B Supplementary Appendix: Not for Publication

B.1 Additional Proofs

Proof of Example 1

For notational simplicity, let $\pi^e_{q,s} \equiv f(q, s|e)$ denote the probability of state (q, s) conditional on effort e, $\bar{\pi}^e_q \equiv \int \pi^e_{q,s} ds$ denote the marginal probability of output q, and $\bar{\Pi}^e_q$ denote the associated cumulative distribution function ("CDF"). Suppose that $\pi^1_{q,s}$ and $\pi^0_{q,s}$ are both independent of s. As in Grossman and Hart (1983), it is convenient to write the principal's program in terms of "utils". Ignoring intermediate effort levels, the program is:

$$\min_V \int h(V(q))\, \bar{\pi}^1_q dq \text{ s.t.}$$
$$\int V(q)\, \bar{\pi}^1_q dq \geq \overline{U} \tag{29}$$
$$\int V(q) \left(\bar{\pi}^1_q - \bar{\pi}^0_q\right) dq \geq 1, \tag{30}$$

where $h = V^{-1}$.

We wish to study conditions under which the solution to this relaxed program also solves the original program – i.e. under which the following omitted ICs are satisfied:

$$\int_S \int_X V(q) \left(\pi^1_{q,s} - \pi^e_{q,s}\right) dq ds \geq 1 - e, \ \forall e.$$

Using the marginal distributions, we can rewrite these constraints as

$$\xi(e) \equiv \int_X V(q) \left(\bar{\pi}^1_q - \bar{\pi}^e_q\right) dq - (1 - e) \geq 0.$$

Note that $\xi(1) = 0$ and, by the binding IC (30), $\xi(0) = 0$. Thus, it suffices to show that ξ is concave.

Applying integration by parts to the solution of the relaxed program, we obtain

$$\int V(q) \left(\bar{\pi}^1_q - \bar{\pi}^e_q\right) dq = \int \dot{V}(q) \left(\bar{\Pi}^e_q - \bar{\Pi}^1_q\right) dq,$$

where $\bar{\Pi}$ is the CDF associated with $\bar{\pi}$. Substituting back in the definition of ξ yields

$$\xi(e) = \int \dot{V}(q) \left(\bar{\Pi}^q_e - \bar{\Pi}^q_1\right) dq + e - 1.$$

Since the likelihood ratio $\bar{\pi}_q^1/\bar{\pi}_q^0$ is non-decreasing in q, the solution of the relaxed program is monotonic: $\dot{V} \geq 0$. Then, since $\bar{\Pi}_q^e$ is a concave function of e, ξ is concave.

Proof of Theorem 1, non-binding IR

This appendix completes the proof of Theorem 1, by considering the case where the IR (17) does not bind. We can thus ignore the IR from the principal's program. The first-order condition with respect to $u_{q,s}$ is

$$-p_{q,s}^{e^*}h'\left(u_{q,s}\right)-\mu_1\left(K(1)p_{q,s}^1-K\left(e^*\right)p_{q,s}^{e^*}\right)-\mu_2\left(K(2)p_{q,s}^2-K\left(e^*\right)p_{q,s}^{e^*}\right)=0 \ \ \forall q,s. \quad (31)$$

For the wage to be independent of the signal, the system of equations (18) and (31) must have as a solution $u_{q,s}=u_q \ \forall \ q,s$. We can write this system of equations using the function $F:\mathbb{R}^{X(1+3S)+5}\to\mathbb{R}^{XS+2}$, where

