Internet Appendix for "Chasing Lemons: Competition for Talent under Asymmetric Information"

July 2019

1. The Planner's Problem

To understand the role of contractual assumptions for the implications of the model, we consider the problem of a social planner who faces no exogenous restrictions on the set of mechanisms that can be chosen. We show that (i) the social planner generally cannot achieve the first best allocation, (ii) any allocation of managers to firms must have a threshold property in which all retained managers have types above a given threshold (as in Lemma 2), and (iii) the threshold chosen by the planner is such that either all managers above $\gamma\mu$ are retained or all managers above μ are poached.

As in the decentralized case, at $t = 0$ there is no meaningful decision problem; each firm should hire one manager from the outside pool. At $t = 1$, because of firm-specific skills, it is never efficient to reallocate managers from one firm to another when both firms are of the same type. Similarly, transferring managers from H firms to L firms is always inefficient. Thus, the planner needs to consider only the possibility of transferring managers from L firms to H firms. To simplify the exposition, we refer to an L firm with an incumbent manager at the beginning of $t = 1$ as an incumbent firm, and to H firms with vacancies as potential poachers.

The timing of decisions in $t = 1$ is significantly simplified. First, the planner offers (and commits to) a mechanism (i.e., a contract) to each incumbent firm. Second, each incumbent firm sends a message $\tau^m \in [0, \overline{\tau}]$.* Third, the allocation is implemented.

The planner's problem is to assign incumbent managers to one of three possible sets: P denotes the set of managers who are assigned to a poacher, R denotes the set of managers who remain with the incumbent firm, and S denotes the set of managers who are unassigned (i.e., they are "sacked").

*Note that by appealing to the revelation principle, we can restrict the set of messages to the set of types.

For expositional simplicity, we restrict the analysis to the case in which, for a given $\widehat{\tau} \in [0, \overline{\tau}]$, all managers with type $\tau < \widehat{\tau}$ are fired (i.e., they are assigned to S) and all managers with type $\tau \geq \widehat{\tau}$ are either retained (i.e., assigned to R) or poached (i.e., assigned to P). Although such a constraint substantially simplifies the presentation, it has no implications for the analysis, because this constraint is not binding in the optimal solution.

Definition 1 *An* ***allocation*** *is a function $p(\tau \mid \widehat{\tau}) : [\widehat{\tau}, \overline{\tau}] \to [0,1]$ where, for a given $\widehat{\tau}$, $p(\tau \mid \widehat{\tau})$ is the probability that a manager with type τ is assigned to set P.*

In other words, we define an allocation as a stochastic assignment rule. The allocation function determines which types of incumbent managers are allocated to L firms, to H firms, or to no firm. Our definition of allocation does not consider feasibility. An allocation $p(\tau \mid \widehat{\tau})$ must meet some market clearing conditions in order for it to be feasible. (A2) is a sufficient condition that guarantees that all allocations as defined above are feasible.

From Proposition 1, we know that the first-best allocation is

$$p^{FB}(\tau \mid \widehat{\tau} = \gamma\mu) = \begin{cases} 1 & \text{if } \tau \in \left[\tau^{\#}, \overline{\tau}\right] \\ 0 & \text{if } \tau \in \left[\gamma\mu, \tau^{\#}\right] \end{cases} . \tag{1}$$

To make information asymmetries relevant, we maintain the assumption that outsiders (including the planner) cannot observe performance outcomes. We assume that the planner can force firms and managers to participate in any mechanism, and also that the planner can assign managers to firms in any way she chooses.* Similarly, we assume that the planner faces no constraints on the transfers she can impose on players, e.g., there are no liquidity or budget-balance constraints. Our planner is thus completely unconstrained in her choices and actions; the only *endogenous* constraint the planner faces is incomplete information about the types of incumbent managers.

Because of (A2), the planner wants to make sure that no H firm with $\tau \geq \gamma\mu$ dismisses its manager, which can be easily accomplished by setting the maximum payoff for H firms who dismiss managers at $\theta\gamma\mu$. Thus, the planner needs to consider as potential poachers only the set of H firms with managers with talent below $\gamma\mu$.

A *mechanism* $\langle p, t\rangle$ is an allocation rule $p(\tau^m \mid \widehat{\tau})$ and a *transfer function* $t(\tau^m)$, where τ^m is a message sent by an L firm. We consider only symmetric mechanisms where the planner offers the same contract to all L firms. Thus, to simplify notation, we omit firm subscripts.

Let $U(\tau, \tau^m \mid p, t)$ denote the payoff of an incumbent firm with type τ from reporting τ^m under mechanism $\langle p, t\rangle$. An allocation p is *implementable* if there exists at least one transfer

*In other words, we do not require the mechanisms to satisfy individual rationality constraints. Our goal in this section is to show that incentive compatibility constraints are the main reason for our results.

function t such that

$$\tau \in \arg\max_{\tau^m \in [0,\overline{\tau}]} U(\tau, \tau^m \mid p, t). \tag{2}$$

In other words, p is implementable if there exists at least one transfer function such that truth-telling is incentive compatible.

The next result restricts the set of implementable allocations:

Result 1 *For any implementable allocation p, if $p(\tau') > p(\tau'')$ for some $\tau', \tau'' \in [\widehat{\tau}, \overline{\tau}]$, then it must be that $\tau' < \tau''$.*

Proof. The revelation principle implies that there is no loss of generality from focusing on truth-telling direct mechanisms. Define an incumbent firm's payoff function under mechanism $\langle p, t\rangle$ as

$$U(\tau, \tau^m \mid p, t) = \begin{cases} (1 - p(\tau^m))\,\tau + p(\tau^m)\,\gamma\mu + t(\tau^m) & \text{if } \tau^m \in [\widehat{\tau}, \overline{\tau}] \\ \gamma\mu + t(\tau^m) & \text{if } \tau^m \in [0, \widehat{\tau}) \end{cases}. \tag{3}$$

Note that an implicit assumption here is that a firm that loses its manager ends up employing a random manager from the outside pool. Suppose that an allocation p with $p(\tau') > p(\tau'')$ for some pair (τ', τ'') is implementable (i.e., it is incentive compatible for the firm to report $\tau^m = \tau$). Incentive compatibility requires

$$\begin{aligned} (1 - p(\tau'))\,\tau' + p(\tau')\,\gamma\mu + t(\tau') &\geq (1 - p(\tau''))\,\tau' + p(\tau'')\,\gamma\mu + t(\tau'') \\ t(\tau') - t(\tau'') &\geq [p(\tau') - p(\tau'')]\,(\tau' - \gamma\mu) \end{aligned} \tag{4}$$

and

$$\begin{aligned} (1 - p(\tau''))\,\tau'' + p(\tau'')\,\gamma\mu + t(\tau'') &\geq (1 - p(\tau'))\,\tau'' + p(\tau')\,\gamma\mu + t(\tau') \\ t(\tau'') - t(\tau') &\geq [p(\tau'') - p(\tau')]\,(\tau'' - \gamma\mu). \end{aligned} \tag{5}$$

Adding both sides of (4) and (5) yields

$$0 \geq [p(\tau') - p(\tau'')]\,(\tau' - \tau'') \tag{6}$$

which implies $\tau' < \tau''$.[†]. ■

Result 1 has a straightforward corollary:

Corollary 1 *There is no mechanism that implements the first-best allocation.*

[†]We cannot have $p(\tau') > p(\tau'')$ for $\tau'' = \tau'$ because p must be a function.

Intuitively, Corollary 1 holds because, under the first-best allocation, the planner has to compensate a firm that risks losing a high-ability manager with a high monetary transfer to induce this firm to truthfully reveal the manager's type. However, if the planner takes this approach, then a firm with a low-ability manager would prefer to pretend to have a high-ability manager in order to receive a higher transfer.[‡]

Although it is unsurprising that non-monotonic allocations are not implementable, in our application, this impossibility leads to an extreme form of inefficiency; the first-best allocation is not implementable, and further, no allocation in which *some* better managers are more likely to move to better firms is implementable.

A class of implementable allocations is the set of allocations that exhibit no matching on types:

Definition 2 *A* ***matching-free allocation*** *is a function such that* $p(\tau \mid \widehat{\tau}) = c \in [0,1]$*, for all* $\tau \in [\widehat{\tau}, \overline{\tau}]$.

Under a matching-free allocation, the planner chooses to ignore the information revealed by firms with managers with types in $[\widehat{\tau}, \overline{\tau}]$ when deciding to assign managers to firms. It is easy to see that matching-free allocations are implementable.[§] We call such an allocation matching-free because for all managers who remain matched (that is, excluding managers who become unemployed), the matching decision is type independent.

