I.S.S.N: 1885-6888

ECONOMIC ANALYSIS
WORKING PAPER SERIES

AGGREGATING JUDGMENTS BY THE MAJORITY METHOD.

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Working paper 10/2006

UNIVERSIDAD AUTONOMA DE MADRID

DEPARTAMENTO DE ANÁLISIS ECONÓMICO:
TEORÍA ECONÓMICA E HISTORIA ECONÓMICA

Aggregating judgments by the majority method.

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Abstract

Judgement aggregation has been receiving increasing attention over recent years. Some typical impossibility results have been proved, about majority and other similar aggregation methods. Those results depend essentially on certain logical constraints borrowed from standard two-valued deductive logic. Nevertheless, the adequacy of these constraints is doubtful. In this paper, we show that by weakening the consistency conditions in a plausible way, such impossibility theorems can be reversed. We also show that the formalism habitually employed in social choice theory may convey a richer setting for analysing this sort of aggregation.

Key Words : Judgement aggregation, majority method, logical constraints on judgment aggregation, discursive dilemma.

JEL Codes: D7, D70, D71.

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It is known that, in aggregating preferences, the majority method can lead to perplexing phenomena such as the voting paradox, also known as Condorcet's paradox. More recently, in the growing literature on judgment aggregation inspired by the discursive dilemma[2] and by the pioneering work of Philip Pettit (2001) and Christian List and Philip Pettit (2002), it has been pointed out that the majority method and other similar aggregation procedures may lead to basic logical inconsistencies. With these methods thus disqualified, other procedures are proposed, even though such procedures may attach to the group judgments that a majority of its members reject.

We suggest in this paper that the apparent logical inconsistencies may arise because the logical restrictions imposed on collective judgments may be too strong. In the above-mentioned literature and in this paper as well, collective logical rationality is understood by analogy to individual logical rationality. However, we suggest that the right analogy for any group which lacks a unanimous point of view is that of an individual who is not certain about his judgments and beliefs, rather than that of a person completely certain on his position. Collective unanimity would be analogous to individual certainty, while lack of unanimity would correspond at the individual level to lack of certainty. If this suggestion is accepted, then it would follow that the logical restrictions required from collective judgments should be weaker than those[1] postulated habitually in the literature.

In addition, we propose certain weaker logical restrictions that are met by the majority method and its variants. Therefore, the main aim of the paper is to recover the majority method and its variants for aggregating judgments when available information about the individual points of view is of a qualitative nature.

[2] Table 4.1 in section 4 shows a version of the dilemma.

The paper assumes a relational framework for representing judgments that is more general than the usual framework on judgment aggregation and is more familiar in social choice theory.[3] This relational setting up allows us to point to certain ambiguities in the habitual framework and conveys some additional outcomes, for instance the coincidence of the majority method under certain circumstances with the procedure based on attaching with each collective judgment the difference between the number of the group's members who accept it and the number of the group's members who reject it. As a consequence, the majority method generates under such circumstances a transitive aggregate relation, and the voting paradox and similar difficulties disappear.

In section 2 and 3, judgment relations, judgment aggregation functions and some variants of the majority method are introduced, and some well-known properties of those variants are enumerated. Section 4 focuses on the impossibility theorem presented by List and Pettit (2002). Doing so exemplifies these kind of results, the role that habitual logical constraints play in them, and the way that the majority method and its variants are disqualified as judgment aggregation procedures. In section 5 and 6, weaker logical constraints are introduced and it is shown that the majority method and its variants hold for them. This circumstance and some other considerations are commented on for justifying the proposed weaker logical restrictions. Section 7 points to the possibility of simplifying the way aggregation is set up in cases such as the discursive dilemma, where aggregation concerns arguments and conclusions. Section 8 presents a summary.

2.- **BASIC NOTIONS**

.- Agenda and language.[4]

$N = \{1, 2, \dots, n\}$ $(n \geq 2)$ is a (finite) group of individuals. Each individual may make a judgement on each of a set of propositions, *p*, *q*, *r*, *s*,….. This set is called the "agenda"

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[3] List and Pettit (2006b) compare both frameworks, that of preference aggregation and that of judgment aggregation.

[4] As far as possible, we try to use the same notions and the same notation as Dietrich (2006).

and is denoted by G. Aggregating the judgments made by the individuals obtains judgements that can be considered the group's collective judgments.

Following Dietrich (2006: 287-8) in the basic logical notions, let "the set of all propositions" *L* (i.e. *the language*) be the (smallest) set such that

- *L* contains a set of atomic (non-descomposable) propositions,
- if *L* contains *p* and *q*, then *L* also contains the negation of both ($\neg p$, $\neg q$) and every proposition obtained by linking *p* and *q* by means of any of the four diadic logical functors ($\wedge$ "and", $\vee$ "or", $\rightarrow$ "if, then" and $\leftrightarrow$ "if and only if", or in short "iff").

Given *L*, the *agenda G* is a non-empty subset of *L*, which contains no double-negated propositions ($\neg\neg p$), and such that for every $p \in G$, also $\sim p \in G$, where $\sim p = \neg p$ if *p* is not itself a negated proposition, and $\sim p = q$ if *p* is the negated proposition $\neg q$. We refer to $\sim p$ as the complementary proposition of *p*, and to the latter one as the complementary proposition of the former.

A *truth-value assignment* is a function assigning the value "true" or "false" (or "0" and "1") to each proposition in *L*, in the standard way.[5] For every $S \subseteq L$, *S* is (*logically*) *consistent* (resp. *inconsistent*) if there exists a (resp. no) truth-value assignment that assigns "true" to each $r \in S$; for every $p \in L$, *S (logically) entails p*, if $S \cup \{\sim p\}$ is inconsistent; it follows that for every pair $p, q \in L$, *p (logically) entails q*, if $\{p, \sim q\}$ is inconsistent.

The agenda *G* is habitually assumed to contain at least two distintct and independent propositions, $p \neq q$, and their negations, $\neg p$ and $\neg q$, where "independent" means that none of them implies the other proposition nor its complementary one. For simplicity, let us assume also that for some pair of independent propositions $p, q \in G$, the agenda contains their conjunction ($p \wedge q$) (and its negation $\neg(p \wedge q)$).

.- Judgment relationships.

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[5] For any $p, q \in L$, $\neg p$ is true iff *p* is false; ($p \wedge q$) is true iff both *p* and *q* are true; ($p \vee q$) is true iff *p* is true or *q* is true; ($p \rightarrow q$) is true iff *p* is false or *q* is true; ; ($p \leftrightarrow q$) is true iff *p* and *q* are both true, or both are false.

A judgment relationship is a binary relation *R* defined in an agenda G; "*pRq*" means that proposition *p* is accepted (by some person or by the group) as firmly, at least, as *q* is.

We define relations *P* and *I* as usually: for every $p, q \in G$,

- *pPq*, iff, *pRq* and not *qRp*;
- *pIq*, iff, *pRq* and *qRp*.

The expresion "*pPq*" means that proposition *p* is more firmly accepted (by some person or by the group) than *q*; when *q*= ~*p*, "*pPq*" means that *p* is accepted and *q* is rejected. The expression "*pIq*" means that proposition *p* is equally firmly accepted (by some person or by the group) as *q*.

Judgment relationships may hold the usual relational properties. For instance, *R* is

- *reflexive* iff for every $p \in G$, *pRp;*
- *weakly connected* iff for every $p, q \in G$, $p \neq q$, *pRq o qRp*;
- *strongly connected or complete* iff for every $p, q \in G$, *pRq o qRp*;
- *transitive* iff for every $p, q, r \in G$, *pRq* and *qRr*, then *pRr*;
- a *complete preordering* iff it is (strongly) *complete* and *transitive.*

The above definitional conditions imply that *I* is *reflexive* (for every $p \in G$, *pIp*) and *symmetrical* (for every $p, q \in G$, if *pIq* then *qIp*). Relation *P*, on the contrary, is *asymmetrical* (for every $p, q \in G$, if *pPq* then ¬*qPp*).

Properties focusing on the relations between complementary propositions may be relevant, as for instance:

- *Vertical or complementary completeness*: for every $p \in G$, *pR*~*p* or ~*pRp*.

- *Vertical restriction*: judgment relation R is *vertically restricted* iff for every $p, q \in G$ such that *pRq* or *qRp*, *q*=~*p* (and *p*=~*q*).

Since judgment relations compare propositions, they may hold properties of a logical character. For instance, the definition of *P* implies these two versions of the *non-contradiction principle*:

- *Restricted non-contradiction principle*: for every *p Î G*, it cannot be that *pP~p* and also *~pPp*.
- *General non-contradiction principle*: for every *p Î G*, it cannot be that *pP~p* and also *~pRp*.

Similarly, *tertio excluso* is the name of another basic *principle* of two-valued standard logic:

- *Tertio excluso principle*: for every *p Î G*, if *pR~p* or *~pRp*, then *pP~p* or *~pPp*.

A judgment on the question raised by proposition *p* may take one of the following mutually exclusive forms: *pPØp*, *pIØp* and *ØpPp*. If the *tertio excluso principle* holds, then the second option is excluded.

Other properties concerning relationships between non-complementary propositions, like the following basic one, may be also relevant:

- *Vertical* or *complementary balance*: for every *p, q Î G*, *pRq* iff *~qR~p*.

If *R* is *transitive*, then *vertical* or *complementary balance* entails basic coherence properties such as these:

(a) if *pP~p* and *qR~q*, then *pP~q* and *qP~p*;
(b) if *pP~p* and *qI~q*, then *pPq*;
(c) if *pI~p* and *qI~q*, then *pIq* and *~pI~q*.[6]

[6] For (a): Imagine that, on the contrary, *~qRp* or *~pRq*. If *~qRp*, *complementary balance* entails that *~pPq*; by transitivity, *~qRq*, contradicting the hypothesis. Analogously, it follows from *~pRq* that *~pPp*.
For (b): Note that if *qPp*, then *qI~q*; and that if *pIq* then *pI~p*.
(c) follows from similar arguments.

As will be clear below, the following property is also relevant for the purposes of this paper:

- *Levelling*: for every $p, q \in G$, if $pP{\sim}p$ and $qP{\sim}q$, then pIq and ${\sim}pI{\sim}q$.

Several additional properties of a logical nature are considered in next sections.

.- Judgment relationships aggregation functions.

We will focus on aggregating sets of individual judgments and model these sets as binary relations. An aggregation procedure starts from a set of individuals's points of view represented by the corresponding judgment relations (one for each person of the group), and gives as output a collective point of view also represented as a binary aggregate relation.

Formally, a *profile of individual judgment relations* $f = (R_1,\ldots, R_n)$ is a n-tuple of judgement relations where R_i denotes the point of view or judgment relation of individual i. A (*relational*) *judgment aggregation function F* is a function $f = (R_1,\ldots, R_n)$ that assigns a judgment relationship (namely, the *aggregate judgment relationship*) to each *profile of individual judgment relationships* in its *domain*, in symbols, to each $f = (R_1,\ldots, R_n) \in Dom(F)$. [7] *Dom (F)* denotes the domain of F, *i. e.* the set of all profiles of *individual judgments relationships* that are considered as admissible. As far as judgment aggregation procedures and functions are concerned, this paper focus on the majority method and on some related aggregation mechanisms.

