6 Online Appendix (Not for Publication)

6.1 Existence of Optimal Deterministic Mechanism

In this appendix, we establish that an optimal deterministic mechanism exists.

Proposition 3 *There exists an optimal deterministic mechanism.*

Proof. It suffices to show that for any $\{r_1, ..., r_L\} \subset \{1, ..., T\}$ there exists an optimal mechanism in which the principal offers the stopping plan where the number of failures born by agents is described by the set $\{r_1, ..., r_L\}$. Notice that an incentive-compatible mechanism $\mathcal{M}$ leads to L mappings $\theta \to (a_i(\theta), b_i(\theta))$ for $i \in \{1, ..., L\}$, where $a_l(\theta)$ stands for for the firm's lump-sum payment and $b_l(\theta)$ for the success bonus of a type $\tau = (\theta, c)$ who chooses a stopping plan with tolerance for r_l failures. Let π^* be the supremum of all payoffs obtained by mechanisms in which the principal offers the action plans $\{r_1, ..., r_L\}$. We will show that there exists a mechanism $\mathcal{M}^*$ that yields the payoff π^* to the principal. For each $n \in \mathbb{N}$, take a sequence of mechanisms $\mathcal{M}_n$ yielding a payoff to the principal at least as large as $\pi^* - n^{-1}$. The mechanism $\mathcal{M}_n$ leads to the mappings $(a_i^n(\theta), b_i^n(\theta))$ for $i \in \{1, ..., L\}$ and to the threshold curves $v_{r_l}^n(\theta)$. Define $\theta_{r_l}^n \equiv \inf\left\{\theta' : v_{r_l}^n(\theta') > 0\right\}$ for $l \in \{1, ..., L\}$. We will use the mappings $(a_i^n(\theta), b_i^n(\theta))$ $(i \in \{1, ..., L\}, n \in \mathbb{N})$ to construct our mechanism $\mathcal{M}^*$.

It is easy to show that we can restrict attention to mechanisms for which $b_i^n(\theta) \geq 0$ for all θ. Next, we claim that for each $m \in \mathbb{N}$ and $i \in \{1, ..., L\}$ we have

$$\sup_n \max_{\theta \in [0, 1-m^{-1}]} b_i^n(\theta) < \infty. \tag{29}$$

Assume towards a contradiction that there is $m \in \mathbb{N}$ and a subsequence for which

$$\sup_n \max_{\theta \in [0, 1-m^{-1}]} b_i^n(\theta) = \infty.$$

Notice that the payoff of any type $\theta' \in \left[1 - \frac{1}{2m}, 1\right]$ from choosing an allocation designed for a type $\theta'' \in \left[0, 1 - \frac{1}{m}\right]$ is at least

$$a_i^n(\theta'') + b_i^n(\theta'') \cdot D_i(\theta') - \left(\sum_{t=1}^{T} \delta^{t-1}\right) \cdot \bar{c},$$

where $D_i(\theta') \equiv \left[\sum_{t=1}^{r_i} \delta^{t-1}\lambda + (1-(1-\lambda)^{r_i}) \cdot \sum_{t=r_i+1}^{T} \delta^{t-1}\lambda\right] \cdot \theta'$. Since $a_i^n(\theta'') + b_i^n(\theta'') \cdot D_i(\theta'') \geq 0$ the expression above is at least as large as

$$b_i^n(\theta'') \cdot \left(D_i(\theta') - D_i(\theta'')\right) - \left(\sum_{t=1}^{T} \delta^{t-1}\lambda\right) \cdot \bar{c} \tag{30}$$

Hence, we conclude that (30) diverges to ∞. Consequently, so does the payoff of all types $\theta' \in \left[1-\frac{1}{2m}, 1\right]$, which automatically implies that the principal obtains a negative payoff whenever n is large enough.

