



Notes

Risk-neutral firms can extract unbounded profits from consumers with prospect theory preferences [☆]

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Received 21 May 2010; final version received 3 June 2011; accepted 4 July 2011

Available online 20 January 2012

Abstract

This paper considers the problem of a risk-neutral firm offering a gamble to consumers with preferences given by prospect theory. Under conditions satisfied by virtually all functional forms used in the literature, firms can extract arbitrarily high expected values from consumers. Moreover, for any given lottery, there exists another lottery that makes both the firm and the consumer better off. As a consequence, equilibria and Pareto optimal allocations do not exist in standard monopolistic or competitive models.

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JEL classification: D81; D03

Keywords: Prospect theory; Dutch books; Equilibrium existence; Insurance markets

1. Introduction

Prospect theory is perhaps the most well-known alternative to expected utility theory. It was originally formulated by Kahneman and Tversky [11] for binary lotteries, and refined by Tversky and Kahneman [17] in order to deal with lotteries with multiple outcomes and prevent violations of first-order stochastic dominance. Prospect theory is able to accommodate behavior consistent

[☆] We thank Nick Barberis, David Dillenberger, Drew Fudenberg, Paulo Natenzon, Wolfgang Pesendorfer, Drazen Prelec, Andrei Shleifer, Wei Xiong, and an associate editor for comments.

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with the paradoxes of Allais [2] and Ellsberg [9], as well as the coexistence of gambling and insurance. It is largely consistent with field and laboratory data.¹

Despite the relative success in explaining empirical regularities, prospect theory is rarely applied to strategic and market environments. This paper presents problems that one necessarily faces when attempting to incorporate prospect theory in those environments.

We show that, under conditions satisfied by virtually all functional forms used in the literature, individuals with prospect theory preferences accept gambles with arbitrarily large negative expected values. This result severely limits the applicability of prospect theory when the supply side of the market is endogenous.

For example, consider an insurance model featuring a risk-neutral monopolistic firm and consumers with prospect theory preferences. The monopolist can charge any arbitrarily large price for an “insurance policy” featuring either unbounded gains or unbounded losses with probability approaching zero. Not only is this outcome counterintuitive (consumers end up facing even more risk with these insurance policies than without them), but it also implies that there is no solution to the firm’s problem.

Moreover, for any insurance policy, there is always another policy that makes both the firm and consumers simultaneously better off (i.e., the set of Pareto optimal allocations is empty). Therefore, assumptions that typically restrict the firm’s ability to extract surplus and prevent Dutch books do not help in this case. If the individual is subject to wealth constraints, the monopolist can extract all the consumer’s wealth with probability approaching one. If consumers are heterogeneous and heterogeneity is private information, the monopolist can extract each consumer’s entire wealth with probability approaching one by offering a menu of contracts. In both cases, the ability to extract the consumers’ wealth with probability approaching (but not being equal to) one prevents a solution from existing. If instead of having a monopoly, we assume that firms compete *à la* Bertrand, no Nash equilibrium exists since firms can profitably deviate from any candidate for an equilibrium.

Additionally, we show that under the value function suggested by Tversky and Kahneman [17] and used by most parameterizations in the literature, for any probability weighting function, either (i) individuals are willing to pay arbitrarily large amounts for lotteries with finite expected value or (ii) individuals refuse all actuarially fair gambles. Case (i) is undesirable because it prevents the existence of equilibrium when the supply side is endogenous. Case (ii) is undesirable because one of the main successes of prospect theory is its ability to simultaneously explain risk-seeking and risk-averse behavior (such as the same person purchasing lottery tickets and insurance policies).

2. General result

Let $\mathcal{L} = (G, p; L, 1 - p)$ denote the lottery that pays $G \geq 0$ with probability p and $-L \leq 0$ with probability $1 - p$. We consider individuals whose preferences over such lotteries are represented by

$$V(\mathcal{L}) = w^+(p)v(G) - \lambda w^-(1 - p)v(L). \quad (1)$$

¹ See, for example, Camerer [6] for a review of the empirical literature.