We now consider the optimal mechanisms. For simplicity, we assume that the planner cares only about the total surplus created by the allocation of managers to firms, not about the transfers. The planner maximizes some function $\mathcal{S}(p, \widehat{\tau})$, which is formally defined in the proof of the next result, over the set of all incentive-compatible mechanisms. This leads to the following result:

Result 2 *The optimal mechanism implements a matching-free allocation* $p^*(\tau \mid \widehat{\tau}) = c^*$ *for* $\tau \in [\widehat{\tau}, \overline{\tau}]$*, such that*

$$\begin{array}{ll} c^* = 1 \text{ and } \widehat{\tau} = \mu, & \text{if } E[\tau \mid \tau \geq \mu] \geq \tau^{\#} + k, \\ c^* = 0 \text{ and } \widehat{\tau} = \gamma\mu, & \text{if } E[\tau \mid \tau \geq \mu] \leq \tau^{\#} + k, \end{array} \tag{7}$$

where

$$k \equiv \frac{\int_{\gamma\mu}^{\mu} (\tau - \gamma\mu) dF(\tau)}{(1 - F(\mu))(\theta\gamma - 1)}. \tag{8}$$

[‡]Formally, Corollary 1 holds because the first best allocation violates the typical monotonicity requirement for implementable decisions (here, for simplicity, we call a decision an allocation) under incomplete information (see, e.g., Fudenberg and Tirole, 1991, p. 260).

[§]To see this, suppose first that $c > 0$ and that the planner sets $t = 0$ for $\tau < \gamma\mu$ and $t = -\varepsilon$, with $\varepsilon > 0$, for $\tau \in [\gamma\mu, \overline{\tau}]$. All types less than $\gamma\mu$ report truthfully because they strictly prefer to replace the worker. All types such that $\tau \geq (\varepsilon/c) + \gamma\mu$ will also report truthfully. As we make $\varepsilon \to 0$, all types in $[\gamma\mu, \overline{\tau}]$ report truthfully. If $c = 0$ instead, then any flat transfer implements the allocation.

Proof. First, we postulate the planner's objective function as:

$$\begin{aligned}\mathcal{S}(p,\widehat{\tau}) &= H\int_{\gamma\mu}^{\overline{\tau}}\theta\tau dF(\tau)+HF(\gamma\mu)\,\theta\gamma\mu+L\int_{\widehat{\tau}}^{\overline{\tau}}p(\tau\mid\widehat{\tau})\,\theta\gamma(\tau-\mu)dF(\tau)\\ &\quad +LF(\widehat{\tau})\,\gamma\mu+L\int_{\widehat{\tau}}^{\overline{\tau}}[p(\tau\mid\widehat{\tau})\,\gamma\mu+(1-p(\tau\mid\widehat{\tau}))\,\tau]dF(\tau). \qquad (9)\end{aligned}$$

To understand this expression, note first that the first line represents the surplus created by H firms. The first term is the surplus created by firms with an incumbent manager with type $\tau \geq \gamma\mu$. Those firms will always retain their managers in an optimal allocation. The second and the third terms represent the surplus created by H firms with vacancies, i.e. those whose incumbent managers have types $\tau < \gamma\mu$. The second term is the minimum surplus created by such firms. The third term is the incremental surplus created by transferring some managers from L firms to H firms. Such transfers occur with probability $p(\tau \mid \widehat{\tau})$. The second line represents the surplus created by L firms. The first term is the surplus created by firms with incumbent managers with types below $\widehat{\tau}$. The second term is the surplus created by L firms with incumbent managers with types above $\widehat{\tau}$. With probability $p(\tau \mid \widehat{\tau})$, L firms lose their managers and produce $\gamma\mu$; otherwise firms retain their incumbent managers. Note that, for simplicity, we assume that the planner only cares about the total surplus created by the allocation of managers to firms, and not about the transfers.

Next, notice that if $\theta\gamma \leq 1$, or $\theta\gamma > 1$ and $(\theta-1)\gamma\mu/(\theta\gamma-1) \geq \overline{\tau}$, (1) implies that the first-best outcome can be achieved by a matching-free allocation with $\widehat{\tau} = \gamma\mu$ and $p(\tau \mid \widehat{\tau} = \gamma\mu) = 0$ for all $\tau \in [\gamma\mu, \overline{\tau}]$.

If $\theta\gamma > 1$ and $(\theta-1)\gamma\mu/(\theta\gamma-1) < \overline{\tau}$, the first best-outcome is not feasible, because from Result 1 any feasible allocation must be non-increasing in $\tau \in [\widehat{\tau}, \overline{\tau}]$. To solve for the optimal mechanism, we proceed in two steps. First, we find the set of optimal mechanisms for a given $\widehat{\tau}$; $m(\widehat{\tau})$ denotes the set of all such mechanisms. Second, we find the $\widehat{\tau}$ that maximizes surplus among all mechanisms in $\{m(\widehat{\tau}) : \widehat{\tau} \in [\gamma\mu, \overline{\tau}]\}$.

Take $\widehat{\tau}$ as given and consider an implementable allocation p. To simplify notation, we write $p(\tau \mid \widehat{\tau})$ as simply $p(\tau)$. For any given τ' we have

$$p(\tau')(\theta\gamma\tau'+\gamma\mu)+(1-p(\tau'))(\tau'+\theta\gamma\mu)=p(\tau')[(\theta\gamma-1)\tau'-(\theta-1)\gamma\mu]+\theta\gamma\mu+\tau', \qquad (10)$$

If $\tau' \in [\widehat{\tau}, \tau^{\#}]$, (10) is decreasing in $p(\tau')$ because $\tau' \leq (\theta-1)\gamma\mu/(\theta\gamma-1)$. Thus, $\mathcal{S}(p,\widehat{\tau})$ can be weakly increased by (pointwise) replacing $p(\tau')$ with $p(\tau^{\#})$ for all $\tau' \in [\widehat{\tau}, \tau^{\#}]$ (recall that p must be non-increasing because of Result 1). By the same argument, if $\tau'' \in [\tau^{\#}, \overline{\tau}]$, the planner can increase surplus by replacing $p(\tau'')$ with $p(\tau^{\#})$. Thus the optimal allocation must be a matching-free allocation $p(\tau) = c$, with surplus

$$\mathcal{S}(p, \widehat{\tau}) = Q + LF(\widehat{\tau})(\gamma\mu + \theta\gamma\mu) + L\int_{\widehat{\tau}}^{\overline{\tau}} (\theta\gamma\mu + \tau)\, dF(\tau) + cL\int_{\widehat{\tau}}^{\overline{\tau}} [(\theta\gamma - 1)\tau - (\theta - 1)\gamma\mu]\, dF(\tau), \tag{11}$$

where Q is a constant given by

$$Q \equiv [F(\gamma\mu) H - L]\theta\gamma\mu + H\int_{\gamma\mu}^{\overline{\tau}} \theta\tau dF(\tau). \tag{12}$$

The optimal choice of c will depend on the last term of function (11), which can be rewritten as

$$cL\int_{\widehat{\tau}}^{\overline{\tau}} [(\theta\gamma - 1)\tau - (\theta - 1)\gamma\mu]\, dF(\tau) = cL(1 - F(\widehat{\tau}))[(\theta\gamma - 1) E(\tau \mid \tau \geq \widehat{\tau}) - (\theta - 1)\gamma\mu], \tag{13}$$

which implies that the optimal choice of c is

$$c^* = \begin{cases} 0 & \text{if } E(\tau \mid \tau \geq \widehat{\tau}) \leq \tau^{\#} \\ 1 & \text{if } E(\tau \mid \tau \geq \widehat{\tau}) \geq \tau^{\#} \end{cases}. \tag{14}$$

Now, if $c^* = 0$, the optimal $\widehat{\tau}$ is $\gamma\mu$, because an incumbent is better off retaining any type above $\gamma\mu$ than hiring from the outside pool. If $c^* = 1$, the optimal $\widehat{\tau}$ is μ, because an H firm with a vacancy is better off employing a manager with type above μ than hiring from the outside pool. Thus, the optimal mechanism requires either $c^* = 0$ and $\widehat{\tau} = \gamma\mu$ or $c^* = 1$ and $\widehat{\tau} = \mu$. The mechanism that implements $c^* = 1$ (all managers above μ poached) is optimal if

$$\int_{\mu}^{\overline{\tau}} (\theta\gamma\tau + \gamma\mu) dF(\tau) + \int_{\gamma\mu}^{\mu} (\gamma\mu + \theta\gamma\mu) dF(\tau) \geq \int_{\gamma\mu}^{\overline{\tau}} (\tau + \theta\gamma\mu) dF(\tau), \tag{15}$$

which can be rewritten as

$$\int_{\mu}^{\overline{\tau}} [(\theta\gamma - 1)\tau - \gamma\mu(\theta - 1)]\, dF(\tau) \geq \int_{\gamma\mu}^{\mu} (\tau - \gamma\mu) dF(\tau), \tag{16}$$

$$(1 - F(\mu)) \left[(\theta\gamma - 1) E(\tau \mid \tau \geq \mu) - \gamma\mu(\theta - 1) - \frac{\int_{\gamma\mu}^{\mu} (\tau - \gamma\mu) dF(\tau)}{1 - F(\mu)} \right] \geq 0, \tag{17}$$

The result then follows by defining

$$k = \frac{\int_{\gamma\mu}^{\mu} (\tau - \gamma\mu) dF(\tau)}{(1 - F(\mu))(\theta\gamma - 1)}. \tag{18}$$

■

The economic intuition behind Result 2 is easier to grasp for the limiting case in which γ is close to 1 and $k \approx 0$. Because the probability of manager mobility must be non-increasing in manager types, the planner ignores the information revealed by incumbent firms and makes her decision by comparing the *expected type* $E[\tau \mid \tau \geq \mu]$ with the *critical type* $\tau^{\#}$. If the expected type is greater than the critical type, the planner assigns all managers with types in $[\mu, \overline{\tau}]$ to H firms. Similarly, if the expected type is lower than the critical type, all managers in $[\mu, \overline{\tau}]$ are retained by incumbent firms.