We will construct a contract $(a_i(\theta), b_i(\theta)) = \{(a_i(\theta), b_i(\theta))\}_{i=1}^{L}$ from the sequence of contracts $\{(a_i^n(\theta), b_i^n(\theta))\}_{i=1}^{L}$. Notice that $b_i^n : [\theta_{r_i}^n, 1) \to \mathbb{R}$ is increasing, while $a_i^n : [\theta_{r_i}^n, 1) \to \mathbb{R}$ is decreasing. Notice that we may extend $(a_i^n(\theta), b_i^n(\theta))$ to $[0,1)$ by letting $(a_i^n(\theta), b_i^n(\theta)) = \left(a_i^{n\prime}(\theta_{r_i}^n), b_i^{n\prime}(\theta_{r_i}^n)\right)$ for all $\theta \leq \theta_{r_i}^{n\prime}$ (notice that zero-measure sets have no impact on payoffs). Notice that (29) imply that $b_i^n(\theta)$ is monotonic and uniformly bounded over the interval $\left[0, 1-m^{-1}\right]$ (for each m) and, thus, Helly's First Theorem (Theorem 6.1.18 in Kannan and Krueger 1996) asserts that there exists a subsequence $b_i^{n_m}(\theta)$ which converges (a.e.) over $\left[0, 1-m^{-1}\right]$. This property is also true for $\left[0, 1-(m+z)^{-1}\right]$ for all $z \in \mathbb{N}$, and hence we can find a subsequence of $b_i^{n_m}(\theta)$, call it $b_i^{n_{m+1}}(\theta)$, which converges over $\left[0, 1-(m+1)^{-1}\right]$. Proceeding inductively (by a diagonal argument) we obtain a subsequence of $b_i^n(\theta)$, call it $b_i^{n\prime}(\theta)$, and an increasing function $b_i(\theta)$ such that $b_i^{n\prime}(\theta) \to b_i(\theta)$ for almost all $\theta \in [0,1)$. Since $(a_i^{n\prime}(\theta))$ is decreasing the same argument implies that we may take a subsequence $(a_i^{n\prime}(\theta))$ of $(a_i^{n\prime}(\theta))$ and a function $a_i(\theta)$ such that $a_i^{n\prime}(\theta) \to a_i(\theta)$ for almost all $\theta \in [0,1)$.

Proceeding analogously for all $i \in \{1, ...L\}$, we obtain $\{(a_1(\theta), b_1(\theta)), ..., (a_L(\theta), b_L(\theta))\}$. We must show that

$$\pi^* = \int_0^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta. \tag{31}$$

Let $S \equiv T\Delta$ and notice that $\Pi(a(\theta), b(\theta)) - S \leq 0$ for all θ. Let $\mathbf{1}_{[\theta \leq 1-m^{-1}]}$ be the indicator function for $\theta \leq 1-m^{-1}$ and define g^m by

$$g^m(\theta) \equiv (\Pi(a(\theta), b(\theta)) - S) \cdot f(\theta) \cdot \mathbf{1}_{[\theta \leq 1-m^{-1}]}(\theta).$$

Notice that g_m is a decreasing sequence of nonpositive functions. Hence by the Lebesgue's monotone convergence theorem:

$$\int_0^1 (\Pi(a(\theta), b(\theta)) - S) f(\theta) d\theta = \lim_m \int_0^1 g_m(\theta) d\theta. \tag{32}$$

We claim that $\int_0^1 (\Pi(a(\theta), b(\theta)) - S) f(\theta) d\theta > -\infty$. Assume towards a contradiction that

$$\int_0^1 (\Pi(a(\theta), b(\theta)) - S) f(\theta) d\theta = -\infty.$$

In this case we can find $\bar{m} \in \mathbb{N}$ such that $\int_0^1 g_{\bar{m}}(\theta) d\theta < -4S$ and hence

$$\int_0^1 \Pi(a(\theta), b(\theta)) \cdot f(\theta) \cdot \mathbf{1}_{[\theta \leq 1-\bar{m}^{-1}]}(\theta) d\theta < -3S.$$

Thus we can find $n^* \in \mathbb{N}$ such that $n > n^*$ implies

$$\int_0^1 \Pi(a^n(\theta), b^n(\theta)) \cdot f(\theta) \cdot \mathbf{1}_{[\theta \leq 1-\bar{m}^{-1}]}(\theta) d\theta < -2S \tag{33}$$

Since $\int_0^1 \Pi(a^n(\theta), b^n(\theta)) \cdot f(\theta) \cdot \mathbf{1}_{[\theta>1-\bar{m}^{-1}]}(\theta)\, d\theta < S$, (33) implies

$$\int_0^1 \Pi(a^n(\theta), b^n(\theta)) \cdot f(\theta)\,(\theta)\, d\theta < -S, \tag{34}$$

which contradicts the assumption that $\int \Pi(a^n(\theta), b^n(\theta)) \cdot f(\theta)\,(\theta)\, d\theta > \pi^* - \frac{1}{n} \geq -\frac{1}{n}$ whenever $n > S^{-1}$. Thus we have $\int_0^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta > -\infty$.