$$F\left(\underbrace{u_1,...,u_X}_{X},\underbrace{\mu_1,\mu_2}_{2};\underbrace{\Theta}_{3},\underbrace{p_{1,1}^e,...,p_{X,S}^e}_{3XS}\right)$$
$$=\left[\begin{array}{c}
p_{1,1}^3h'\left(u_1\right)+\mu_1(K(1)p_{1,1}^1-K(3)p_{1,1}^3)+\mu_2(K(2)p_{1,1}^2-K(3)p_{1,1}^3)\\
\vdots\\
p_{1,S}^3h'\left(u_1\right)+\mu_1(K(1)p_{1,S}^1-K(3)p_{1,S}^3)+\mu_2(K(2)p_{1,S}^2-K(3)p_{1,S}^3)\\
\vdots\\
p_{X,1}^3h'\left(u_X\right)+\mu_1(K(1)p_{X,1}^1-K(3)p_{X,1}^3)+\mu_2(K(2)p_{X,1}^2-K(3)p_{X,1}^3)\\
\vdots\\
p_{X,S}^3h'\left(u_X\right)+\mu_1(K(1)p_{X,S}^1-K(3)p_{X,S}^3)+\mu_2(K(2)p_{X,S}^2-K(3)p_{X,S}^3)\\
\sum_{q=1}^X u_q\left(K(2)\sum_s p_{q,s}^2-K(3)\sum_s p_{q,s}^3\right)+G(2)-G(3)\\
\sum_{q=1}^X u_q\left(K(1)\sum_s p_{q,s}^1-K(3)\sum_s p_{q,s}^3\right)+G(1)-G(3)
\end{array}\right].$$

To apply Corollary 1, we need to show that DF has full row rank. It is given by:

$$DF=\left[\begin{array}{cccccc}
A_{XS\times X} & C_{XS\times 2} & D_\Theta & H_{XS\times XS}^3 & H_{XS\times XS}^2 & H_{XS\times XS}^1\\
B_{2\times X} & \mathbf{0}_{2\times 2} & E_\Theta & J_{2\times XS}^3 & J_{2\times XS}^2 & J_{2\times XS}^1
\end{array}\right].$$

Matrices $A_{XS\times X}$ and $B_{2\times X}$ are, respectively, the derivative of the first XS equations and the last 2 equations (ICs) with respect to $\mathbf{u}$:

$$\begin{aligned}
A_{XS\times X} &= \begin{bmatrix} h''(u_1)\mathbf{P}_1^3 & 0 & ... & 0 \\ 0 & h''(u_2)\mathbf{P}_2^3 & ... & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & ... & h''(u_X)\mathbf{P}_X^3 \end{bmatrix}, \\
B_{2\times X} &= \begin{bmatrix} K(2)\mathbf{P}_1^2\cdot\mathbf{1}_S - K(3)\mathbf{P}_1^3\cdot\mathbf{1}_S & ... & K(2)\mathbf{P}_S^2\cdot\mathbf{1}_S - K(3)\mathbf{P}_X^3\cdot\mathbf{1}_S \\ K(1)\mathbf{P}_1^1\cdot\mathbf{1}_S - K(3)\mathbf{P}_1^3\cdot\mathbf{1}_S & ... & K(1)\mathbf{P}_S^1\cdot\mathbf{1}_S - K(3)\mathbf{P}_X^3\cdot\mathbf{1}_S \end{bmatrix}.
\end{aligned}$$

The derivatives with respect to the multipliers μ_1 and μ_2 are, respectively,

$$C_{XS\times 2} = \begin{bmatrix} K(1)p_{1,1}^1 - K(3)p_{1,1}^3 & K(2)p_{1,1}^2 - K(3)p_{1,1}^3 \\ \vdots & \\ K(1)p_{1,S}^1 - K(3)p_{1,S}^3 & K(2)p_{1,S}^2 - K(3)p_{1,S}^3 \\ \vdots & \\ K(1)p_{X,1}^1 - K(3)p_{X,1}^3 & K(2)p_{X,1}^2 - K(3)p_{X,1}^3 \\ \vdots & \\ K(1)p_{X,S}^1 - K(3)p_{X,S}^3 & K(2)p_{X,S}^2 - K(3)p_{X,S}^3 \end{bmatrix} \tag{32}$$

and the null matrix $\mathbf{0}_{2\times 2}$. The derivatives with respect to $\{G(3), G(2), G(1)\}$ are, respectively, $\mathbf{0}_{XS\times 3}$ and

$$E_{\mathbf{G}} = \begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}.$$

Thus, if $\mathbf{K}$ is constant, $\Theta = \mathbf{G}$, and we have $D_\Theta = D_{\mathbf{G}} = \mathbf{0}_{XS\times 3}$ and $E_\Theta = E_{\mathbf{G}}$.

