Research Note

Generous Legislators? A Description of Vote Trading Agreements∗

Rafael Hortala-Vallve

Government Department — London School of Economics;
R.Hortala-Vallve@ac.uk

ABSTRACT

Legislators trade influence to attain the approval of their most preferred bills. A classic example can be found in pork barrel politics with concentrated benefits and diffuse costs, in which logrolling agreements can load costs onto legislators excluded from winning coalitions. I model the bargaining amongst legislators as a repeated game and show that the outcome depends on the voters’ relative valuations toward each bill. Most interestingly, I shed light on a vote trading outcome that has so far been overlooked in the literature; legislators most affected by logrolling agreements (those who bear costs with no benefit) may break such coalitions. Specifically, in equilibrium some legislators “generously” offer their support for bills that are not to their benefit, and obtain nothing in exchange.

Legislators usually need to reach decisions on multiple bills and attempt to gain greater influence over those that are more important to them. However, legislators can only cast a single vote in each decision so the only way for an individual to exert more influence on a particular bill is by trading votes with

∗ I thank Torun Dewan, David Myatt, Kevin Roberts, Thomas Stratmann, two anonymous referees, and seminar participants at various conferences. A version of this paper was previously circulated with the title: “Legislative Bargaining (over discrete bills)”.

MS submitted 12 May 2010; final version received 5 July 2011
ISSN 1554-0626; DOI 10.1561/100.00010034
© 2011 R. Hortala-Vallve
fellow legislators. This trade of influence is often referred to as logrolling — if you’ll help me roll my logs, I’ll help you roll yours. The classical example is found in pork barrel politics with concentrated benefits and diffuse costs where agreements involving a majority of legislators allow the approval of a set of bills, each one only benefiting a minority.\footnote{A logrolling situation exists when $x \succ y$ and $v \succ w$, but $yw \succ xv$ ($\succ$ denotes the group preference).} Despite the prevalence of such arrangements, we still lack a full theoretical characterization of this phenomenon.

It has been shown (see Riker and Brams, 1973; Bernholz, 1975) that whenever a logrolling agreement can occur, group preferences are cyclical and no coalitional equilibrium exists. In such circumstances it is suggested that legislators will continuously amend the status quo whilst never settling on any alternative. However, Roberts (2007) shows that Condorcet cycles may be avoided in an intertemporal setting when some legislators are patient enough and prefer settling on their second best alternative rather than continuously cycling over three alternatives (one of which may be highly disliked). Similarly, Penn (2009) analyzes the incentives of farsighted legislators when implementing a continuing program: in a world in which decisions taken today become the status quo tomorrow, legislators need to consider “not only the utility they receive from a given policy today, but also the utility they will receive from policies likely to replace that policy in the future”. Both papers show that an equilibrium generally exists when legislators vote probabilistically. The probabilistic nature of these equilibria can be reconciled with pure strategy equilibria in a Bayesian framework where legislators have private and payoff relevant information (Proposition 4 in Penn, 2009). Bernheim and Slavov (2009) look at a non game theoretic repeated majoritarian election and characterize the sequence of history dependent outcomes that win all pairwise comparisons: they show that history dependency may avoid nonexistence of Condorcet winners when increasing the dimensionality of the policy space. Vartanian (2011) complements this by showing that randomization is not required to ensure the existence of dynamic Condorcet winners.

In order to gain a better understanding of vote trading agreements and how these depend on the legislators’ relative valuation of each bill, I consider a repeated game where the status quo is amended by farsighted legislators. I show that whenever the costs of bills under consideration are too large relative to their concentrated benefits, no vote trading occurs and the
majoritarian will is implemented. On the other hand, when the benefits of a set of bills are large enough, it is very likely to observe the logrolling outcome where all bills that provide benefits to a majority of legislators are approved. Finally, and most interestingly, whenever the costs of the cheapest bill(s) included in a logrolling agreement are low enough, the legislators excluded from the logrolling outcome may avoid this outcome by offering their support for the cheapest bill(s) of the logrolling coalition. This behavior appears irrational as they appear to generously offer their support and receive nothing in exchange, yet at the same time they prevent the approval of other (more expensive) bills.

The insights of my model are illustrated in Section 3 where I revisit the analysis of the 1985 Farm Bill used by Stratmann (1992) to empirically prove the existence of vote trading agreements. Stratmann’s data shows that a logrolling coalition could have occurred between legislators with peanut interests and those with wheat interests. However, legislators with dairy interests generously offered their support to the peanut bill and the wheat bill was never approved. One conjecture is that legislators with dairy interests sought to avoid the approval of both bills by offering their support toward a (conceivably) cheaper bill that benefited peanut farmers. This is a simple yet, to my knowledge, unexplored idea in the literature. Such behavior would not constitute an equilibrium according to the classic reference on logrolling: in Ferejohn et al., (1987) only agreements that are preferred to the status quo by a majority of legislators are placed on the agenda; the peanut bill did not satisfy that requirement and so its passage in the absence of a logrolling agreement cannot arise in an equilibrium of that model. By contrast I offer a more general characterization of vote trading agreements. I do not assume a deterministic agenda at the onset of the game and so any alternative can eventually become the challenger to the status quo. As a consequence, and as I will show, an intuitively appealing equilibrium in which legislators with dairy interests offer gratuitous support for the wheat bill that can be sustained.