$w^+, w^- : [0, 1] \rightarrow [0, 1]$ are called probability weighting functions, and $v : \mathbb{R}_+ \rightarrow \mathbb{R}$ is called a value function.² The parameter $\lambda \geq 1$ represents the degree of loss aversion. We make the following technical assumptions:

Assumption. v is continuous, strictly increasing, and $v(0) = 0$. w^- and w^+ are continuous, strictly increasing, $w^-(0) = w^+(0) = 0$, and $w^-(1) = w^+(1) = 1$.

We consider the problem of a risk-neutral firm designing a lottery \mathcal{L} to sell to individuals with preferences represented by (1). For simplicity, we will assume that the firm is a monopolist and has perfect information about the individuals' preferences.

Let $\pi(\mathcal{L}) = (1 - p)L - pG$ denote the firm's expected profit from the lottery $\mathcal{L} = (G, p; L, 1 - p)$. The firm maximizes its expected profit subject to the individuals' acceptance to participate, $V(\mathcal{L}) \geq 0$. Denote the greatest profit the firm can extract by $\Pi(V)$.³ We will show that profits are unbounded under fairly general conditions for V .

Proposition 1. *Suppose one of the following conditions is satisfied:*

- (1) $\lim_{p \rightarrow 0} w^+(p)v(1/p) = \infty$, or
- (2) $\lim_{p \rightarrow 0} w^-(p)v(1/p) = 0$.

Then, for any K_1 and K_2 , there exists a lottery \mathcal{L} such that $\pi(\mathcal{L}) > K_1$ and $V(\mathcal{L}) > v(K_2)$. In particular, the firm's possible profits are unbounded, $\Pi(V) = \infty$.

These conditions are related to how individuals evaluate lotteries with a small probability of a large gain or loss. Condition (1) can be stated in terms of a lottery that pays a large sum $1/p$ with probability p and 0 otherwise. This lottery has expected payment 1 but the gain $1/p$ grows unboundedly as the probability of winning p approaches zero. The condition states that the individuals would be willing to pay arbitrarily large amounts in order to participate in this lottery when p is small enough. Then, because the lottery has expected value 1, the firm can achieve infinite expected profits while still offering a lottery with arbitrarily large certainty equivalents. Such a lottery would resemble a lottery ticket with an extremely small probability of winning.

Condition (2) can be stated in terms of the inverse of the previous lottery: it features a large loss $1/p$ with very small probability p . This lottery has expected payment -1 but the loss $1/p$ grows unboundedly as the probability of losing approaches zero. Under condition (2), the risk-premium charged by the individual in order to participate in this lottery can be made arbitrarily close to zero by making the probability p of losing small enough. Intuitively, because individuals are willing to participate in lotteries with negative expected payoff for arbitrarily small prices, we can simultaneously achieve infinite expected profits for the firm and arbitrarily high utilities to the individuals by having the firm buy these lotteries from them. Such lotteries would resemble catastrophe bonds with prices below the actuarially fair price.

² More generally, we could have considered different value functions v^+ and v^- for the gain and loss domains. We use a single value function to simplify notation but the result holds in the general case.

³ That is, $\Pi(V) = \sup_{p,G,L} (1 - p)L - pG$ subject to $w^+(p)v(G) - \lambda \cdot w^-(1 - p)v(L) \geq 0$.

The firm can obtain unbounded profits if either of the conditions of [Proposition 1](#) is satisfied. Moreover, for any lottery that the firm offers, it is always possible to find another lottery that simultaneously yields a higher utility for the individuals and a greater profit for the firm. Therefore, the set of Pareto optimal lotteries is empty and existence of equilibrium is problematic.

Avoiding the result from [Proposition 1](#) requires joint restrictions on the value and probability weighting functions. For example, suppose that $w^+ = w^- = w$. Then, if the firm cannot obtain unbounded profits, conditions (1) and (2) imply that $0 < \lim_{p \rightarrow 0} w(p)v(1/p) < +\infty$. For a given weighting function w , v cannot grow too fast or too slow. Thus, one cannot pick these functions independently.

