-\sigma)u_{priv,1}]}\right\}\left[\sigma u_{priv,2}+(1-\sigma)u_{priv,1}\right] \\ &= \sigma u_{priv,2}+(1-\sigma)u_{priv,1} - \frac{I}{c_{priv}}. \end{aligned} \tag{A21}$$

To prove this Lemma, we have to consider three different cases.

(1) Suppose that $k \geq k_1$. Thus, from Lemma 6, we have that $b=1$. Define

$$u_1(\sigma,k=1) \equiv \mu V_{priv}(\sigma,1)+(1-\mu)\big[(1-p)V_{priv}(\sigma,1)+p(1+p)\big], \tag{A22}$$

$$u_2(\sigma,k=1) \equiv \mu V_{priv}(\sigma,1)+(1-\mu)\big[(1-\delta p)V_{priv}(\sigma,1)+\delta p(1+\theta p)\big]. \tag{A23}$$

These are the ex ante utilities if k is 1. Thus,

$$u_{priv,1} = u_1(\sigma,k=1)-[\mu+(1-\mu)(1-p)](1-k)V_{priv}(\sigma,1), \tag{A24}$$

$$u_{priv,2} = u_2(\sigma,k=1)-[\mu+(1-\mu)(1-\delta p)](1-k)V_{priv}(\sigma,1). \tag{A25}$$

The value of one share held by the insider is thus

$$\begin{aligned} \sigma u_{priv,2}+(1-\sigma)u_{priv,1} &= \sigma u_2(\sigma,k=1)+(1-\sigma)u_1(\sigma,k=1)-[\mu+(1-\mu)] \\ &\quad [\sigma(1-\delta p)+(1-\sigma)(1-p)](1-k)V_{priv}(\sigma,1). \end{aligned} \tag{A26}$$

From Lemma 3, we know that $\sigma u_2(\sigma,k=1)+(1-\sigma)u_1(\sigma,k=1)=\sigma v_2+(1-\sigma)v_1$. Thus, from (A21), we have

$$\begin{aligned} W_{priv} &= \sigma v_2+(1-\sigma)v_1 - \frac{I}{c_{priv}} \\ &\quad -[\mu+(1-\mu)][\sigma(1-\delta p)+(1-\sigma)(1-p)](1-k)V_{priv}(\sigma,1). \end{aligned} \tag{A27}$$

From

$$V_{priv}(\sigma,b) = \frac{\mu[v_1+\sigma(v_2-v_1)]+(1-\mu)[1-p+\sigma p(1-\delta)]pb}{\mu+(1-\mu)[1-p+\sigma p(1-\delta)]b}, \tag{A28}$$

we get

$$\begin{aligned} W_{priv} &= \sigma v_2+(1-\sigma)v_1 - \frac{I}{c_{priv}} \\ &\quad -(1-k)\{\mu[\sigma v_2+(1-\sigma)v_1]+(1-\mu)[1-p+\sigma p(1-\delta)]p\}. \end{aligned} \tag{A29}$$

(2) Suppose that $k \in (k_2,k_1)$. In this case, the insider is indifferent between selling and not selling, and thus $kV(\sigma,b(\sigma))=p$. We then have

$$u_{priv,1} \equiv \mu p+(1-\mu)[(1-p)p+p(1+p)] = \mu p+(1-\mu)v_1, \tag{A30}$$

$$u_{priv,2} \equiv \mu p+(1-\mu)[(1-\delta p)p+\delta p(1+\theta p)] = \mu p+(1-\mu)v_2. \tag{A31}$$

Thus,

$$\sigma u_{priv,2}+(1-\sigma)u_{priv,1}=\sigma v_2+(1-\sigma)v_1-\mu[\sigma v_2+(1-\sigma)v_1-p]. \tag{A32}$$