A Simple Logrolling Model

Consider a logrolling situation with three voters and two bills: voter 1 is the only voter who wants the first bill approved; voter 2 is the only one who wants the second bill approved; and, both voters prefer both bills approved
The payoffs in Table 1 capture this situation when \( m_1, m_2 \in (0, 1) \) — without loss of generality assume that \( m_3 \in (0, 1) \). The payoff when a bill is dismissed is 0 and total payoffs are the sum of payoffs on each bill. To have two bills that can be approved or dismissed implies that there are four possible alternatives. The set of alternatives is denoted \( \mathcal{A} := \{aa, dd, ad, da\} \) where the first (second) character denotes whether the first (second) bill is approved or dismissed — e.g., \( ad \) is the alternative where the first bill is approved and the second dismissed. I normalize the previous payoffs, \( \lambda_i := \frac{m_i}{1 + m_i} \in (0, \frac{1}{2}) \), so that the payoff voters derive from the four alternatives are given in Table 2.

As noted in the introduction, the possibility of logrolling implies nontransitive group preferences — this is shown in Figure 1 where the direction of each arrow indicates the alternative that wins the pairwise vote.

Following on from the work of Roberts (2007) and Penn (2009), I consider a dynamic game where farsighted voters interact repeatedly: at each period the status quo alternative is implemented; voters decide by majority whether the status quo remains until the next period or is replaced by an alternative that has been selected with equal probability from the remaining

---

**Table 1.** Payoffs when each of the bills is approved.

<table>
<thead>
<tr>
<th></th>
<th>first bill approved</th>
<th>second bill approved</th>
</tr>
</thead>
<tbody>
<tr>
<td>voter 1</td>
<td>1</td>
<td>(-m_1)</td>
</tr>
<tr>
<td>voter 2</td>
<td>(-m_2)</td>
<td>1</td>
</tr>
<tr>
<td>voter 3</td>
<td>(-m_3)</td>
<td>(-1)</td>
</tr>
</tbody>
</table>

**Table 2.** Preferences over the four possible alternatives.

<table>
<thead>
<tr>
<th></th>
<th>ad</th>
<th>aa</th>
<th>dd</th>
<th>da</th>
</tr>
</thead>
<tbody>
<tr>
<td>voter 1</td>
<td>ad</td>
<td>aa</td>
<td>dd</td>
<td>da</td>
</tr>
<tr>
<td>voter 2</td>
<td>da</td>
<td>aa</td>
<td>dd</td>
<td>ad</td>
</tr>
<tr>
<td>voter 3</td>
<td>dd</td>
<td>ad</td>
<td>da</td>
<td>aa</td>
</tr>
</tbody>
</table>

obtained utility: 1 \( 1 - \lambda_i \) \( \lambda_i \) 0
alternatives. The horizon is infinite and voters discount the future by \( \delta \in (0, 1) \).

I restrict my analysis to Markovian pure strategies by specifying voters intentions in each pairwise comparison of alternatives. Given a set of strategies, I denote the majoritarian winner in the pairwise vote between alternatives \( x \) and \( y \) as \( v^{xy} \). The expected continuation utility for a farsighted voter at alternative \( x \) depends on the strategies followed by all voters, \((v_1, v_2, v_3)\), and reads as follows:

\[
U_i(x \mid v_1, v_2, v_3) = u_i(x) + \frac{\delta}{3} [U_i(v^{xy} \mid v_1, v_2, v_3) + U_i(v^{xz} \mid v_1, v_2, v_3) + U_i(v^{xt} \mid v_1, v_2, v_3)]
\]

where \( x, y, z, \) and \( t \) are different alternatives from \( A \). The continuation utility at alternative \( x \) is the instantaneous utility at \( x \) (\( u_i(x) \) as described in Table 2) plus the sum of the continuation utilities of the alternatives that win each pairwise vote (this last term is discounted and multiplied by the probability that each alternative becomes the challenger to the status quo).

For notational simplicity, hereafter I omit the dependence of the continuation utility on the voters’ strategies.

My objective is to characterize stationary Markovian (perfect) equilibrium strategies under a weak dominance requirement (i.e., voters act as if their decision is pivotal); a set of strategies for each voter so that given any two

---

2 Penn (2009) allows for a nonuniform probability that any alternative challenges the status quo when legislators implement a policy at each period. This may be relevant when analyzing vote trading agreements but it adds an extra layer of complexity that I want to avoid at this stage.

