



# Contractual structure in agriculture with endogenous matching<sup>☆</sup>

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## ARTICLE INFO

### Article history:

Received 1 December 2011

Received in revised form 6 May 2013

Accepted 22 May 2013

Available online 30 May 2013

### Keywords:

Tenancy

Sharecropping

Endogenous matching

## ABSTRACT

We analyze optimal contractual forms and equilibrium matching in a double-sided moral hazard model of sharecropping similar to Eswaran and Kotwal (1985). We show that, with endogenous matching, the presence of moral hazard can reverse the matching pattern relative to the first best, and that even if sharecropping is optimal for an exogenously given pair of agent types, it may not be observed in equilibrium with endogenous matching. The economy with endogenous matching features less sharecropping compared to an economy with agent types drawn at random from the same distribution. This suggests that studies of agency costs in sharecropping may underestimate their extent if focusing only on the intensive margin and ignoring the extensive margin.

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## 1. Introduction

The choice of contractual form in agriculture, in particular, the choice between wage, fixed rent, or sharecropping contracts has important implications for productive efficiency. Alternative theories of choice of contractual form generate sharply different predictions about the efficiency of sharecropping and the potential effects of policies such as tenancy or land reform.<sup>1</sup> For example, in models that involve moral hazard and wealth constraints (e.g., Banerjee et al., 2002 or Mookherjee, 1997), a tenancy or land reform program that gives a higher share or transfers ownership rights to tenants would unambiguously improve productive efficiency. On the other hand, Eswaran and Kotwal (1985) develop a model of contractual choice where sharecropping is a way of pooling non-contractible inputs of landowners and tenants, and in the context of this model, a tenancy reform would reduce rather than increase productive efficiency by limiting the efficient division of labor.

<sup>☆</sup> We would like to thank Sandeep Baliga, Kaniska Dam, Priyanka Pandey, Canice Prendergast, Eric Maskin, Martin Wittenberg, and especially two anonymous referees for helpful feedback, and Devaki Ghose for research assistance. The first author would like to thank the Institute for Advanced Study, Princeton for hospitality during the academic year 2000–01 when the first draft of this paper was written. A more recent version was circulated as “Contractual Structure and Endogenous Matching in Partnerships”, C.E.P.R. Discussion Paper 8298, 2010. The second author thanks the Social Sciences and Humanities Research Council of Canada.

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<sup>1</sup> See Singh (1989) for a review of the literature on sharecropping tenancy and Banerjee (1999) for a review of the literature on land reform. More recent empirical contributions on the efficiency effects of tenancy reforms include those of Banerjee et al. (2002) and Bardhan and Mookherjee (2011).

These alternative theories, usually, take the pattern of matching between landlords and tenants (and land quality and tenant ability) as exogenously given. A more recent literature has started to pay attention to the important issue of endogenous matching and its implications for the observed distribution of contractual forms and efficiency (e.g., Akerberg and Botticini, 2002; Alonso-Pauli and Perez-Castrillo, 2012; Dam and Perez-Castrillo, 2006; Legros and Newman, 1996, 2002, 2007; Serfes, 2005).<sup>2</sup>

We contribute to the literature on contractual choice in agriculture by introducing endogenous matching in a double-sided moral hazard model similar to the classic Eswaran and Kotwal (1985) model. We extend and recast their analysis in an equilibrium matching setting and show that selection effects matter for contractual choices. For example, we show that if sharecropping is optimal for an exogenously given pair of types it may not be observed in equilibrium with endogenous matching. We compare two economies, both with the same distribution of types of landlords and tenants (e.g., ability levels), in terms of the likelihood of observing sharecropping – one economy where matching is endogenous, and one where landlord and tenant types are drawn at random. We show that sharecropping is less likely with endogenous matching.

We model the interplay of two economic forces in the context of organizing agricultural production, namely incentive provision under double-sided moral hazard and matching. The choice of contractual form is influenced by these two important factors. On the one hand, it is shaped by incentive considerations since, more often than not in reality, the characteristics or actions of contract parties are unobservable and need to be revealed or implemented through constrained-optimal

<sup>2</sup> We discuss this literature and our contribution with respect to it below.

contract design. On the other hand, the organizational structure depends on the relative importance of the inputs that contracting parties bring in. By pooling various inputs organizations can exploit specialization and absolute advantage.<sup>3</sup> In general, the gains from specialization have to be balanced against the agency costs stemming from incentive problems.

To fix ideas, suppose agricultural production depends on the effort level and ability (henceforth also called the agent's *type*) of each of two parties, the landlord and the tenant. Individuals differ in their types. As *Eswaran and Kotwal (1985)* put it, sharecropping can be viewed as a partnership in which each partner provides the unmarketed input in which he is better endowed or has an absolute advantage. For example, landowners may have an absolute advantage in managerial skills related to technology and market information while tenants may have an absolute advantage in labor supervision. Sharecropping pools ability but is subject to agency problems in effort provision. To give more incentives to one party means giving less incentives to the other. If ability and effort are complements in production, the higher the relative ability of one agent, the higher should be his output share, and consequently, the lower is the share of his partner. This means that he might be optimally matched with a low-ability agent. However, this undermines the gains from trade due to specialization, since a match with a high-ability person would produce more output. If the incentive costs outweigh the gains from specialization, sharecropping may not be observed in equilibrium and the high-ability agent may instead simply hire a low-ability agent from the market to work for him or produce on his own (corresponding to fixed-wage or fixed-rent contracts). That is, simply pooling ability is not a sufficient argument for sharecropping — sufficiently strong complementarities between the qualities of the supplied inputs (the agents' abilities) are also needed, otherwise with endogenous matching sharecropping may not be observed in equilibrium.

We show that the presence of double-sided moral hazard combined with weak complementarity in input quality in production leads to the possibility of agent types being strategic substitutes in joint surplus and to a reversal of the equilibrium matching pattern relative to the first best, from positive to negative assortative. In contrast, without incentive problems, even a small amount of complementarity between the qualities of inputs implies positive assortative matching. This means that when individuals are heterogeneous in their types, allowing for endogenous matching between landlords and tenants can affect the optimal choice of contractual form (sharecropping vs. sole production in fixed rent or wage contract) relative to the setting of *Eswaran and Kotwal (1985)* where isolated landlord–tenant pairs are considered. Specifically, there exist situations in which sharecropping is the optimal contractual form for exogenously given pairs of types but, with endogenous sorting, the landlord or tenant performs both tasks in equilibrium. This has the implication that empirical studies on agency costs (e.g., see *Chiappori and Salanie, 2003* for a review of the contracting literature, including sharecropping) may underestimate their full extent by focusing on the intensive margin and ignoring the extensive margin. This is because sharecropping may not arise at all and the consequent welfare loss in terms of efficient matching between landowners and tenants, as opposed to the loss of efficiency due to moral hazard when sharecropping is actually observed.<sup>4</sup>

In our theoretical analysis we use the general approach and results from *Legros and Newman (2002)*, who study matching in transferable-utility economies with or without market imperfections and provide weaker conditions for monotone matching than the standard sub- and

super-modularity conditions from the earlier literature (a companion paper, *Legros and Newman, 2007*, analyzes the non-transferable utility case). *Legros and Newman* also give examples of payoff functions in settings with market imperfections in which the equilibrium matching pattern is reversed (e.g., from positive to negative assortative) by changing a model parameter which could be interpreted as the “strength” of the market imperfection. In our application to contractual choice in agriculture we echo this by showing that the fact that a market imperfection exists, here arising endogenously from the moral hazard problem, can reverse the matching pattern relative to the first best by affecting the optimal effort choices made by the match parties.

The classical optimal sorting literature (e.g., *Becker, 1993; Sattinger, 1975*) has argued that the sub- or super-modularity of the payoff function is closely related to whether the agents' characteristics over which matching occurs (here, the types) are substitutes or complements. With substitutes, negative assortative matching arises, while with complements positive assortative matching results. As pointed out by *Legros and Newman (2002)*, however, the link between the degree of substitutability of agents' characteristics and the modularity of the payoff function may break down in more general environments featuring credit market imperfections (e.g., *Legros and Newman, 1996; Sadoulet, 1998*); restrictions on how output is shared (e.g., *Farrell and Scotchmer, 1998*); or particular production technologies (e.g., *Kremer and Maskin, 1996*). Our application to contract choice in agriculture with double-sided moral hazard fits into this list.

This paper is also related to the empirical literature on agricultural tenancy and the role of matching. *Ackerberg and Botticini (2002)* pose the puzzle that output shares of tenants cultivating risky crops tend to be higher than those of tenants cultivating less risky crops, which is contrary to the predictions of standard models of sharecropping under moral hazard in which greater output variance would lead to higher demand for insurance by the risk-averse tenant and, therefore, lower crop share (e.g., *Stiglitz, 1974*). *Ackerberg and Botticini* argue that risk-neutral tenants are more likely to select riskier crops (and therefore, in equilibrium, their observed shares could be higher) and provide empirical evidence of matching between landowners and tenants consistent with this hypothesis. The argument has been formalized in *Serfes (2005)*.

More generally, our work is related to previous theoretical literature on organizational choice with endogenous matching. *Legros and Newman (1996)* build a general equilibrium model with moral hazard, endogenous firm formation by agents heterogeneous in wealth, and a credit market with costly state verification problem. They show that, when borrowing is not costly, firms are efficiently organized and essentially consist of one type of agents but, when the information problem is sufficiently severe, firm organization and production efficiency depend on the distribution of wealth. *Dam and Perez-Castrillo (2006)* study a principal–agent model with risk-neutral principals and agents who are heterogeneous in initial wealth, moral hazard in effort choice, and limited liability. They analyze the simultaneous determination of stable pairwise matching and the contracts signed and show that the contracts in a stable outcome of their economy are more efficient than principal–agent contracts in which the principals obtain the full surplus.<sup>5</sup> *Alonso-Pauli and Perez-Castrillo (2012)* study endogenous matching between heterogeneous managers and shareholders who can adopt an incentive-based contract or a “code of best practice” in a setting with moral hazard. They show that firms with the best projects tend to adopt a code when managers are not too heterogeneous while, when projects are similar, the best managers tend to be hired via incentive

<sup>3</sup> As in *Eswaran and Kotwal (1985)*, absolute advantage is key to specialization by task in sharecropping. Unlike in trade models, comparative advantage without absolute advantage (e.g., one party better at both tasks) does not lead to a partnership.