The general (non-limiting) case of γ not close to 1 is slightly different because of an additional trade-off: if $c^* = 1$, the optimal $\widehat{\tau}$ is μ, because an H firm with a vacancy is better off employing a manager with type above μ than hiring from the outside pool. Thus, if $c^* = 1$, there is inefficient firing of types in $[\gamma\mu, \mu]$, and therefore, the planner compares the expected type with the critical type *plus* some adjustment for the cost of inefficient firing, here measured by k.

Result 2 implies that the planner has to choose between the lesser of two evils: the planner either chooses to assign all incumbent managers with types greater than $\widehat{\tau}$ to L firms, or chooses to assign all such managers to H firms. Fine tuning the allocation of talent to efficiently match managers and firms is not possible. The first solution displays inefficient retention of the best managers – managers in $\left[\tau^{\#}, \overline{\tau}\right]$ are retained but should have been poached. The second solution displays inefficient poaching of the mediocre managers – managers in $\left[\mu, \tau^{\#}\right]$ are poached but should have been retained, and there is also inefficient firing of managers in $[\gamma\mu, \mu]$.

Result 2 may provide a justification for banning contracts in which firms own labor – i.e., quasi-slavery contracts. Even if managers voluntarily enter such contracts, these contracts generate externalities because there will be too much retention of high types. If the planner would like to set $c^* = 1$ but can use only regulatory tools, the planner may choose to ban non-compete clauses or other contracts that effectively give incumbent firms rights to retain their managers under most circumstances.[¶]

2. Mixed-strategy Equilibria

We relax Assumption A1 to allow for the possibility of mixed-strategy equilibria. In a mixed-strategy equilibrium, a type-τ_i manager who is indifferent between accepting or rejecting a

[¶]Even when it is optimal to ban bonding contracts, incumbent firms may still choose to write such contracts. In Section 4 of this Internet Appendix, we present a setting in which a firm commits in $t = 0$ to a deferred compensation contract in which a worker is paid only at the end of the game, conditional on the worker not (voluntarily) quitting the firm. We show that such contracts, even when feasible, may not be voluntarily adopted by firms.

poaching offer (i.e., an offer such that $w^p(w_i) = w_i$) rejects the poaching offer with probability $p_i(w_i)$. We then obtain the following result:

Result 3 *In any equilibrium, $p_i(w_i)$ is non-decreasing in w_i.*

Proof. Suppose that there is an equilibrium in which $w_i' = w^p(w_i') > w_i = w^p(w_i)$. In such an equilibrium,

$$E[\tau_i \mid w_i'] \equiv \int_0^{\overline{\tau}} \tau dF(\tau \mid w_i') > \int_0^{\overline{\tau}} \tau dF(\tau \mid w_i) \equiv E[\tau_i \mid w_i], \tag{19}$$

(because of $w^p(w, i, W) = \theta\gamma\left(\int_0^{\overline{\tau}} \tau dF^W(\tau \mid w, i) - \mu\right)$ and Bayesian rationality on the equilibrium path). Suppose now that $p_i(w_i') < p_i(w_i)$. Then an incumbent firm facing a manager with type $\tau_i' \geq E[\tau_i \mid w_i']$ could deviate from the equilibrium and offer this manager w_i. The manager has now a strictly lower probability of being poached and receives a strictly lower wage if retained. The incumbent firm is strictly better off after this deviation. Thus, $p_i(w_i)$ must be non-decreasing in equilibrium. ■

Result 3 implies that higher types are more likely to be retained in any equilibrium. Result 3 implies that mixed-strategy equilibria also typically involve the inefficient poaching of mediocre managers, and therefore implies that mixed-strategy equilibria are also talent-allocation inefficient. Thus, allowing for mixed-strategy equilibria does not restore efficiency, and our qualitative results are not affected by Assumption A1.

Now we fully characterize equilibria involving strictly mixed strategies in the case in which $1 \geq \theta\gamma$. For brevity, we only characterize the equilibrium poaching of managers from l firms, and thus to simplify notation we now drop the subscript $i = l$. From

$$\tau^{\#} = \begin{cases} \overline{\tau} & \text{if } \theta\gamma \leq 1 \\ \min\{(\theta - 1)\gamma\mu/(\theta\gamma - 1), \overline{\tau}\} & \text{if } \theta\gamma > 1 \end{cases}. \tag{20}$$

we have that $\tau^{\#} = \overline{\tau}$, thus poaching is always inefficient. Because equilibria in which managers play strictly-mixed strategies must involve some poaching, it follows trivially that such equilibria will also be inefficient. Furthermore, the source of inefficiency is the same as in the pure-strategy equilibria: there is too much poaching. Thus, the policy implications are also unchanged.

Although the equilibrium still involves excessive poaching, mixed strategies may improve allocational efficiency by allowing for the retention of some types in $[\gamma\mu, \tilde{\tau}]$ with some positive probability (but not with probability 1).

An equilibrium is characterized in the same way as in the pure-strategy case, except that we now need to describe the equilibrium behavior of a manager who faces two equivalent offers. Whenever an equilibrium with strictly-mixed strategies exists, there exists a function $p(w)$ that maps incumbent wage offers into probabilities of acceptance. Here we describe the equilibrium properties of this function.

Define $w(\tau)$ as the equilibrium wage offer that an incumbent makes to a manager of type τ and let $p(\tau) \equiv p(w(\tau))$. Result (3) shows that $p(w)$ is nondecreasing in w, which trivially implies that $p(\tau)$ is also non-decreasing in τ. Another equilibrium property of $p(\tau)$ is as follows:

Result 4 *Function $p(\tau)$ is continuous for all τ such that $p(\tau) > 0$.*

Proof. Consider τ' and let $\lim_{\varepsilon \to 0} p(\tau') - p(\tau' - \varepsilon) \equiv \delta$. For a deviation not to be profitable, we need

$$p(\tau')(\tau' - \varepsilon - \gamma\mu - w(\tau')) \leq p(\tau' - \varepsilon)(\tau' - \varepsilon - \gamma\mu - w(\tau' - \varepsilon)) \tag{21}$$

and

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \geq p(\tau' - \varepsilon)(\tau' - \gamma\mu - w(\tau' - \varepsilon)) \tag{22}$$

We take the limit as $\varepsilon \to 0$ and let $\tilde{w}(\tau') \equiv \lim_{\varepsilon \to 0} w(\tau' - \varepsilon)$. Then

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \leq (p(\tau') - \delta)(\tau' - \gamma\mu - \tilde{w}(\tau')) \tag{23}$$

and

$$p(\tau')(\tau' - \gamma\mu - w(\tau')) \geq (p(\tau') - \delta)(\tau' - \gamma\mu - \tilde{w}(\tau')), \tag{24}$$

which implies that $\delta = 0$, i.e., $p(\tau)$ must be continuous. ∎

The next result follows directly from Results 3 and 4:

Corollary 2 *For $\tau \in [\tau', \overline{\tau}]$ such that $p(\tau') > 0$, we can find sets $A_1, A_2, ...$ such that $\bigcup_i A_i = [\tau', \overline{\tau}]$ and that, for each A_i, either $p(\tau)$ is constant for $\tau \in A_i$ or $p(\tau)$ is strictly increasing for $\tau \in A_i$.*

In other words, $p(\tau)$ is defined over regions of *pooling* (i.e., $p(\tau)$ is constant over an interval) and *fully-revealing separation* (i.e., $p(\tau)$ is strictly increasing over an interval, so that types in this interval are fully revealed in equilibrium).