3 My setting could be reformulated with no discount and an exogenous probability of termination like in Diermeier and Fong (2009). In that reformulation, voters decide whether to replace the status quo by a randomly selected alternative while knowing that at a given exogenous probability the current status quo can be the implemented outcome.
alternatives \(x\) and \(y\), voter \(i\) votes in favor of \(x\) only when the continuation utility at alternative \(x\) is higher than the continuation utility at alternative \(y\) (i.e., \(u^x_i = x\) if \(U_i(x) > U_i(y)\) for \(i = 1, 2, 3\) and \(x, y \in \mathcal{A}\)).

**Equilibria**

I describe equilibria in terms of the group preferences in each pairwise vote (instead of individual voting decisions) and I think of them as directed graphs: each node is an alternative, and the direction of the arrow between any two nodes captures the alternative that receives majoritarian support. I consider all possible configurations in the directed graph and find that the set of parameter values for which each pairwise comparison has majoritarian support. I am interested in finding the conditions under which the dynamic version of the static game with Condorcet cycles yields a single outcome with probability close to 1. Given voters’ strategies, I call a *Farsighted Condorcet Winner* (FCW) any alternative that wins all pairwise comparisons. In what follows I characterize the equilibria with FCW and discuss the existence of equilibria without FCW.

When equilibrium voting decisions yield a FCW, group preferences amongst the remaining alternatives may contain a cycle or a *Farsighted Condorcet Loser* (I define an alternative to a be a *Farsighted Condorcet Loser*, FCL, when it loses all pairwise comparisons). I analyze these two cases separately. All proofs can be found in the Appendix.

**Farsighted Condorcet Winner and Cycle**

By carefully analyzing the payoffs in Table 2 I can discard two alternatives from being a FCW when the remaining alternatives form a cycle.

First, when dismissing both bills (\(dd\)) is the FCW, the remaining three alternatives cannot form a cycle because approving the first bill only (\(ad\)) is the most preferred alternative in the cycle by voters 1 and 3; these voters have no incentives to vote in favor of alternatives where both bills or only the preferred bill of the second voter are approved (\(aa\) or \(da\)) as this should only decrease the utility they derive from only approving the first

---

4 There may be situations in which voters are indifferent between any two alternatives. These situations occur for a set of parameters with zero measure and are not analyzed in the subsequent sections for the sake of simplicity.
Generous Legislators? A Description of Vote Trading Agreements

A consequence of this argument is that, whenever there is a cycle and a FCW there can be no Condorcet winner among the alternatives in the cycle.

Second, only approving the second bill \((da)\) is the most preferred alternative of voter 2 and the least of voter 1. When such alternative is the FCW, voter 2 will always vote for it and voter 1 will always vote against it. It follows that this alternative can only be a FCW if voter 3 votes for it in all pairwise votes. However, voter 3 cannot vote in favor of implementing his third preferred alternative in all subsequent periods. To show this I write the continuation utilities for voter \(i\) when \(x\) is a FCW and \(y \succ z \succ t \succ y\) (\(\succ\) denotes the group preference relation and should be read as “has majority support over”):

\[
U_i(x) = u_i(x) + \frac{\delta}{3} [U_i(x) + U_i(x) + U_i(x)] \\
U_i(y) = u_i(y) + \frac{\delta}{3} [U_i(x) + U_i(y) + U_i(t)] \\
U_i(z) = u_i(z) + \frac{\delta}{3} [U_i(x) + U_i(z) + U_i(y)] \\
U_i(t) = u_i(t) + \frac{\delta}{3} [U_i(x) + U_i(t) + U_i(z)]
\]

A necessary condition for voter 3 to favor alternative \(da\) in all pairwise votes is that \(3U_3(da) > U_3(dd) + U_3(aa) + U_3(ad)\). Using (1) I can rewrite this expression as \(3u_3(da) > u_3(dd) + u_3(aa) + u_3(ad)\); i.e., the instantaneous utility at the FCW needs to be higher than the average instantaneous utility among the alternatives cycle. The last inequality does not hold true for any value of \(\lambda_3 \in (0, 1/2)\).

The next Proposition summarizes the conditions under which approving both bills or only approving the first bill can be a FCW.

**Proposition 1** An equilibrium exists where the approval of both bills is a FCW while a cycle exists among the remaining alternatives when (1) voters 1 and 2 do not strongly dislike bills 2 and 1, respectively and (2) voter 3 derives a similar disutility for any of the two bills. Formally, equilibrium voting decisions yield a FCW in \(aa\) and \(ad \succ dd \succ da \succ ad\) when \(\lambda_1 < \frac{3\delta}{9 - 3\delta + 2\delta^2}; \lambda_2 < \frac{\delta}{3}; \lambda_3 > \frac{3 - \delta}{6 - \delta}\); instead, the same FCW is accompanied by the cycle \(ad \succ da \succ dd \succ ad\) when \(\lambda_1 < \frac{\delta}{3}; \lambda_2 < \frac{3\delta}{2\delta^2 - 3\delta + 9}; \lambda_3 > \frac{\delta}{3 + \delta}\).
For illustrative purposes consider the bounds as $\delta \to 1$:

$$\lambda_1 < \frac{3}{8}, \lambda_2 < \frac{1}{3}, \lambda_3 > \frac{2}{5} \quad \text{and} \quad \lambda_1 < \frac{1}{3}, \lambda_2 < \frac{3}{8}, \lambda_3 > \frac{1}{4}.$$ 