Thus, from (A21), we have

$$W_{priv}=\sigma v_2+(1-\sigma)v_1-\frac{I}{c_{priv}}-\mu[\sigma v_2+(1-\sigma)v_1-p]. \tag{A33}$$

(3) Suppose that $k\leq k_2$. From Lemma 6, $b=0$. Thus,

$$u_{priv,1}\equiv\mu k V_{priv}(\sigma,0)+(1-\mu)v_1, \tag{A34}$$

$$u_{priv,2}\equiv\mu k V_{priv}(\sigma,0)+(1-\mu)v_2. \tag{A35}$$

Thus,

$$\begin{aligned}\sigma u_{priv,2}+(1-\sigma)u_{priv,1}&=\sigma v_2+(1-\sigma)v_1-\mu\left[\sigma v_2+(1-\sigma)v_1-kV_{priv}(\sigma,0)\right]\\&=\sigma v_2+(1-\sigma)v_1-\mu(1-k)[\sigma v_2+(1-\sigma)v_1].\end{aligned} \tag{A36}$$

Thus, from (A21), we have

$$W_{priv}=\sigma v_2+(1-\sigma)v_1-\frac{I}{c_{priv}}-\mu(1-k)[\sigma v_2+(1-\sigma)v_1].$$

■

Proposition 8.

Proof. Define $w(k)\equiv W_{priv}-W_{pub}$. The insider chooses the private structure if $w(k)>0$. With $c_{priv}=c_{pub}$, the expression of $w(k)$ simplifies to $w(k)=u_{priv}-u_{pub}$. Notice that the value of u_{pub} does not depend on k, as k affects the sale of shares only in the private case.

Part 1. If $v_1\geq v_2$, by Proposition 3, $\sigma^*_{pub}=0$, then $u_{pub}=v_1$. If $k<1$, the insider is strictly worse off by choosing the private structure, because under the private structure $\sigma^*_{priv}\geq 0$ and the illiquidity cost $L(k)$ is strictly positive.

Part 2. If $v_2>v_1$ then, by Proposition 3, after some algebra,

$$\sigma^*_{pub}=\begin{cases}\frac{v_2-v_1-\delta\mu p^2(\theta-1)}{(v_2-v_1)(1-\delta\mu)}, & \text{if } \mu<\mu_L\equiv\frac{v_2-v_1}{\delta p^2(\theta-1)},\\ 0, & \text{if } \mu\geq\mu_L.\end{cases} \tag{A37}$$

By Proposition 6, $\sigma^*_{priv}=1$. Thus, $u_{priv}=v_2-L(k)$. To show that there exists a $k^*\in(0,1)$ such that the insider chooses the private structure iff $k\geq k^*$, we need to show that $w(k)$ is nondecreasing and that a unique $k^*\in(0,1)$ exists such that $w(k^*)=0$. To prove the existence of at least one $k^*\in(0,1)$ such that $w(k^*)=0$, it suffices to show that the function $w(k)$ has the following properties: $w(k)$ is continuous in k, $w(0)<0$, and $w(1)>0$. Existence thus follows from the Intermediate Value Theorem. Continuity of $w(k)$ is easily verified by inspection of the function $L(k)$.

(i) $w(0)<0$. Proof: Consider first the case of $\mu\geq\mu_L$. In such a case, $w(0)=W_{priv}-W_{pub}=(1-\mu)v_2-v_1$. Because this function is decreasing in μ, it achieves a maximum at $\mu=\mu_L$, in which case it becomes

$$\left[1-\frac{v_2-v_1}{\delta p^2(\theta-1)}\right]v_2-v_1=v_2-v_1-\frac{v_2(v_2-v_1)}{\delta p^2(\theta-1)} \tag{A38}$$

$$=-(v_2-v_1)\frac{(1+\delta)}{\delta p(\theta-1)}<0. \tag{A39}$$

Thus, $w(0)$ is also negative for any $\mu>\mu_L$.