3.- AGGREGATING JUDGMENTS BY MEANS OF THE MAJORITY METHOD AND SIMILAR PROCEDURES.

[7] There are also judgment aggregation functions of a quantitative (non-relational) kind. For these, see for instance Dietrich (2006).

.- Categorical judgment relations.[8]

As List and Pettit (2002: 90) remark, judgments are ‘modelled on acts of assent or dissent, assertion or denial, and differ from credences in not allowing *degrees* of confidence’. Hence, judgments are frequently modelled at the individual level as (categorical) truth valuations of the propositions in the agenda made by each of the persons of the group. Analogously, judgments are also modelled at the aggregate level as collective or aggregate truth valuations of these same propositions. Alternatively, aggregation may start from the set of choices made by each individual between each proposition in the agenda and its complementary proposition, arriving at a correponding set of choices supposedly made by the group or simply attributed to it (see Dietrich 2006). In any case, such favorable truth valuations and choices are interpreted as acceptance of the propositions concerned, and as rejection of the negatively valuated or non chosen propositions.

When trying to formalize such judgments by means of binary relations, they may be modelized in two alternative ways. First, individual judgment relations may be conceived as *complete*, *vertically balanced* and *levelled preorderings* that satisfy the *tertio excluso principle*. Such relations represent the world of standard (categorical deductive) logic, and contain only categorical judgments about the truth and falsehood of the propositions in the agenda.

Aggregation procedures considered may also be interpreted in another way. It may be contended, for instance, that truth-valuation does not presuppose nor imply that any two true (or any two false) propositions are at the same “level”. It may also be contended that, in many cases, it is only possible to get yes or no answers for questions of acceptance or rejection of judgments, independently of how certain people themselves feel about their answers. For these and other reasons, individual judgments expressed by truth values may also be represented by *vertically complete* and *vertically restricted* relations. We assume that, like categorical relations, these relations also meet the *tertio excluso principle*.

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[8] According to my use of the word “categorical”, a categorical judgment is a judgment made or stated ‘with certainty and firmness’ (*cf. Collins Cobuild English Dictionary*. London, Harper Collins Publishers, 1995).

According to the habitual model of judgment aggregation, individuals follow standard logical constraints (List and Pettit (2002: 90); Dietrich (2006: 288)). In our framework, relations satisfying one of the following two logical properties can represent this type of behaviour:

- *Set-wise (logical) deductive closure*: for every *S Í G* and every *p Î G*, if *S* (logically) entails *~p*, then (1) if every *s Î S*, *sP~s* then *~pPp*, and (2) if every *s Î S*, *sR~s*, then *~pRp*.[9]

- *Set-wise deductive transfer*: for every *S Í G* and every *p Î G*, if *S* logically entails *p*, then it cannot happen that for every *q Î S*, *qPp*.

It is easy to check that if *R* is *transitive*, *complete*, and *vertically balanced*, then *set-wise deductive transfer* implies *set-wise (logical) deductive closure*.[10] Analogously, if *R* is *transitive, complete, vertically balanced* and *levelled,* then *set-wise deductive closure* implies *set-wise deductive transfer*.[11]

Let us call "*levelled relations of categorical judgment*" all *complete*, *vertically balanced* and *levelled preorderings* defined on *G* that satisfy the *tertio excluso principle*, *set-wise deductive transfer* and, therefore, *set-wise deductive closure*, and let ***A*** be the set of all such judgment relations. The corresponding universal domain for aggregation is the set of all profiles of such relations, that is, the set $\boldsymbol{A}^n$.

Analogously, let us call "*segmented relation of categorical judgments*" any *vertically complete* and *vertically restricted* relation defined on *G*, that also meet the *tertio excluso*

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[9] This is the same property postulated by List and Pettit (2002), enlarged for allowing that *sI~s*.

[10] If *S* logically entails *p*, for *set-wise deductive transfer* it cannot happen that for every *s Î S*, *sPp*. Then, for *completeness*, there is some *s Î S* such that *pRs* and, for *vertical balance*, *~sR~p*. Then, *for transitivity*, if every *s Î S*, *sR~s*, then *pR~p*, and if for every *s Î S*, *sP~s*, then *pP~p*.

[11] Suppose that, on the contrary, *S* logically entails *p* and for every *q Î S*, *qPp*. Then, by *levelling*, for every *q Î S*, *qR~q*, and for set-wise deductive closure, *pR~p*. Then, given that for every *q Î S*, *qPp*, and it would not be possible that for every *q Î S*, *qI~q*. Then for some *q Î S*, *qP~q*. But even then, it cannot be that for every *q Î S*, *qPp*.

principle and *set-wise deductive closure*, and let $\boldsymbol{B}$ be the set of all of them. The corresponding universal domain for aggregation is the set of all profiles of such relations, that is, the set $\boldsymbol{B}^n$. It should be noted that *set-wise deductive transfer* lacks meaning in regard to judgment relations of this sort.

List and Pettit (2002: 97; 2006:) postulate about three habitual logical constraints: *completeness*, *consistency*, and *deductive closure*. The first of these three constraints corresponds, in our terminology, to *vertical completeness*. The second corresponds to the *restricted non-contradiction principle*. The third could be translated by the following formulation: for every set $S \subseteq G$ and every $p \in G$, if set S logically entails p, and if for every $q \in S$ it happens that $qP{\sim}q$, then $pP{\sim}p$. Therefore, if R is *vertically complete*, then *set-wise deductive closure* entails *deductive closure*. In addition, if R is *transitive*, *complete*, and *vertically balanced*, then *set-wise deductive transfer* implies (*set-wise (logical) deductive closure*) and *deductive closure*.

.- **Adapting the majority method**

Given that acceptance and rejection are understood as acts that do not allow degrees, and given that the aggregation method is not applied to binary relations, as is the usual case in social choice theory, the way the majority method is habitually understood and applied to judgment aggregation may differ slightly from the usual understanding of this method in social choice theory.

It is also habitually presupposed that the aggregation output has to be, for every individual profile, a judgment relation of the same sort as individual judgment relations. That means that if individual points of view are modelled as categorical *judgment levelled relations*, the aggregate or collective point of view should also be a categorical *judgment levelled relation*. If on the contrary, individual points of view are represented as categorical *judgment segmented relations*, the aggregate or collective point of view should be a relation of this same kind. It is habitually also assumed that the *tertio excluso principle* holds at both levels, individual and aggregate. Judgment acceptance and rejection are understood at the aggregate level, as well, as acts that do not allow degrees. This assumption induces a curious circumstance: the majority method is not an appropriate aggregation procedure when there are ties.

In any case, the *majority method* (*MMD*) can be defined as the judgment relational aggregation function F that asigns to every profile of individual judgment relations f= =(R_1,…, R_n) in its domain the aggregate relation R^J such that for every two propositions p and q in the agenda, pR^Jq iff $|N_{(p,q),f}| \geq |N_{(q,p),f}|$, where $|N_{(r,s),f}|$ is the number of individuals i for which rP_is.

Even when individual judgment relations are *levelled*, as well as when they are *segmented*, the majority method has to be adapted. In the latter case, because the only propositions compared under the individual segmented relations are complementary propositions. Imagine that the majority method is applied to the pair formed by two propositions p and q that are not complementary. Then, $|N_{(p,q),f}| = |N_{(q,p),f}|$ =0 and pI^Jq. Therefore, we have to apply some variant of this method that avoids such outcomes. Let us call the "*vertical restriction of the majority method*" (*VMD*) the judgment-relational aggregation function F that asigns to every profile of individual judgment relations f= =(R_1,…, R_n) in its domain the aggregate relation R^V such that for every two propositions p and q in the agenda, pR^Vq iff, (a) $|N_{(p,q),f}| \geq |N_{(q,p),f}|$ and (b) q =~p (and p =~q). As is obvious, R^V compares only complementary propositions, that is, if pR^Vq or qR^Vp, then q =~p (and p =~q).

If, on the contrary, individual judgment relations are *levelled*, peculiarities in understanding and in applying the majority method increase. The following example illustrates this point.

Table 3.1

	Proposition *p*	*Øp*	*q*	*Øq*
Person 1	T	F	T	F
Person 2	T	F	T	F
Person 3	F	F	T	F
Majority	T	F	T	F

According to table 3.1, the propositions p and q are equally valued as "true" or equally "accepted" at the aggregate level. However, according to the notion of majority method that is usual in social choice theory, proposition q must be considered higher placed than p (insofar as being true or accepted reveals a higher level than being false or rejected). Specifically, $pP_{1,2}Øp$, $qP_{1,2}Øq$, $ØpP_3p$ and $qP_3Øq$. So, it is natural to assume that qP_3p. But then, $|N_{(p,q)}|=0$, $|N_{(q,p)}|=1$, and qP^Jp. Therefore, if it is understood that two collectivelly "accepted" judgments are "equally accepted" or "equally firmly accepted", then the majority method has to be modified accordingly.

Let us call the *"levelled variant of the mayority method"* (*LMD*) the judgment relational aggregation function F that assigns to every profile of individual judgment relations $f=$ $=(R_1,\ldots, R_n)$ in its domain the aggregate relation R^L such that

(a) for every two complementary propositions p and $\sim p$ in the agenda,

$pR^L\sim p$ iff $|N_{(p,\sim p),f}|\geq|N_{(\sim p,p),f}|$; and

(b) for every propositions p and q in the agenda such that $p¹\sim q$ (and, hence, $\sim p¹q$),

(b.1) if $pP^L\sim p$ and $qP^L\sim q$, then pI^Lq and $\sim pI^L\sim q$;

(b.2) if $pP^L\sim p$ and $qI^L\sim q$, then pP^Lq;

(b.3) if $pI^L\sim p$ and $qI^L\sim q$, then pI^Lq and $\sim pI^L\sim q$.

What differences might arise if the majority method is applied to profiles of individual judgment levelled relations instead of being applied to its levelled variant? This question is approached below.

.- Some properties of the majority method and its variants.

As it is well known, the majority method and its *levelled variant* (LMD) meet several desirable properties. Let us denote by ***U*** the set of all of the *complete, transitive* and *vertically balanced* relations that can be defined on G. In addition, let ***UT*** be the subset of ***U*** that includes exactly all the relations in ***U*** which satisfy the *tertio excluso principle*.

If the individual relations are taken from ***U*** or ***UT***, MMD and LMD both hold two of the known conditions postulated in May's theorem, namely, *anonimity*[12] and *neutrality*[13].