<sup>4</sup> This fits Bastiat's general dictum that an economist must take into account both what is seen and what is not seen (*Bastiat, 1873*).

<sup>5</sup> *Besley and Ghatak (2005)* consider a similar setting but allow agents to be intrinsically motivated and principals and agents to differ in terms of their ‘mission’ preferences which affect the level of agent motivation. They analyze the effect of endogenous matching on efficiency, and show that it may reduce incentive pay since agent and principal mission preferences are better aligned.

contracts. They also show the possibility of reversal in the optimal contract compared to the case of an exogenous principal–agent pair. Our paper focuses exclusively on incentive problems with double-sided moral hazard and two-sided matching. In our model there are no limited-liability constraints and utility is fully transferable. Comparative advantage and not wealth is thus the key source of heterogeneity that we focus on.<sup>6</sup>

The rest of the paper proceeds as follows. Section 2 presents the model and characterizes the equilibrium matching and joint surplus maximizing organization forms that will be observed in the first-best and under double-sided moral hazard allowing for a varying degree of complementarity in agents' characteristics. We demonstrate the possibility for matching pattern reversal between the first-best and under moral hazard. Several empirically testable implications of the model are discussed in Section 3. The final section makes some concluding observations.

## 2. Model

Output depends on the effort levels,  $e_1 \geq 0$  and  $e_2 \geq 0$ , supplied in two distinct tasks. Following Eswaran and Kotwal (1985) we call them 'management' and 'effective labor effort' (which may include supervision of wage labor) or more simply, 'task 1' and 'task 2'. Production also requires one unit of land. There are two kinds of economic agents: those who are better at task 1 and those who are better at task 2. This will be formalized below. Suppose landlords are good at supplying managerial inputs (task 1) while tenants (who are assumed to be landless) are good at labor supervision (task 2). Each landlord has a single unit of land. A landowner (task 1 specialist) could work on both tasks on his own although he is not very good at task 2. Alternatively, he can rent out the land to a tenant who can work on both tasks in exchange for fixed rent. Finally, the landowner can enter a contract with a task 2 specialist in a sharecropping arrangement. This allows gains from specialization to be realized but is subject to incentive problems when efforts are unobservable.

The type of a task 1 specialist (the landlord) is  $\theta_1 \geq 1$  and the type of a task 2 specialist (the tenant) is  $\theta_2 \geq 1$ . One possible interpretation of  $\theta_1$  and  $\theta_2$  that we use here is ability in performing the task in which a person specializes. Both  $\theta_1$  and  $\theta_2$  take values on the set  $\Theta = [1, \theta^{max}]$  where  $\theta^{max} > 1$  can be arbitrarily large but finite. The types,  $\theta_1$  and  $\theta_2$  are observable and contractible. In contrast, the effort levels  $e_1$  and  $e_2$  may or may not be contractible. All agents are risk-neutral and there are no limited liability or wealth constraints.<sup>7</sup> This means that, if it is efficient for the task 2 specialist to become the full residual claimant (e.g., through a fixed-rent contract), then there is nothing preventing that contractual form being chosen, in contrast to many models of tenancy where a rent-extraction vs. incentives trade-off exists due to a limited liability constraint, and therefore, even if fixed-rent contracts are efficient and can achieve the first-best (e.g., when both parties are risk-neutral) they will not be chosen.<sup>8</sup>

Each agent has an additively separable payoff in their consumption,  $c$  and their effort in one or both tasks,  $u = c - \gamma(e_1) - \gamma(e_2)$ . The disutility cost of providing effort is assumed to be quadratic,  $\gamma(e_1) = \frac{1}{2}e_1^2$  and  $\gamma(e_2) = \frac{1}{2}e_2^2$ . The reservation payoff of each agent is set to 0. If a task 1 specialist of type  $\theta_1$  and a task 2 specialist of type  $\theta_2$  engage in joint production, output is:

$$q^J(e_1, e_2) = \alpha\theta_1\theta_2 + \theta_1e_1 + \theta_2e_2 + \varepsilon \tag{1}$$

where  $\varepsilon$  is a random variable with  $E(\varepsilon) = 0$  and finite variance and  $\alpha$  is a parameter capturing the extent of the types' complementarity in production.

Assume that if a task  $i$  ( $i = 1, 2$ ) specialist of type  $\theta_i$  undertakes production on her own she can perform task  $k$  ( $k = 1, 2$  with  $k \neq i$ ) just as well as task- $k$  specialist of the lowest possible type,  $\theta_k = 1$ . For example, a landlord (task 1 specialist) can produce on his own but will be only as productive as the lowest-type tenant at task 2. This puts a natural lower bound on  $\theta_i$  – otherwise there is no advantage in producing jointly. Given this, if a task  $i$  specialist of type  $\theta_i$  undertakes sole production (that is, she works on both tasks either using own or rented land), output is:

$$q^S(e_i, e_k) = \alpha\theta_i + \theta_i e_i + e_k + \varepsilon. \tag{2}$$

Using Eqs. (1) and (2), we can write the production function, for any organization type between agents of types  $\theta_i$  and  $\theta_k$  in  $\Theta$  as:

$$q(e_i, e_k) = \alpha\theta_i z_k + \theta_i e_i + z_k e_k + \varepsilon \tag{3}$$

where  $z_k = \theta_k$  if production is done jointly and  $z_k = 1$  if the task  $i$  specialist is sole producer, for  $i, k = 1, 2$  with  $i \neq k$ . Assume  $\alpha \geq 0$ .<sup>9</sup> If  $\alpha = 0$  agents' types are substitutes in production, while their complementarity increases in  $\alpha$  for  $\alpha > 0$ .

Suppose that each of  $\theta_1$  and  $\theta_2$  can take just two values,  $\sqrt{a}$  and  $\sqrt{b}$  in the set  $\Theta$  with  $a \geq b$ . Hereafter, with some abuse of notation, we refer to  $a$  as the "high type" and to  $b$  as the "low type". We assume transferable utility, i.e., agents can make any side payments or transfers among each other. Therefore, there are no restrictions on the level of the wage or fixed rent coming from wealth or liquidity constraints, unlike in models with limited liability. Below, we use the qualifier "optimal" to refer to organization/contract forms that maximize expected joint payoff (total surplus). To determine the equilibrium matching pattern in our economy, we follow Legros and Newman (2002), henceforth LN (2002) and use the core as our equilibrium concept.

There are continuum of measure one of task-1 specialists and continuum of measure one task-2 specialists. Since land is essential in production, and since there are measure one land units in total, following LN (2002), we therefore define an equilibrium in our model economy as consisting of a pairwise matching correspondence between landlords and tenants and payoff allocation for each pair (see LN (2002, p. 929) for details). Since utility is transferable, we only keep track of the total expected payoff in each pair which, as well as aggregate surplus, will be maximized in equilibrium – see LN (2002), Proposition 1. Assume for now that there is an equal measure,  $1/2$  of each type,  $a$  and  $b$  for both the landlords and tenants (this will be relaxed in Section 2.3.2).

Given our assumption of two possible types for each specialist,  $a \geq b$ , let  $J(i, i)$ ,  $i = a, b$  denote the expected total surplus when a landlord and a tenant both from type  $i$  produce jointly; let  $J(a, b)$  denote the expected total surplus when a landlord and tenant of different types produce jointly; and let  $S(i)$ ,  $i = a, b$  denote the expected total surplus when an agent of type  $i$  engages in sole production (with own or rented land). We can thus define the possible values for expected total surplus in any landlord–tenant pair as follows:

$$\begin{aligned} \pi(a, a) &\equiv \max\{J(a, a), S(a)\} \\ \pi(a, b) &\equiv \max\{J(a, b), S(a)\} \\ \pi(b, b) &\equiv \max\{J(b, b), S(b)\}. \end{aligned} \tag{4}$$

<sup>6</sup> A related recent paper by Dam (2010) analyzes a two-sided matching model involving entrepreneurs who vary in wealth and financial intermediaries who vary in monitoring ability and shows that, with endogenous matching, more efficient monitors lend to borrowers with lower wealth.

<sup>7</sup> We briefly discuss the case of wealth constraints at the end of Section 2.3.1 below.

<sup>8</sup> See for example, Banerjee et al. (2002) and Mookherjee (1997).

<sup>9</sup> We rule out the case  $\alpha < 0$  in which higher partner type reduces the joint payoff. In this case, if  $\alpha$  is sufficiently negative, sole production always dominates joint production.

In writing the expression for  $\pi(a,b)$  in Eq. (4) we used the fact that  $q^J$  in Eq. (1) is symmetric in  $\theta_1$  and  $\theta_2$  and that  $S(a) \geq S(b)$ , i.e., sole production by the low-type in an  $(a, b)$  pair is never optimal – a low-type landlord can always rent his land to the high-type tenant while a high-type landlord can produce alone (the low-type tenant he is matched to can be thought of sitting idle). To see this, note that

$$S(i) \equiv \max_{e_i, e_k} E(q^S(e_i, e_k)) - \gamma(e_i) - \gamma(e_k)$$

which, given the assumed functional forms for output  $q^S$  in Eq. (2) and effort costs  $\gamma(\cdot)$ , yields optimal effort choices  $e_i^S = \theta_i$  and  $e_k^S = 1$ , and so the maximized expected joint surplus with sole production is

$$S(a) = \alpha\sqrt{a} + \frac{1}{2}(a + 1) \text{ and } S(b) = \alpha\sqrt{b} + \frac{1}{2}(b + 1). \tag{5}$$

Clearly,  $S(a) \geq S(b)$  whenever  $a \geq b$ .