Suppose that the interval $[a, b]$ is an equilibrium pooling region with $p(\tau) \in (0, 1)$ for $\tau \in [a, b]$, and assume that this interval is not contained in any other pooling interval. The equilibrium wage must be

$$w(\tau) = w^p = \theta\gamma \left(\int_a^b \frac{\tau f(\tau)}{F(b) - F(a)} d\tau - \mu \right) \text{ for } \tau \in [a, b]. \tag{25}$$

To find $p(\tau)$ for $\tau \in [a,b]$ notice there must exist at least one separating interval to the right or to the left of $[a,b]$. From continuity,

$$\lim_{\tau \to a} p(\tau) = \lim_{\tau \to b} p(\tau), \tag{26}$$

which implies that we can characterize $p(\tau)$ for $\tau \in [a,b]$ by the limit of $p(\tau)$ over any fully-revealing separation region in the neighborhood of $[a,b]$. This implies that it suffices to characterize $p(\tau)$ over separation regions.

Let $[c,d]$ denote a fully-revealing separation interval, so that type $\tau \in [c,d]$ is fully revealed in equilibrium. Due to competition among poachers, $w^p(w(\tau)) = \theta\gamma(\tau - \mu)$. In order to obtain separation, the probability schedule must be such that it prevents an incumbent employer with a manager of type τ from pretending that the manager is of type $\hat{\tau} \in [c,d]$ and $\hat{\tau} \neq \tau$. Thus, the following incentive compatibility constraint must hold for any such $\hat{\tau}$:

$$p(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] \geq p(\hat{\tau})(\left[\tau - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)\right]. \tag{27}$$

Define

$$U(\tau) = \max_{x \in [c,d]} p(x)\left[\tau - \gamma\mu - \theta\gamma(x - \mu)\right] + \gamma\mu. \tag{28}$$

By the envelope theorem we obtain:

$$\frac{\partial U(\tau)}{\partial \tau} = p(x^*) = p(\tau), \tag{29}$$

where the second equality follows from the IC condition in (27): If τ is fully revealed in equilibrium, then $x^* = \tau$.

Integrating (29) yields

$$U(\tau) = U(d) - \int_{\tau}^{d} p(x)dx. \tag{30}$$

For simplicity we assume that the function $p(\tau)$ is twice differentiable over the interval $[c,d]$. Then the next result allows us to solve for $p(\tau)$.

Result 5 *All incentive constraints are satisfied if and only if the following two sets of constraints hold:*

(i) Local incentive compatibility:

$$p'(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] - \theta\gamma p(\tau) = 0 \tag{31}$$

(ii) Monotonicity:

$$p'(\tau) \geq 0. \tag{32}$$

Proof. Assume first that all incentive compatibility constraints are satisfied, then it must be that the following first and second order conditions are satisfied at $x^* = \tau$

$$FOC: \; p'(x^*)\left[\tau - \gamma\mu - \theta\gamma(x^* - \mu)\right] - \theta\gamma p(x^*) = 0 \tag{33}$$

$$SOC: \; p''(x^*)\left[\tau - \gamma\mu - \theta\gamma(x^* - \mu)\right] - 2\theta\gamma p'(x^*) \leq 0 \tag{34}$$

Replacing x^* with τ and totally differentiating the local incentive compatibility constraint with respect to τ, we obtain:

$$p''(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] - 2\theta\gamma p'(\tau) + p'(\tau) = 0. \tag{35}$$

From the second order condition, this equation implies that $p'(\tau) \geq 0$.

Now, suppose that both the monotonicity and local incentive compatibility conditions hold. This must imply that all incentive compatibility constraints are satisfied:

$$p(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] \geq p(\hat{\tau})\left[\tau - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)\right] \text{ for any } \tau \neq \hat{\tau}. \tag{36}$$

This equation can be rewritten as:

$$p(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] \geq p(\hat{\tau})\left[\hat{\tau} - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)\right] - (\hat{\tau} - \tau)p(\hat{\tau})$$

or

$$(\hat{\tau} - \tau)p(\hat{\tau}) \geq p(\tau)\left[\tau - \gamma\mu - \theta\gamma(\tau - \mu)\right] - p(\hat{\tau})\left[\hat{\tau} - \gamma\mu - \theta\gamma(\hat{\tau} - \mu)\right], \tag{37}$$

which implies

$$\begin{aligned} \int_{\tau}^{\hat{\tau}} p(\hat{\tau})dx \;\; \geq \;\; & \int_{\tau}^{d} \left\{p(x) + p'(x)\left[x - \gamma\mu - \theta\gamma(x - \mu)\right] - \theta\gamma p(x)\right\} dx \\ & - \int_{\hat{\tau}}^{d} \left\{p(x) + p'(x)\left[x - \gamma\mu - \theta\gamma(x - \mu)\right] - \theta\gamma p(x)\right\} dx. \end{aligned} \tag{38}$$

If the local incentive compatibility constraint holds and $\hat{\tau} \geq \tau$, this condition becomes:

$$\int_{\tau}^{\hat{\tau}} p(\hat{\tau})dx \geq \int_{\tau}^{\hat{\tau}} p(x)dx, \tag{39}$$

which always holds for $p'(\tau) \geq 0$. If $\hat{\tau} < \tau$, the condition becomes:

$$\int_{\hat{\tau}}^{\tau} p(x)dx \geq \int_{\hat{\tau}}^{\tau} p(\hat{\tau})dx, \tag{40}$$

which always holds for $p'(\tau) \geq 0$. ■

This result allows us to characterize $p(\tau)$ by solving the differential equation in (31):

Corollary 3 *In any mixed-strategy equilibrium, the probability that type τ is retained is*

$$p(\tau) = K\left[(1-\theta\gamma)\tau + \gamma\mu(\theta-1)\right]^{\frac{\theta\gamma}{1-\theta\gamma}}, \tag{41}$$

where $K \geq 0$ is a constant.

The constant K is pinned down by the boundaries of $[c, d]$. The indeterminacy of K reflects the potential multiplicity of equilibria. Once a boundary condition is chosen, K is uniquely determined. For example, if $d = \overline{\tau}$ and type $\overline{\tau}$ is retained with probability 1, then

$$K = \left[(1-\theta\gamma)\overline{\tau} + \theta\mu(\theta-1)\right]^{\frac{\theta\gamma}{\theta\gamma-1}}. \tag{42}$$

3. Changing the Timing of the Offers

In the paper, the timing of the game is such that the uninformed party (the poacher) moves last. We now introduce the case in which the informed party (the incumbent) moves last.

We modify the original timing slightly by adding a date between Dates 2 and 3:

Date $2\frac{1}{2}$. Each firm i independently makes a counter offer w_i^c.

At Date 3, a manager from a firm i who holds an initial offer w_i, a poaching offer $w^p(w_i)$, and a counter offer w_i^c, accepts the poaching offer if and only if $w^p(w_i) > \max\{w_i, w_i^c\}$.

We now characterize the equilibrium under this modified timing. For the sake of brevity, we focus only on the equilibrium that displays the maximum amount of retention by the incumbent firm.[‖] First, define the set $Y_i \equiv \{y \in Y_i : H_i(y) = 0\}$ where

$$H_i(y) \equiv y - \frac{\theta\gamma}{i}\left(\frac{\int_{\gamma\mu}^{y}\tau dF(\tau)}{F(y) - F(\gamma\mu)} - \mu\right) - \gamma\mu. \tag{43}$$

We then have the following result:

Result 6 *The (maximum-retention) equilibrium has the following properties:*

[‖]In the original game, the most-efficient equilibrium is also the equilibrium that maximizes retention. By contrast, in the modified game, these two properties ("most-efficient" and "maximum-retention") may not lead to the same equilibrium. For comparing the two games, we choose the maximum retention criterion as the most natural. However, our conclusions are not sensitive to using alternative equilibrium-selection criteria.

1. *There is a unique* $\tilde{\tau}_i' \in [\gamma\mu, \overline{\tau}]$ *such that all types* $\tau_i \geq \tilde{\tau}_i'$ *are retained. Threshold* $\tilde{\tau}_i'$ *is given by*

$$\tilde{\tau}_i' = \begin{cases} \text{the largest element in } \{\gamma\mu\} \cup Y_i & \text{if } H_i(\overline{\tau}) \geq 0 \\ \overline{\tau} & \text{if } H_i(\overline{\tau}) \leq 0 \end{cases}. \tag{44}$$

All retained managers are offered wage

$$w_i^{*\prime} = \max\left\{\theta\gamma\left(\frac{\int_{\gamma\mu}^{\tilde{\tau}'_i} \tau dF(\tau)}{F(\tilde{\tau}_i') - F(\gamma\mu)} - \mu\right), 0\right\}. \tag{45}$$

2. *All types* $\tau_i \in [0, \gamma\mu]$ *are fired in equilibrium.*

3. *All types* $\tau_i \in [\gamma\mu, \tilde{\tau}_i']$ *are poached in equilibrium.*

Proof. As before, we assume that E1 and E2 hold.

To find the equilibrium, we work backwards. At Date $2\frac{1}{2}$, the incumbent observes a poaching wage w_i^p. The incumbent pays the poaching wage and retains type τ if and only if $\tau - \frac{w_i^p}{i} \geq \gamma\mu$.