The derivatives with respect to $\{K(3), K(2), K(1)\}$ are, respectively:

$$\begin{aligned}
D_{\mathbf{K}} &= \begin{bmatrix} -\mu_1 p_{1,1}^3 - \mu_2 p_{1,1}^3 & \mu_2 p_{1,1}^2 & \mu_1 p_{1,1}^1 \\ \vdots & & \\ -\mu_1 p_{1,S}^3 - \mu_2 p_{1,S}^3 & \mu_2 p_{1,S}^2 & \mu_1 p_{1,S}^1 \\ \vdots & & \\ -\mu_1 p_{X,1}^3 - \mu_2 p_{X,1}^3 & \mu_2 p_{X,1}^2 & \mu_1 p_{X,1}^1 \\ \vdots & & \\ -\mu_1 p_{X,S}^3 - \mu_2 p_{X,S}^3 & \mu_2 p_{X,S}^2 & \mu_1 p_{X,S}^1 \end{bmatrix}, \\
E_{\mathbf{K}} &= \begin{bmatrix} -\sum_{q=1}^X u_q \sum_s p_{q,s}^3 & \sum_{q=1}^X u_q \sum_s p_{q,s}^2 & 0 \\ -\sum_{q=1}^X u_q \sum_s p_{q,s}^3 & 0 & \sum_{q=1}^X u_q \sum_s p_{q,s}^1 \end{bmatrix}.
\end{aligned}$$

Thus, if $\mathbf{G}$ is constant, $\Theta = \mathbf{K}$, and we have $D_\Theta = D_{\mathbf{K}}$, and $E_\Theta = E_{\mathbf{K}}$.

The derivatives with respect to $\left(p^3_{q,s}\right)$, $\left(p^2_{q,s}\right)$, and $\left(p^1_{q,s}\right)$ are, respectively:

$$
\begin{aligned}
H^3_{XS\times XS} &= \begin{bmatrix} \left[h'\left(u_1\right) - K(3)(\mu_1+\mu_2)\right]\mathbf{I}_S & \mathbf{0}_{S\times S} & ... & \mathbf{0}_{S\times S} \\ \mathbf{0}_{S\times S} & \ddots & ... & \mathbf{0}_{S\times S} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{S\times S} & \mathbf{0}_{S\times S} & ... & \left[h'\left(u_X\right) - K(3)(\mu_1+\mu_2)\right]\mathbf{I}_S \end{bmatrix} \\
J^3_{2\times XS} &= \begin{bmatrix} -u_1K(3)\mathbf{1}_S & ... & -u_XK(3)\mathbf{1}_S \\ -u_1K(3)\mathbf{1}_S & ... & -u_XK(3)\mathbf{1}_S \end{bmatrix},
\end{aligned}
$$

$$
\begin{aligned}
H^2_{XS\times XS} &= \begin{bmatrix} \mu_2K(2)\mathbf{I}_S & \mathbf{0}_{S\times S} & ... & \mathbf{0}_{S\times S} \\ \mathbf{0}_{S\times S} & \mu_2K(2)\mathbf{I}_S & ... & \mathbf{0}_{S\times S} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0}_{S\times S} & \mathbf{0}_{S\times S} & ... & \mu_2K(2)\mathbf{I}_S \end{bmatrix} = \mu_2\mathbf{I}_{XS}, \\
J^2_{2\times XS} &= \begin{bmatrix} u_1K(2)\mathbf{1}_S & ... & u_XK(2)\mathbf{1}_S \\ \mathbf{0}_S & ... & \mathbf{0}_S \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
H^1_{XS\times XS} &= \mu_1K(1)\mathbf{I}_{XS} \\
J^1_{2\times XS} &= \begin{bmatrix} \mathbf{0}_S & ... & \mathbf{0}_S \\ u_1K(1)\mathbf{1}_S & ... & u_XK(1)\mathbf{1}_S \end{bmatrix}.
\end{aligned}
$$