The expected total surplus under joint-production,  $J(a, a)$ ,  $J(a, b)$  and  $J(b, b)$  will be determined below since its form depends on the contractibility of effort.

2.1. First best

As a benchmark, we first characterize the first best, that is, the case when both efforts  $e_1$  and  $e_2$  are contractible. Under joint production and no incentive problems,  $e_1$  and  $e_2$  are chosen by the agents to maximize total expected surplus, i.e., we have,

$$J(i,j) \equiv \max_{e_1, e_2} E(q^J(e_1, e_2)) - \gamma(e_1) - \gamma(e_2)$$

which, given our assumed functional forms for output  $q^J(\dots)$  and effort costs  $\gamma(\cdot)$ , yields first-best optimal efforts  $e_1^J = \theta_1$  and  $e_2^J = \theta_2$  where  $\theta_i, i = 1, 2$  denotes the type of the type- $i$  specialist in the pair. Given that there are no agency problems by assumption, we would expect joint production to dominate sole production in this case, due to assumed gains from specialization and absolute advantage. Plugging in, and using the superscript *FB* to denote the first best, we obtain:

$$J^{FB}(a, a) = \alpha a + a, \quad J^{FB}(b, b) = \alpha b + b$$

$$\text{and } J^{FB}(a, b) = \alpha\sqrt{ab} + \frac{1}{2}(a + b).$$

We have the following straightforward result:

**Lemma 1.** *In the first best, joint production is the joint-surplus maximizing organization form for any agent pair. That is,  $J^{FB}(i, i) \geq S(i)$  for  $i = a, b$  and  $\pi^{FB}(i,j) = J^{FB}(i,j)$  for  $(i,j) \in \{(a,a), (a,b), (b,b)\}$ .*

**Proof.** We simply check all three cases.  $J^{FB}(a,a) \geq S(a)$  is equivalent to  $\alpha a + a \geq \alpha\sqrt{a} + \frac{1}{2}(a + 1)$  which is true since  $a \geq 1$ . Similarly for  $b$ . We also have  $J^{FB}(a, b) = \alpha\sqrt{ab} + \frac{1}{2}(a + b) \geq \alpha\sqrt{a} + \frac{1}{2}(a + 1) = S(a)$  since  $b \geq 1$ . ■

Using Lemma 1, we next characterize the equilibrium matching pattern in the first best.

**Proposition 1.** First best.

*Suppose agent efforts are contractible. Then, for any  $\alpha \geq 0$ , in equilibrium there is joint production with positive assortative matching of the ‘segregation’ type,  $(a, a)$  and  $(b, b)$ .*

**Proof.** We already showed in Lemma 1 that  $\pi^{FB}(i,j) = J^{FB}(i,j)$  for  $(i,j) \in \{(a,a), (a,b), (b,b)\}$ . Following LN (2002), define the surplus function,

$\sigma^{FB}$  as:

$$\sigma^{FB}(a, b) \equiv \max \left\{ 0, \pi^{FB}(a, b) - \frac{1}{2}(\pi^{FB}(a, a) + \pi^{FB}(b, b)) \right\}$$

$$= \max \left\{ 0, J^{FB}(a, b) - \frac{1}{2}(J^{FB}(a, a) + J^{FB}(b, b)) \right\}$$

$$= \frac{1}{2} \max \left\{ 0, 2\alpha\sqrt{ab} + a + b - \alpha a - a - \alpha b - b \right\}$$

$$= \frac{1}{2} \max \left\{ 0, -\alpha(\sqrt{a} - \sqrt{b})^2 \right\}$$

$$= 0 \text{ for all } a, b \text{ and } \alpha \geq 0.$$

The proposition statement then follows directly from Proposition 4 in LN (2002, p. 932). ■

As pointed out in LN (2002), the surplus function  $\sigma^{FB}$  being identically zero is a much weaker condition for segregation than strict super-modularity of the joint payoff function,  $\pi^{FB}$  (note, super-modularity does not hold here for  $\alpha = 0$ ).

The intuition for the result in Proposition 1 is similar to that in Becker’s model of the marriage market – when agents’ types are complements in the joint payoff, positive assortative matching (henceforth, PAM) results (Becker, 1993, ch. 4). Clearly, this would also hold when there are more than two types (assuming measure consistency) – if there were any heterogeneous pairs, a profitable deviation to segregation PAM supported by appropriate transfers would be possible.

2.2. Double-sided moral hazard

Now suppose the efforts  $e_1$  and  $e_2$  are non-contractible, that is, joint production has the advantage of exploiting gains from specialization (the fact that both agents have types  $\theta_1$  and  $\theta_2$  strictly larger than 1) but this has to be traded-off against the agency costs that arise from the double-sided moral hazard problem. Consistent with the tenancy literature, we restrict attention to linear contracts of the form  $(s,R)$  where  $s \in [0,1]$  represents the output share of the task 2 specialist (the ‘tenant’) and  $R$  represents a transfer from the task 1 specialist (the ‘landlord’) to the tenant which could be negative (a ‘rent’) or positive (a ‘wage’).<sup>10</sup> Using the superscript *MH* to denote this setting with moral hazard, given  $(s,R)$  the agents’ optimal effort choices are:

$$e_1^{MH} = \arg \max_{e_1} \left( (1-s)(\alpha\theta_1\theta_2 + \theta_1 e_1 + \theta_2 e_2) - R - \frac{e_1^2}{2} \right) = (1-s)\theta_1$$

$$e_2^{MH} = \arg \max_{e_2} \left( s(\alpha\theta_1\theta_2 + \theta_1 e_1 + \theta_2 e_2) + R - \frac{e_2^2}{2} \right) = s\theta_2.$$

Note that these effort levels are (weakly) lower than the corresponding first-best levels. Expected indirect total surplus when agents with types  $\theta_1$  and  $\theta_2$  produce jointly (sharecrop) using contract  $(s,R)$  is then

$$J^{MH}(s) = \alpha\theta_1\theta_2 + \theta_1 e_1^{MH} + \theta_2 e_2^{MH} - \frac{(e_1^{MH})^2}{2} - \frac{(e_2^{MH})^2}{2} =$$

$$= \alpha\theta_1\theta_2 + \theta_1^2 \left( (1-s) - \frac{1}{2}(1-s)^2 \right) + \theta_2^2 \left( s - \frac{1}{2}s^2 \right). \tag{6}$$

<sup>10</sup> See Bhattacharya and Lafontaine (1995) for sufficient conditions under which the optimal sharing rule under double-sided moral hazard can be represented by a linear contract without loss of generality. These conditions are satisfied in our setting. Additionally, we assume that there is no budget-breaker and the first-best cannot be achieved as in Holmstrom (1982).



The optimal sharing rule,  $s^*$  (the tenant's crop share) is chosen to maximize indirect joint surplus (Eq. (6)), which incorporates the parties' incentive compatible efforts,  $e_i^{MH}$ .<sup>11</sup> It is:

$$s^* = \frac{\theta_2^2}{\theta_1^2 + \theta_2^2}. \tag{7}$$

Intuitively, the optimal tenant share  $s^*$  is the greater the tenant's type relative to the landlord's type. This follows from the fact that type (e.g., ability) and effort are complements in the production function, and so the higher is one's type, the greater is effort for the same share. The marginal gain from raising the share to elicit greater effort is therefore increasing in the agent's type. Since our assumptions on the range of  $\theta_i$  imply  $s^* \in (0,1)$ , we interpret the optimal contract in joint production under moral hazard as a sharecropping agreement.

Substituting  $s^*$  in Eq. (6) and simplifying, we obtain the following expression for the maximized joint surplus under sharecropping by agents with types  $\theta_1$  and  $\theta_2$ ,

$$J^{MH}(\theta_1, \theta_2) = \alpha\theta_1\theta_2 + \frac{1}{2} \left\{ \theta_1^2 + \theta_2^2 - \frac{\theta_1^2\theta_2^2}{\theta_1^2 + \theta_2^2} \right\}. \tag{8}$$

Note that, because of the incentive problem, the modularity of indirect joint surplus under moral hazard,  $J^{MH}$  cannot be inferred directly from the substitutability or separability of agent characteristics in the production function  $q(\dots)$  as we did in the first-best (or, as in Becker, 1973; Sattinger, 1975). Now it also depends on the endogenous choices by the parties and the sharing rule. The optimal share of each party is function of the magnitude of her type relative to the other party's type. The higher is  $\theta_1$ , the share of the task 1 specialist goes up while the share of the task 2 specialist goes down, and as a result,  $e_1$  goes up and  $e_2$  goes down. The positive effect of an increase in  $\theta_2$  on the share  $s^*$  and  $e_2$  (relative to  $e_1$ ) is therefore lower the higher is  $\theta_1$ . In other words, given that type and effort are complements in production, a high-type agent's ability is best utilized if he also has strong incentives, but the higher the type of the other agent, this will not be the case. This is the intuition for the fact that the types of the agents engaged in sharecropping under moral hazard can be substitutes in joint surplus – see Eq. (9).

Finally, if a task  $i = 1,2$  specialist performs both tasks (sole production), since there are no incentive problems, total surplus is still  $S(i)$  as obtained in Eq. (5) in the previous section.