At Date 2, a manager with a wage offer w_i receives a poaching offer equal to

$$\theta\gamma\left(\int_0^{\overline{\tau}} \tau dF(\tau \mid w_i, i) - \mu\right). \tag{46}$$

The beliefs represented by $F(\tau \mid w_i, i)$ must be Bayesian on the equilibrium path and consistent with E2.

At Date 1, the incumbent chooses w_i. We argue that an incumbent offers a unique wage $w_i = 0$ to any retained employee, i.e., an employee with talent $\tau_i \geq \gamma\mu$. The argument is similar to the one used to prove Lemma 1. Suppose that there are two types $\tau' > \tau''$ and that an incumbent i wants to retain both of them. Suppose the incumbent offers two different wages $w_i' > w_i''$ and suppose the poacher's offers are $w^p(w_i') > w^p(w_i'')$. Then, there is a profitable deviation for the incumbent, which is to offer w_i'' to both types. Now, suppose that $w_i > 0$. Then, the incumbent could deviate and offer $w_i' = 0$; Assumption E2 implies that $w^p(0) < w^p(w_i)$. Thus, $w_i = 0$. E1 implies that all $\tau < \gamma\mu$ receive negative offers. Maximum retention implies that the incumbent offers $w_i = 0$ to all $\tau_i \geq \gamma\mu$. This proves Part 2 of the result and that there is a unique $\tilde{\tau}_i' \in [\gamma\mu, \overline{\tau}]$ such that all types $\tau_i > \tilde{\tau}_i'$ are retained. Then, it follows that the equilibrium poaching wage is given by

$$w_i^p = \theta\gamma\left(\frac{\int_{\gamma\mu}^{\tilde{\tau}'_i} \tau dF(\tau)}{F(\tilde{\tau}_i') - F(\gamma\mu)} - \mu\right), \tag{47}$$

and thus all retained managers are offered wage

$$w_i^{*\prime} = \max\left\{\theta\gamma\left(\frac{\int_{\gamma\mu}^{\tilde{\tau}_i^{\prime}} \tau dF(\tau)}{F(\tilde{\tau}_i^{\prime}) - F(\gamma\mu)} - \mu\right), 0\right\}, \tag{48}$$

because the incumbent only needs to offer $w_i^c = \max\{w_i^p, 0\}$. If w_i^p is strictly positive, then clearly all types $\tau_i \in (\gamma\mu, \tilde{\tau}_i^{\prime})$ are poached in equilibrium. If $w_i^p \leq 0$, then no one is poached and thus $\tilde{\tau}_i^{\prime} = \gamma\mu$. This proves Part 3.

To prove Part 1, suppose first that $H_i(\overline{\tau}) < 0$. Then, the incumbent does not wish to retain any type, implying that $\tilde{\tau}_i^{\prime} = \overline{\tau}$.

Suppose now that $H_i(\overline{\tau}) \geq 0$. If $H_i(\tau_i) \geq 0$ for all τ_i, then the incumbent can retain any type for a given equilibrium w_i^p and still make a net profit. Thus, all types higher than $\gamma\mu$ are retained. Finally, if $H_i(\tau_i) < 0$ for some τ_i, then the set Y_i is non-empty and the equilibrium threshold must be in Y_i (which has at least two elements because $H_i(0) > 0$). Consider a candidate equilibrium threshold $\tau_i^* \in Y_i$, with respective equilibrium poaching wage w_i^{p*}, and assume that τ_i^* is not the largest element of Y_i. Then, a single poacher may deviate and offer an alternative poaching wage equal to

$$w_i^{p\prime} = \tilde{\tau}_i^{\prime} - \alpha - \gamma\mu, \tag{49}$$

where $\tilde{\tau}_i^{\prime}$ is the largest element in Y_i and $\alpha > 0$ is sufficiently small so that $w_i^{p*} < w_i^{p\prime}$. This poacher would be successful at poaching all types $[\gamma\mu, \tilde{\tau}_i^{\prime} - \alpha)$ at a wage that is strictly lower than the one implied by the zero net profit condition. Thus, this deviation is profitable. Thus, the equilibrium threshold must be $\tilde{\tau}_i^{\prime}$, i.e., the largest element of Y_i. ■

The equilibrium outcome is qualitatively similar to the outcome in Proposition 2: All types above a threshold are retained, and only mediocre types are poached. Thus, our main result that asymmetric information creates inefficiencies in talent allocation does not depend on whether the informed party moves last or not. In particular, we note that not only inefficient retention is possible, but also that inefficient poaching will often occur because at least a subset of types in $[\gamma\mu, \tilde{\tau}_i^{\prime}]$ should be retained in the first-best allocation.

An important property of this equilibrium is as follows:

Result 7 *In the modified game in which the incumbent moves last, fewer types are poached in equilibrium:*

$$\tilde{\tau}_i^{\prime} \leq \tilde{\tau}_i. \tag{50}$$

Proof. Threshold $\tilde{\tau}_i$ is defined by the lowest value that solves

$$\tilde{\tau}_i - \gamma\mu = \frac{\theta\gamma}{i}\left(\frac{\int_{\tilde{\tau}_i}^{\overline{\tau}} \tau dF(\tau)}{1 - F(\tilde{\tau}_i)} - \mu\right). \tag{51}$$

(We assume an interior solution for simplicity; if the solution is not interior, then there is no retention, and the result is trivially proven).

Threshold $\tilde{\tau}_i'$ is defined by the largest value that solves

$$\tilde{\tau}_i' - \gamma\mu = \frac{\theta\gamma}{i}\left(\frac{\int_{\gamma\mu}^{\tilde{\tau}_i'} \tau dF(\tau)}{F(\tilde{\tau}_i') - F(\gamma\mu)} - \mu\right). \tag{52}$$

Suppose that $\tilde{\tau}_i' > \tilde{\tau}_i$. Then, it must be that

$$\frac{\int_{\gamma\mu}^{\tilde{\tau}_i'} \tau dF(\tau)}{F(\tilde{\tau}_i') - F(\gamma\mu)} > \frac{\int_{\tilde{\tau}_i}^{\overline{\tau}} \tau dF(\tau)}{1 - F(\tilde{\tau}_i)}, \tag{53}$$

which cannot be true. ■

This result demonstrates that when the incumbent has the option to make the last offer, it is able to retain the manager more often. This result is unsurprising because this modified timing gives more market power to the incumbent. One interpretation for this timing of offers is that if the manager accepts the incumbent's offer at date $2\frac{1}{2}$, this offer becomes binding and the manager can no longer accept a poaching offer.

Because of (50), the modified game is more likely to display inefficient retention than the original game. The modified game is less likely to display inefficient poaching than the original game for the same reason. Thus, by giving the incumbent the option to make a final binding offer, poaching inefficiencies can be reduced and sometimes eliminated.

4. Deferred compensation

The solution to the planner's problem reveals that the inefficient allocation of talent result is a consequence of information asymmetries alone and not of any artificial restriction on the space of contracts. It is nevertheless instructive to consider the case in which the incumbent may use deferred compensation as a means to reduce mobility.

Result 2 immediately implies that, from a social welfare perspective, such bonding contracts may either improve or worsen efficiency. However, even when it is optimal to ban these contracts, incumbent firms may still choose to write such contracts. Here we show that such contracts, even when feasible, may not be voluntarily adopted by firms.

Consider the following contract: Before the incumbent firm learns its manager's type (at $t = 0$), the firm commits to a fixed wage $\overline{w}_i$ to be paid at the end of the game, but only if the manager remains with the firm or if the manager is fired. To retain types $\tau_i \geq \gamma\mu$, the lowest wage that must be offered is $\overline{w}_i = w^p(\overline{w}_i) = \theta\gamma\left(\int_{\gamma\mu}^{\overline{\tau}} \frac{\tau f(\tau)}{1-F(\gamma\mu)} d\tau - \mu\right)$. Under commitment to $\overline{w}_i$, expected profit (at $t = 1$) to the incumbent is thus

$$E[\pi_{ic}] = F(\gamma\mu)\gamma\mu + [1 - F(\gamma\mu)] \int_{\gamma\mu}^{\overline{\tau}} \frac{\tau f(\tau)}{1 - F(\gamma\mu)} d\tau - \overline{w}_i. \quad (54)$$

Without commitment, we know that the equilibrium implies that the incumbent chooses some $\tilde{\tau}_i \geq \gamma\mu$, and thus its expected profit at $t = 1$ is

$$E[\pi_{inc}] = F(\tilde{\tau}_i)\gamma\mu + [1 - F(\tilde{\tau}_i)] \left[\int_{\tilde{\tau}_i}^{\overline{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau}_i)} d\tau - \theta\gamma\left(\int_{\tilde{\tau}_i}^{\overline{\tau}} \frac{\tau f(\tau)}{1 - F(\tilde{\tau}_i)} d\tau - \mu\right)\right]. \quad (55)$$

It can be shown, through simple examples, that $E[\pi_{inc}] \lesseqgtr E[\pi_{ic}]$ depending on the parameters. The intuition for this result is that deferred compensation schemes (such as restricted shares or vesting of stock options) are costly to the firm because some managers who are fired are still paid $\overline{w}_i$, which may leave rents to dismissed managers (for example, if wages at $t = 0$ cannot be negative). Thus, the expected excess cost of such a scheme is $F(\gamma\mu)\overline{w}_i$. Without such a scheme, the overall surplus may be higher or lower, but the profit could still be larger *even when the surplus is lower*. Hence, deferred compensation contracts may not be chosen by firms even when they are feasible.