[12] A relational judgment aggregation function F holds *anonimity* iff for every two profiles $f'=(R_1,\ldots, R_n)$, $f''=(R_{r(1)},\ldots, R_{r(n)})\,Î\,Dom(F)$, where $r:\{1,\ldots,n\}\rightarrow\{1,\ldots,n\}$ is any permutation of the individuals, $F(f')=F(f'')$.

They also meet most of the known conditions of Arrow's theorem; specifically, they satify *unanimity*,[14] and *independence of irrelevant alternatives*[15], the latter property being a formal consequence of *neutrality.*

It is worth noticing that, in argumental situations such as those exemplified by the doctrinal paradox or the discursive dilemma, *independece of irrelevant alternatives* avoids the possibility of changing the outcome by altering the agenda. In general, if any two agendas include some subset S of propostions, and MMD or LMD is applied to S, then for any profile f Î $\boldsymbol{U^n}$, the restriction of R^L to S is the same in both cases (on agenda manipulation, *cf*. Dietrich 2006: 288-9).

If the aggregation domain is "large enough", both methods also satisfy *non-dictatorship*[16] and *non-oligarchy*.[17]

The majority method also meets *positive responsiveness* [18] and the (*strong*) *Pareto condition*.[19] LMD does not. In terms of responsiveness, LMD satisfies only *non-*

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13 A relational judgment aggregation function F holds *neutrality* iff For every two profiles $f'=(R'_1,\ldots, R'_n)$, $f''=(R''_1,\ldots, R''_n)$ Î $Dom(F)$, and every p^1, p^2, p^3 $p^4 \in G$,
if for every i Î N, ($p^1 R'_i\, p^2$ syss $p^3 R''_i\, p^4$) and ($p^2 R'_i\, p^1$ syss $p^4 R''_i p^3$),
then, ($p^1 R'\, p^2$ syss $p^3 R''\, p^4$) and (($p^2 R'\, p^1$ syss $p^4 R'' p^3$).

14 A relational judgment aggregation function F holds *unanimity* iff for every profile $f=(R_1,\ldots, R_n)$, and every $p, q \in G$, if for every individual i Î N, pP_iq, then pP^Fq.

15 A relational judgment aggregation function F holds *independence of irrelevant alternatives* iff for every two profiles $f'=(R'_1,\ldots, R'_n)$, $f''=(R''_1,\ldots, R''_n)$ Î $Dom(F)$, and every $p, q \in G$, if for every i Î N, ($pR'_i\, q$ syss $pR''_i\, q$), then ($pR'\, q$ syss $pR''q$).

16 A relational judgment aggregation function F holds *non-dictatorship* iff there is no individual k such that for every profile $f=(R_1,\ldots, R_n)$ and every $p, q \in G$, if pP_kq, then $pP^F\, q$.

17 It is also easily shown that with MMD or LMD there is nobody with veto power. A person h has *veto power* iff for every profile $f=(R_1,\ldots, R_n)$ and every $p, q \in G$, if pP_hq, then not qPp. In turn, that implies that there is no *oligarchy*, where an *oligarchy* is any group of individuals N' such that for every profile $f=(R_1,\ldots, R_n)$, and every $p, q \in G$, (a) if for every i Î N' it happens that pP_iq, then pP^Fq; and (b) every j Î N' has veto power.

18 A relational judgment aggregation function F holds *positive responsiveness* iff for every two profiles $f'=(R'_1,\ldots, R'_n)$, $f''=(R''_1,\ldots, R''_n)$ Î $Dom(F)$, and every $p, q \in G$,
if it happens that (1) for every individual i Î N, if $pI'_i\, q$ then $pR''_i\, q$, and if $pP'_i\, q$ then $pP''_i\, q$,
and (2) for some individual j Î N, $pI'_j\, q$ and $pP''_j\, q$, or, $qP'_j\, p$ and $pR''_j\, q$,
then, if $pI'\, q$ then $pP''q$.

negative responsiveness.[20] The following table illustrates how and why this method may fail to hold *positive responsiveness*.

Table 3.2

	p/f	*q/f*		*p/f'*	*q/f'*
Person 1	T	T		T	T
Person 2	T	T		T	T
Person 3	F	F		T	F
LMD	T	T		T	T

Similarly, instead of meeting the (*strong*) *Pareto condition*, LMD satisfies only its vertical version, which becomes also the vertical version of *unanimity* when individual judgment relations satisfy the *tertio excluso principle*.[21] The following table illustrates how and why LMD may fail to meet the (*strong*) *Pareto condition*.

Table 3.3

Person 1	pP_1q
Person 2	pI_2q
Person 3	pI_3q
LMD	pI^Lq

In addition, even when operating in the domain $\boldsymbol{U}^n$, LMD always generates an aggregate judgment relation R^L that is *complete*, *transitive, vertically balanced* and *levelled*, and such that P^L is *asymmetrical* and I^L *reflexive* and *symmetrical*. As far as trantitivity is concerned, this performance contrasts with that of MMD. It is well known that when aggregating preferences MMD may generate aggegate relations that are not even

[19] A relational judgment aggregation function *F* holds the *strong Pareto condition* iff for every profile f=(R_1,…, R_n) in its domain and every *p*, *q* Î *G*, if for every individual pR_iq and for some of them pP_jq, then *pPq*.

[20] If the expression "pR"$_i$ q" replaces "pP"$_i$ q" in the last line of the *positive responsiveness* formulation, the resulting property is called *non-negative responsiveness*. For obtaining the vertical versions of both, it suffices to substitute ~ *p* for *q*.

[21] For obtaining the vertical version of the (*strong*) *Pareto condition* it suffices to substitute ~ *p* for *q*.

acyclical, though they are always *complete.*[22] If its domain of aggregation is $\boldsymbol{U}^n$ *or* $\boldsymbol{UT}^n$, MMD performs similarly.

With regard to VMD within the domain $\boldsymbol{V}^n$ or $\boldsymbol{VT}^n$, it should be noted that R^V links only complementary propositions. Consequently, within the domain $\boldsymbol{V}^n$, R^V is only *vertically complete* and satifies vacuously *transitivity* and *vertical balance*. VMD also meets *anonimity*, *neutrality*, *independence of irrelevant alternatives*, *positive responsiveness*, *unanimity*, and the *strong Pareto condition*. It should be borne in mind, however, that as far as the comparisons of non-complementary propositions is concerned, these properties are vacuously satisfied. VMD also meets *non-dictatorship* and *non-oligarchy*.[23]

VMD may be also applied to the profiles of individual relations in $\boldsymbol{U}^n$. In this case, R^V satisfies only the vertical versions of *neutrality*, *independence of irrelevant alternatives*, *positive responsiveness*, *unanimity* and the *strong Pareto condition*, *non-dictatorship* and *non-oligarchy*. These last two properties, however, are met in a trivial way. Imagine any profile such that for some individual pP_iq for a pair of non-complementary propositions in the agenda. By definition, pP^Vq *is not possible*, because since p and q are non-complementary, neither pR^Vq, nor qR^Vp. So nobody can be a dictator, but for the wrong reasons. Let us say that k is a vertical dictator for the aggregation function F if for every profile and any pair of complementary propositions p and $\sim p$ in the agenda, if $pP_i\sim p$, then $pP^F\sim p$. It can be easily checked that applying VMD avoids the existence of vertical dictators. Similarly, VMD also meets the analogous condition of *non-oligarchy*.

In addition, since there can be ties between the individuals that accept and those that reject any (non logically contradictory) proposition, the aggregate relations generated by applying MMD or LMD or VMD do not meet the *tertio excluso principle* (even if

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[22] A relation R is *acyclical* on the agenda G if for any sequence $p^1, p^2, \ldots, p^n$ such that p^iPp^{i+1}, it happens that p^1Rp^n.

[23] This is the reason why it may be meaningful to define the appropriate notions of dictatorship and oligarchy. A vertical dictator imposes his point of view at the collective level when he chooses a proposition against its complementary proposition. The correponding property may be called *non-vertical dictatorship*. R^V also satisfies this new property and the analogous *non-vertical oligarchy*.

individual relations do) if the aggregation domain is large enough. It should also borne in mind that the majority method and its variants are not *strategy-proof*.

.- The net majority method.

Even if the individual judgment relations are vertically restricted, there is an aggregation procedure, also based on the numbers of individuals supporting each judgment, which conveys richer information than the two former variants of the majority method.

Let us call the *"net majority function"* the function $d(p|f)$ that, given any profile of individual judgment relationships $f = (R_1,\ldots, R_n)$, attaches to any proposition in the agenda the difference between the number of individuals for which $pP_i \sim p$ minus the number of individuals for which $\sim pP_i\, p$. In symbols, $d(p|f) = |N_{(p,\sim p),f}| - |N_{(\sim p,p),f}|$.

The *net majority function* induces the *net majority relation*: for every $p,q \in G$, $pR^S q$ iff $d(p|f) = d(q|f)$. We denote as NMMD (*net majority method*) the aggregation function that is based on the *net majority function* and generates the *net majority relation*.

If the aggregation domain is $\boldsymbol{V^n}$, $\boldsymbol{VT^n}$ or any subset of one of them, NMMD conveys information not supplied by VMD, namely, whether the net number of individuals supporting a judgment is higher, equal or lower than the net number of individuals supporting an other judgment. It should be noted also that R^S is *complete* (not only vertically complete), *transitive* and *vertically balanced*, whether its domain is a subset of $\boldsymbol{U^n}$, $\boldsymbol{UT^n}$, $\boldsymbol{V^n}$ or of $\boldsymbol{VT^n}$.[24] In addition, NMMD satifies *anonimity*, *neutrality*, *positive responsiveness*, *unanimity*, the (*strong*) *Pareto condition*, *independence of irrelevant alternatives*, *non-dictatorship*, and there is nobody with *veto power:* therefore, NMMD is *non-oligarchical.* And as is obvious, R^S does not usually meet *levelling* or the *tertio excluso principle.*

[24] With regard to the property of *vertical balance*, as it obvious, $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| = (-1)\,[\,|N_{(\sim p,p),f}| - |N_{(p,\sim p),f}|\,]$. Therefore, $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| \geq |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$, iff, $|N_{(\sim q,q),f}| - |N_{(q,\sim q),f}| \geq |N_{(\sim p,p),f}| - - |N_{(p,\sim p),f}|$.

But the most salient novelties concerning NMMD and MMD arise when the aggregation domain is $\boldsymbol{UL^n}$ or $\boldsymbol{UTL^n}$ (or any subset of one of them), where $\boldsymbol{UL}$ is the set of all the *levelled* relations in $\boldsymbol{U}$ and, similarly, $\boldsymbol{UTL}$ is the set of all the *levelled* relations in $\boldsymbol{UT}$. In those cases, NMMD conveys the same information as within $\boldsymbol{V^n}$ and $\boldsymbol{VT^n}$, and also satisfies the above properties. It is more informative than LMD, and meets a more satisfactory set of properties than this latter variant of the majority method.