In this equilibria, the FCW (approving both bills) is the second most preferred alternative of voters 1 and 2 and the least preferred of voter 3. It follows that voters 1 and 2 should favor such an alternative in all pairwise votes. In particular, they should favor it over their most preferred alternative but this can only happen when implementing both bills is not too inferior to only implementing their preferred bill, i.e., when $\lambda_1$ and $\lambda_2$ are bounded above. Note that the bounds switch roles depending on the direction of the cycle. When $ad \succ dd \succ da \succ ad$, voter 1’s most preferred alternative in the cycle is followed by his least preferred alternative; instead, voter 2’s most preferred alternative is followed by his third ranked alternative. This implies that voter 1’s expected utility at his most preferred alternative of the cycle is lower than the expected utility of voter 2 at his most preferred alternative, therefore voter 1 is (slightly) more inclined to vote in favor of the FCW than voter 2. When group preferences are $ad \succ da \succ dd \succ ad$, this reasoning is reversed and so are the bounds for voters 1 and 2. Finally, the bounds on voter 3 are needed to ensure that the majoritarian pairwise comparisons are satisfied.

The logrolling outcome can be sustained in a dynamic setting with far-sighted voters. For this outcome to hold we need each voter in the majority valuing highly enough the logrolling outcome (i.e., not bearing high costs from approving the bill that benefits the other majority member). In addition, we need the voter who only bears the costs of the logrolling outcome to equally dislike all bills that are passed. The bound on voter 3’s preferences is key in order to sustain this equilibrium. In the following Proposition, I show that when this voter dislikes one bill much more than the other, he is able to avoid the logrolling outcome.

---

5 In the first case voter 3 needs to favor $da$ over $ad$. This vote allows him to move toward his most preferred alternative ($dd$) but it is a vote against his instantaneous utilities. It follows that the costs in instantaneous utility need not be too large: $u_3(da) = \lambda_3$ and $u_3(ad) = (1 - \lambda_3)$ need to be close enough or $\lambda_3$ bounded below. In the second case, voter 3’s bound is relaxed because the inequalities he needs to satisfy ($U_3(dd) > U_3(ad) > U_3(da)$) coincide with his instantaneous valuations ($u_3(dd) > u_3(ad) > u_3(da)$). However, $\lambda_3$ cannot be too small, otherwise the gains from moving toward his most preferred alternative would not outweigh the costs of being closer to his least preferred one.
Proposition 2  An equilibrium exists where the approval of only the first bill is a FCW while a cycle exists among the remaining alternatives when (1) voter 1 does not strongly dislike the second bill, (2) voter 2 strongly dislikes the first bill; and (3) voter 3 much prefers the approval of the first bill to the approval of the second one. Formally, equilibrium voting decisions yield a FCW in $ad$ and $aa \succ dd \succ da \succ aa$ when $\lambda_1 < \frac{3-\delta}{6-\delta}, \lambda_2 > \frac{\delta}{3+\delta}, \lambda_3 < \frac{3\delta}{2\delta - 3\delta + 9}$; instead, the same FCW is accompanied by the cycle $aa \succ da \succ dd \succ aa$ when $\lambda_1 < \frac{\delta}{3+\delta}, \lambda_2 > \frac{3-\delta}{6-\delta}, \lambda_3 < \frac{4}{3}$.

A similar reasoning to that above allows us to understand the different bounds on the relative valuations. Consider the bounds as $\delta \to 1$:

$$\lambda_1 < \frac{2}{5}, \lambda_2 > \frac{1}{4}, \lambda_3 < \frac{3}{8} \quad \text{and} \quad \lambda_1 < \frac{1}{4}, \lambda_2 > \frac{2}{5}, \lambda_3 < \frac{1}{3}.$$  

The FCW is voter 1’s most preferred alternative and voter 2’s least preferred alternative: the former always favors such an alternative and the latter always opposes it. This implies that voter 3 is pivotal in all pairwise votes involving the FCW. However, voter 3 is only willing to favor the approval of the first bill when its costs are small enough (i.e., when $\lambda_3$ is bounded above). The bound when $aa \succ da \succ dd \succ aa$ is larger than when the cycle runs in the opposite direction due to voter 3’s most preferred alternative in the cycle being followed by his least preferred one (instead of his second least preferred alternative as is the case when $aa \succ da \succ dd \succ aa$). Finally, the conditions on voters 1 and 2 ensure that each pairwise comparison among the three alternatives in the cycle is supported by a majority of voters.