5. An Infinite-Horizon Model: Symmetric Learning

Under symmetric learning, all firms have the same information about an old manager's type, i.e., they learn the employed young managers' types at Date 1 of each period. As the equilibrium will be time-invariant, for simplicity we ignore time subscripts. At Date 1 of each period, a type-i firm with an incumbent manager who is of a known type τ offers the wage:

$$w_i^S = \begin{cases} \text{any } w < 0 & \tau \leq \underline{\tau}_i \\ 0 & \tau \in \left[\underline{\tau}_i, \hat{\tau}_i^S\right] \\ w^{pS}(\tau) & \tau \in \left[\hat{\tau}_i^S, \tau_i^{\#}\right] \\ \text{any } w < w^{pS}(\tau) & \tau \in \left[\tau_i^{\#}, \overline{\tau}\right) \end{cases}, \quad (56)$$

where $\underline{\tau}_i$, $\hat{\tau}_i$, $\tau_i^{\#}$ and function $w^{pS}(\tau)$ are to be determined in equilibrium.

Because poachers compete à la Bertrand, their equilibrium value function, $V_h^{pS}(\tau)$, when poaching a manager of type τ should be equal to the value they derive from hiring a young manager, V_h^{yS}:

$$V_h^{pS}(\tau) - V_h^{yS} = 0, \tag{57}$$

where

$$V_h^{pS}(\tau) = \theta\gamma\tau - w^{pS}(\tau) + \delta \max\left\{V_h^{yS}, V_h^{pS}(\tau)\right\}, \tag{58}$$

$$V_h^{yS} = \theta\gamma\mu - w^{yS} + \delta V_h^{oS}, \tag{59}$$

and

$$V_h^{oS} = F(\underline{\tau}_h) \max\left\{V_h^{yS}, V_h^{pS}(\tau)\right\} + \theta \int_{\underline{\tau}_h}^{\overline{\tau}} \tau f(\tau) d\tau - \int_{\hat{\tau}_h^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}. \tag{60}$$

By replacing (58) and (59) into (57), we obtain the following expression for the poaching wage (recall that this is only defined for non-negative wages):

$$w^{pS}(\tau) = \theta\gamma(\tau - \mu) + w^{yS} - \delta(V_h^{oS} - V_h^{yS}). \tag{61}$$

The threshold $\hat{\tau}_i$ corresponds to the level of talent above which a poacher offers a positive wage to a manager of type $\tau > \hat{\tau}_i$. Because information is symmetric, the poaching wage depends only on a manager's talent, therefore we set $\hat{\tau}_l^S = \hat{\tau}_h^S = \hat{\tau}^S$, and thus threshold $\hat{\tau}^S$ is given by $w^{pS}(\hat{\tau}^S) = 0$.

Using (59) in (60), we obtain:

$$\begin{aligned} V_h^{oS} &= F(\underline{\tau}_h)\left[\theta\gamma\mu - w^{yS} + \delta V_h^{oS}\right] + \theta \int_{\underline{\tau}_h}^{\overline{\tau}} \tau f(\tau) d\tau \\ &\quad - \int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}. \end{aligned} \tag{62}$$

Subtracting V_h^{yS} from both sides yields

$$\begin{aligned} V_h^{oS} - V_h^{yS} &= -\left[1 - F(\underline{\tau}_h)\right]\left[\theta\gamma\mu - w^{yS} + \delta V_h^{oS}\right] + \theta \int_{\underline{\tau}_h}^{\overline{\tau}} \tau f(\tau) d\tau \\ &\quad - \int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau + \delta(1 - F(\underline{\tau}_h)) V_h^{yS}, \end{aligned} \tag{63}$$

or

$$V_h^{oS} - V_h^{yS} = \frac{\int_{\underline{\tau}_h}^{\overline{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau - \int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau + (1 - F(\underline{\tau}_h)) w^{yS}}{1 + \delta(1 - F(\underline{\tau}_h))}. \tag{64}$$

The first-period wage of a young manager is given by the first-period participation constraint:[**]

$$w^{yS} = -\delta \int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau. \tag{65}$$

Therefore, we can replace $\int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau$ by $-w^{yS}/\delta$ in (64) to obtain:

$$V_h^{oS} - V_h^{yS} = \frac{\int_{\underline{\tau}_h}^{\overline{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau}{1 + \delta(1 - F(\underline{\tau}_h))} + \frac{w^{yS}}{\delta}. \tag{66}$$

Now, plug (66) into (61) to find the poaching wage offered to a manager with talent τ (this function is defined only for values of τ such that $w^{pS}(\tau) \geq 0$):

$$w^{pS}(\tau) = \theta\gamma\,(\tau - \mu) - \frac{\delta}{1 + \delta(1 - F(\underline{\tau}_h))} \int_{\underline{\tau}_h}^{\overline{\tau}} (\theta\tau - \theta\gamma\mu) f(\tau) d\tau. \tag{67}$$

In the infinite-horizon model, for a given τ, the offer made by a poacher is lower than that in the two-period model. In the infinite-horizon setting, hiring a young manager has an option value: once the firm learns the manager's type it has the option to retain this manager for the subsequent period. The value of this option is given by the second term on the right-hand side of (67). Thus, poaching an old manager comes at an opportunity cost, which is the value of this option.

The first-period wage w^{yS} of a young manager is given by

$$w^{yS} = -\delta \int_{\hat{\tau}^S}^{\overline{\tau}} w^{pS}(\tau) f(\tau) d\tau. \tag{68}$$

Note that this wage is always negative and equal to the discounted expected wage received by this manager in the second period. In other words, young managers have zero expected surplus. This result is a consequence of our assumptions that the manager's outside option is zero and that there is no limited liability. We know from Terviö (2009) that, in a dynamic model with symmetric learning, limited liability creates inefficiencies: There is excessive retention of mediocre types. Because we want to isolate the effect of asymmetric learning on welfare, we choose not to impose limited liability, which also implies that, unlike Terviö (2009), the first-best allocation is obtained in our benchmark model with symmetric learning.

Threshold $\underline{\tau}_i$ from (56) is determined by

$$V_i^{oS}(\underline{\tau}_i) - V_i^{yS} = 0, \tag{69}$$

where $V_i^{oS}(\tau)$ is the value function a type-i firm from retaining an incumbent (old) manager with talent τ, and V_i^{yS} is the value from hiring a young manager. For a type-h firm, this is

[**] Note that the wage of a young worker is independent of the type of the firm.

given by

$$V_h^{oS}(\underline{\tau}_h) - V_h^{yS} = 0, \tag{70}$$

where

$$V_h^{oS}(\underline{\tau}_h) = \theta\underline{\tau}_h + \delta V_h^{yS}, \tag{71}$$

and V_h^{yS} is given by equation (59) (Recall that in equilibrium $V_h^{yS} = V^{pS}(\tau)$ for any $\tau \geq \hat{\tau}^S$). We can rewrite (70) as

$$\begin{aligned}
&\theta\underline{\tau}_h - \theta\gamma\mu + w^{yS} - \delta(V_h^{oS} - V_h^{yS}) = 0 \\
\Longleftrightarrow\ &\theta\underline{\tau}_h - \theta\gamma\mu - \frac{\delta}{1+\delta(1-F(\underline{\tau}_h))}\int_{\underline{\tau}_h}^{\overline{\tau}}(\theta\tau - \theta\gamma\mu)f(\tau)d\tau = 0 \\
\Longleftrightarrow\ &\underline{\tau}_h - \gamma\mu - \delta\int_{\underline{\tau}_h}^{\overline{\tau}}(\tau - \underline{\tau}_h)f(\tau)\,d\tau = 0. \qquad (72)
\end{aligned}$$

The equilibrium threshold $\underline{\tau}_h$ is given by the unique solution to (72) (note that the left-hand side of (72) is increasing in $\underline{\tau}_h$ and is negative for $\underline{\tau}_h = 0$ and positive for $\underline{\tau}_h = \overline{\tau}$). Then, we have a closed form solution for the poaching wage in (67). By setting $w^{pS}(\widehat{\tau}^S) = 0$ in (67), we then obtain a unique equilibrium value for $\hat{\tau}^S$.