In addition to the concerns raised in [Proposition 1](#), a different problem arises under the most commonly used value function. Most of the prospect theory literature assumes a power value function: $v(x) = x^\alpha$, $0 < \alpha \leq 1$. This value function is homogeneous of degree α . Consequently, for any lotteries $\mathcal{L} = (G, p; L, 1 - p)$ and $c\mathcal{L} = (cG, p; cL, 1 - p)$, where c is a positive constant, we have $V(c\mathcal{L}) = c^\alpha V(\mathcal{L})$. Therefore, V satisfies the following property: for any $c > 1$, we have

$$V(c\mathcal{L}) \geq V(\mathcal{L}).$$

That is, if individuals are willing to accept a lottery \mathcal{L} , then they must also be willing to accept any lottery $c\mathcal{L}$ in which original payments are proportionately scaled by a factor $c > 1$. As a result, if there exists a lottery \mathcal{L} yielding expected profit π , the firm is able to obtain profit $c\pi$ by offering $c\mathcal{L}$ and the individuals would still be willing to participate. Hence, if the seller can obtain some positive profit, it can obtain any positive profit. That is, either $\Pi(V) = 0$ or ∞ .

If an individual with a power value function is willing to pay a strictly positive sum for an actuarially fair lottery, we have that $\Pi(V) > 0$ and, therefore, $\Pi(V) = \infty$. Moreover, if an individual accepts an actuarially unfair gamble (such as lottery tickets or most insurance policies), the firm can attain unbounded profits regardless of the probability weighting function.⁴

3. Examples

In this section, we show that the firm can achieve unbounded profits under virtually all functional forms that have been proposed in the literature.

Example 1. Prelec [14] provides an axiomatic foundation for the weighting functions

$$w^+(p) = \exp\{-\beta^+(-\ln p)^\sigma\},$$

$$w^-(p) = \exp\{-\beta^-(-\ln p)^\sigma\}$$

where $\sigma \in (0, 1)$, and $\beta^+, \beta^- \in (0, +\infty)$, and a power value function. Since $\lim_{p \rightarrow 0} w^+(p)v(1/p) = \infty$, this functional form satisfies condition (1) from [Proposition 1](#).

⁴ To see this formally, suppose there exists an actuarially fair lottery \mathcal{L} such that $V(\mathcal{L}) > 0$. By continuity, there exists a lottery \mathcal{L}' with strictly negative expected value and $V(\mathcal{L}') > 0$. Therefore the firm can attain unboundedly high profits by offering a lottery of the form $c\mathcal{L}'$, where $c > 1$.

Table 1
Optimal seller profits in functional forms estimated in the literature.

	$\hat{\alpha}$	$\hat{\gamma}$	$\hat{\delta}$	$\Pi(V)$
Tversky and Kahneman [17]	0.88	0.61	0.69	∞
Camerer and Ho [7]	0.37	0.56	0.56	∞
Tversky and Fox [16]	0.88	0.69	0.69	∞
Wu and Gonzalez [18] (gains only)	0.52	0.71	–	0 or ∞
Abdellaoui [1]	(0.89, 0.92)	0.60	0.70	∞
Bleichrodt and Pinto [5] (gains only)	0.77	0.67	–	∞

Example 2. Tversky and Kahneman [17] proposed the probability weighting function

$$w^+(p) = \frac{p^\gamma}{(p^\gamma + (1 - p)^\gamma)^{1/\gamma}}, \quad w^-(p) = \frac{p^\delta}{(p^\delta + (1 - p)^\delta)^{1/\delta}}, \tag{2}$$

where $\alpha, \gamma, \delta \in (0, 1)$ and $\lambda > 1$, and a power value function.⁵

Proposition 1 implies that $\Pi(V) = \infty$ if either $\gamma < \alpha$ or $\alpha < \delta$. Table 1 shows that these conditions are satisfied by all empirical estimates of these parameters we found, except possibly Wu and Gonzalez [18].⁶

If we impose the additional restriction that $w^+(p) = w^-(p)$ (as in the original prospect theory formulation of Kahneman and Tversky [11]), it follows that $\Pi(V) = \infty$ for almost all parameters. The only case in which we may have $\Pi(V) < \infty$ is when $\gamma = \delta = \alpha$.

Example 3. Wu and Gonzalez [19] propose the following linear in log-odds weighting function

$$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1 - p)^\gamma},$$

and assume a power value function. As with the Kahneman and Tversky weighting function, we have $\Pi(V) = \infty$ if $\alpha > \gamma$. They estimate the parameters for domain of gains only and obtain $\hat{\alpha} = 0.49$ and $\hat{\gamma} = 0.44$, which again satisfies the conditions of Proposition 1.