Note that $DF_{\mathbf{P}} = \begin{bmatrix} H^3_{XS\times XS} & H^2_{XS\times XS} & H^1_{XS\times XS} \\ J^3_{2\times XS} & J^2_{2\times XS} & J^1_{2\times XS} \end{bmatrix}$ has $XS+2$ rows and $3XS$ columns. Since $XS+2 < 3XS$, it suffices to show that $DF_{\mathbf{P}}$ has full row rank to establish that DF has full row rank. We thus need to show that for any vector $\mathbf{y} \in \mathbb{R}^{XS+2}$,

$$
\underbrace{\mathbf{y}}_{1\times(XS+2)} \times \underbrace{DF_{\mathbf{P}}}_{(XS+2)\times 3XS} = \underbrace{\mathbf{0}}_{1\times 3XS} \implies \mathbf{y} = \underbrace{\mathbf{0}}_{1\times(XS+2)}.
$$

Let $DF_{\mathbf{P}_i} = \begin{bmatrix} H^i_{XS\times XS} \\ J^i_{2\times XS} \end{bmatrix}$. First, expanding $\mathbf{y} \times DF_{\mathbf{P}_2} = \mathbf{0}$ gives:

$$
\begin{aligned}
&\mu_2K(2)y_1 + u_1K(2)y_{XS+1} = ... = \mu_2K(2)y_S + u_1K(2)y_{XS+1} = 0 \\
&\mu_2K(2)y_{S+1} + u_2K(2)y_{XS+1} = ... = \mu_2K(2)y_{2S} + u_2K(2)y_{XS+1} = 0 \\
&\vdots \\
&\mu_2K(2)y_{S(X-1)+1} + u_XK(2)y_{XS+1} = ... = \mu_2K(2)y_{XS} + u_XK(2)y_{XS+1} = 0.
\end{aligned}
$$

Dividing through by $K(2) > 0$ and rearranging gives:

$$\begin{aligned} \mu_2 y_1 &= \ldots = \mu_2 y_S = -u_1 y_{XS+1} \\ \mu_2 y_{S+1} &= \ldots = \mu_2 y_{2S} = -u_2 y_{XS+1} \\ &\vdots \\ \mu_2 y_{S(X-1)+1} &= \ldots = \mu_2 y_{XS} = -u_X y_{XS+1}. \end{aligned} \tag{33}$$

Similarly, expanding $\mathbf{y} \times DF_{\mathbf{P}_1} = \mathbf{0}$ yields

$$\begin{aligned} \mu_1 K(1) y_1 &= \ldots = \mu_1 K(1) y_S = -u_1 K(1) y_{XS+2} \\ \mu_1 K(1) y_{S+1} &= \ldots = \mu_1 K(1) y_{2S} = -u_2 K(1) y_{XS+2} \\ &\vdots \\ \mu_1 K(1) y_{S(X-1)+1} &= \ldots = \mu_1 K(1) y_{XS} = -u_X K(1) y_{XS+2} \end{aligned} \tag{34}$$

with $K(1) > 0$. Recall that $\mu_1 \geq 0$ and $\mu_2 \geq 0$ and at least one of them is strict. Thus,

$$\begin{aligned} y_1 &= \ldots = y_S =: \bar{y}^1 \\ y_{S+1} &= \ldots = y_{2S} =: \bar{y}^2 \\ &\vdots \\ y_{S(X-1)+1} &= \ldots = y_{XS} =: \bar{y}^X. \end{aligned}$$

From equation (33), we have:

$$\begin{aligned} \mu_2 \bar{y}^1 &= -u_1 y_{XS+1} \\ &\vdots \\ \mu_2 \bar{y}^X &= -u_X y_{XS+1} \end{aligned} \tag{35}$$

Second, recall that $DF_{(\mu_1,\mu_2)} = \begin{bmatrix} C_{XS\times 2} \\ \mathbf{0}_{2\times 2} \end{bmatrix}$. Thus, $\mathbf{y} \times DF_{(\mu_1,\mu_2)} = \mathbf{0}$ gives

$$\sum_{q,s} \bar{y}^q \left[K(1)p^1_{q,s} - K(3)p^3_{q,s}\right] = 0, \quad \sum_{q,s} \bar{y}^q \left[K(2)p^2_{q,s} - K(3)p^3_{q,s}\right] = 0, \quad \forall q. \tag{36}$$