Take $\varepsilon > 0$. We must show that

$$\int_0^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta \geq \pi^* - \varepsilon \tag{35}$$

to establish (31). Since $\int_0^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta > -\infty$, (32) implies that there is $m_1 \in \mathbb{N}$ such that $n > m_1$ implies

$$\int_{1-m_1^{-1}}^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta > -\frac{\varepsilon}{4}.$$

Notice also that

$$\int_0^1 \Pi(a^n(\theta), b^n(\theta)) \cdot f(\theta) \cdot \mathbf{1}_{[\theta>1-m^{-1}]}(\theta)\, d\theta < S \cdot \left(1 - F(1-m^{-1})\right).$$

Thus we take $m_2 \geq m_1$ such that $S \cdot \left(1 - F(1-m_2^{-1})\right) < \frac{\varepsilon}{4}$ and $n^* \in \mathbb{N}$ such that $n^{*-1} < -\frac{\varepsilon}{4}$. Take $n^{**} > n^*$ such that

$$\left| \int_0^{1-m_2^{-1}} \Pi(a^{n^{**}}(\theta), b^{n^{**}}(\theta)) f(\theta) d\theta - \int_0^{1-m_2^{-1}} \Pi(a(\theta), b(\theta)) f(\theta) d\theta \right| < \frac{\varepsilon}{4}.$$

We have

$$\begin{aligned}
& \int_0^1 \Pi(a(\theta), b(\theta)) f(\theta) d\theta \\
\geq \; & \int_0^{1-m_2^{-1}} \Pi(a(\theta), b(\theta)) f(\theta) d\theta - \frac{\varepsilon}{4} \\
\geq \; & \int_0^{1-m_2^{-1}} \Pi(a^{n^{**}}(\theta), b^{n^{**}}(\theta)) f(\theta) d\theta - \frac{\varepsilon}{2} \\
\geq \; & \int_0^1 \Pi(a^{n^{**}}(\theta), b^{n^{**}}(\theta)) f(\theta) d\theta - \frac{3\varepsilon}{4} \\
\geq \; & \pi^* - n^{**-1} - \frac{3\varepsilon}{4} \\
> \; & \pi^* - \varepsilon,
\end{aligned}$$

which establishes (35) and completes the proof. ■

6.2 Correlation

In this appendix, we show that the result on the irrelevance of expertise regarding project quality (Proposition 1) does not rely on the independence between θ and c. First, we construct and example where expertise is irrelevant even though θ and c are correlated. Second, we specialize the model to $T = 1$, and derive sufficient conditions for expertise to be irrelevant (therefore extending Proposition 1 to environments with correlation). Third, we show that Proposition 1 is robust to small perturbations away from independence (in the form of mixture distributions).

Consider the following example.

Example A 1 ***[Conditional Distribution is Mirrored Generalized Pareto]*** *Let the distribution of the safe project payoff conditional on the quality of the risky project be*

$$H(c|\theta) = \left(\frac{c - \underline{c}}{\bar{c} - \underline{c}}\right)^{\eta\theta}, \qquad \textit{where} \qquad \eta > 0.$$

Notice that the conditional distribution $H(\cdot|\theta)$ increases in the sense of first-order stochastic dominance as θ increases, in which case θ and c are positively correlated. In this case, the conditional reverse hazard rate is $\gamma(c|\theta) \equiv \frac{H(c|\theta)}{h(c|\theta)} = \frac{c-\underline{c}}{\eta\theta}$. Consider the action plan ϕ^e as described Lemma 3, after replacing the unconditional reverse hazard rate $\gamma(c)$ by its conditional counterpart $\gamma(c|\theta)$, and let $P^e(\tau)$ be the expected payments induced by the expert-investor optimal mechanism (formula 9).

Following the same reasoning as in the proof of Proposition 1, it follows that the action plan ϕ^e is implementable by a menu of linear contracts with lump-sum payments and success bonuses:

$$a^*(\tau) = \frac{\Phi_{k^*(\tau)}(\theta)}{\eta + 1} \cdot (\underline{c} - \eta\theta K) \qquad \textit{and} \qquad b^*(\tau) = \triangle \cdot \left(1 + \frac{1}{\eta\theta}\right)^{-1}.$$

To understand the implementability claim, note that ICS is satisfied by construction, ICR_1 holds as $b^*(\tau)$ is increasing in θ, and ICR_2 holds by the same argument as in the proof of Proposition 1. Because, by construction, expected payments under two-dimensional asymmetric information equal $P^e(\tau)$, it follows that expertise (about θ) is irrelevant for payoffs.

The example above presents a parametric case exhibiting correlation where the result and proof technique of Proposition 1 readily apply. In what follows, we will derive sufficient conditions for expertise to be irrelevant. For tractability, we will assume that $T = 1$. As in the example above, let $H(c|\theta)$ denote the conditional cumulative distribution of c given θ, let $h(c|\theta) = \frac{\partial H}{\partial c}(c|\theta)$ denote its density, and let $\gamma(c|\theta) \equiv \frac{H(c|\theta)}{h(c|\theta)}$ denote its associated reverse hazard rate. As in the case of independence, we assume that $H(c|\theta)$ is log-concave in c for each θ, so that $\frac{\partial\gamma}{\partial c}(c|\theta) \geq 0$.