Example 4. Rieger and Wang [15] note that under the functional forms of Kahneman and Tversky and under the parameters typically obtained in empirical estimates, individuals succumb to St. Petersburg’s paradox: lotteries with finite expected value may yield infinite utility. In order to ensure the finiteness of utility for every lottery with finite expected value, they suggest either the use of bounded value functions or the use of the following polynomial weighting function:

$$w(p) = \frac{3 - 3b}{a^2 - a + 1} (p^3 - (a + 1)p^2 + ap) + p. \tag{3}$$

Condition (2) from Proposition 1 is automatically satisfied if the value function is bounded. Individuals are always willing to pay arbitrarily large amounts for some lotteries with finite

⁵ As noted by Ingersoll [10], another unfortunate property of Kahneman and Tversky’s weighting function is that weights are negative when $\gamma, \delta \in (0, 0.279)$. However, this is not particularly problematic since this condition is not satisfied under all empirical estimates of γ and δ .

⁶ Whether the monopolist can attain 0 or ∞ profits under the estimates of Wu and Gonzalez [18] is indeterminate because they only estimate preferences for gains, but the conclusion also depends on the parameters in the losses region.

expected values when the value function is bounded. Therefore, profits are unbounded. Moreover, condition (1) is true under the weighting function (3) and a power utility function, which again leads to unbounded profits.

Example 5. De Giorgi and Hens [8] propose the following value function:

$$v(x) = \exp(-\alpha x) - 1,$$

where $0 \leq \alpha \leq 1$. Because this value function is bounded, the firm obtains unbounded profits as in the previous example.

4. Extensions

Section 2 considered a monopolist facing identical consumers with prospect theory preferences. This section discusses the robustness of the results to variations in the model.⁷

Competition. Competition among rational agents sometimes limits the extent to which they can extract rents from agents with behavioral biases.⁸ This is not the case in the present model. Proposition 1 implies that for any lottery, there exists another lottery that makes both firms and consumers better off. Therefore, even when consumers have all the bargaining power, they can always find a new lottery that improves upon their original one and gives nonnegative profits to firms. Similarly, in a model in which firms compete *à la* Bertrand, for any offers by other firms, a seller can always find a lottery that generates both a higher profit and a higher utility to the consumers. Thus, no Nash equilibrium exists. In fact, since the set of Pareto optimal allocations is empty, in any model of competition in which equilibria are necessarily Pareto optimal, no equilibrium exists.

Heterogeneity. In general, consumer heterogeneity may reduce the monopolist's ability to extract surplus. However, because the monopolist can simultaneously obtain arbitrarily large profits and provide arbitrarily large utility to consumers when the conditions from Proposition 1 are satisfied, consumer heterogeneity does not prevent the monopolist from obtaining unbounded profits.⁹

Wealth constraints. A potentially more serious limitation of our analysis is the presence of wealth constraints. If consumers have a limited amount of wealth $B > 0$, they cannot accept lotteries with unbounded losses. Then, the monopolist's profit per-consumer is bounded above by B .

If condition (1) from Proposition 1 is satisfied, the monopolist is still able to obtain any expected profit arbitrarily close to B by considering lotteries that lead to a loss of B with probability

⁷ For a formal analysis of these points, see the working paper version of this article available on the authors' home pages (irm.wharton.upenn.edu/dgott/ or www.people.fas.harvard.edu/~azevedo/papers/).

⁸ This general point is a response to the argument that rational agents eventually drive irrational agents out of the market by offering 'Dutch book' schemes. Laibson and Yariv [13] argue that in a competitive equilibrium rational agents compete with each other, thereby exhausting the gains from such schemes.