So, the threshold is given by:

$$\underline{\tau}_h = \gamma\mu + \delta\int_{\underline{\tau}_h}^{\overline{\tau}}(\tau - \underline{\tau}_h)f(\tau)d\tau. \tag{73}$$

The decision to retain a manager is given by the following trade-off. The left-hand side of (73) is the immediate gain from retaining an old manager of type $\underline{\tau}_h$; the right-hand side is the benefit from hiring a young manager from the outside pool. This benefit has two components. First, a young manager from the outside pool produces (in expectation) $\gamma\mu$ during the first year of employment. Second, hiring a young manager again gives the firm the option to retain this manager in the subsequent period. The value of this option is given by the second term on the right-hand side of (73).

We now need to find threshold $\underline{\tau}_l$. An l-firm is willing to retain a manager of type $\underline{\tau}_l$ for a wage of zero if the following condition holds:

$$V_l^{oS}(\underline{\tau}_l) - V_l^{yS} = 0,$$

where

$$V_l^{oS}(\underline{\tau}_l) = \underline{\tau}_l + \delta V_l^{yS}, \tag{74}$$

$$V_l^{yS} = \gamma\mu - w^{yS} + \delta V_l^{oS}, \tag{75}$$

and

$$\begin{aligned} V_l^{oS} &= (F(\underline{\tau}_l) + 1 - F(\tau_l^{\#}))V_l^{yS} + \int_{\underline{\tau}_l}^{\tau_l^{\#}} \tau f(\tau)d\tau \\ &- \int_{\widehat{\tau}_l}^{\tau_l^{\#}} w^{pS}(\tau)f(\tau)d\tau + \delta(F(\tau_l^{\#}) - F(\underline{\tau}_l))V_l^{yS}. \end{aligned} \tag{76}$$

We use (74), (75), and (76) to obtain:

$$V_l^{oS}(\underline{\tau}_l) - V_l^{yS} = 0 \;\; \Leftrightarrow \underline{\tau}_l - \gamma\mu - \frac{\delta\left(\int_{\underline{\tau}_l}^{\tau_l^{\#}}(\tau-\gamma\mu)f(\tau)d\tau + \int_{\tau_l^{\#}}^{\overline{\tau}} w^p(\tau)f(\tau)d\tau\right)}{1+\delta(F(\tau_l^{\#})-F(\underline{\tau}_l))} = 0 \tag{77}$$

$$\Leftrightarrow \underline{\tau}_l - \gamma\mu - \delta\int_{\underline{\tau}_l}^{\tau_l^{\#}}(\tau - \underline{\tau}_l)f(\tau)d\tau - \delta\int_{\tau_l^{\#}}^{\overline{\tau}} w^p(\tau)f(\tau)d\tau = 0, \tag{78}$$

which again determines a unique $\underline{\tau}_l$ for a given $\tau_l^{\#}$.

For a type-l firm the retention threshold $\underline{\tau}_l$ is thus:

$$\underline{\tau}_l = \gamma\mu + \delta\int_{\underline{\tau}_l}^{\tau_l^{\#}}(\tau - \underline{\tau}_l)f(\tau)d\tau + \delta\int_{\tau_l^{\#}}^{\overline{\tau}} w^{pS}(\tau)f(\tau)d\tau. \tag{79}$$

The first two terms on the right-hand side of (79) are analogous to those in (73). The key difference between these two conditions is the last term on the right-hand side of (79), which represents the present value of the wages paid to those managers who are poached in equilibrium in the second year of employment. A firm of type l is able to capture such surplus by offering a negative wage to young managers. Thus, these firms are compensated for being talent discoverers; even if their best managers leave to work for other firms, type-l firms capture all the surplus generated by an efficient allocation of talent.

Now, we only need to find $\tau_l^{\#}$. Poaching exists only if the incremental surplus to the poacher is larger than the net loss to the incumbent firm:

$$V_l^{oS}(\tau_l) - V_l^{yS} \leq V_h^{pS}(\tau_l) - V_h^{yS}. \tag{80}$$

To see that this must hold in any equilibrium with poaching, note that if it did not hold, the incumbent could offer a slightly larger wage and profitably prevent poaching. Thus, if an interior $\tau_l^{\#}$ exists, it is determined by one of the solutions to (80) with equality, which yields:

$$\tau_l^{\#} - \gamma\mu - \frac{\delta\int_{\underline{\tau}_l}^{\tau_l^{\#}}(\tau-\gamma\mu)f(\tau)d\tau + \delta\int_{\tau_l^{\#}}^{\overline{\tau}} w^p(\tau)f(\tau)d\tau}{1+\delta(F(\tau_l^{\#})-F(\underline{\tau}_l))} = \theta\gamma(\tau_l^{\#}-\mu) - \frac{\delta\int_{\underline{\tau}_h}^{\overline{\tau}}(\theta\tau-\theta\gamma\mu)f(\tau)d\tau}{1+\delta(1-F(\underline{\tau}_h))}. \tag{81}$$

(If there is no interior solution, the equilibrium is such that no one is poached). If there is more than one solution, only one of such solutions is an equilibrium. To see this, note that if τ_l is poached in any equilibrium, then $\tau_l' > \tau_l$ will also be poached because $\tau_l - \gamma\mu - \theta\gamma(\tau_l - \mu)$ is strictly decreasing in τ_l (note that the value of future options do not change with τ_l). Thus, there is a unique set of values $\left(\underline{\tau}_l, \underline{\tau}_h, \tau_l^{\#}, \widehat{\tau}^S, w^{yS}\right)$ and function $w^{pS}(\tau)$ that characterize the equilibrium.

We now discuss two important properties of the equilibrium. First, we have the following result:

Result 8 $\underline{\tau}_l \geq \underline{\tau}_h$.

Proof. Begin by rewriting (78) as

$$\underline{\tau}_l - \gamma\mu = \delta\int_{\underline{\tau}_l}^{\overline{\tau}}(\tau-\underline{\tau}_l)f(\tau)d\tau + \delta\int_{\tau_l^{\#}}^{\overline{\tau}}(w^p(\tau)-\tau+\underline{\tau}_l)f(\tau)d\tau. \tag{82}$$

The left-hand side of equation (82) increases with $\underline{\tau}_l$ and the right-hand side (RHS) decreases with $\underline{\tau}_l$. If $\tau_l^{\#} = \overline{\tau}$, then the conditions defined by equations (82) and (72) are the same and $\underline{\tau}_l = \underline{\tau}_h$. If $\tau_l^{\#} < \overline{\tau}$, then $\delta\int_{\tau_l^{\#}}^{\overline{\tau}}(w^p(\tau)-\tau+\underline{\tau}_l)f(\tau)d\tau > 0$, which increases the RHS and thus increases the value for $\underline{\tau}_l$. ■

This result indicates that l-firms are more likely to fire managers with low talent than are h-firms. The intuition is as follows: It is more efficient for l-firms to act as talent discoverers than as producers because l-firms are as efficient as h-firms in discovering talent, but less efficient at producing output. Thus, l-firms have a comparative (but not absolute) advantage at discovering talent and should thus do more of it in an efficient allocation.

We also have the following result:

Result 9 *The unique equilibrium under symmetric learning is efficient (in the Kaldor-Hicks sense).*

To prove this result formally, we proceed as follows. We first state the necessary and sufficient conditions for an allocation, here fully characterized by thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$, to be (Kaldor-Hicks) efficient (i.e., to maximize a social welfare function with equal weights to all players). We then show that we can construct a set of prices (wages) that sustains such an allocation as a decentralized equilibrium of our game. Thus, an efficient allocation is also a decentralized equilibrium. Because the decentralized equilibrium is unique, it is thus always efficient.

Proof. For simplicity, without loss of generality we consider only symmetric allocations in which all firms and managers of the same type and in identical situations are assigned the same surplus by a hypothetical social planner. Under this assumption, to derive the efficiency conditions we can work with an alternative interpretation of the model in which there is only one firm of each type.

Consider an allocation associated with the thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$. Let $S^*(\tau_l, \tau_h)$ denote the total surplus generated by this allocation, conditional on knowing the incumbent managers' types (τ_l, τ_h) (if one or both firms do not have incumbent managers, define the surplus accordingly as being conditional only on the type of the existing incumbent manager, if any). This allocation is efficient if and only if, for any other allocation with conditional surplus $S'(\tau_l, \tau_h)$,

$$S^*(\tau_l, \tau_h) \geq S'(\tau_l, \tau_h) \text{ for all } (\tau_l, \tau_h). \tag{83}$$

We can focus on conditional surplus because, under the current interpretation, there are only two firms and at most two incumbent managers.