The alternative described as the generous outcome in the introduction where two voters approve a bill that only benefits one member of the winning coalition can be a FCW. A necessary condition for this to hold true is that voter 1 need not strongly dislike the second bill so he is willing to reach a logrolling agreement with voter 2. It is precisely the threat of a logrolling coalition together with voter 3 not terribly disliking the first bill which makes voter 3 willing to vote in favour of the alternative most preferred by voter 1. The FCW in this equilibrium is precisely the vote trading outcome overlooked in the literature: voter 3, in light of the risk of seeing both bills passed, and taking into account that one of the bills is very cheap ($\lambda_3$ is bounded above), avoids the logrolling outcome.
Farsighted Condorcet Winner and Farsighted Condorcet Loser

I started the previous subsection by showing that dismissing both bills cannot be a FCW when there is a cycle among the remaining alternatives: when \( dd \) is the FCW, a Condorcet winner exists among the remaining three alternatives thus farsighted legislators do not cycle among these alternatives. However, I now show that \( dd \) may indeed be a FCW when there is no cycle among the remaining alternatives.

**Proposition 3** An equilibrium exists where the dismissal of both bills is a FCW while a FCL exists among the remaining alternatives when both voters 1 and 2 strongly dislike bills 2 and 1, respectively. Formally, equilibrium voting decisions yield a FCW in \( dd \) when \( \lambda_1 > \frac{\delta}{3-\delta} \) and \( \lambda_2 > \frac{3(3-2\delta)}{2(3-\delta)^2} \) (in such case, \( aa \) is a FCL and \( ad \succ da \)).

Farsighted voters prefer to implement the issue-by-issue majoritarian outcome when there is no much to gain from a logrolling coalition. The Proposition shows that a logrolling coalition may not always succeed even when a majority of farsighted legislators strictly prefer such agreements to the majoritarian issue-by-issue outcome — by construction (see payoff tables above) a logrolling coalition is always strictly preferred by voters 1 and 2 to having no bills approved. This result holds true when legislators are farsighted because the gain voters 1 and 2 may obtain from a logrolling coalition are outweighed by the risks of implementing any of the alternatives each voter most dislikes.

The generous outcome can also constitute a FCW in the presence of a FCL:

**Proposition 4** An equilibrium exists where the approval of only the first bill is a FCW while a FCL exists among the remaining alternatives when voter 2 strongly dislikes the first bill and voter 3 much prefers the approval of the first bill to the approval of the second one. Formally, equilibrium voting decisions yield a FCW in \( ad \) when \( \lambda_2 < \frac{\delta}{3+\delta} \) and \( \lambda_3 < \frac{\delta}{3-\delta} \) (in such case, \( da \) is a FCL and \( aa \succ dd \)).

Once again, the generous outcome in which one of the voters gives his support for a bill he does not benefit from is a FCW. The necessary conditions in this case are similar to these in Proposition 2: voter 3 has strong incentives to avoid a logrolling coalition and can do so “cheaply” given that he does not strongly dislike the approval of the first bill, and the risk of a
logrolling coalition exists because voter 2 does not strongly dislike the first bill so he is willing to reach a logrolling agreement with voter 1. While in Proposition 2 the threat of a logrolling coalition came from voter 1, it now comes from voter 2. The other main difference between these two propositions is that when there is no cycle, no constraint is required from the voter who most prefers the FCW.

In common with the previous subsection, I end this section by stating that only approving the second bill cannot be a FCW: voter 3 should be pivotal in all pairwise votes involving da but he does not always want to favor his third preferred alternative. The logrolling outcome cannot be a FCW either. The intuition behind this last result is a little more elaborate. On the one hand, voter 1 is pivotal in the pairwise vote between the potential FCW (approving both bills) and his preferred option (approving only the first bill), which implies that he should not dislike the approval of the second bill too much ($\lambda_1$ bounded above). On the other hand, voter 1 is pivotal in the pairwise vote between only approving the second bill and approving none; this vote goes in favor of his instantaneous utility but implies that his preferred alternative will never be reached thus his instantaneous gain should be large enough ($\lambda_1$ bounded below). The proof relies on showing that voter 1 cannot be pivotal in both pairwise votes in the way that sustains the logrolling outcome being the FCW.

**Corollary 1** No equilibrium exists in which alternatives aa or ad are a FCW while a FCL exists in the remaining alternatives.

**Equilibria with no Farsighted Condorcet Winners**

So far I have analyzed equilibria with FCW. These equilibria are a natural focal point in my analysis because when they are played I can predict the outcome of the legislative process almost with certainty. However, there are many more equilibria where no FCW exists. For instance for a large set of parameters, there is an equilibrium in which the group preference of the repeated game replicates the group preferences in the one shot game (see Figure 1). Other equilibria exist where an alternative is a FCL and there is a cycle among the remaining three alternatives.

In a previous version of this paper (Hortala-Vallve, 2009), I considered a slightly different game where policy would only be implemented when voters decided to no longer amend the status quo. In that setting it can be shown
that there is always a voter who has incentives to amend the status quo unless the status quo is a FCW. In other words, equilibria with no FCW have voters amending the status quo indefinitely; instead, equilibria with FCW lead to the implementation of such alternative. Considering an extended model where disagreement is explicitly modelled as the worst possible scenario (Eliaz et al., 2004), legislators would never play equilibria without a FCW.