### 2.3. Matching and organization forms under moral hazard

Calling  $\pi^{MH}(i,j) \equiv \max\{S(i)J^{MH}(i,j)\}$  for all possible type pairs  $(i,j) \in \{(a,a),(a,b),(b,b)\}$  as before, with  $a \geq b \geq 1$  and using Eqs. (5) and (8), the expressions for indirect expected total surplus under moral hazard are:

$$\begin{aligned} \pi^{MH}(a, a) &= \frac{1}{2} \max \left\{ 2\alpha\sqrt{a} + a + 1, 2\alpha a + \frac{3}{2}a \right\} \\ \pi^{MH}(a, b) &= \frac{1}{2} \max \left\{ 2\alpha\sqrt{a} + a + 1, 2\alpha\sqrt{ab} + a + b - \frac{ab}{a+b} \right\} \\ \pi^{MH}(b, b) &= \frac{1}{2} \max \left\{ 2\alpha\sqrt{b} + b + 1, 2\alpha b + \frac{3}{2}b \right\}. \end{aligned} \tag{9}$$

The equilibrium organization forms and matching pattern now depends on the values of  $a, b$  and  $\alpha$ , unlike in the first best. Below, we first analyze the case  $\alpha = 0$  in which we show that the equilibrium displays negative assortative matching (henceforth, NAM) consisting

of heterogeneous agent type pairs  $a, b$  with  $a \geq b$ . We also characterize the joint-surplus maximizing organization form depending on the values of  $a$  and  $b$ . For most of the analysis we assume that the two types differ, i.e.,  $a > b$ . We analyze the special case  $a = b$  separately.

#### 2.3.1. Moral hazard and negative assortative matching

Assume  $\alpha = 0$  in this section. Therefore we have, from Eq. (9):

$$\begin{aligned} \pi^{MH}(a, a) &= \frac{1}{2} \max \left\{ a + 1, \frac{3}{2}a \right\}, \pi^{MH}(a, b) = \frac{1}{2} \left\{ a + 1, a + b - \frac{ab}{a+b} \right\} \\ \text{and } \pi^{MH}(b, b) &= \frac{1}{2} \max \left\{ b + 1, \frac{3}{2}b \right\}. \end{aligned} \tag{10}$$

We have the following result.

**Proposition 2.** NAM under moral hazard.

Let  $\alpha = 0$ . For any  $a > b \geq 1$ ,

$$\pi^{MH}(a, b) > \frac{1}{2} [\pi^{MH}(a, a) + \pi^{MH}(b, b)], \tag{*}$$

that is, the indirect expected joint payoff function  $\pi^{MH}$  is strictly sub-modular and the equilibrium matching is negative assortative (NAM).

**Proof.** There are four cases to consider:

- Case (a)**  $b < a < 2$ . Then, from Eq. (10),  $\pi^{MH}(a, a) = \frac{a+1}{2}$  and  $\pi^{MH}(b, b) = \frac{b+1}{2}$  since  $a, b < 2$ . Also,  $\pi^{MH}(a, b) = \frac{a+1}{2}$  since  $a + 1 > a + b - \frac{ab}{a+b}$  is equivalent to  $a > b^2 - b$ . The latter is true since  $b^2 - b < b$  for  $b < 2$  and so  $a > b > b^2 - b$ . Thus, in Case (a) sole production is optimal in all pairs of types. Inequality (\*) holds since it is equivalent to  $a > b$ .
- Case (b)**  $b < 2 \leq a$ . Then, from Eq. (10), using similar arguments as in Case (a), we have:  $\pi^{MH}(a, a) = \frac{3}{4}a$ ,  $\pi^{MH}(b, b) = \frac{b+1}{2}$  and  $\pi^{MH}(a, b) = \frac{a+1}{2}$ . Thus, (\*) is equivalent to  $a + 1 > \frac{3}{4}a + \frac{b+1}{2}$  or  $\frac{1}{4} + \frac{1}{2} > \frac{b}{2}$ , which is true since  $a \geq 2$  and  $b < 2$ .
- Case (c)**  $2 \leq b < a$  and  $a \leq b^2 - b$ . Then, from Eq. (10),  $\pi^{MH}(a, a) = \frac{3}{4}a$ ,  $\pi^{MH}(b, b) = \frac{3}{4}b$  and  $\pi^{MH}(a, b) = \frac{1}{2}(a + b - \frac{ab}{a+b})$ . In this case for  $a, b$  joint production is surplus-maximizing for all pairs of types. Inequality (\*) is now equivalent to  $a + b - \frac{ab}{a+b} > \frac{3}{4}(a + b)$  which, after simplifying, reduces to  $(a - b)^2 > 0$ .
- Case (d)**  $2 \leq b < a$  and  $a > b^2 - b$ . Then, from Eq. (10),  $\pi^{MH}(a, a) = \frac{3}{4}a$ ,  $\pi^{MH}(b, b) = \frac{3}{4}b$  and  $\pi^{MH}(a, b) = \frac{a+1}{2} > \frac{1}{2}(a + b - \frac{ab}{a+b})$  (see Case (a) for the last inequality). Thus,

$$\begin{aligned} \pi^{MH}(a, b) &> \frac{1}{2} \left( a + b - \frac{ab}{a+b} \right) > \frac{3}{8}a + \frac{3}{8}b \\ &= \frac{1}{2} (\pi^{MH}(a, a) + \pi^{MH}(b, b)) \end{aligned}$$

using the same argument as in Case (c) to show the second inequality.

Overall, we showed that inequality (\*) holds for all  $a > b \geq 1$ . It corresponds to Condition H in LN (2002), which implies perfectly heterogeneous matching in equilibrium (see LN, 2002, p. 933). Since there are only two possible types in our model, inequality (\*) also implies that the indirect joint payoff function  $\pi^{MH}$  is strictly sub-modular, i.e., since  $\pi^{MH}$  is also symmetric, by Proposition 7 in LN (2002), the equilibrium sorting pattern is NAM.■

Comparing with the result from Proposition 1, Proposition 2 implies that the presence of agency costs can reverse the equilibrium

<sup>11</sup> The same optimal sharing rule obtains if we maximize one party's payoff subject to a participation constraint by the other.

matching pattern relative to the first best.<sup>12</sup> Specifically, for  $\alpha = 0$  we showed that PAM obtains in the first best but NAM obtains under moral hazard — the presence of moral hazard not only implies the usual loss of surplus from agency costs but also the opposite sorting pattern. Suppose we have sharecropping in heterogeneous,  $(a, b)$  pairs in equilibrium under moral hazard (see Corollary A below for conditions) and would like to assess the welfare loss relative to the first-best outcome. If we took the  $(a, b)$  pairs as exogenously given, in isolation, then resolving the moral hazard problem would indicate a smaller increase in welfare compared to the gains that would obtain when the parties are allowed to endogenously re-match in PAM pattern, as they optimally would in the first best (Proposition 1). Conversely, if we started from the first-best benchmark, the welfare loss due to moral hazard may be overstated if, keeping the matching fixed, we were to compute the agency cost due to moral hazard. Therefore, agency costs are higher under exogenous matching compared to under endogenous matching.

**Corollary A to Proposition 2.** Optimal organization forms with moral hazard and NAM.

Let  $\alpha = 0$ .

- (a) If  $2 \leq b < a$  and  $a \leq b^2 - b$ , then the optimal organizational form is sharecropping in heterogeneous pairs,  $(a, b)$ . For all other  $a, b$  with  $a > b \geq 1$  the optimal organization form is sole production by the high-type agents with own land (“fixed wage”) or with rented land (“fixed rent”).
- (b) If  $b = a$ , then for  $b < 2$  sole production by all agents is optimal and for  $b \geq 2$  sharecropping by all in homogeneous pairs is optimal.

**Proof.** The results follow directly from the discussion of the four cases listed in the proof of Proposition 2. ■

Proposition 2 and its Corollary A imply that, with double-sided moral hazard and endogenous matching, when joint surplus is sub-modular, sharecropping will be observed in equilibrium only if the types of both parties are: (i) sufficiently large — so that gains from specialization are large, and (ii) relatively similar — since, due to NAM, with type-heterogeneity sole production is more likely to dominate joint production. Specifically, Corollary A, part (a) implies that a necessary condition to observe sharecropping in equilibrium is that the lower type  $b$  be larger or equal to 2. Also, note we can re-write the condition  $a \leq b^2 - b$  as  $a - b \leq b^2 - 2b$ . Since  $b^2 - 2b$  is increasing in  $b$  for  $b \geq 1$ , the smaller is  $b$ , the smaller the type difference  $a - b$  must be for sharecropping to be optimally chosen in equilibrium.

*Discussion and comparison with Eswaran and Kotwal (1985).* Eswaran and Kotwal (1985) study the choice of organization form in agriculture with double-sided moral hazard by taking pairs of agents with certain given types in isolation. We extend their analysis by recasting the basic model in a matching setting and study the choice of optimal organization form and equilibrium sorting pattern with endogenous matching between many agents of different types.

We first derive conditions in our setting analogous to those in EK (1985), for an exogenously given pair of agents (agent 1, a landlord and agent 2, a tenant) with types  $\theta_1$  and  $\theta_2$  in  $\Theta$ , respectively. Maintain the assumption  $\alpha = 0$  for this sub-section. As seen before, there are three

possible organization forms: (i) the agents produce jointly subject to the double-sided moral hazard problem (‘sharecropping’); (ii) Agent 1, the landlord, is sole producer (paying unskilled workers fixed wage); and (iii) Agent 2, the tenant, is sole producer paying the landlord a fixed rent. Since output is increasing in the type in forms (ii) and (iii), it is optimal that the higher-type agent be the sole producer.