From the same argument as in the text, when there is symmetric information about θ, the agent experiments if $c \leq v(\theta)$, where $v(\theta)$ is the implicit solution of:

$$\theta \cdot \lambda \cdot \triangle - K = v(\theta) + \gamma\left(v(\theta)|\theta\right). \tag{36}$$

Differentiating (36), gives

$$v'(\theta) = \frac{\lambda \cdot \triangle - \frac{\partial\gamma}{\partial\theta}\left(v(\theta)|\theta\right)}{1 + \frac{\partial\gamma}{\partial c}\left(v(\theta)|\theta\right)}. \tag{37}$$

Notice that the log-concavity of $H(c|\theta)$ guarantees that the denominator above is positive. When there is correlation, in order to guarantee that $v'(\theta) \geq 0$, we also need that $\frac{\partial\gamma}{\partial\theta} \leq \lambda \cdot \triangle$, which we assume from now on. This condition limits by how much an increase in θ shifts the conditional reverse hazard rate of c. In intuitive terms, it requires that the correlation between θ and c is not too negative.

As in the case of independence, the expert-investor optimal mechanism is incentive compatible when θ is private information if and only if $v(\theta)$ is convex. When c and θ are independent, we have $\frac{\partial\gamma}{\partial\theta} = 0$ and, therefore, v is convex if and only if $\frac{\partial\gamma}{\partial c}$ is decreasing, i.e., the reverse hazard rate is weakly concave (Condition C).

For the general case, differentiate (37) again to obtain:

$$v''(\theta) = -\frac{\frac{\partial^2\gamma}{\partial\theta^2}\left(v(\theta)|\theta\right) + v'(\theta)\left[\frac{\partial^2\gamma}{\partial c^2}\left(v(\theta)|\theta\right)v'(\theta) + 2\frac{\partial^2\gamma}{\partial c\partial\theta}\left(v(\theta)|\theta\right)\right]}{1 + \frac{\partial\gamma}{\partial c}\left(v(\theta)|\theta\right)}.$$

Therefore, the following are sufficient conditions for expertise to be irrelevant: $\frac{\partial^2\gamma}{\partial c^2}, \frac{\partial^2\gamma}{\partial\theta^2}, \frac{\partial^2\gamma}{\partial\theta\partial c} \leq 0$. As such, in the case of correlation, expertise is irrelevant whenever $\gamma(c|\theta)$ is concave in each of its arguments and submodular.

Next, we show that our main result is robust to small perturbations away from independence.

Let $H(c)$ be a log-concave distribution with a strictly concave reverse hazard rate. Let $\Upsilon(c,\theta)$ be a joint distribution with a smooth density that is bounded away from zero in its support $[\underline{c},\bar{c}] \times [0,1]$.

Consider the mixture distribution

$$Q^\alpha(c,\theta) := \alpha H(c) + (1-\alpha)\Upsilon(c,\theta),$$

where $0 \leq \alpha \leq 1$. As α approaches 1, this distribution converges to H. For each θ, let $c \to Q^\alpha(c \mid \theta)$ represent the marginal distribution associated with Υ. Let $q(c \mid \theta)$ denote its density and $\gamma^\alpha(c,\theta) := \frac{Q^\alpha(c|\theta)}{q^\alpha(c|\theta)}$ denote its reverse hazard rate.

For each α, let $v(\theta,\alpha)$ denote the implicit solution of (36), and let

$$\vartheta(\theta,\alpha) \equiv -\frac{\frac{\partial^2\gamma^\alpha}{\partial\theta^2} + \frac{\lambda\Delta - \frac{\partial\gamma^\alpha}{\partial\theta}}{1+\frac{\partial\gamma^\alpha}{\partial c}}\left[\frac{\partial^2\gamma^\alpha}{\partial c^2} \cdot \frac{\lambda\Delta - \frac{\partial\gamma^\alpha}{\partial\theta}}{1+\frac{\partial\gamma^\alpha}{\partial c}} + 2\frac{\partial^2\gamma^\alpha}{\partial\theta\partial c}\right]}{1 + \frac{\partial\gamma^\alpha}{\partial c}},$$

where we omit the $(v(\theta,\alpha),\theta)$ from all functions on the right-hand side for notational simplicity. As argued previously, expertise is irrelevant when $\vartheta(\theta,\alpha) \geq 0$ for all θ (for a fixed α). We claim that there exists $\alpha^* \in (0,1)$ such that $\vartheta(\theta,\alpha) > 0$ for all $\alpha > \alpha^*$. Since $(\theta,c) \to v(\theta,\alpha)$ is smooth, so is $(\theta,\alpha) \to \vartheta(\theta,\alpha)$. Moreover, by our assumption on H, $\vartheta(\theta,1) > 0$ for all θ. The result then follows by uniform continuity.

References

[1] Kannan, R. and Krueger, C. K. (1996): *Advanced Analysis on the Real Line*, Springer, New York.