Multiplying both sides of the first equation in (36) by $\mu_2 \geq 0$:

$$\mu_2 \sum_{q,s} \bar{y}^q \left[K(1)p^1_{q,s} - K(3)p^3_{q,s}\right] = K(1) \sum_{q,s} \left(\mu_2 \bar{y}^q\right) p^1_{q,s} - K(3) \sum_{q,s} \left(\mu_2 \bar{y}^q\right) p^3_{q,s} = 0. \tag{37}$$

However, from equation (35), we have

$$\begin{aligned}
& K(1)\sum_{q,s}\left(\mu_2\bar{y}^q\right)p^1_{q,s} - K(3)\sum_{q,s}\left(\mu_2\bar{y}^q\right)p^3_{q,s} \\
= \quad & -y_{XS+1}\left[K(1)\sum_{q,s}u_q p^1_{q,s} - K(3)\sum_{q,s}u_q p^3_{q,s}\right] = -y_{XS+1}(G(3)-G(1)), \quad (38)
\end{aligned}$$

where the last equality follows from the binding IC for $e = 1$. Let $G(3) \neq G(1)$ (the set of parameters for which $G(3) = G(1)$ have zero Lebesgue measure). Then, (37) and (38) imply $y_{XS+1} = 0$. Applying this logic to the second equation in (36) yields $y_{XS+2} = 0$.

Third, recall from equations (33) and (34) that, $\forall\ q$,

$$\mu_2\bar{y}^q = -u_q y_{XS+1} \text{ and } \mu_1\bar{y}^q = -u_q y_{XS+2}.$$

Moreover, $\mu_1 \geq 0$ and $\mu_2 \geq 0$ with at least one of them strict. Since $y_{XS+1} = y_{XS+2} = 0$, we have $\mu_1\bar{y}^q = \mu_2\bar{y}^q = 0$. Since either $\mu_1 \neq 0$ or $\mu_2 \neq 0$, this implies $\bar{y}^q = 0\ \forall\ q$. Thus, $\mathbf{y} \times DF_{\mathbf{P}} = \mathbf{0} \implies \mathbf{y} = \mathbf{0}$, i.e., $DF_{\mathbf{P}}$ has full row rank.

B.2 Multiple Binding ICs

This appendix shows that the case in which multiple ICs simultaneously bind is not knife-edge. The problem of implementing effort e at minimum cost is:

$$\min_{\{u_{q,s}\}} \sum_{q=q_1}^{q_X}\sum_{s=1}^{S} p^e_{q,s} b\left(u_{q,s}\right)$$

subject to

$$\begin{aligned}
\sum_{q=q_1}^{q_X}\sum_{s=1}^{S} p^e_{q,s}u_{q,s} - c_e \quad & \geq \quad \bar{U} \\
\sum_{q=q_1}^{q_X}\sum_{s=1}^{S}\left(p^e_{q,s} - p^{\tilde{e}}_{q,s}\right)u_{q,s} \quad & \geq \quad c_e - c_{\tilde{e}} \quad \forall\tilde{e}.
\end{aligned}$$

We study the case of three effort levels and three states. This is the simplest environment to study multiple binding ICs. With two effort levels, there is only one IC; with two states, wages are two-dimensional and, since the IR and at least one IC must bind for any effort except the least costly one, we generically can only have one binding IC.

Let $\mathcal{S} = \{1, 2, 3\}$ and $\mathcal{E} = \{1, 2, 3\}$, and take the utility function $u(c) = \sqrt{c + K}$, where $K > 0$ allows for negative wages. The inverse utility function is then

$$h(u) = u^2 - K.$$

Without loss of generality, let $e = 2$ denote the implemented effort. The program is:

$$\min_{\{u_s\}} \sum_{s=1,2,3} p_s^2 u_s^2$$

subject to

$$\begin{aligned} \sum_{s=1,2,3} p_s^2 u_s &\geq c_2 \\ \sum_{s=1,2,3} \left(p_s^2 - p_s^1\right) u_s &\geq c_2 - c_1 \\ \sum_{s=1,2,3} \left(p_s^2 - p_s^3\right) u_s &\geq c_2 - c_3 \end{aligned}$$