The next proposition re-states Eswaran and Kotwal’s (1985) results in our context:

**Proposition 3.** Let  $\alpha = 0$ . An exogenously given pair of agents with types  $\theta_1, \theta_2 \in \Theta$  with  $\theta_1 \geq \theta_2$  would optimally use sharecropping if and only if  $\theta_2 \geq \sqrt{2}$  and  $\theta_1^2 \leq \theta_2^4 - \theta_2^2$ . Otherwise, sole production by the high type,  $\theta_1$  (a fixed rent or fixed wage arrangement) maximizes expected joint surplus. If the agents’ types are equal,  $\theta_1 = \theta_2 = \theta$ , then sharecropping is joint-surplus maximizing if and only if  $\theta \geq \sqrt{2}$  and sole production is joint-surplus maximizing otherwise.

**Proof.** (see Appendix A).

To illustrate the importance of endogenous sorting, suppose we took an isolated single pair of agents of the same type,  $\theta_1 = \theta_2 = \theta$ . By Proposition 3, sharecropping would be optimally chosen if and only if

$$\theta^2 \geq 2. \quad (11)$$

Note that condition (11) is stronger than the corresponding condition for the first best ( $\theta \geq 1$ ) derived in Lemma 1, which is intuitive given the presence of agency costs.

If, as before, there are two types,  $\sqrt{a}$  and  $\sqrt{b}$  in  $\Theta$  that each satisfy Eq. (11), i.e., we have  $a \geq b \geq 2$ , and if two  $(a, a)$  and  $(b, b)$  homogeneous pairs were given exogenously, in isolation, we would conclude that sharecropping would be the only contractual form observed in the economy. For example, if we think of these agent pairs as living in two separate villages, we would expect to observe only sharecropping and no fixed wage or rent in each location. By taking landlord–tenant pairs in isolation and inferring the contractual form they would choose we, however, ignore the possibility that such pair of agents may never match in equilibrium with endogenous sorting. To see this, suppose there are a couple of tenants and a couple of landlords from each type  $a$  and  $b$  — that is, four agents in total. If they can match with any partner they want to, sharecropping may no longer be joint-surplus maximizing even if it were optimal when the pairs  $(a, a)$  and  $(b, b)$  are studied in isolation. Indeed, the sub-modularity of indirect joint surplus under moral hazard shown in Proposition 2 implies that if sharecropping is optimal agent pairs must take the heterogeneous form  $(a, b)$ . Therefore, with endogenous matching, it is no longer the case that sharecropping is joint-surplus maximizing for any  $a$  and  $b$  satisfying Eq. (11), that is,  $a, b \geq 2$ . Instead, Proposition 2 and Corollary A imply that, for any  $a > b \geq 2$ , if the high type is sufficiently productive so that  $a > b^2 - b$  holds, then sole production by the high-type agents achieves higher expected joint surplus than sharecropping (see Proposition 3). For instance,  $a > b^2 - b$  always holds when  $b = 2$ . Hence, with endogenous matching, we would not observe sharecropping in our economy with double-sided moral hazard populated with two agents of type  $b = 2$  and two of type equal to any  $a > 2$ .

Graphically, for any landlord and tenant types  $\theta_1$  and  $\theta_2$  in  $\Theta$ , the  $(\theta_1^2, \theta_2^2)$  parameter space can be divided into three areas depicted on Fig. 1.

- A1 ‘Fixed wage’ area corresponding to the set  $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 > \theta_2^4 - \theta_2^2\}$ .
- A2 ‘Fixed rent’ area corresponding to the set  $\{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 > \theta_1^4 - \theta_1^2\}$ .
- A3 ‘Sharecropping’ area corresponding to the set  $\{(\theta_1, \theta_2) : \theta_1 \geq \theta_2 \text{ and } \theta_1^2 \leq \theta_2^4 - \theta_2^2\} \cup \{(\theta_1, \theta_2) : \theta_2 \geq \theta_1 \text{ and } \theta_2^2 \leq \theta_1^4 - \theta_1^2\}$ .

<sup>12</sup> See also Alonso-Pauli and Perez-Castrillo (2012) and Legros and Newman (1996) for related results and also Kaya and Vereshchagina (2010) who study matching with double-sided moral hazard and show that, depending on whether types and efforts are substitutes or complements, positive or negative assortative matching results.

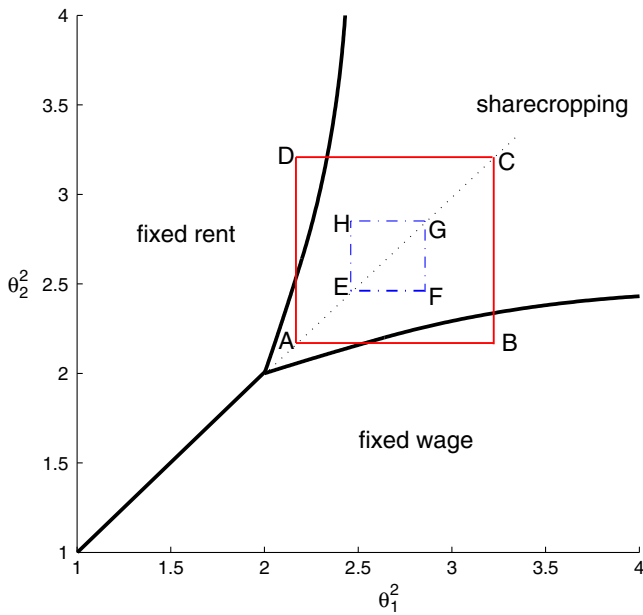


Fig. 1. Contract Forms.

Looking at Fig. 1, sharecropping is optimal (maximizes expected joint surplus) only when both agents have relatively high (here, larger than  $\sqrt{2}$ ) and not too dissimilar types  $\theta_1, \theta_2$ . This is similar to Eswaran and Kotwal (1985). However, Eswaran and Kotwal focus on the contract that maximizes total surplus for any given pair of types, i.e., they characterize the “contractual frontier” for all possible agent pairs but are silent on which matches and what organization forms are expected to arise in equilibrium. We extend and complement their analysis by embedding the double-moral hazard setting in a matching model and show that sharecropping may no longer be optimal for the same parameter values. For instance, on Fig. 1 suppose we take two agents in isolation at either point A (both low-type) or at point C (both high-type). Since Eq. (11) is satisfied at both points A and C, taking each of these pairs in isolation, we would conclude that sharecropping will be observed in both. If we consider, instead, an economy with all four agents together and allowed to match endogenously, the joint expected surplus from two heterogeneous sharecropping pairs, corresponding to points B and D is higher than that from the two homogeneous pairs at points A and C (see Proposition 2). However, for the types corresponding to points B and D, sharecropping in heterogeneous pair is dominated by other contractual forms (fixed wage or fixed rent) in which the high-type agent is a sole producer. That is, with endogenous matching no sharecropping would be observed in this economy.

Contrast the above discussion with another scenario in which we start with two isolated pairs at points E and G on Fig. 1 (i.e., when the difference between the high and low types is smaller than that in A and C). With exogenously given homogeneous pairs at E and G, by Eq. (11), we have sharecropping as joint-surplus maximizing. Unlike in the A, B, C and D example, however, with all four agents together free to match with anyone they like, Proposition 2 and Corollary A imply that we would still observe sharecropping in equilibrium but in heterogeneous pairs corresponding to the points H and F. This has implications about the optimal crop share (we come back to this in Section 3).

*Expected output.* Using our previous results – expression (6) and the discussion right after, it is easy to compute expected total output in equilibrium with double-sided moral hazard depending on the agent types, the parameters  $a$  and  $b$  (we still hold  $\alpha = 0$ ). Under

sole production by type  $k = a, b$ , it is easy to verify that expected total output is  $q_S^{MH}(k, k) = k + 1$ ; in a homogeneous sharecropping pair with types  $k$ , it is  $q_J^{MH}(k, k) = k$ ; and in a heterogeneous sharecropping pair, expected total output is  $q_J^{MH}(a, b) = \frac{a^2 + b^2}{a + b}$ .

Look back at Fig. 1, points A, B, C and D (the case  $2 \leq b < a$  and  $a > b^2 - b$ , calling  $b$  the type at A and  $a$  that at C). With exogenously given agent pairs at points A and C (homogeneous sharecropping pairs), total expected output is  $a + b$ . With endogenous matching, by Proposition 2 and Corollary A, sole production by the high-types (at points B and D) is optimal, yielding total expected output  $2a + 2 - b$  – strictly higher. Note that the difference in expected output between the exogenous and endogenous cases is increasing in the type difference  $a - b$ .

Consider now Fig. 1, points E, F, G and H (the case  $2 \leq b < a$  and  $a \leq b^2 - b$ ). With isolated pairs at points E and G (homogeneous sharecropping pairs), total expected output is  $a + b$ . With endogenous matching, by Proposition 2 and Corollary A, sharecropping in heterogeneous pairs at points F and H is optimal, yielding total expected output  $2(\frac{a^2 + b^2}{a + b})$ . The difference in expected output between the two cases is  $2(\frac{a^2 + b^2}{a + b}) - (a + b)$  which equals  $\frac{(a - b)^2}{a + b}$ . Thus, total expected output is once again higher under endogenous matching and the output difference is increasing in  $a - b$ .

*Wealth constraints*

In the above analysis it was assumed that all three organization forms (sharecropping, fixed wage, fixed rent) are feasible for all types of agents. In particular, if sole production by high-type tenants (“fixed rent” contract) occurs in equilibrium (see Corollary A, part (a)), it was assumed that these tenants have sufficient wealth to pay the rent. However, if they do not have the resources to rent the land (e.g., suppose they are poor and paying the rent up-front is required), then this organization form is infeasible and the next best form has to be chosen, which is either sharecropping or sole production by the low-type landlord. Notice that this introduces an asymmetry in the joint payoff  $\pi^{MH}(a, b)$  since a high-type landlord can still engage in sole production. The equilibrium outcome under endogenous matching can be thus affected by the presence of wealth constraints. The full analysis of the possibility of wealth constraints remains outside the scope of our paper but we address it here by the following example.