Furthermore, it can be easily shown that in those aggregation domains, MMD also satisfies all of the above properties. In particular, R^J is always *complete* and *transitive*. This means that for every profile of *complete*, *transitive*, *vertically balanced* and *levelled* individual judgment relations, R^J is *complete* and *transitive*, and well-known difficulties like the voting paradox disappear. The reason is very simple: if the aggregation domain is $\boldsymbol{UL^n}$ or $\boldsymbol{UTL^n}$ or any subset of one of them, then $R^J = R^S$, that is, MMD and NMD always generates the same aggregate relation.[25]

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[25] .- CLAIM 4.1.- If individual relations of categorical judgments are levelled, that is, if the domain for MMD and for NMMD is $\boldsymbol{A^n}$ or any subset of it,
then for every profile f and every $p, q \hat{I} G$, $pR^S q$ iff $pR^J q$, *i.e.*
$|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| = |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$ iff $|N_{(p,q),f}| \geq |N_{(q,p),f}|$.

PROOF.- Let $N_{(r),(s),f} = \{i \hat{I} N: rP_i \sim r$ and $sP_i \sim s\}$, that is, the set of all the persons who accept both propositions r and s. Then, because R_i satisfies the *tertio excluso principle*,
$N_{(p,\sim p),f} = N_{(p),(q),f} \cup N_{(p),(\sim q),f}$; $N_{(\sim p,p),f} = N_{(\sim p),(\sim q),f} \cup N_{(\sim p),(q),f}$;
$N_{(q,\sim q),f} = N_{(p),(q),f} \cup N_{(\sim p),(q),f}$; and $N_{(\sim q,q),f} = N_{(\sim p),(\sim q),f} \cup N_{(p),(\sim q),f}$;

Let us prove first that:
(1.1) if $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| = |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$, then $|N_{(p),(\sim q),f}| = |N_{(\sim p),(q),f}|$;
(1.2) if $|N_{(p),(\sim q),f}| = |N_{(\sim p),(q),f}|$, then $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| = |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$;
(2.1) if $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| > |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$, then $|N_{(p),(\sim q),f}| > |N_{(\sim p),(q),f}|$;
(2.2) if $|N_{(p),(\sim q),f}| > |N_{(\sim p),(q),f}|$, then $|N_{(p,\sim p),f}| - |N_{(\sim p,p),f}| > |N_{(q,\sim q),f}| - |N_{(\sim q,q),f}|$.

For 1. - By definition, $|N_{(p),(q),f} \cup N_{(p),(\sim q),f}| - |N_{(\sim p),(\sim q),f} \cup N_{(\sim p),(q),f}| =$
$= |N_{(p),(q),f} \cup N_{(\sim p),(q),f}| - |N_{(\sim p),(\sim q),f} \cup N_{(p),(\sim q),f}|$. Hence, $|N_{(p),(\sim q),f}| - |N_{(\sim p),(q),f}| =$
$= |N_{(\sim p),(q),f}| - |N_{(p),(\sim q),f}|$. Therefore, $|N_{(p),(\sim q),f}| = |N_{(\sim p),(q),f}|$ (and viceversa/ vice versa).

For 2.1.- By definition, $|N_{(p),(q),f} \cup N_{(p),(\sim q),f}| - |N_{(\sim p),(\sim q),f} \cup N_{(\sim p),(q),f}| >$
$> |N_{(p),(q),f} \cup N_{(\sim p),(q),f}| - |N_{(\sim p),(\sim q),f} \cup N_{(p),(\sim q),f}|$. Hence, $|N_{(p),(q),f} \cup N_{(p),(\sim q),f}| +$
$+ |N_{(\sim p),(\sim q),f} \cup N_{(p),(\sim q),f}| > |N_{(p),(q),f} \cup N_{(\sim p),(q),f}| + |N_{(\sim p),(\sim q),f} \cup N_{(\sim p),(q),f}|$. Hence, $|N_{(p),(\sim q),f}| + |N_{(p),(\sim q),f}| >$
$|N_{(\sim p),(q),f}| + |N_{(\sim p),(q),f}|$. Therefore, $|N_{(p),(\sim q),f}| > |N_{(\sim p),(q),f}|$ (and vice versa).

It should be noted also that:
(a) $|N_{(p,q),s}| = |N_{(q,p),s}|$, iff, $|N_{(p),(\sim q),f}| = |N_{(\sim p),(q),f}|$; and
(b) $|N_{(p,q),s}| > |N_{(q,p),s}|$, iff, $|N_{(p),(\sim q),f}| > |N_{(\sim p),(q),f}|$.

Given that R_i is a *levelled* preordering, for every $i \hat{I} N_{(p),(\sim q),f}$, pP_iq;
for every $j \hat{I} N_{(\sim p),(q),f}$, qP_jp; and for every $h \hat{I} N_{(p),(q),f} \cup N_{(\sim p),(\sim q),f}$, pI_hq.
Hence, $|N_{(p),(\sim q),f}| = |N_{(p,q),f}|$ and $|N_{(\sim p),(q),f}| = |N_{(q,p),f}|$. The rest is obvious. Q.E.D.

To sum up, let us imagine first that the aggregation domain is $\boldsymbol{B^n}$, and that therefore, individual relations are segmented and categorical, and let us imagine also that VMD is applied. In this framework, R^V is *vertically complete*, (vacuously) *transitive*, and *vertically balanced*. In addition, VMD satisfies *anonimity*, *neutrality*, *independence of irrelevant alternatives*, *positive responsiveness*, *unanimity*, the *Pareto condition*, *non-dictatorship* and *non-oligarchy*. It should be noted, however, that with regard to the comparisons of non-complementary propositions, VMD satisfies these properties vacuously.

In the same domain $\boldsymbol{B^n}$, NMMD conveys a richer information than VMD does. It also satisfies all those properties, and R^S is always *complete*, (non-vacuously) *transitive* and *vertically balanced*. It should also be noted that since R^S compares every pair of propositions, it always holds the above aggregation properties in a non-vacuos way.

If, on the contrary, the individual relations are levelled and categorical and the aggregation domain is $\boldsymbol{A^n}$, MMD and NMMD are both more informative methods than LMD, both always induce the same aggregate relation; this aggregate relation is always *complete*, *transitive* and *vertically balanced*; and both methods hold a very satisfactory set of aggregation properties (the same as NMMD meets when its domain is $\boldsymbol{B^n}$). LMD also holds these properties, except *positive responsiveness*. It meets *non-negative responsiveness* instead.

However, the performance of these methods finds an seemingly decisive limit. As the following table shows, preplexing situations like the discursive dilemma may be generated applying any of the four former procedures: the aggregate outcome on a conjunction such as (*pÙq*) may be its rejection, while its components are both accepted at the aggregate level.

Table 3.4

	p	*Q*	(*pÙq*)
Person 1	T	T	T

Person 2	T	F	F
Person 3	F	T	F
VMD, LMD and MMD	T	T	F
SMD	1; T	1; T	-1; F

In other words, the aggregate relations R^V, R^L, R^J and R^S generated respectively by VMD, LMD, MMD and NMMD may fail to hold *set-wise deductive closure* and, therefore, LMD, MMD and NMMD may fail also to hold *set-wise deductive transfer*, even if individual judgment relations hold these logical constraints.

4.- **THE ROLE OF LOGICAL CONSTRAINTS**

The discursive dilemma is a specific impossibility. It has inspired other more general impossibility results that theorem 1 presented by List and Pettit (2002: 100) may exemplify.[26]

We will offer a version of this theorem in our terms to illustrate the role played by each restriction in these impossibility results. For the sake of simplicity, we assume that the individual and the aggregate judgment relations are segmented.

List and Pettit consider that judgment aggregation functions assign to each profile of individual points of view a collective or aggregate set of judgments of the same kind as individual sets. Thus, if the domain of the aggregation function F is the set $\boldsymbol{B^n}$ of all the profiles where a *vertically complete* and *vertically restricted* relation that holds the *tertio excluso principle*, and *set-wise deductive closure* is associated with each individual, then $R^F = F(f)$ is also, for all profile f Î $\boldsymbol{B^n}$, a *segmented relation categorical of categorical judgments* that holds those same properties, *i.e.* $F(f)$ Î $\boldsymbol{B}$.

We should remember that a relational judgment aggregation function F holds *anonimity* iff for every two profiles $f' = (R_1, \ldots, R_n)$, $f'' = (R_{r(1)}, \ldots, R_{r(n)})$ Î $Dom(F)$, where $r: \{1, \ldots, n\} \rightarrow \{1, \ldots, n\}$ is any permutation of the individuals, $F(f') = F(f'')$.

[26] *Cf.* Pauly and van Hees (2003), Dietrich (2006) and Gärdenfors (2006) for other general impossibility results.

In addition, a relational judgment aggregation function F is *vertically neutral* iff for every two profiles $f'=(R'_1,\ldots, R'_n)$, $f'' =(R''_1,\ldots, R''_n)$ Î $Dom(F)$, and every p, $\sim p$, q, $\sim q \in G$, if for every i Î N, ($pR'_i{\sim}p$ iff $qR''_i{\sim}q$) and ($\sim pR'_i\, p$ iff $\sim qR''_i q$),
then ($pR'{\sim}p$ iff $qR''{\sim}q$) and ($\sim pR'\, p$ iff $\sim qR''q$).

We can now restate List and Pettit's theorem thus:

> *List and Pettit's theorem adapted version*: There is no *anonimous* and *vertically neutral*[27] judgment aggregation function F such that $Dom(F)= \boldsymbol{B^n}$, and such that $F(f)$ Î $\boldsymbol{B}$ for every profile f Î $\boldsymbol{B^n}$.[28]

It should be noted, first, that for every *anonimous* and *vertically neutral F* and for every p, $q \in G$, if $|N_{(p,\sim p),f}| = |N_{(q,\sim q),f}|$, then $pR^F{\sim}p$ iff $qR^F{\sim}q$. Since R^F satisfies the *tertio excluso principle*, there are only two possibilities: (1) $pP^F{\sim}p$ and $qP^F{\sim}q$, or (2) $\sim pP^F p$ and $\sim qP^F q$.[29]

If the number N of individuals in the group is even, then there is a profile $\boldsymbol{B^n}$ such that for some $r \in G$, $|N_{(r,\sim r),f}| = |N_{(r,\sim r),f}|$. But then, if $rR^F{\sim}r$ or $\sim rR^F r$, then $rP^F{\sim}r$ and $\sim rP^F r$, which is excluded by the asymmetry of P^F. Hence, R^F would be not vertically complete for this profile.

[27] Instead of neutrality, List and Pettit postulate a different condition: *systematicity*. But this (last) condition and *anonimity* together entail *vertical neutrality*.

[28] For the original proof, see List and Pettit (2002: 109-10).

[29] To prove that if $pP^F{\sim}p$ then $qP^F{\sim}q$, imagine that, on the contrary, $pP^F{\sim}p$ and $\sim qP^F q$.
It should be noted first that if $|N_{(p,\sim p),f}| = |N_{(q,\sim q),f}|$, then the number of individuals for which $pP_i{\sim}p$ and $\sim qP_i q$, equals the number of individuals for which $\sim pP_i\, p$ and $qP_i{\sim}q$.