Example

Stratmann (1992) provides empirical evidence of the existence of logrolling agreements in the Congressional amendments to the 1985 Farm Bill. He shows that legislators with dairy interests favored the peanut bill when the legislators with peanut interests were not necessary for the approval of the dairy bill. My model may help understand why some legislators generously favored a bill that would only impose costs on them.

I illustrate the aspect above with a simplified Congress with only three legislators and Table 3 shows their payoffs.

Benefits are concentrated and costs evenly distributed, thus a majority of the legislators wish for the dismissal of each bill. However, the peanut and wheat legislators have incentives to logroll by jointly favoring each other’s bill. The dairy legislator dislikes this outcome and may try to break a wheat–peanut coalition by appearing generous and supporting the peanut bill. By normalizing the payoff matrix I can apply the findings in Section 2: $m_1 = -0.64, m_2 = -0.4, m_3 = -0.29$, and using the transformation $\lambda_i = \frac{m_i}{1 + m_i}$ we have that $\lambda_1 = 0.38, \lambda_2 = 0.29$, and $\lambda_3 = 0.22$.

Table 3. Payoff table illustrating an oversimplified Congress.

<table>
<thead>
<tr>
<th></th>
<th>peanut bill approved</th>
<th>wheat bill approved</th>
</tr>
</thead>
<tbody>
<tr>
<td>peanut legislator</td>
<td>13 – 2</td>
<td>–7</td>
</tr>
<tr>
<td>wheat legislator</td>
<td>–2</td>
<td>12 – 7</td>
</tr>
<tr>
<td>dairy legislator</td>
<td>–2</td>
<td>–7</td>
</tr>
</tbody>
</table>

Irwin and Kroszner (1996) and Elvik (1995) also show the presence of logrolling agreements in the Smoot-Hawly Tariff Act and highway expenditures across Norway, respectively.
When legislators are arbitrarily patient (i.e., $\delta \to 1$) (1) there can be no equilibrium where $aa$ is the FCW and a cycle exists among the remaining alternatives ($\lambda_3$ is too small), (2) there is an equilibrium where $ad$ is the FCW only when the cycle among remaining alternatives is such that $aa \succ dd \succ da \succ aa$, and (3) there is no equilibrium with a FCW and a FCL.

With this toy example, I have shown that there are situations in which vote trading agreements involve legislators generously offering their support for packages of bills they do not benefit from. In my example the peanut bill is much cheaper than the wheat bill (total costs are 6 instead of 21) and it is in the dairy legislator’s interests to favor the approval of the peanut bill so as to avoid a logrolling coalition where both bills are approved.

Concluding Remarks

Vote trading games are notoriously difficult and existing results are not fully satisfactory. By considering the repeated interaction of legislators I have avoided well-known problems of equilibrium existence. In the light of my analysis, it is worth emphasizing the differences with the well-known paper on logrolling by Ferejohn et al. (1987). Their work shows that strategic legislators only select the Condorcet elements from the set of alternatives preferred to the status quo. The idea behind their analysis is simple: in a world where legislators anticipate the behavior of their fellow legislators, only the best alternatives among those superior to the status quo can be selected. Their rationale is not able to explain the generous outcome because there is not a majority of voters who prefer this alternative over the issue-by-issue majoritarian outcome. Instead, the generous outcome is supported in my game because the threat of any alternative challenging the status quo allows the possibility of alternatives initially inferior to the status quo to be implemented. Finally note that my results are consistent with previous work on coalitional bargaining where it is shown that the uncovered set contains the outcomes that can be implemented under a variety of institutional

---

7 The seminal work of Buchanan and Tullock (1962) started a long normative discussion on whether vote trading is welfare improving. Positive analysis of vote trading have assumed a centralised market for votes with perfectly tradeable and divisible votes (e.g., Mueller, 1967; Wilson, 1969; Phillipson and Snyder, 1996). More recently Casella et al. (2010) have shown that a competitive market for votes always results in dictatorship and that such a market generates welfare losses. These works offer interesting insights but fail to fully explain observed vote trading agreements.
sets up: approving only the second bill (da) can never be a FCW and this is the only alternative not included in the uncovered set of my model.

This paper is not a comprehensive theory of vote trading: there are equilibria with no FCW and for some parameter values there exists a multiplicity of equilibria with FCW. It demonstrates however the strength in considering farsighted legislators. By adding institutional structure, this model may help us to understand the vote trading agreements that are usually forged in our legislative bodies.