To maximize (conditional) surplus, we list three necessary conditions:[††]

(1) For any given τ_l, firm l retains this type instead of hiring a young manager if and only if:

$$U_l^o(\tau_l) + u(\tau_l) + U_h^{ol} + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + u^{yl} + U_h^{yl} + u_f(\tau_h, \tau_l), \tag{84}$$

where $U_i^o(\tau_i)$ is the expected payoff to i of retaining τ_i under the allocation, $u(\tau_i)$ is the expected payoff to manager τ_i of being retained by i, U_h^{ol} is the expected payoff to h of l retaining τ_l, $u(\tau_h, \tau_j)$ is the expected payoff to manager τ_h, who currently works for firm h, if manager τ_l is retained by l (if h has no incumbent manager, we set this value to zero), U_i^y is the expected payoff to i of hiring a young manager, $u_f(\tau_l)$ is the expected payoff to a manager of type τ_l of being fired by l, u^{yi} is the expected payoff to a young manager of being hired by i, $U_h^{yl}$is the expected payoff to h of l hiring a young manager, and $u_f(\tau_h, \tau_l)$ is the expected payoff to manager τ_h if manager τ_l is fired by firm l (if firm h has no incumbent manager, we set this value to zero).

(2) If firm h has a vacancy, h poaches a manager of type τ_l instead of hiring a young

[††]In what follows, for simplicity we assume that all workers who remain unemployed are assigned zero net surplus by the social planner. This is without loss of generality. In addition, in line with the previous assumption that only h firms can be poachers, we focus on the cases where there are no job transitions from h to l.

manager if and only if:

$$U_h^p(\tau_l) + u^h(\tau_l) + U_l^y + u^{yl} \geq U_h^y + u^{yh} + \max\left\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\right\}. \tag{85}$$

where $U_h^p(\tau_l)$ is the expected payoff to h of poaching τ_l and $u^h(\tau_l)$ is the expected payoff to manager τ_l of being hired by h.

(3) For any given τ_h and τ_l, firm h retains this type if and only if:

$$\begin{aligned} U_h^o(\tau_h) + u(\tau_h) + \max\left\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\right\} \geq \\ \max\left\{U_h^y + u^{yh} + \max\left\{U_l^o(\tau_l) + u(\tau_l), U_l^y + u^{yl}\right\}, U_h^p(\tau_l) + u^h(\tau_l) + U_l^y + u^{yl}\right\}. \end{aligned} \tag{86}$$

Now, consider the efficient allocation, which is determined by the thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$. Note first that these thresholds fully determine the following wages:

$$w^{p*}(\tau) = \theta\gamma\tau - \theta\gamma\mu - \frac{\delta}{1+\delta(1-F(\underline{\tau}_h^*))}\int_{\underline{\tau}_h^*}^{\overline{\tau}}(\theta\tau - \theta\gamma\mu)f(\tau)d\tau, \tag{87}$$

$$w^{y^*} = -\delta\int_{\hat{\tau}^*}^{\overline{\tau}} w^{pS}(\tau)f(\tau)d\tau, \tag{88}$$

where $\hat{\tau}^*$ is the threshold for which $w^{p*}(\hat{\tau}^*) = 0$. Given these wages, then we can easily verify that we can uniquely define $V_h^{p*}(\tau), V_h^{o*}(\tau), V_h^{y*}, V_h^{o*}, V_l^{o*}(\tau), V_l^{y*}$, and V_l^{o*} as the value functions as before, but taking the thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$ as given.

We now need to show that such wages can sustain a decentralized equilibrium such that Conditions (1)-(3) hold. Start with (84). First, if $u^{yl} \neq 0$, then use (positive or negative) lump-sum transfers from the manager to firm l to create a new allocation on the right-hand side of (84), without changing its total surplus, so that U_l^y under this new allocation is equal to the old U_l^y plus the old u^{yl}, and thus the new u^{yl} becomes zero:

$$U_l^o(\tau_l) + u(\tau_l) + U_h^{ol} + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + U_h^{yl} + u_f(\tau_h, \tau_l). \tag{89}$$

Second, consider U_h^{ol}. Suppose that h has a vacancy. If $U_h^{ol} \neq V_h^{y*}$, make transfers to or from all the other players until $U_h^{ol} = V_h^{y*}$ and the surplus on left-hand side is unchanged.[‡‡] Make similar transfers in the analogous case in which h has a manager of type τ_h until $U_h^{ol} = \max\{V_h^{y*}, V_h^{o*}(\tau_h)\}$. Make similar transfers on the right-hand side until $U_h^{yl} = V_h^{y*}$ or $U_h^{yl} = \max\{V_h^{y*}, V_h^{o*}(\tau_h)\}$, depending on which case is relevant. Then, we can rewrite the

[‡‡] Notice that such transfers can always be made because the initial allocation is assumed to be efficient and thus has the maximum possible conditional surplus. If, counterfactually, V_h^{y*} was higher than the maximum surplus, an allocation that delivered V_h^{y*} (which is possible by construction) would be superior to the efficient allocation, which is a contradiction.

condition above as

$$U_l^o(\tau_l) + u(\tau_l) + u(\tau_h, \tau_l) \geq U_l^y + u_f(\tau_l) + u_f(\tau_h, \tau_l). \tag{90}$$

Third, consider $u(\tau_h, \tau_l)$. This term is zero if h has a vacancy. If instead h has an incumbent who is retained (i.e., if if $\tau_h \geq \widetilde{\tau}_h^*$), then $u(\tau_h, \tau_l) = u(\tau_h)$. Suppose in this case that $\tau_h \leq \hat{\tau}^*$. If $u(\tau_h) \neq 0$, make transfers so that $u(\tau_h) = 0$. If instead $\tau_h > \hat{\tau}^*$, if $u(\tau_h) \neq V_h^{p*}(\tau_h) - V_h^{y*}$, make transfers until $u(\tau_h) = V_h^{p*}(\tau_h) - V_h^{y*}$. Similarly, we have that $u_f(\tau_h, \tau_l) = u(\tau_h)$ if $\tau_h \geq \widetilde{\tau}_h^*$ and $u_f(\tau_h, \tau_l) = 0$ otherwise. Make transfers on the right-hand side so that $u(\tau_h) = 0$ or $u(\tau_h) = V_h^{p*}(\tau_h) - V_h^{y*}$, depending on which case is relevant. Then, we can rewrite the condition above as

$$U_l^o(\tau_l) + u(\tau_l) \geq U_l^y + u_f(\tau_l). \tag{91}$$

Finally, suppose first that $\tau_l \leq \hat{\tau}^*$. If $u(\tau_l) \neq 0$, make transfers to or from l so that $u^l(\tau_l) = 0$. Suppose now that $\tau_l > \hat{\tau}^*$. If $u(\tau_l) \neq V_h^{p*}(\tau_l) - V_h^{y*}$, make transfers to or from l so that $u(\tau_l) = V_h^{p*}(\tau_l) - V_h^{y*}$. Similarly, make transfers on the right-hand side so that $u_f(\tau_l) = 0$ or $u_f(\tau_l) = V_h^{p*}(\tau_l) - V_h^{y*}$, depending on which case is relevant. Then, we can rewrite the condition above as

$$U_l^o(\tau_l) \geq U_l^y, \tag{92}$$

which by construction is equivalent to $V_l^{o*}(\tau_l) \geq V_l^{y*}$. But this is also a necessary condition for the retention of type τ_l in a competitive equilibrium given thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$. Thus, condition (84) is compatible with a decentralized equilibrium with thresholds $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$.

It is possible to replicate this argument for the other two conditions (i.e., (85) and (86)), and similarly show that none of these conditions impose restrictions on the equilibrium. The steps are tedious but simple; we omit them here for brevity.

We then conclude that, for any given efficient allocation $\left(\tau_l^{\#*}, \underline{\tau}_l^*, \underline{\tau}_h^*\right)$, it is possible to construct prices (i.e., wages) that support this allocation as a decentralized equilibrium. Because we showed earlier that the decentralized equilibrium is unique, then this equilibrium must be efficient. ■

The intuition for this result is straightforward. As there are no labor market frictions, perfect competition for talent implies that the allocation of managers to firms is efficient. Thus, the only potential source of inefficiency is the choice between the retention of an old manager and the hiring of a young manager. Hiring a young manager is a potential source of externalities, as everyone learns about the talent of a young manager, which increases the number of options available to all players. However, because a firm that hires a young manager can extract all of the manager's surplus by charging a negative wage, and because

Bertrand competition implies that poachers obtain zero net surplus from their poaching activity, a firm extracts all of the expected surplus from its decision to hire a young manager. Thus, the firm internalizes all of the potential costs and benefits of such a decision, and thus the firm's optimal private decision is also socially optimal.

Although it is not surprising that under symmetric learning the first-best outcome is achieved, we note that, unlike the static case, a hypothetical social planner has to consider two different trade-offs. First, we require an efficient allocation of managers to firms. As discussed above, the social planner would then choose the poaching threshold $\tau^{\#}$ by trading off the loss in firm-specific skills and the gain from assigning a manager to a more productive firm. Second, the social planer must find the optimal rate of talent discovery. The social planner chooses the retention threshold $\underline{\tau}_i$ by trading off the loss in firm-specific skills and the gain from sampling a young manager and learning about the manager's type in the subsequent period.