Let now f' be a profile such that for every individual i Î N,
if in the profile f, $pP_i{\sim}p$ and $qP_i{\sim}q$, then in the second profile f', $pP'_i{\sim}p$ and $qP'_i{\sim}q$,
if in the profile f, $pP_i{\sim}p$ and $\sim qP_i q$, then in the profile f', $\sim pP'_i\, p$ and $qP'_i{\sim}q$,
if in the profile f, $\sim pP_i\, p$ and $qP_i{\sim}q$, then in the profile f', $pP'_i{\sim}p$ and $\sim qP'_i q$,
if in the profile f, $\sim pP_i\, p$ and $\sim qP_i q$, then in the profile f', $\sim pP'_i\, p$ and $\sim qP'_i q$.

Anonimity entails that $pP'^F{\sim}p$ and $\sim qP'^F q$. But every individual evaluates q against $\sim q$ in the new profile f' exactly like he evaluates p against $\sim p$ in the old profile f'. Hence, by vertical neutrality $qP'^F{\sim}q$, contradicting the former conclusion. Therefore, if $pP^F{\sim}p$ then $qP^F{\sim}q$. By an analogous argument, if $qP^F{\sim}q$ then $pP^F{\sim}p$. Q.E.D.

If on the contrary, N is odd, there is a profile $\boldsymbol{B^n}$ such that in for some $r \in G$, $|N_{(p,\sim p),f}| = |N_{(q,\sim q),f}| = |N_{(Ø(pÙ q),\ (pÙ q)),f}|$.[30] Only two cases are possible: (1) $pP^F\neg p$, $qP^F Øq$, and $\neg(pÙq)P^F(pÙq)$, or (2) $\neg pP^F p$, $\neg qP^F q$, and $(pÙq)P^F\neg(pÙq)$. In both cases R^F fails to hold *set-wise deductive closure* because the set {*p*, *q*} entails the proposition (*pÙq*), and this (last) proposition entails *p* and entails *q*.

Summarizing, besides *anonimity*, *vertical neutrality* and the hypothesis made over the domain of *F*, two logical restrictions play a decisive role: the *tertio excluso principle* and *set-wise deductive closure*.

This and similar impossibility results have been interpreted as disqualifying the majority method and similar "proposition-wise" aggregation procedures. Consequently, there has been a search for results using aggregation procedures of another kind, namely, "set-wise" (instead of "proposition-wise") aggregation procedures like the "premise-based procedure", the best known example.[31]

The latter procedure is habitually illustrated in reference to some instance of the discursive dilemma, as in the following table. The propositions *p* and *q* are premises, the proposition *r* is the potential conclusion, and $[r \leftrightarrow (p \wedge q)]$ express a rule, doctrine, scientific law or regularity accepted as true by everybody in the group. It should be noted that given $[r \leftrightarrow (p \wedge q)]$, the conclusion *r* holds only if the premises *p* and *q* hold.

Table 4.1

	p	*q*	$[r \leftrightarrow (p \wedge q)]$	*r*
Person 1	T	T	T	T
Person 2	T	F	T	F
Person 3	F	T	T	F
Majority	T	T	T	F
Deductive closure				T

[30] By hypothesis, *G* includes, at least *p*, *q*, (*pÙq*) and the negations of these three propositions.

[31] List and Pettit (2006a) make an informal survey of this kind of procedure. They talk of "propotion-wise" supervenience for referring to those cases and procedures according to which the group judgment on each proposition in the agenda is a (possibly different) function of the individual judgments on that proposition (p. 10). On the contrary, "set-wise" supervenience means that the set of group judgments on all the propositions in the agenda is a (possibly different) function of the individual sets of judgments on (some or all o(p)f) these propostions (p. 12).

The dilemma arises because there seem to be two alternative procedures for reaching the conclusion that could be attached to the group. One is that, *r* is rejected by a majority of the group, so if the group members vote on this proposition, its negation $\neg r$ would win. The other is that each of the premises *p* and *q* and $[r \leftrightarrow (p \wedge q)]$ are supported by a majority of persons. Hence, if the conclusion is deductively inferred from the premises that are accepted by a majority of the group, *r* should be accepted as the collective conclusion. This latter procedure is the so called the "premise-based" procedure. As shown, it consists in choosing as the collective conclusion the option *r* that can be deductively inferred from the premises voted by a majority of individuals, *i. e*. the premises *p*, *q* and [*r* iff (*p* and *q*)].

This procedure can easily be generalized for any set of premises *P*. Dietrich (2006: 296), for instance, generalizes it in the following way. Any set of premises *P* is a subset of the agenda such that (a) if *p* Î *P* then also ~*p* Î *P*, and (b) for each pair of complementary propositions in the agenda *q* and ~*q*, the set of premises *P* implies one of them. The premise-based procedure (for a set of premises *P*) is the aggregation function *F* such that for each $f=(R_1,\ldots, R_n)$ Î *Dom*(*F*), if *R*= *F*(*f*), then *qP~ q* for every *q* that is implied by the set of the premises that are each of them supported by a majority of individuals.[32]

It should be noted that all such "set-wise" procedures induce collective judgments that, at least for some profiles of individual points of view, are different from those collective judgments generated by the majority method. As a consequence these procedures may attach to the group as its own collective judgments some judgments that are rejected by a majority of the persons in the group.

The main lesson behind these impossibility and possibility results would be that "set-wise" aggregation procedures should be employed instead of applying the disqualified "proposition-wise" methods like the majority method and its variants. But is this really a lesson that should be obeyed? Is there some way of recovering the possibility of

[32] Setting aside the difficulties induced by ties, Dietrich (2006: 296), for instance, accepts by convention that in such cases the negated proposition should win over the asserted one.

employing the majority and similar methods for aggregating judgments? The majority method, its levelled and its restricted variants and the net majority method meet a large set of attractive properties and, last but not least, they are very simple to apply. In addition, “set-wise” aggregation procedures, as the example above illustrates, lead to a clearly anti-intuitive behavior. They lead to attaching to the group as part of its point of view collective judgments that are rejected by a majority of the group members.

5.- THE LOGICAL CONSTRAINTS QUESTION

.- Weakening logical constraints.

Individuals can not always make a categorical judgment about each issue at stake. Let us imagine that a person can assign a definite probability to each of all of the *segmented (or levelled) relations of categorical judgment* that can be logically defined on the agenda G, and that he manages probabilities according to probability calculus. Such a probability distribution induces the following probability assignment *p*(*p*) to every proposition *p Î G* in the agenda: *p*(*p*) is the sum of the probabilities of all the *categorical judgment relations* for which *p* is true, that is, of all the *categorical judgment relations* for which *pP~p*. In addition, given *p*(*p*), the induced *believed judgment relation R** may be defined in the following way: for every *p, q Î G, pR*q* iff *p*(*p*) ≥ *p*(*q*).

It is known that *R** does not necessarily hold *set-wise deductive closure*, nor *set-wise deductive transfer*. If *p* and *q* are statistically or logically independent, it may happen that *p*(*p*)> 0.5, *p*(*q*)> 0.5, and *p*(*pÙq*)< 0.5. That would mean that *pP*Øp* , *qP*Øq*, and ¬(*pÙq*)*P**(*pÙq*). This is the reason why we introduce below two weakened logical constraints, called *proposition-wise deductive closure*, and *weak* or *proposition-wise deductive transfer*.

On the other hand, if the agenda includes some set S of propostions such that *p*(*s*)= 1 or *p*(*s*)= 0 for all *s Î S*, *R** satisfies *set-wise deductive transfer* and *set-wise deductive closure* with regard to them. To represent this, we also introduce below two additional conditions, labelled *combined deductive closure*, and *combined deductive transfer*.

- *Proposition-wise deductive closure*:

For every $p, q \in G$,

if p logically entails q and $qR{\sim}q$ or ${\sim}qRq$,

then, if $pP{\sim}p$ then $qP{\sim}q$, and, if $pR{\sim}p$ then $qR{\sim}q$.

- *Proposition-wise deductive transfer*:

For every $p, q \in G$, if p logically entails q, then not pPq.

By analogous arguments to that developed regarding the relationships between *set-wise deductive closure* and *set-wise deductive transfer*, it can be easily shown (1) that if R is *transitive*, *complete*, and *vertically balanced*, then *proposition-wise deductive transfer* implies *proposition-wise deductive closure*, and (2) that if R is *transitive*, *complete*, *vertically balanced* and *levelled*, then *proposition-wise deductive closure* implies *proposition-wise deductive transfer*.

A salient fact regarding these two proposition-wise conditions is that they are transmitted to the aggregate level by every variant of the majority method.

To show this, let ***UP*** be the set of all of the *vertically balanced* and *complete preordering*s defined on G that hold *proposition-wise deductive transfer*. Analogously, let ***VP*** be the set all of of the *vertically complete* and *vertically balanced* relations defined on G that hold *proposition-wise deductive transfer*.

CLAIM 5.1.- If F is LMD, MMD or NMMD, and $Dom(F) \subseteq \boldsymbol{UP}^n$, then R^F holds *proposition-wise deductive transfer*.[33]

Corollary of Claim 5.1.- If F is LMD, MMD or NMMD, and $Dom(F) = \boldsymbol{A}^n$, then R^F holds *proposition-wise deductive transfer*.

[33] PROOF of CLAIM 5.1.- Notice that if p logically entails q, then for every individual i, qR_ip. Hence, qR^Jp. In addition, if $pP_i{\sim}p$, then $qP_i{\sim}q$, and if $pI_i{\sim}p$, then $qR_i{\sim}q$. Thus qR^Sp, and in regard with LMD, (2) if $pP^L{\sim}p$, then $qP^L{\sim}q$, and (3) if $pI^L{\sim}p$, then $qR^L{\sim}q$. Then, qR^Lp. Q.E.D.

CLAIM 5.2.- If F is VMD or NMMD, and $Dom(F)= \boldsymbol{VP}^n$ (or if F is VMD and $Dom(F)= =\boldsymbol{UP}^n$), then R^F holds *proposition-wise deductive closure*.[34]

Corollary of Claim 5.2.- If F is VMD or NMMD, and $Dom(F)= \boldsymbol{B}^n$, then R^F holds *proposition-wise deductive closure*.

Let us now say that p is unanimously judged iff for every $i \in N$, $pP_i{\sim}p$, or for every $i \in N$, $pI_i{\sim}p$, or for every $i \in N$, ${\sim}pP_ip$. In symbols, $UN(G)= \{p \in G: p$ is judged unanimously$\}$, that is,

$UN(G | f)^{+}= \{p \in G:$ for every $i \in N, pP_i{\sim}p\}$,

$UN(G | f)^{\sim}= \{p \in G:$ for every $i \in N, {\sim}pP_ip\}$,

$UN(G | f)^{\text{»}}= \{p \in G:$ for every $i \in N, pI_i{\sim}p\}$,

$UN(G | f)= UN(G | f)^{+} \cup UN(G | f)^{\sim} \cup UN(G | f)^{\text{»}}$.