*Example A – Wealth constraints*

Suppose the model parameters are as in Case (a) in the proof of Proposition 2, i.e.,  $b < a < 2$ . If wealth constraints are not present, we know from Proposition 2 and Corollary A that sole production (fixed rent or wage) by the high-type landlords or tenants is joint-surplus maximizing. Suppose, however, wealth constraints prevent high-type tenants from sole production (fixed-rent contract). High-type landlords can still produce on their own (fixed wage). Thus, the form of  $\pi^{MH}(a, b)$  now depends on whether the high-type party is a landlord or a tenant. We have  $\pi^{MH}(a, b) = \pi_L^{MH}(a, b) \equiv \frac{1}{2} \max\{a + 1, a + b - \frac{ab}{a+b}\}$  if the high-type agent is landlord (as before), but  $\pi^{MH}(a, b) = \pi_T^{MH}(a, b) \equiv \frac{1}{2}(a + b - \frac{ab}{a+b})$  if the high-type agent is tenant since, with wealth constraints, the only option for a high-type tenant is to sharecrop with a low-type landlord. Hence, in Case (a) in Proposition 2, for  $b < a < 2$ , if

$$\pi_L^{MH}(a, b) + \pi_T^{MH}(a, b) > \pi^{MH}(a, a) + \pi^{MH}(b, b), \tag{12}$$

which is equivalent to  $a + 1 + a + b - \frac{ab}{a+b} > a + 1 + b + 1$ , i.e., to  $a^2 > a + b$ , then there is sole production by high-type landlords and sharecropping between low-type landlords and high-type tenants in equilibrium (instead of sole production by high-types in the case without wealth constraints). For instance, this holds for  $a = 1.9$  and  $b = 1.1$ . If instead,  $b < a < 2$  but  $a^2 < a + b$  (e.g.,  $a = 1.2$  and  $b = 1.1$ ), then Eq. (12) does not hold and NAM no longer obtains in equilibrium for this case.



In contrast, take Case (c) in Proposition 2 ( $2 \leq b < a$  and  $a \leq b^2 - b$ ). Now, the presence of wealth constraints does not affect our previous results since the equilibrium consists of sharecropping in heterogeneous pairs which is still feasible.

2.3.2. Comparison with an economy with randomly drawn types

In this sub-section we still maintain the assumption  $\alpha = 0$  (thus, Proposition 2 applies) but allow the measure of agents to differ by type, as well as by task-specialization, i.e., tenants vs. landlords. We compare the fraction of sharecropping units in our economy (hereafter, “the GK economy”) with that which would obtain in an economy similar to Eswaran and Kotwal (1985), in which agents of different types are drawn at random from type distributions with the same support and probability (hereafter, “the EK economy”).

Formally, let  $\lambda_k \in (0,1)$  denote the fraction of low-type agents (type  $b$ ) who specialize in task  $k = 1,2$  (landlords and tenants respectively). Assume first  $a > b$ ; the case  $a = b$  is analyzed separately at the end of this section. We study the equilibrium matching pattern depending on the values of  $a$  and  $b$  (the support of the type distribution) and the agents’ distribution by type and task-specialization, namely, the values  $\lambda_k$ . There are four relevant cases for  $a$  and  $b$ , as listed in the proof of Proposition 2:

- Case (a):  $b < a < 2$ .
- Case (b):  $b < 2 \leq a$ .
- Case (c):  $2 \leq b < a$  and  $a \leq b^2 - b$ .
- Case (d):  $2 \leq b < a$  and  $a > b^2 - b$ .

The case  $\lambda_1 = \lambda_2 = 1/2$

Begin with the case of equal measure of each type for both tenants and landlords,  $\lambda_1 = \lambda_2 = 1/2$ . This is the parameterization used in Section 2.2 and Fig. 1. Proposition 2 implies that the equilibrium matching pattern is always NAM no matter which organization form is optimally chosen. Thus, we have that in cases (a), (b) or (d) for  $a, b$  (note case (d) corresponds to example A, B, C and D on Fig. 1), then  $\pi^{MH}(a,b) = S(a)$  – sole production by the high-types is optimal. Thus, in equilibrium we have high-type landlords producing alone and high-type tenants producing alone renting land from the low-type landlords. There is no sharecropping. In case (c) for  $a, b$  (which corresponds to example E, F, G and H on Fig. 1) we have  $\pi^{MH}(a,b) = J^{MH}(a,b)$  – sharecropping in heterogeneous pairs obtains in equilibrium due to NAM and the fact that  $S(a)$  is dominated. All agents are involved in sharecropping.

Compare these results with randomly drawing a landlord and a tenant from the corresponding type distributions (the same  $a, b, \lambda_1$  and  $\lambda_2$ ) and looking at them in isolation as in Eswaran and Kotwal (1985). Sharecropping would be chosen if we draw values  $\theta_1$  and  $\theta_2$  that are both in the “triangular” region in the middle of Fig. 1. Thus, in case (a) for  $a, b$  there is no sharecropping in both the GK economy (our model with endogenous matching) and the EK economy (the setting with isolated pairs). In case (b) the probability of sharecropping in the EK economy is  $1/4$  (the probability of drawing two  $a$  types), while, as explained above, in the GK economy the fraction of sharecropping contracts in equilibrium is zero. In case (c) the fraction of sharecropping in both the EK and GK economies equals 1. Finally, in case (d) the probability of sharecropping in the EK economy is  $1/2$  (the probability of drawing two  $a$  types or two  $b$  types), whereas in the GK economy the fraction of sharecropping pairs is zero.

We next look at the general case  $\lambda_1, \lambda_2 \in (0,1)$  and obtain the following result.

**Proposition 4.** *In an economy in which a pair of types is drawn at random (the “EK economy”), there is a (weakly) higher likelihood of sharecropping than in an economy in which agents have types whose measures and supports coincide with the probability distribution over types in the EK economy*

and in which agents can match endogenously (the “GK economy”). This holds for any types  $a, b \in [1, (\theta^{\max})^2]$  with  $a \geq b$  and any probability/measure distribution  $\lambda_1, \lambda_2 \in (0,1)$  over the two types where  $\lambda_i, i = 1,2$  denotes the probability/measure of low-type task- $i$  specialists.

**Proof.** (see Appendix A).

Proposition 4 shows that in an economy in which agents with different types are drawn at random there is a higher likelihood of sharecropping being the optimally chosen (surplus-maximizing) contractual form than in an economy with the same distribution of types but in which there is endogenous matching. This is a consequence of the NAM result shown earlier. Proposition 4 holds for any distribution of types over agent types which further emphasizes the general message that selection effects are crucial to understand the nature of equilibrium contracts.

2.3.3. Moral hazard and positive assortative matching

In Proposition 2 we showed that for  $\alpha = 0$ , for any  $a > b \geq 1$  the inequality  $\pi^{MH}(a, b) > \frac{1}{2}[\pi^{MH}(a, a) + \pi^{MH}(b, b)]$  holds, and hence there is negative assortative matching in equilibrium. By continuity, the same result holds for sufficiently small and positive  $a$ . In this section we show that for  $a$  sufficiently large there can be positive assortative matching (PAM) in equilibrium under moral hazard. We are able to provide sufficient conditions for PAM:

**Proposition 5.** PAM under moral hazard – sufficient conditions.

Suppose the following sufficient conditions hold: (i)  $\alpha \geq 1/2$ ; (ii)  $a > b \geq 2$ ; and (iii)  $a \leq b^2 - b$ . Then

$$\pi^{MH}(a, b) < \frac{1}{2} [\pi^{MH}(a, a) + \pi^{MH}(b, b)] \tag{**}$$

which implies that the equilibrium matching pattern in the economy is positive assortative of the segregation type.

Given  $\alpha \geq 1/2$ , weaker but more complicated forms of the sufficient conditions (ii) and (iii) in the Proposition statement can be derived using Eqs. (15) and (16) from the proof of Proposition 5 in Appendix A. Specifically, for  $\alpha \geq 1/2$ , the left-hand side of Eq. (15),  $(\frac{1}{2} - 1) + 2\alpha(k - \sqrt{k}) \geq \frac{1}{2}k - \sqrt{k} - 1$  which is non-negative for  $\sqrt{k} \geq \frac{1+\sqrt{7}}{3} \approx 1.22$ , i.e.,  $b \geq (1.22)^2$  is a weaker version of condition (ii). Similarly, for  $\alpha \geq 1/2$ , Eq. (16) implies that  $b^2 \geq (a+b)[1 - \sqrt{a}(\sqrt{b}-1)]$  is a weaker version of condition (iii).

Suppose the sufficient conditions (i)–(iii) in Proposition 5 are satisfied. Then, the optimal organization form is sharecropping in homogeneous pairs, ( $a, a$ ) and ( $b, b$ ). Now the logic from the A, B, C and D example depicted in Fig. 1 goes the other way. In particular, if we started with two heterogeneous agent pairs at points B and D in isolation, we would conclude that no sharecropping would be observed. However, with endogenous matching allowed, sharecropping in homogeneous pairs corresponding to points A and C is the equilibrium outcome.

3. Discussion and implications

In this section we discuss some applications and potentially testable implications of the theoretical analysis in Section 2.