⁹ More precisely, suppose the firm faces $N \geq 1$ types of consumers with different utility functionals V_i , $i = 1, \dots, N$. Each type of consumer has mass $m_i > 0$, and at least one of the types satisfies condition (1) or (2). Then, the monopolist can extract arbitrarily large profits from this type of consumers. Because there is a positive mass of each type, the monopoly can obtain unbounded profits and no solution exists.

approaching 1. However, because the monopolist cannot extract B with probability 1, the monopolist's program has no solution. Under condition (2), however, it may no longer be possible for the monopolist to extract arbitrarily large profits from consumers since wealth constraints do not allow for lotteries with unbounded losses that occur with probability approaching zero. Recall from Section 3 that the most common functional forms in the literature satisfy condition (1) for the relevant range of parameters found in empirical estimates. Therefore, a solution does not exist under these functional forms even when consumers face wealth constraints.

Reference points. We have followed Kahneman and Tversky [11] in defining the origin as the reference point. However, it is immediate to generalize Proposition 1 for any fixed reference point.

5. Conclusion

Essentially all functional forms used in the prospect theory literature imply that risk-neutral firms can extract arbitrarily high expected values from consumers. This happens because, under the usual assumptions, consumers are either willing to pay a very large sum for gambles that pay a large prize with small probability (such as a lottery ticket), or willing to accept a trivial sum for gambles that result in large losses with a small probability (such as writing a catastrophe insurance policy).

It is well known that most models of choice do not provide a good description of behavior in the realm of extremely small probabilities.¹⁰ Nevertheless, the functional forms typically used in prospect theory are particularly ill behaved in this range, preventing the model from being applicable to situations in which the supply side of the market is endogenous. In particular, when some consumers have prospect theory preferences satisfying some weak conditions, no equilibrium exists in standard monopolistic or competitive models.

While we view prospect theory as providing important insights for decision making under risk, our results suggest that risk-seeking behavior is unlikely to hold over the whole domain of losses. In particular, we find it unlikely that individuals would be willing to pay arbitrarily large amounts for lotteries with finite expected values and with arbitrarily small probabilities of winning. One possible way of solving the unboundedness problem while still retaining some of the insights from prospect theory is to adopt the global-plus-local functional form of Barberis and Huang [3] and Barberis et al. [4].¹¹ This functional form consists of an expected utility term defined over final wealth and the prospect theory utility term. If the expected utility function is “sufficiently risk averse,” individuals will not accept lotteries in which they lose their whole income with probability approaching one.

Appendix A. Proof of Proposition 1

Proof. Suppose condition (1) is satisfied and consider the lottery $(\frac{1}{p}, p; \frac{1+\pi}{1-p}, 1-p)$. By construction, the expected profit is π . Moreover,

¹⁰ Indeed, Kahneman and Tversky [11] themselves did not extend their graph of the probability weighting function to values close to the boundary.

¹¹ Koszegi and Rabin [12] present a rational expectations model of reference points under a global-plus-local functional form.

$$\begin{aligned} & \lim_{p \rightarrow 0} w^+(p)v\left(\frac{1}{p}\right) - \lambda w^-(1-p)v\left(\frac{1+\pi}{1-p}\right) \\ & \geq \lim_{p \rightarrow 0} w^+(p)v\left(\frac{1}{p}\right) - \lambda v(1+\pi) = \infty. \end{aligned}$$

Therefore, the firm can attain any profit $\pi > 0$ and provide an arbitrarily high utility to the consumer by taking p sufficiently small.

It is straightforward to show that $\lim_{p \rightarrow 0} w^-(p)v(1/p) = 0$ if and only if, for any $c > 0$, $\lim_{p \rightarrow 0} w^-(p)v(c/p) = 0$. Suppose condition (2) is satisfied, and consider the lottery $(\frac{u}{p}, p; \frac{\pi+u}{1-p}, 1-p)$, where $u, \pi \geq 0$. The lottery seller gets an expected profit of π , and the consumer obtains utility

$$w^+(p)v\left(\frac{u}{p}\right) - \lambda w^-(1-p)v\left(\frac{\pi+u}{1-p}\right).$$

Taking the limit as $p \rightarrow 1$, the consumer's utility converges to

$$v(u) - \lambda \cdot \lim_{p \rightarrow 0} \left[w^-(p)v\left(\frac{\pi+u}{p}\right) \right] = v(u).$$

Thus, it is possible to obtain any profit $\pi \geq 0$ and provide any utility $v(u)$ in $[0, v(\infty))$ by taking p close enough to 1. \square

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