Consider now the following restriction:

- *Collectively combined deductive transfer*: For every $S \subseteq G$ and every $p \in G$,
if for every $s \in S$, $s \in UN(G | f)$, then
if $\{p\} \cup S$ logically entails q, then it cannot happen that for every $r \in \{p\} \cup S$, rPq.

Let $\boldsymbol{US}$ be the set of all of the *verticaly balanced* and *complete preordering*s defined on G that hold *set-wise deductive transfer*.

CLAIM 5.3.- If F is LMD, MMD or NMMD, and $Dom(F)= \boldsymbol{US}^n$, then R^F holds *combined deductive transfer*.[35]

The following is the corresponding version for deductive closure:

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[34] PROOF of CLAIM 5.2.- Given that every R_i is vertically complete, if p logically entails q, then for every individual i, if $pP_i{\sim}p$, then $qP_i{\sim}q$, and if $pI_i{\sim}p$, then $qR_i{\sim}q$. Hence, $|N_{(q,\sim q),f}| \geq |N_{(p,\sim p),f}|$. The rest is (even more) obvious. Q.E.D.

[35] PROOF of CLAIM 5.3.- For every individual i, if for some $s \in S$, ${\sim}sP_is$, then for every individual j, ${\sim}sP_js$. Then, qR^Ls, qR^Js, and qR^Ss. Let us suppose, therefore, that for every $s \in S$ and every individual, $sP_i {\sim}s$. In this case, given that R_i satisfies *set-wise deductive transfer*, if $\{p\} \cup S$ logically entails q, then for every individual i, qR_ip. (The argument continues as in the proof of the Claim 5.1).

- *Colectively combined deductive closure*: For every *S Í G* and every *p Î G*,
if for every *s Î S, s Î UN(G | f)*, {*p*}∪*S* logically entails *q* and *qR~q* or *~qRq*, then
if for every *r Î* {*p*}∪*S*, *rP~r*, then *qP~q*,
and, if for every *r Î* {*p*}∪*S*, *rR~r*, then *qR~q*.

Let now ***VS*** be the set all of of the *vertically complete* and *vertically balanced* relations defined on *G* that hold *set-wise deductive closure*.

CLAIM 5.4.- If *F* is VMD or NMMD, and *Dom*(*F*)= $\boldsymbol{VS}^n$ (or if *F* is VMD and *Dom*(*F*)= $\boldsymbol{US}^n$), then R^F holds *combined deductive closure*.[36]

It should also be noted that if *R* is *transitive* and *complete*, then *combined deductive transfer* implies *combined deductive closure.*[37]

.- **Possibility results for individual categorical two -valued judgment relations.**

The following four possibility results are easily inferred from the former four claims, on aggregation procedures that take as inputs individual *levelled* or *segmented relations of categorical judgments* (*i.e.* relations in the set ***A***, or in the set ***B***).

.- CLAIM 5.5.- There is a relational judgment aggregation function *F* (namely, VMD), such that (1) *Dom*(*F*)= $\boldsymbol{B}^{\boldsymbol{n}}$ (or *Dom*(*F*)= $\boldsymbol{A}^{\boldsymbol{n}}$);
(2) for every for every *f*=(R_1,.., R_n) *Î* *Dom*(*F*), *F*(*f*) is *vertically complete*, *vertically restricted,* (vacuously) *transitive*, and (vacuously) *vertically balanced*,
(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *positive responsive*, *unanimous*, (*strongly*) *paretian*, *non-dictatorial* and *non-oligarchical*,

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[36] PROOF of CLAIM 5.4.- For every individual *i*, if for some *s Î S*, $\sim sP_is$, *combined deductive closure* holds vacuously. Let us suppose, then, that for every *s Î S* and every individual, $sR_i\sim s$. In this case, given that R_i satisfies *set-wise deductive closure*, if {*p*}∪*S* logically entails *q*, then for every individual *i*, if $pP_i\sim p$, then $qP_i\sim q$, and if $pI_i\sim p$, then $qR_i\sim q$. (The argument continues as in the proof of the Claim 5.2).

[37] Suppose that for every *s Î S*, *s Î UN(G | f)*, and {*p*}∪*S* logically entails *q*. *R* is *complete*, hence *qR~q* or *~qRq* on the one hand, and *combined deductive transfer* implies that for some *r Î* {*p*}∪*S*, *qRr*, on the other. Therefore, for *transitivity* of *R*, if for every *r Î* {*p*}∪*S*, *rP~r*, then *qP~q*, and, if for every *r Î* {*p*}∪*S*, *rR~r*, then *qR~q*. Q.E.D.

(4) and every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) satisfies *proposition-wise deductive closure* and *combined deductive closure*.

.- CLAIM 5.6.- There is a relational judgment aggregation function *F* (namely, NMMD), such that:

(1) *Dom*(*F*)= $\boldsymbol{B}^n$,

(2) for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) is *complete*, (vacuously *transitive*, and *vertically balanced*,

(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *positive responsive*, *unanimous*, (*strongly*) *paretian*, *non-dictatorial* and *non-oligarchical*,

(4) and for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) satisfies *proposition-wise deductive closure* and *combined deductive closure*.

.- CLAIM 5.7.- There is a relational judgmente aggregation function *F* (namely, LMD), such that:

(1) *Dom*(*F*)= $\boldsymbol{A}^n$,

(2) for every for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) is *complete*, *transitive*, *levelled* and *vertically balanced*,

(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *non-negative responsive*, *unanimous*, *non-dictatorial* and *non-oligarchical*,

(4) and for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) satisfies *proposition-wise deductive closure* and *proposition-wise deductive closure with unanimity.*

.- CLAIM 5.8.- There is a relational judgment aggregation function *F* (namely, MMD and NMMD), such that:

(1) *Dom*(*F*)= $\boldsymbol{A}^n$,

(2) for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) is *complete*, *transitive*, and *vertically balanced: (,)*

(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *positive responsive*, *unanimous*, (*strongly*) *paretian*, *non-dictatorial* and *non-oligarchical*,

(4) and for every $f=(R_1,.., R_n)$ Î *Dom*(*F*), *F*(*f*) satisfies *proposition-wise deductive closure* and *proposition-wise deductive closure with unanimity.*

.- **Which logical constraints are to be met?**

These results show the decisive role played by the strong logical restrictions imposed on aggregation functions in deriving the impossibility theorems obtained in the judgment aggregation literature. This raises the question whether those restrictions are so compelling. Might they, on the contrary, be avoided or, at least, weakened as we have done?

.- Deductive transfer and deductive closure.

A typical feature of the literature is to consider that strong restrictions like set-wise deductive closure and set-wise deductive transfer are completely compelling. List and Pettit (2002: 95), for instance, make them the measuring rod of rationality in judgment making and in judgment aggregation, and Dietrich (2006:293) calls the corresponding condition "*collective* rationality". Pauly and van Hees (2003: 3), to give another example, view logical consistency restrictions as more fundamental than contraints on individual preferences.

Nevertheless, as shown above, those restrictions go too far in the case of a person managing his beliefs according to the probability calculus. I contend that this sort of behavior is perfectly rational, not only when people may make quantitative probability judgments, but also when, being only able to make comparisons of reliability or likelihood, they follow the qualitative restrictions derived from the theory of probability when making such judgments. What notion of rationality would forbid this sort of epistemic behavior?

It is true that the strong logical restrictions generate impossibility results when, instead of being imposed on individuals, they are required from the aggregation precedures. They mainly restrain the behavior of the group. But we do not see any clear justification for imposing on groups more stringent logical restrictions than those that might reasonably be imposed on individuals.

If there are no disagreements inside the group and if individuals obey strong logical constraints in making judgments, the majority method guarantees that the group also displays this sort of strongly or set-wise consistent behavior. But what if the members of the group disagree, so that aggregation cannot proceed by unanimity? Again, we do not see any clear justification for imposing more stringent logical restrictions on groups, especially when their members disagree, than on individuals who have doubts or who are not certain about the truth or the falsehood of their beliefs and judgments.

In addition, imposing set-wise logical restrictions on judgment aggregation procedures leads one to reject the majority method and similar mechanisms, and to accept some of the so called "set-wise procedures", like the "premise-based" one. These procedures, in turn, lead one to include as part of the group's collective point of view judgments that are rejected by a majority of its members. This sort of behavior seems clearly unnatural when aggregating judgments.[38] In addition, the possibility results given above suggest that this behavior may lack the justification usually supposed for it. Given those results, the question is now whether substituting the combined logical restrictions for the corresponding set-wise variants is a more or less justified option than assigning to the group as its own collective judgments positions that the majority of the group members reject. We suggest that this latter way of doing it is far more unnatural than implementing the substitution.

In conclusion, we contend that it is plainly plausible to relax the logical restrictions that are habitually required from judgment aggregation functions, and replace them with conditions such as *proposition-wise deductive closure*, *combined deductive closure*, *proposition-wise deductive transfer* and *combined deductive transfer with unanimity*. It should be noted, on the one hand, that these less demanding conditions are met by individuals who behave according to the rules of probability or likelihood. If this weakening of logical restrictions were rejected, the group's collective point of view would include judgments that are rejected by a majority of its component persons.

Something similar may be said about the *tertio excluso* restriction. Any individual is allowed to doubt and assign the same probability to a proposition as to its negation. Why is this possibility forbidden for groups?

[38] When aggregating preferences, things may be different. It may happen, for instance, that (some) justice, equity or similar notions, critera or features are considered relevant for the aggregate outcome. On the other hand, analysis that focus on the properties of some aggregation methods shoud be distinguished from analysis of the choice of such methods. On this latter framework, see Zamora (2006).

In the literature on the aggregation of judgments it is frequently remarked that judgments are answers to yes or no questions, or yes or no assertions, and that in contrast with beliefs, they do not admit degrees. But whatever the proposed characterization of judgments, it is an indisputable fact that many judgments made by persons express only beliefs and are only supported by those persons as beliefs. The act of expressing a judgment may be a yes or no act, but this does not imply that the involved person feels himself sure or certain enough to think that it is true. He may think, for instance, that it is more likely or more reliable than its negation. This would be a good reason to accept the judgment or proposition and reject its negation, as least until better information is available.

Of course, this does not exclude paying attention to judgments that are backed by true propositions or by categorical convictions. But in the case of judgments dealt with as true assertions, it seems unreasonable to postulate in addition that such judgments are always made, whatever the question. In more formal terms, in such a case judgment sets or judgment relations should not be assumed always as vertically complete.

6.- **INDIVIDUAL JUDGMENTS BASED ON BELIEFS**.