We know that IR binds. Substituting the binding IR into the two ICs, the IR and two ICs now become:

$$\sum_{s=1,2,3} p_s^2 u_s = c_2$$

$$\sum_{s=1,2,3} p_s^1 u_s \leq c_1 \tag{39}$$

$$\sum_{s=1,2,3} p_s^3 u_s \leq c_3 \tag{40}$$

An economy is parametrized by conditional distributions and costs: $\{p_1^e, p_2^e, c_e\}_{e=1,2,3}$ (p_3^e is given by $p_3^e = 1 - p_2^e - p_1^e$). We claim that there exists an open neighborhood of parameters in which both ICs (39) and (40) bind. To show this, we will study the maximization program where we ignore one of them. If the ignored IC is satisfied at the solution of this "relaxed program," this solution solves the principal's program. We will show that, for some open set of parameter values, each of these two constraints (39 and 40) fails to hold when it is ignored, so they both simultaneously bind.

First, consider the relaxed program where we omit (40). The Lagrangian is

$$L = -p_1^2 u_1^2 - p_2^2 u_2^2 - p_3^2 u_3^2 + \lambda\left(p_1^2 u_1 + p_2^2 u_2 + p_3^2 u_3 - c_2\right) + \mu\left(p_1^1 u_1 + p_2^1 u_2 + p_3^1 u_3 - c_1\right),$$

which has as first-order conditions the following linear system:

$$\begin{aligned}
2u_1 &= \lambda + \mu \frac{p_1^1}{p_1^2},\ 2u_2 = \lambda + \mu \frac{p_2^1}{p_2^2},\ 2u_3 = \lambda + \mu \frac{p_3^1}{p_3^2}, \\
p_1^2 u_1 + p_2^2 u_2 + p_3^2 u_3 &= c_2, \\
p_1^1 u_1 + p_2^1 u_2 + p_3^1 u_3 &= c_1.
\end{aligned}$$

We will now combine the first three equations into one by eliminating λ. From the first equation, we have $2u_1 - \mu \frac{p_1^1}{p_1^2} = \lambda$. Substituting into the second and third and combining yields the following linear system with three equations and three unknowns:

$$\begin{bmatrix} \left(\frac{p_2^1}{p_2^2} - \frac{p_1^1}{p_1^2}\right) & \left(\frac{p_1^1}{p_1^2} - \frac{p_3^1}{p_3^2}\right) & \left(\frac{p_3^1}{p_3^2} - \frac{p_2^1}{p_2^2}\right) \\ p_3^2 & p_2^2 & p_1^2 \\ p_3^1 & p_2^1 & p_1^1 \end{bmatrix} \begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 \\ c_1 \end{bmatrix},$$

which characterizes the solution of the relaxed program where we ignore (40).

Similarly, the solution of the relaxed program where we ignore (39) is given by:

$$\begin{bmatrix} \left(\frac{p_2^3}{p_2^2} - \frac{p_1^3}{p_1^2}\right) & \left(\frac{p_1^3}{p_1^2} - \frac{p_3^3}{p_3^2}\right) & \left(\frac{p_3^3}{p_3^2} - \frac{p_2^3}{p_2^2}\right) \\ p_3^2 & p_2^2 & p_1^2 \\ p_3^3 & p_2^3 & p_1^3 \end{bmatrix} \begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 \\ c_3 \end{bmatrix}.$$

It is easy to apply Cramer's rule to obtain a closed-form solution.

Use the following vector notation: $\mathbf{p}^e \equiv (p_1^e, p_2^e, p_3^e)$. Consider $\mathbf{p}^1 = (0.1,\ 0.28,\ 0.62)$, $\mathbf{p}^2 = (0.2,\ 0.15,\ 0.65)$, $\mathbf{p}^3 = (0.3,\ 0.1,\ 0.6)$, $c_1 = 0.75$, $c_2 = 1$, $c_3 = 0.5$.