What about $\mu < \mu_L$? In this case, we have $u_{pub} = v_1 + \sigma^*_{pub}(v_2 - v_1)$, or

$$u_{pub} = v_1 + \frac{v_2 - v_1 - \delta\mu p^2(\theta-1)}{(1-\delta\mu)}, \tag{A40}$$

which implies

$$w(0) = (1-\mu)v_2 - v_1 + \frac{v_2 - v_1 - \delta\mu p^2(\theta-1)}{(1-\delta\mu)} \tag{A41}$$

$$= (1-\mu)v_2 - v_1 + p\frac{p\delta(\theta-1)(1-\mu)-(1-\delta)}{(1-\delta\mu)}. \tag{A42}$$

Differentiating this expression with respect to μ yields

$$\frac{\partial w(0)}{\partial \mu} = -v_2 + p\delta\frac{p\delta(\theta-1)(1-\mu)-(1-\delta)-p(\theta-1)(1-\delta\mu)}{(1-\delta\mu)^2} \tag{A43}$$

$$= -p\delta\frac{[p(\theta-1)+1](1-\delta)}{(1-\delta\mu)^2} < 0. \tag{A44}$$

Thus, the highest value of $w(0)$ occurs when $\mu \to 0$. As $\lim_{\mu\to 0} w(0) = v_2 - v_2 = 0$, then $w(0) < 0$ for all $\mu > 0$.

(ii) $w(1) > 0$. Proof: This is trivially verified: $w(1) = v_2 - \sigma^*_{pub}v_2 - \left(1-\sigma^*_{pub}\right)v_1$. Proposition 3 implies that $\sigma^*_{pub} < 1$; thus, $w(1) > 0$ if $v_2 > v_1$.

As a result, there exists at least one $k^* \in (0,1)$ such that $w(k^*) = 0$.

Now we need to show that $w(k)$ is nondecreasing. We have to consider the different regions in which $b=0$, $0<b<1$, and $b=1$. In Region 3 ($k \le k_2$), we have $u_{priv} = v_2 - (1-k)\mu v_2$ (recall that $u_{priv} = W_{priv} + \frac{I}{c_{priv}}$; see Lemma 7 for the expressions for W_{priv} in each case), which is strictly increasing in k. In Region 2 ($k_2 < k < k_1$), we have $u_{priv} = \mu p + (1-\mu)v_2$, which is constant in k. In Region 1 ($k \ge k_1$), we have $u_{priv} = v_2 - (1-k)[\mu v_2 + (1-\mu)(1-\delta p)p]$, which is strictly increasing in k. Thus, u_{priv} is increasing in regions 1 and 3, and constant in region 2. Therefore, $w(k)$ is nondecreasing in k.

Finally, to prove uniqueness, we have to rule out $w(k) = 0$ for $k \in [k_2, k_1]$. As u_{priv} is constant in k in this region, it suffices to show that $w(k_2) < 0$. If $\mu \ge \mu_L$, then $u_{pub} = v_1$. Thus, $w(k_2) = \mu p + (1-\mu)v_2 - v_1$, which is decreasing in μ. Substituting the expression of μ_L, we have that $w(k_2)|_{\mu=\mu_L} = -\frac{v_2-v_1}{p(\theta-1)} < 0$. If $\mu < \mu_L$, then $w(k_2) = \mu p + (1-\mu)v_2 - v_1 - \frac{(v_2-v_1)-\delta p^2(\theta-1)\mu}{(1-\delta\mu)}$. We have $w(k_2) = 0$ trivially if $\mu = 0$, which is ruled out by assumption. For $\mu > 0$, we have $w(k_2) < 0 \Leftrightarrow \mu < 1$, which is always true. Therefore, $w(k_2) < 0$ for all μ, which implies that $k^* > k_2$. As $w(k)$ is constant in $[k_2, k_1]$, then $k^* > k_1$. Because $w(k)$ is strictly increasing for $k > k_1$, we have a unique k^*. ■

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