The final considerations in the preceeding section support as plausible the idea that aggregate judgment relations may not obey the *tertio excluso principle*. In this way, a tie seems to be the natural outcome when the group's members points of view are divided, for instance, in half. As it has been emphazised above, this eventuality is habitually excluded in the specialized literature. However, it is an analogous situation to that of a person who assigns the same probability to the ocurrence and to the non ocurrence of an event.

But reasons of the same sort suggest the relevance of considering how judgments may be aggregated when individual judgments express only beliefs, rather than categorical convictions. Individuals can cannot habitually make a categorical judgment about every issue at stake. Bearing this frequent situation in mind, let us introduce relations which allow individuals to make judgments at diferent levels of support, likeness, reliability, or truth, the latter being the case of Pauly and van Hees (2003: 3), who generalize the habitual judgment aggregation framework introducing the use of many-valued logic. ‘Since *t* [the number of truth values] may be larger than 2, we allow individuals as well as the group as a whole to express degrees of acceptance and rejection’.

This can be done in different ways depending on the available information about the categorical character of individual judgments.

.- **Aggregating individual segmented relations of non-categorical judgments.**

Imagine first that individuals are not certain about some of the issues at stake, but it is not known on which issues they can make a categorical judgment, and on which they cannot. Assuming that individual judgment relations are *vertically complete*, *vertically restricted* and hold *proposition-wise deductive closure* offers a way of formally representing this kind of situation. But then, Claim 5.2 tells us that if the aggregation domain is $\boldsymbol{VP}^n$ (the set of all the *segmented relations of non-categorical judgments*) or any its subsets, the aggregate relation R^V generated by VMD, and the analogous relation R^S induced by NMMD both hold *proposition-wise deductive closure*. Hence, new possibly results, for this kind of case can be formulated and easily checked:

.- CLAIM 6.1.- There exists a relational judgment aggregation function *F* (namely, VMD), such that (1) $Dom(F)= \boldsymbol{VP^n}$,

(2) for every for every $f=(R_1,.., R_n)$ Î $Dom(F)$, $F(f)$ is *vertically complete*, *vertically restricted*, (vacuously) *transitive*, and (vacuously) *vertically balanced*,

(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *positive responsive*, *unanimous*, (*strongly*) *paretian*, *non-dictatorial* and *non-oligarchical: (,)*

(4) (and) every $f=(R_1,.., R_n)$ Î $Dom(F)$, $F(f)$ satisfies *proposition-wise deductive closure*.

.- CLAIM 6.2.- There is a relational judgment aggregation function *F* (namely, NMMD), such that:

(1) $Dom(F)= \boldsymbol{VP^n}$,

(2) for every for every $f=(R_1,.., R_n)$ Î $Dom(F)$, $F(f)$ is *complete*, *transitive*, and *vertically balanced*,

(3) *F* is *anonimous*, *neutral*, *independendent of irrelevant alternatives*, *positive responsive*, *unanimous*, (*strongly*) *paretian*, *non-dictatorial* and *non-oligarchical: (*,)

(4) and for every *f* =(R_1,.., R_n) Î *Dom*(*F*), *F*(*f*) satisfies *proposition-wise deductive closure*.

It should be noted, however, that if the agenda is not exceedingly poor, the assumption made would not be the case. For instance, if the agenda includes a simple logical contradiction like (*pÙØp*) it does not seem easy to avoid making the assumption that individuals make categorical beliefs about it.

.- **Aggregating individual segmented relations of partitioned judgments.**

Imagine now that there is additional information available about the issues on which each individual makes a categorical judgment. For processing this new information we need for each individual the corresponding partition of the agenda q_i =(GC_i; GT_i), where GC_i is the subset of G of all the propositions and their negations about which (it is known or it can be assumed that) individual *i* makes (or is able to make) a categorical judgment. GT_i is the subset all the propositions in the agenda and their negations about which individual *i* is only able to make a tentative or non-categorical judgment.

Given the partition of the agenda corresponding to individual *i*, R_i must hold the *tertio excluso principle* in the *i*-categorical part of the agenda:

- R_i holds the *restricted tertio-excluso principle* iff
for every *p* Î GC_i, if $pR_i{\sim}p$ or ${\sim}pR_ip$, then $pP_i{\sim}p$ or ${\sim}pP_ip$.

Similarly, *deductive closure* may be reformulated in the following terms:

- *Individually combined deductive closure* (*given the partition* q_i =(GC_i; GT_i)):
For every *S* Í *G* and every *p* Î *G*,
if for every *s* Î *S*, *s* Î GC_i, {*p*}∪*S* logically entails *q* and $qR_i{\sim}q$ or ${\sim}qR_iq$, then
if for every *r* Î {*p*}∪*S*, $rP_i{\sim}r$, then $qP_i{\sim}q$,
and, if for every *r* Î {*p*}∪*S*, $rR_i{\sim}r$, then $qR_i{\sim}q$.

Given a partition q_i for each individual, the profile of the individual partitions can be defined as the *n*-tuple *Q*=(q_1, q_2,.., q_n). In turn, given the profile *Q*=(q_1, q_2,.., q_n), we denote by ***V**Q* the set of all the *vertically complete* and *vertically restricted* relations defined on G that, given the partition q_i, hold the corresponding version of the *restricted tertio excluso principle*, and the corresponding condition of *deductive closure*.

For realizing the analogous task in regard with the aggregate level, we first have to define a new sort of sets of unanimous and categorical judgments.

Let us say that *p* is unanimously and categorically judged iff (1) *p Î* GC_i for every *i Î N*, (2) and pP_i~*p* for every *i Î N*, ~pP_ip or for every *i Î N*. In symbols, *UNC(G)*= {*p Î G: p* is judged unanimously and is categorical for every person}, that is,

UNC(G | *f)*$^+$= {*p Î G:* for every *i Î N, p Î* GC_i, and pP_i~*p*},

UNC(G | *f)*$^\sim$= {*p Î G:* for every *i Î N, p Î* GC_i, ~pP_ip},

UNC(G | *f)*= *UN(G* | *f)*$^+$ *È UN(G* | *f)*$^\sim$.

The following are the corresponding versions for the *tertio excluso principle* and for *deductive closure*:

- *Aggregate restricted tertio-excluso principle* (*given the profile of individual partitions* *Q*=(q_1, q_2,.., q_n)):

for every *p Î UNC(G* | *f)*, if pR_i~*p* or ~pR_ip, then pP_i~*p* or ~pP_ip.

- *Doubly combined deductive closure* (*given the profile of individual partitions* *Q*=(q_1, q_2,.., q_n)):

For every *S Í G* and every *p Î G*,

if for every *s Î S, s Î UNC(G* | *f)*, {*p*}∪*S* logically entails *q* and *qR*~*q* or ~*qRq*, then

if for every *r Î* {*p*}∪*S*, *rP*~*r*, then *qP*~*q*,

and, if for every *r Î* {*p*}∪*S*, *rR*~*r*, then *qR*~*q*.

CLAIM 6.3.- If F is VMD or NMMD, and $Dom(F)= \boldsymbol{VQ}^n$, then R^F holds *aggregate restricted tertio-excluso principle*[39] and *doubly combined deductive closure.*[40]

In turn, this claim allows us to state two new possibility results concerning VMD or NMMD operating in the aggregation domain $\boldsymbol{VQ}^n$ that are entirely analogous to claims 6.1 and 6.2 above. For their formulation, it is enough to substitute $\boldsymbol{VQ}^n$ for $\boldsymbol{VP}^n$ and add that R^V and R^S satisfy the *aggregate restricted tertio-excluso principle*.

.- Aggregating judgment regular individual relations. The voting paradox reappears.

Moving a step further, imagine that additional information is provided about what comparisons individuals make between non-complementary propositions according their higher or lower likelihood or reliability, and about how they resolve them. If individuals make all of these comparisons, we need a complete and non-levelled judgment relation for representing the point of view of each person, reflecting the fact that every pair of propositions p and q in the agenda, complementary or not, are compared by every individual, and that it may the the case that pP_iq, pI_iq, or qP_ip.

Even if this is not the actual situation, analysing such points of view may be also relevant. Imagine that only levelled or segmented individual relations of categorical judgment relations can be assumed from individual points of view. The analysis of the potential beliefs that might underlie these relations may highlight eventual divergences between these relations and the underlying beliefs.

Let R be a *complete* and *vertically balanced preordering* defined on G. Such a relation defines the following partition of the agenda $\theta_R =(GC_R; GT_R)$: $GC_R=\{p \in G: pRq$ for every $q \in G$, or qRp for every $q \in G\}$; GT_R= G\\GC_R (G_R= G-GC_R).

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[39] It is obvious that R^V and R^S satisfy the *aggregate tertio-excluso principle* because if $p \in UNC(G|\varphi)$, then $pP_i{\sim}p$ for every individual, or ${\sim}pP_ip$ for every individual.

[40] The argument regarding *doubly combined deductive closure* is entirely analogous to that of Claim 5.4 in the last section.

Given the partition of the agenda q_R, two new properties can be formulated:

- *Autorestricted tertio-excluso principle* (*given the profile of individual partitions* $Q=(q_1, q_2,.., q_n)$):

for every $p \in GC_i$, if $pR_i{\sim}p$ or ${\sim}pR_ip$, then $pP_i{\sim}p$ or ${\sim}pP_ip$.

- *Individually combined deductive transfer* (*given the partition* $q_i =(GC_i; GT_i)$):

For every $S \subseteq G$ and every $p \in G$,

if for every $s \in S$, GC_i, and $\{p\}\cup S$ logically entails q,

then it cannot be that for every $r \in \{p\}\cup S$, rPq.

Let $\boldsymbol{Z}$ be the set of all the *complete* and *vertically balanced preorderings R* defined on G that (given the partition q_R) satisfy the *autorestricted tertio excluso principle* and *combined deductive transfer,* and let us call *judgment general relations* to all the relations in $\boldsymbol{Z}$.

¿How do the majority method and its variants VMD and NMMD perform when aggregating individual relations of this last kind?[41] ¿How do they behave in this more general setting?

For an argument entirely analogous to that regarding Claim 6.3, the following similar claim can be stated:

CLAIM 6.4.- If F is VMD and $Dom(F)= \boldsymbol{Z}^n$, then R^F holds the *aggregate restricted tertio-excluso principle* and *doubly combined deductive closure*.

Let us introduce now the corresponding version of *deductive transfer* for aggregate relations.

- *Doubly combined deductive transfer* (*given the profile of individual partitions* $Q=(q_1, q_2,.., q_n)$):

[41] The levelled variant LMD of the majority method lacks a clear meaning in this new framework.

For every *S Í G* and every *p Î G*,

if for every *s Î S, s Î UNC(G | f),* {*p*}∪*S* logically entails *q*,

then it cannot be that for every *r Î* {*p*}∪*S, rPq*.

It is easy to check that the following claim holds.

CLAIM 6.5.- If *F* is MMD or NMMD, and *Dom*(*F*)= $\boldsymbol{Z}^n$, then R^F holds the *aggregate restricted tertio-excluso principle* and *doubly combined deductive transfer*.