1. Share equality and/or uniformity

Our results imply that, with double-sided moral hazard and endogenous matching, the optimal shares observed in sharecropping would typically be equal to or close to  $1/2$ . In the PAM case supposing there



are multiple agents of each type and the conditions given in Proposition 5 hold, then with endogenous matching we would observe only sharecropping pairs between agents of equal types and hence optimal share  $s^*$  from Eq. (7) equal to 1/2 in each.<sup>13</sup> No other sharing rules would be observed. In addition, as long as the lowest type satisfies Eq. (11), all agents would use sharecropping and all will use a share of one-half – a uniform sharing rule across all production units independent of types. Therefore, in the PAM case, our model gives an alternative explanation for why the observed share in most studies of sharecropping is clustered around 50:50.<sup>14</sup>

In the NAM case (see Proposition 2 and discussion afterwards), Eq. (7) implies that the optimal shares in an  $(a, b)$  sharecropping pair are  $\frac{a}{a+b}$  and  $\frac{b}{a+b}$ . From the discussion after Fig. 1, the optimal share when sharecropping is observed in equilibrium would be close to an equal split. The intuition is the same as in Eswaran and Kotwal (1985) – if the difference between agents' types is too high, any gains from specialization are smaller and cannot offset the agency costs. This is an attractive feature of theirs and our model given the predominance of such shares in data. What is new here, is that accounting for endogenous matching makes the dispersion in types for which sharecropping is optimal narrower than if agent pairs were taken in isolation (e.g., see Fig. 1, points E and G vs. points A and C). More specifically, an implication of our model is that the lower the low-type value  $b$ , the lower is the maximum possible deviation between 1/2 and the optimal share with endogenous matching. Indeed, by Corollary A, given  $b$ , the maximum high-type agent share that can be observed in equilibrium is  $s_{max} = \frac{b^2-b}{b^2} = 1 - \frac{1}{b}$ . It is obtained by setting  $a = b^2 - b$ . Obviously, for low values of  $b$  the value  $s_{max}$  is close to 1/2 (e.g., if  $b = 2$ , sharecropping can only be observed for  $a = b$  and then  $s^* = s_{max} = 1/2$ ).<sup>15</sup> Only if the minimum type  $b$  in the economy is large can shares substantially differ from 1/2 in equilibrium.

2. Within vs. across group heterogeneity

Suppose NAM obtains under double-sided moral hazard (e.g., the case  $\alpha = 0$ ). This means that there is more heterogeneity within groups (sharecropping pairs), but less heterogeneity across pairs. In our simple two-type, four-agent example on Fig. 1, the joint surplus in both pairs H and F is the same. While we do not determine the exact individual payoffs, we showed that the optimal output shares in a heterogeneous pair are unequal which would result in general in unequal surplus shares. If, instead, we had positive sorting as in Proposition 5, there is no intra-group inequality but there is inter-group inequality. The endogenous sorting pattern can therefore have implications about inequality in the economy as a whole.

3. Unobserved type

Negative sorting implies that landlord–tenant pairs are heterogeneous in equilibrium (Proposition 2). If type is not directly measured but instead proxied (e.g., by wealth), such heterogeneity is often interpreted as suggestive of risk-sharing – the relatively less risk-averse (richer) party insures the more risk-averse (poorer) party. We provide an alternative explanation for heterogeneous matching which does not rely on risk aversion and insurance motives.

4. Variance of observed shares

A prediction of our model is that, holding other things equal, the observed distribution of crop shares would have greater variance under negative sorting than under positive sorting.

5. Variance of output

As shown earlier, total output in a sharecropping pair of agents with types  $\theta_1$  and  $\theta_2$  under double-sided moral hazard equals:

$$\alpha\theta_1\theta_2 + \frac{\theta_1^4 + \theta_2^4}{\theta_1^2 + \theta_2^2} = \alpha\theta_1\theta_2 + \theta_1^2 + \theta_2^2 - \frac{2\theta_1^2\theta_2^2}{(\theta_1^2 + \theta_2^2)}.$$

Hence, if  $\alpha = 0$  or positive but not too high, like we did with joint surplus  $\pi^{MH}$  (see Proposition 2), it can be shown that total output is sub-modular in  $\theta_1$  and  $\theta_2$ . This implies that the variance of output across production units will be lower with negative sorting compared to with positive sorting. This observation, together with point 4 above, could be potentially useful to infer the likely matching pattern in sharecropping data when types are unobservable to the researcher.

6. Skewness of the income distribution

With positive sorting Kremer (1993) shows that the income distribution is skewed to the right. His model has no agency problems, just a production function with complementarity in abilities,  $y = A\theta_1\theta_2$  which leads to PAM and homogeneous groups (with  $\theta_1 = \theta_2$ ) being formed. Kremer shows that a given ability gap, in our notation  $\sqrt{a} - \sqrt{b}$ , translates into an income gap proportional to  $a - b$ . Hence, with positive sorting, the income distribution in Kremer's economy is more skewed than the ability distribution. Similar logic applies in our setting under moral hazard. Suppose  $\alpha = 0$  and look at sharecropping in an exogenously given  $(\theta, \theta)$  pair with  $\theta \geq \sqrt{2}$ . Total surplus is  $\pi_{pos} \equiv \frac{3}{8}\theta^2$  – see Eq. (8), where  $\theta$  equals  $\sqrt{a}$  or  $\sqrt{b}$ . Assuming equal surplus sharing, each party earns income  $\frac{3}{16}\theta^2$  which is proportional to the square of her ability  $\theta$ , as in Kremer's model. This implies that, in our model, the income difference across agents of different types would be  $\frac{3}{8}(a-b)$  if we took the  $(a,a)$  and  $(b,b)$  pairs in isolation.

The income distribution is in fact more skewed under negative endogenous sorting for the same  $a > b \geq 2$  and  $\alpha = 0$ . Suppose the conditions in Corollary A are satisfied so that heterogeneous sharecropping units are optimal ( $a$  and  $b$  are as in Case (c) in the proof of Proposition 2). From Eq. (8), each such unit has total surplus  $\pi_{neg} \equiv \frac{1}{2}(a + b - \frac{ab}{a+b})$ . The optimal share of the high-type party is  $s^* = \frac{a}{a+b}$ . Thus, assuming no other transfers, she earns expected income  $s^*\pi_{neg}$  while the low-type agent earns  $(1 - s^*)\pi_{neg}$ . Therefore, the difference between the incomes of the high- and low-type agents is:

$$(2s^* - 1)\pi_{neg} = \frac{1}{2}(a-b) \left[ 1 - \frac{ab}{(a+b)^2} \right] > \frac{3}{8}(a-b)$$

since  $\frac{ab}{(a+b)^2} < \frac{1}{4}$ . Thus, the income distribution is more skewed under negative endogenous sorting compared to if agents were exogenously paired into homogeneous  $(a, a)$  and  $(b, b)$  units. Intuitively, the unequal share  $s^*$  further biases income in favor of the high-type agents – incentive reasons dictate that they earn higher share of total surplus, which enhances the rewards to ability in addition to what comes solely from technology. The reversal in equilibrium matching from positive to negative in the presence of moral hazard thus may increase income inequality above and beyond the purely technological complementarity reasons identified by Kremer.

4. Conclusions

We analyze contractual arrangements between risk-neutral landowners and tenants taking as benchmark the classic model of Eswaran and Kotwal (1985) in which sharecropping exploits absolute advantage in two separate tasks but is subject to double-sided moral hazard. We

<sup>13</sup> This result applies more generally, for more than two types – PAM results in 'more homogeneous' matches and thus in optimal shares closer to 1/2 relative to NAM (see Ghatak and Karaivanov, 2011).

<sup>14</sup> See Young and Burke (2001) for a theory based on conformity to custom.

<sup>15</sup> Remember, we must have  $b \geq 2$  in order to have sharecropping in equilibrium and so  $s_{max} \in [\frac{1}{2}, 1)$ .

study the consequences of allowing endogenous matching of landowners and tenants. We show that the presence of moral hazard can reverse the equilibrium matching pattern relative to the first best. Even if sharecropping is optimal (joint-surplus maximizing) for an exogenously given agent pair, with endogenous matching sharecropping may not be observed in equilibrium. An important implication of these results is that, in the distribution of possible contractual forms, sharecropping is less likely to be observed in equilibrium if matching is endogenous. This involves an efficiency loss because some optimal sharecropping arrangements do not form due to agency costs which must be taken into account, in addition to the usual efficiency loss coming directly from agency costs in an exogenously given sharecropping contract.

In the working paper version (Ghatak and Karaivanov, 2011) we investigate conditions that lead to sub- or super-modularity of the indirect joint surplus function under double-sided moral hazard and endogenous matching under general forms for the production and effort cost functions. In the double-sided moral hazard setting, the connection between substitutability of the types and modularity of joint surplus is not clear ex-ante due to the endogenous choice of effort levels and optimal sharing rule. We demonstrate that the modularity of indirect joint surplus depends on the degree of substitutability in the types of agents but the degree of substitutability in agent efforts plays an additional important role.

## Appendix A

**Proof of Proposition 3.** Call  $m = \theta_1^2$  and  $n = \theta_2^2$ . Given Eq. (8), for any  $\theta_1 \geq \theta_2$  joint production would be chosen if and only if it yields higher expected total surplus, i.e., if

$$\frac{1}{2} \left( m + n - \frac{mn}{m+n} \right) \geq \frac{1}{2} (m + 1). \quad (13)$$

Simplifying this, the condition for sharecropping to maximize expected joint surplus is:

$$m \leq n^2 - n. \quad (14)$$

By assumption,  $n \leq m$ . For Eq. (14) to hold, we need  $\theta_2 = \sqrt{n} \geq \sqrt{2}$  (otherwise,  $n^2 - n < n$ ) and  $m \leq n^2 - n$  for any such  $\sqrt{2} \leq \theta_2 \leq \theta_1$ . Conversely, if  $\theta_2 \geq \sqrt{2}$  ( $n \geq 2$ ), then  $n^2 - n \geq n$ , and so, for any  $m \geq n$  with  $m \leq n^2 - n$  (such  $m$  exist for  $n \geq 2$ ) inequality (13) holds, i.e., sharecropping with share  $s^*$  from Eq. (7) achieves higher expected joint surplus than sole production by the high-type agent. If  $\theta_2 = \sqrt{n} < \sqrt{2}$  (which implies  $n^2 - n < n \leq m$ ), or if  $\theta_2 \geq \sqrt{2}$  but  $m > n^2 - n$ , then by Eq. (13) sole production by the high-type agent achieves higher expected total surplus. For the case of equal types, note that if  $\theta_1 = \theta_2 = \theta$  (i.e.,  $m = n = \theta^2$ ) then Eq. (13) is equivalent to  $m \geq 2$  or  $\theta \geq \sqrt{2}$ . ■

**Proof of Proposition 4.** We already showed the proposition result for the case  $\lambda_1 = \lambda_2 = 1/2$  in the main text. We consider the remaining possibilities for  $\lambda_1, \lambda_2 \in (0, 1)$  in turn. We first analyze the unequal types case  $a > b$  in points 1–4 below. The special case of equal types,  $a = b$  is considered separately in point 5.