Appendix

Lemma 1 Assume $x$ is a FCW and cycle exists among the remaining alternatives $(y, z, \text{ and } t)$.

$$U_i(y) > U_i(z) > U_i(t) \Leftrightarrow u_i(y) > \frac{1}{3-\delta}(3u_i(z) - \delta u_i(t)) \quad \text{and}$$

$$u_i(y) > \frac{1}{\delta}(3u_i(t) - (3-\delta)u_i(z)).$$

Proof: This is an immediate consequence of using the nonrecursive formulation of the continuation utilities in (1):

$$U_i(x) = \frac{1}{1-\delta}u_i(x)$$

$$U_i(y) = \frac{\delta}{2\delta^2 - 5\delta + 3}u_i(x) + \frac{3}{(3-2\delta)(\delta^2-3\delta+9)}((3-\delta)^2u_i(y) + \delta(3-\delta)u_i(t) + \delta^2u_i(z))$$

$$U_i(z) = \frac{\delta}{2\delta^2 - 5\delta + 3}u_i(x) + \frac{3}{(3-2\delta)(\delta^2-3\delta+9)}((3-\delta)^2u_i(z) + \delta(3-\delta)u_i(y) + \delta^2u_i(t))$$

$$U_i(t) = \frac{\delta}{2\delta^2 - 5\delta + 3}u_i(x) + \frac{3}{(3-2\delta)(\delta^2-3\delta+9)}((3-\delta)^2u_i(t) + \delta(3-\delta)u_i(z) + \delta^2u_i(y)).$$

\[\blacksquare\]

* See Miller (1980), Shepsle and Weingast (1984), and Vartianen (2011).
Proof of proposition 1. In any two pairwise comparisons, there is always a voter that belongs to the majority in both votes: there always exists a voter whose preferences over two pairwise comparisons coincide with those of the group. When group preferences are such that \( y \succ z \succ t \succ y \), each player should satisfy one of the following chains of inequalities:

\[
U_i(y) > U_i(z) > U_i(t) \\
U_j(z) > U_j(t) > U_j(y) \\
U_k(t) > U_k(y) > U_k(z)
\]  

(2)

Using the expression for the continuation utilities (1) I can rewrite \( U_i(y) > U_i(z) \) as \( u_i(y) + \frac{\delta}{4} U_i(t) > u_i(z) + \frac{\delta}{4} U_i(y) \). The latter inequality only holds when \( u_i(y) > u_i(z) \) because in (2) I assumed \( U_i(y) > U_i(t) \). Analogous calculations show that a necessary condition for inequality \( U_i(z) > U_i(t) \) is that \( u_i(y) > u_i(t) \). In other words, amongst the alternatives in the cycle, the most highly ranked alternative (in terms of continuation utilities) yields highest instantaneous utility. When \( aa \) is a FCW and \( ad \succ dd \succ da \succ ad \) the following inequalities should hold:

\[
U_1(ad) > U_1(dd) > U_1(da) \\
U_2(da) > U_2(ad) > U_2(dd) \\
U_3(dd) > U_3(da) > U_3(ad)
\]  

(3)

By using Lemma 1 on the previous three chains of inequalities together with the FCW winning all pairwise comparisons (i.e. \( U_1(aa) > U_1(ad) \) and \( U_2(aa) > U_2(da) \)), I obtain the conditions \( \lambda_1 < \frac{3\delta}{9-3\delta+3\delta}, \lambda_2 < \frac{\delta}{3}, \lambda_3 > \lambda_3 \geq \frac{3-\delta}{6-\delta} \). The second instance where group preferences are such that \( ad \succ da \succ dd \succ ad \) is proved analogously.

Proof of proposition 2. When \( ad \) is a FCW and \( aa \succ dd \succ da \succ aa \) the following inequalities should hold:

\[
U_1(aa) > U_1(dd) > U_1(da) \\
U_2(da) > U_2(aa) > U_2(dd) \\
U_3(dd) > U_3(da) > U_3(aa)
\]  

(4)

As in the proof of Proposition 1, the FCW being voter 2’s least preferred alternative implies that voters 1 and 3 should always favor the FCW: \( U_1(ad) > U_1(aa) \) and \( U_3(dd) > U_3(da) \). This together with (4) implies \( \lambda_1 < \frac{3-\delta}{6-\delta}, \lambda_2 > \frac{\delta}{3+\delta}, \lambda_3 < \frac{3\delta}{27\delta - 3\delta + 9} \). When group preferences are such that \( aa \succ da \succ dd \succ aa \) the proof is analogous.
Lemma 2  Equilibrium voting decisions imply that \( x \) FCW, \( t \) is a FCL and \( y \succ z \) if and only if a majority of voters satisfies the following inequalities:

\[
\begin{align*}
1 - u_i(x) > u_i(y) & \quad 4 - u_i(x) > \frac{\delta}{3 - \delta} u_i(y) + \frac{3 - 2\delta}{3 - \delta} u_i(z) \\
2 - u_i(y) > u_i(z) & \quad 5 - u_i(x) > \frac{\delta}{3 - \delta} u_i(y) + \frac{\delta(3 - 2\delta)}{3(3 - \delta)} u_i(z) + \frac{3 - 2\delta}{3} u_i(t) \\
3 - u_i(z) > u_i(t) & \quad 6 - u_i(y) > \frac{\delta}{3} u_i(z) + \frac{3 - \delta}{3} u_i(t)
\end{align*}
\]