The matrix in the relaxed program where we omit (40) is:

$$A_1 \equiv \begin{bmatrix} \left(\frac{p_2^1}{p_2^2} - \frac{p_1^1}{p_1^2}\right) & \left(\frac{p_1^1}{p_1^2} - \frac{p_3^1}{p_3^2}\right) & \left(\frac{p_3^1}{p_3^2} - \frac{p_2^1}{p_2^2}\right) \\ p_3^2 & p_2^2 & p_1^2 \\ p_3^1 & p_2^1 & p_1^1 \end{bmatrix} = \begin{bmatrix} 1.3667 & -0.4538 & -0.9128 \\ 0.65 & 0.15 & 0.2 \\ 0.62 & 0.28 & 0.1 \end{bmatrix}.$$

The solution is

$$\begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = (A_1)^{-1} \begin{bmatrix} 0 \\ c_2 \\ c_1 \end{bmatrix} = \begin{bmatrix} 1.0703 \\ -0.3207 \\ 1.7620 \end{bmatrix},$$

where we used the fact that

$$(A_1)^{-1} = \begin{bmatrix} 0.2499 & 1.2813 & -0.2813 \\ -0.3596 & -4.2829 & 5.2829 \\ -0.5425 & 4.0478 & -3.0478 \end{bmatrix}.$$

Since A_1 has full rank, the solution is continuous in its parameters (conditional probabilities and costs) around these parameter values. Substituting in (40) gives

$$p_3^3 u_3 + p_2^3 u_2 + p_1^3 u_1 - c_3 = 0.6 \times 1.0703 + 0.1 \times (-0.3207) + 0.3 \times 1.7629 - 0.5 = 0.6387 > 0.$$

Thus, (40) fails to hold. Since the expression $p_3^3 u_3 + p_2^3 u_2 + p_1^3 u_1 - c_3$ is a continuous function of conditional probabilities, utilities, and costs, and utility is itself a continuous function of costs and probabilities, it follows that this expression is a continuous function of probabilities and costs. Thus, for parameter values in a neighborhood of the ones considered here, it is also the case that (40) fails to hold.

The matrix in the relaxed program where we omit (39) is:

$$A_3 = \begin{bmatrix} \left(\frac{p_2^3}{p_2^2} - \frac{p_1^3}{p_1^2}\right) & \left(\frac{p_1^3}{p_1^2} - \frac{p_3^3}{p_3^2}\right) & \left(\frac{p_3^3}{p_3^2} - \frac{p_2^3}{p_2^2}\right) \\ p_3^2 & p_2^2 & p_1^2 \\ p_3^3 & p_2^3 & p_1^3 \end{bmatrix} = \begin{bmatrix} -0.8333 & 0.5769 & 0.2564 \\ 0.65 & 0.15 & 0.2 \\ 0.6 & 0.1 & 0.3 \end{bmatrix},$$

which has inverse

$$(A_3)^{-1} = \begin{bmatrix} -0.3545 & 2.0909 & -1.0909 \\ 1.0626 & 5.7273 & -4.7273 \\ 0.3545 & -6.0909 & 7.0909 \end{bmatrix}.$$

The solution of the relaxed program is then

$$\begin{bmatrix} u_3 \\ u_2 \\ u_1 \end{bmatrix} = (A_3)^{-1} \begin{bmatrix} 0 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1.5455 \\ 3.3636 \\ -2.5455 \end{bmatrix}.$$

Again, the solution is continuous in the parameters in a neighborhood of the parameters selected here. Substituting in the omitted IC gives:

$$p_3^1 u_3 + p_2^1 u_2 + p_1^1 u_1 - c_1 = 0.62 \times 1.5455 + 0.28 \times 3.3636 + 0.1 \times (-2.5455) - 0.75 = 0.8955 > 0.$$

Thus, (39) fails to hold. As before, by continuity, this is true for all parameter values in a neighborhood of the ones chosen here.

To summarize, for all parameter values in a neighborhood of the ones chosen here, both ICs simultaneously hold. Thus it is not true that generically only one IC binds.

References

[1] Holmström, Bengt (1982): "Moral hazard in teams." *Bell Journal of Economics* 13, 326–340.