1.  $\lambda_1 = \lambda_2 = \lambda < 1/2$  – equal measure of each type per agent occupation (landlord or tenant) but less low-type than high-type agents overall.

Proposition 2 still implies NAM in equilibrium in the GK economy. However, in this case there are not enough low-type agents to match all agents in NAM pairs. Hence, the equilibrium sorting pattern is to match as many as possible agents in NAM pairs and the rest,  $1 - 2\lambda$  in homogeneous  $(a, a)$  pairs. By Proposition 2, any other matching will be blocked. Existence of any  $(b, b)$  pairs in equilibrium or of more than measure  $1 - 2\lambda$  of  $(a, a)$  pairs

(which would imply the existence of  $(b, b)$  pairs), leads to a contradiction as two  $(a, b)$  pairs achieve higher joint surplus than two homogeneous pairs  $(a, a)$  and  $(b, b)$ . Therefore, we have the following results depending on the values  $a$  and  $b$  and using analogous arguments as in the case  $\lambda_1 = \lambda_2 = 1/2$ .

In case (a) for  $a$  and  $b$  there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is  $(1 - \lambda)^2$  (the probability of drawing two  $a$  type agents) while in the GK economy the fraction of sharecropping pairs is strictly lower,  $(1 - 2\lambda)^2$  (since in case (b) sharecropping is joint surplus maximizing for  $(a, a)$  pairs, see Corollary A to Proposition 2). In case (c) the probability/fraction of sharecropping in both the EK and GK economies is equal to 1. In case (d) the probability of sharecropping in the EK economy is  $\lambda^2 + (1 - \lambda)^2$  (see Proposition 3), while in the GK economy the fraction of sharecropping pairs is strictly lower,  $(1 - 2\lambda)^2$ , as in case (b).

2.  $\lambda_1 = \lambda_2 = \lambda > 1/2$  – equal measure of each type per agent task-specialization (landlord or tenant) but more low-type than high-type agents overall.

Proceed analogously to point 1 above. Now there are not enough  $a$  agents to match everyone in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest,  $2\lambda - 1$  in homogeneous  $(b, b)$  pairs. In case (a) for  $a$  and  $b$  there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is  $(1 - \lambda)^2$  while in the GK economy the fraction of sharecropping in equilibrium is strictly lower, equal to zero (since sole production is joint surplus maximizing for  $(b, b)$  pairs, see Corollary A). In case (c) the probability/fraction of sharecropping in both EK and GK economies is 1. In case (d) the probability of sharecropping in the EK economy is  $\lambda^2 + (1 - \lambda)^2$  while in the GK economy the corresponding fraction is lower,  $(2\lambda - 1)^2$  by the same arguments as in point 1 above.

3.  $\lambda_1 \neq \lambda_2$  with  $\lambda_1 + \lambda_2 \leq 1$ .

Now there are not enough agents of one of the low-type task-specializations to match all agents in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest,  $1 - \lambda_1 - \lambda_2$  (this could be zero) in homogeneous high-type  $(a, a)$  pairs. Note that  $\lambda_1 \leq 1 - \lambda_2$  and  $\lambda_2 \leq 1 - \lambda_1$  so  $\lambda_1 + \lambda_2$  heterogeneous pairs can always be formed. Proceeding as before, we have that, in case (a) for  $a$  and  $b$ , there is no sharecropping in both the EK and GK economies. In case (b), the probability of sharecropping in the EK economy is  $(1 - \lambda_1)(1 - \lambda_2)$  while in the GK economy the corresponding fraction is strictly lower,  $(1 - \lambda_1 - \lambda_2)^2$ . This is true since  $(1 - \lambda_1)(1 - \lambda_2) > (1 - \lambda_1 - \lambda_2)^2$  is equivalent to  $(\lambda_1 + \lambda_2)(1 - \lambda_1 - \lambda_2) + \lambda_1\lambda_2 > 0$ . In case (c), the probability/fraction of sharecropping in both the EK and GK economies is 1. In case (d), by Proposition 3, the probability of sharecropping in the EK economy is  $\lambda_1\lambda_2 + (1 - \lambda_1)(1 - \lambda_2)$  while in the GK economy the corresponding fraction is strictly lower,  $(1 - \lambda_1 - \lambda_2)^2$ .

4.  $\lambda_1 \neq \lambda_2$  with  $\lambda_1 + \lambda_2 > 1$ .

This is analogous to point 3 above but now there are not enough agents of one of the high-type task-specializations to match all agents in NAM pairs. Hence, the equilibrium sorting is to match as many as possible agents in NAM pairs and the rest,  $\lambda_1 + \lambda_2 - 1$  in homogeneous low-type  $(b, b)$  pairs. In case (a) for  $a$  and  $b$ , there is no sharecropping in both the EK and GK economies. In case (b) the probability of sharecropping in the EK economy is  $(1 - \lambda_1)(1 - \lambda_2)$ , while in the GK economy the corresponding fraction is 0 since by Proposition 2 sole production is optimal for  $(b, b)$  tenant-landlord pair. In case (c) the probability/fraction of sharecropping in both the EK and GK economies is 1. In case (d), the probability of sharecropping in the EK economy is  $\lambda_1\lambda_2 + (1 - \lambda_1)(1 - \lambda_2)$  while in the GK economy the corresponding fraction is strictly lower,  $(\lambda_1 + \lambda_2 - 1)^2$  (this is shown as in point 3 above).

5. The special case  $a = b$  and  $\lambda_1, \lambda_2 \in (0, 1)$ .

In this case the measure of both landlords and tenants is one and all agents have the same type,  $b$ . We thus have  $\pi^{MH}(b, b) = \max\{S(b), J^{MH}(b, b)\} = \frac{1}{2}\max\{b + 1, \frac{3}{2}b\}$ . By Corollary A, part (b), for  $b < 2$  there is sole production by all agents in equilibrium and for  $b \geq 2$  there is sharecropping by all agents in both the EK and GK economies.

**Proof of Proposition 5.** Consider the expressions for expected joint surplus under moral hazard, Eq. (9). Note that  $J^{MH}(k, k) \geq S(k)$  for  $k = a, b$  is equivalent to

$$2\alpha k + \frac{3}{2}k \geq 2\alpha\sqrt{k} + k + 1 \text{ or} \tag{15}$$

$$\left(\frac{k}{2} - 1\right) + 2\alpha(k - \sqrt{k}) \geq 0.$$

It is clear from Eq. (15) that  $k \geq 2$  is a sufficient condition for  $J^{MH}(k, k) \geq S(k)$  for any  $\alpha \geq 0$ . Thus, under condition (ii) in the proposition statement, sharecropping is optimally chosen in homogeneous pairs, i.e.,  $J^{MH}$  achieves the maxima in  $\pi^{MH}(a, a)$  and  $\pi^{MH}(b, b)$ .

Next,  $J^{MH}(a, b) \geq S(a)$  is equivalent to

$$2\alpha\sqrt{ab} + b - \frac{ab}{a+b} \geq 2\alpha\sqrt{a} + 1 \text{ or} \tag{16}$$

$$b^2 \geq (a+b) \left[1 - 2\alpha\sqrt{a}(\sqrt{b}-1)\right].$$

Condition (iii) in the Proposition statement implies that Eq. (16) holds for any  $\alpha \geq 0$ , i.e.,  $J^{MH}(a, b) \geq S(a)$  for any  $a, b \geq 1$  satisfying  $a \leq b^2 - b$ .

The above results ensure that  $J^{MH}(ij) \geq S(i)$  for  $(ij) \in \{(a, a), (b, b), (a, b)\}$  – sharecropping is optimal for any pair of types. Thus,  $\pi^{MH}(ij) = J^{MH}(ij)$  for all  $ij$  and so to show Eq. (\*\*\*) we only need to ensure that

$$J^{MH}(a, b) - \frac{1}{2} [J^{MH}(a, a) + J^{MH}(b, b)] < 0$$

which, using Eq. (9) and simplifying, is equivalent to

$$\frac{1}{2} (\sqrt{a} - \sqrt{b})^2 \left( -\frac{\alpha}{2} + \frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} \right) < 0.$$

Note first that  $\frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} < 1/4$  is equivalent to  $(\sqrt{a} - \sqrt{b})^2 > 0$  which is always true for  $a > b$ . Thus, for any  $\alpha \geq 1/2$  we have  $\frac{(\sqrt{a} + \sqrt{b})^2}{8(a+b)} < 1/4 \leq \frac{\alpha}{2}$  i.e.,  $J^{MH}(a, b) - \frac{1}{2} [J^{MH}(a, a) + J^{MH}(b, b)] < 0$  for any  $a, b \geq 1$  and  $\alpha$  satisfying (i)–(iii).

The above results imply that the surplus function  $\sigma^{MH}(a, b) \equiv \max\{0, \pi^{MH}(a, b) - \frac{1}{2} [\pi^{MH}(a, a) + \pi^{MH}(b, b)]\}$  equals zero for  $a, b, \alpha$  satisfying (i)–(iii). Thus, as in Proposition 1, by Proposition 4 in LN (2002), we obtain PAM of the segregation type in equilibrium. ■

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