Proof:  The continuation utilities read as follows:

\[
\begin{align*}
U_i(x) &= u_i(x) + \frac{\delta}{3} [U_i(x) + U_i(x) + U_i(x)] \\
U_i(y) &= u_i(y) + \frac{\delta}{3} [U_i(x) + U_i(y) + U_i(y)] \\
U_i(z) &= u_i(z) + \frac{\delta}{3} [U_i(x) + U_i(y) + U_i(z)] \\
U_i(t) &= u_i(t) + \frac{\delta}{3} [U_i(x) + U_i(y) + U_i(z)].
\end{align*}
\]

(5)

\( x \) is a FCW only when \( U_i(x) > U_i(y) \) for a majority of voters: by rewriting the first two expressions in (5) I see that this inequality holds only when \( u_i(x) > u_i(y) \). Similarly, it can be shown that \( U_i(y) > U_i(z) \) holds only when \( u_i(y) > u_i(z) \), and finally \( U_i(z) > U_i(t) \) holds only when \( u_i(z) > u_i(t) \).

I need three further pairwise comparisons for the previous equilibrium to hold: \( U_i(x) > U_i(z), U_i(x) > U_i(t), \) and \( U_i(y) > U_i(t) \). These three inequalities are equivalent to conditions 4, 5, and 6. To prove this I just need to use the nonrecursive formulation of continuation utilities

\[
\begin{align*}
U_i(x) &= \frac{1}{1 - \delta} u_i(x) \\
U_i(y) &= \frac{\delta}{(3 - 2\delta)(1 - \delta)} u_i(x) + \frac{3 - 2\delta}{3 - 2\delta} u_i(y) \\
U_i(z) &= \frac{\delta}{(3 - 2\delta)(1 - \delta)} u_i(x) + \frac{3\delta}{(3 - \delta)(3 - 2\delta)} u_i(y) + \frac{3}{3 - \delta} u_i(z) \\
U_i(t) &= \frac{\delta}{(3 - 2\delta)(1 - \delta)} u_i(x) + \frac{3\delta}{(3 - \delta)(3 - 2\delta)} u_i(y) + \frac{\delta}{3 - \delta} u_i(z) + u_i(t).
\end{align*}
\]

(6)
Proof of proposition 3. When $dd$ is the FCW, conditions 1, 2, and 3 in Lemma 2 imply that $y = ad$, $z = da$, and $t = aa$. A majority of voters should satisfy each of the conditions 4, 5, and 6 in Lemma 2. Inequality in (4) is never met by voter 2, always by voter 3 and is met by voter 1 when $\lambda_1 > \frac{\delta}{3-\delta}$; inequality in (5) is never met by voter 1, always by voter 3 and is met by voter 2 when $\lambda_2 > \frac{3(3-2\delta)}{2(3-\delta)}$; inequality in (6) is always met by voters 1 and 3.

Proof of proposition 4. When $ad$ is the FCW, conditions 1, 2, and 3 in Lemma 2 imply that there are two possible scenarios: (1) $y = da$, $z = aa$, and $t = dd$; and (2) $y = aa$, $z = dd$, and $t = da$. The first scenario cannot constitute an equilibrium because the sixth inequality in Lemma 2 is only satisfied by voter 2. The second scenario is an equilibrium when each inequality in conditions 4, 5, and 6 (Lemma 2) is satisfied by a majority of voters: inequality in (4) is always met by voter 1, never by voter 2 and by voter 3 only when $\lambda_3 < \frac{\delta}{3-\delta}$; inequality in (5) is always met by voters 1 and 3, and never by voter 2; inequality in (6) is never met by voter 1, always by voter 3, and by voter 2 only when $\lambda_2 < \frac{\delta}{3+\delta}$.

Proof of corollary 1. When $da$ is the FCW, conditions 1, 2, and 3 in Lemma 2 imply that $ad$ is FCL loser and $aa \succ dd$. Having a FCW in $da$ implies that voter 3 is pivotal in all decisions involving $da$, in particular: $U_3(da) > U_3(dd)$. Using (6) I can rewrite this inequality as $\lambda_3 > \frac{9-6\delta}{3+\delta}$ which is never satisfied when $\lambda_3 \in (0, \frac{1}{2})$, i.e., $da$ cannot be a FCW.

When $aa$ is the FCW, conditions 1, 2, and 3 in Lemma 2 imply that $y = dd$, $z = ad$, and $t = da$. Equilibrium voting decisions yield the prescribed pairwise comparisons only when conditions 4, 5, and 6 in Lemma 2 are satisfied: voter 3 always satisfies the fourth but never the sixth; voter 2 always satisfies the sixth but never the fourth; therefore voter 1 is pivotal in both decisions but there is no $\lambda_1$ that satisfies both inequalities simultaneously.

References