To sum up, VMD, MMD and NMMD also preserve in this setting the corresponding version of the weakened logical constraints.

This is good news, but there is bad news coming. In this setting, VMD, and NMMD as well, loose much information about the individual points of view. So, applying MMD may be more appropriate than formerly. But, changing and extending the domain of aggregation to $\boldsymbol{Z}^n$ may lead to MMD and NMMD diverging in comparing non-complementary propositions, as the following example shows.

Table 6.1

Person 1	$pP_1{\sim}p$,	$qP_1{\sim}q$,	qP_1p.
Person 2	$pP_2{\sim}p$,	$qP_2{\sim}q$,	qP_2p.
Person 3	$pP_3{\sim}p$,	${\sim}qP_3q$,	pP_3q.
MMD	$pP^j{\sim}p$,	$qP^j{\sim}q$,	qP^Jp.
NMMD	$pP^S{\sim}p$,	$qP^S{\sim}q$,	pP^Sq.

It should be noted that this divergence is a simple consequence of one elementary fact. As far as the comparisons of non-complementary propositions are concerned, MMD and NMMD collect and convey information of different kinds.

As a consequence, with regard to the relationships between non-complemnetary propositions NMMD may fail to meet some properties like *neutrality*, *independence of irrelevant alternatives* and *positive responsiveness,* as the following two tables illustrate.

Table 6.2

Person 1	$pP_1{\sim}p$,	pI_1q,	$qP_1{\sim}q$.		$pP'_1{\sim}p$,	pI'_1q,	$qP'_1{\sim}q$.
Person 2	$pI_2{\sim}p$,	pI_2q,	$qI_2{\sim}q$.		$pI'_2{\sim}p$,	pI'_2q,	$qI'_2{\sim}q$.
Person 3	${\sim}pP_3p$,	qP_3p,	${\sim}qP_3q$.		${\sim}pP'_3p$,	qP'_3p,	$qP'_3{\sim}q$.
NMMD	$pI^S{\sim}p$,	qI^Sq,	qI^Sp.		$pI^{S'}{\sim}p$,	$qP^{S'}p$,	$qP^{S'}{\sim}q$.

Table 6.3

Person 1	$pP_1{\sim}p$,	pI_1q,	$qP_1{\sim}q$.		$pP'_1{\sim}p$,	pP'_1q,	$qP'_1{\sim}q$.
Person 2	$pP_2{\sim}p$,	pI_2q,	$qP_2{\sim}q$.		$pP'_2{\sim}p$,	pP'_2q,	$qP'_2{\sim}q$.
Person 3	${\sim}pP_3p$,	qI_3p,	${\sim}qP_3q$.		${\sim}pP'_3p$,	qI'_3p,	${\sim}qP'_3q$.
NMMD	$pP^S{\sim}p$	pI^Sq	qP^Sp.		$pP^{S'}{\sim}p$,	$qI^{S'}p$,	$qI^{S'}{\sim}q$.

With regard to MMD, perhaps the most salient point is that the voting paradox, and similar phenomena, may reappear:[42]

Table 6.4

Person 1	$pP_1{\sim}p$,	${\sim}pP_1q$,	pP_1q.
Person 2	${\sim}pP_2p$,	${\sim}pP_2q$,	qP_2p.
Person 3	$pP_3{\sim}p$,	$qP_3{\sim}p$,	qP_3p.
MMD	$pP^J{\sim}p$,	${\sim}pP^Jq$,	qP^Jp.

It should be noted, in addition, that when dealing with judgments, the voting paradox may give rise to situations which lack logical coherence in a very elementary sense.

Table 6.5

Person 1	$pP_1{\sim}p$,	${\sim}pP_1q$,	pP_1q,	${\sim}qP_1q$.
Person 2	${\sim}pP_2p$,	${\sim}pP_2q$,	qP_2p,	$qP_2{\sim}q$ or ${\sim}qP_2q$.

[42] The following table illustrates how the voting paradox disappears when individual judgment relations are categorical.

Person 1	$pP_1{\sim}p$,	${\sim}pI_1q$,	pP_1q.
Person 2	${\sim}pP_2p$,	${\sim}pP_2q$,	qI_2p.
Person 3	$pP_3{\sim}p$,	$qP_3{\sim}p$,	qI_3p.
MMD	$pP^J{\sim}p$,	${\sim}pP^Jq$,	qP^Jp.

Person 3	$pP_3{\sim}p$,	$qP_3{\sim}p$,	qP_3p,	$qP_3{\sim}q$ or ${\sim}qP_3q$.
MMD	$pP^J{\sim}p$,	${\sim}pP^Jq$,	qP^Jp,	$qP^J{\sim}q$ or ${\sim}qP^J q$.

In this example, qP^Jp while $pP^J{\sim}p$. Elementary coherence (and *vertical balance*) would require both that $pP^J{\sim}q$ and ${\sim}pP^J{\sim}q$, or at least, $pP^J{\sim}q$, and $qP^J{\sim}q$ and not ${\sim}pP^J q$. Nevertheless, ${\sim}pP^J q$ and it may be the case that ${\sim}qP^J q$.

.- Many truth-values

It has been mentioned above thet Pauly and van Hees (2003) generalize the judgment aggregation habitual framework introducing the use of many-valued logic. *Complete* and *vertically balanced preorderings* satisfying *set-wise deductive transfer* may represent in our terms many individual truth-valued points of view. Let $\boldsymbol{H}$ be the set of all such relations.

Of course, the difficulties regarding the fulfilment of the set-wise logical constraints at the aggregate level, as those exemplified by the discursive dilemma, return. However, applying Claim 5.3 it is easy to show that if F is NMMD or MMD and $Dom(F)= \boldsymbol{H}^n$, then the aggregate judgment relation always satisfies *collective combined deductive transfer* and, therefore, *combined deductive closure*. Similarly, applying Claim 5.4 it is easy to show that if F is VMD and $Dom(F)= \boldsymbol{H}^n$, then the aggregate judgment relation always satisfies *collective combined deductive closure*.

However, difficulties like those presented in the preceeding section regarding NMMD and MMD are not avoided in this new domain, because they are originated independently from which logical constraints are postulated or satisfied.

.- Summing degrees of support

Finally, imagine that the degrees of support ($s_i \in [0,1]$) which individuals attach to each of their judgments are known, and that the degrees support from different individuals are considered interpersonally comparable. Given a proposition p we can calculate its aggregate degree of support $s(p)= \sum_{i\in N} s_i(p)$ as the sum of all the individual degrees of support. In addition, given the aggregate degree of support of each proposition in the agenda, we can compare such propositions as we would apply the utilitarian criterion, pR^Uq iff $s(p) \geq s(q)$, giving rise to a new aggregate judgment relation with the same good formal properties as the utilitarian aggregate function.

If this procedure is possible and meaningful, R^U avoids the above difficulties with MMD and NMMD when individual relations are regular relations of categorical or of non-categorical judgments. But at the same time, it can happen that a collective judgment obtained by the majority method or by any of its variants is changed if the degrees of support summing method is applied. Suppose, for instance, that $s_i(p)= 0.6$ for $i= 1, 2, 3$, and that $s_j(\neg p)= 1$ for $j= 4, 5$. Then, applying VMD, or LMD, or MMD or NMMD, we obtain that $pR^F Øp$, while summing the degrees of support we have that $ØpP^Up$. This example and the analysis made in this paper show how heavily the choice of the appropriate judgment aggregation method and the outcome reached in each case depend on the information available about the judgments that individuals hold and the firmness that they attach to them.

7.- **AGGREGATING JUDGMENTS OR ARGUMENTS**?

In principle, aggregating individual judgments is a task not restricted to any kind of individual judgment. But the existing literature on aggregating judgments habitually focuses on the agregation of interconnected judgments such as, in the simplest cases, "reasons" or "premises" on the one hand, and "conclusions" or "outcomes" on the other. In other words, the habitual framework would be not so much focused on aggregating judgments, as on aggregating arguments and their potential conclusions.[43] But if this is the case, then problems and situations of this sort may be set up in a much simpler way. In addition, setting up such problems and situations in this simpler way avoids difficulties like those analysed in the preceeding pages.

Imagine that according to a scientific law, if phenomenon *a* and phenomenon *b* both take place, then and only then an event *c* will occur. What should be asked of a single scientist on his opinion about the possibility of event *c*?

Table 7.1

a	*b*	*Øa*	*Øb*	(*aÙb*)	(*aÙØb*)	(*ØaÙb*)	(*ØaÙØb*)	[(*aÙb*) ↔*c*]	*c*	*Øc*
0.6	0.6	0.4	0.4	0.2	0.4	0.4	0	1	0.2	0.8

There are four relevant states of the world: [*a*; *b*; (and then *c*)]; [*a*; *not b*; (and then *not c*)]; [*not a*; *b*; (and then *not c*)]; [*not a*; *not b*; (and then *not c*)]. If asking the scientist on the conjunction (*aÙb*), his answer gives the necessary and sufficient conditions for expecting *c* or for expecting *not c*. If however, we ask about *a* separately, we are asking his position about *a* and *not a*, that is, between the possibility that the state [*a*; *b*; (and then *c*)] or the state [*a*; *not b*; (and then *not c*)] occurs, and the possibility that the state [*not a*; *b*; (and then *c*)] or the state [*not a*; *not b*; (and then *not c*)] occurs. If we ask about *b* separately, an analogous situation occurs. Therefore, if he is not certain, he may answer for *a* and for *b*. But this means only that he thinks that the eventuality that the state [*a*; *b*; (and then *c*)] or the state [*a*; *not b*; (and then *not c*)] occurs is more likely than the possibility that the state [*not a*; *b*; and then *c*)] or the state [*not a*; *not b*; (and then *not c*)] occurs, and similarly with regard to the possiblility of state [*a*; *b*; (and then *c*)] or state [*not a*; *b*; (and then *not c*)] against the possibility of state [*a*; *not b*; and then *c*)] or state [*not a*; *not b*; (and then *not c*)]. In any case, it does not follow that *c* will occur, nor that its ocurrence is more likely than its non-occurrence. As shown in table 7.1, he might assign a probability higher than 0.5 to *a* and to *b*, but he might also assign a probablity lower than 0.5 to the joint ocurrence of *a* and *b*, thus considering ($a \wedge b$) improbable ($a \wedge b$), and thus considering it improbable that *c*.

So, if the scientist is simply asked whether (*a* and *b*) or not, then we can conclude that *c* is more likely than *not c* if the answer is affirmative, or we can conclude that it is less likely if the answer is negative. But if this is the case with a single scientist, why must it be different when asking a committee of scientists? Why aggregate judgments on *a* and *b* separately? Why not ask and aggregrate judgments on (*aÙb*) when they must operate jointly to produce the possible outcome?

One of the (many) possible ways of formalizing these ideas is selecting essential elements of the agenda or reducing the agenda to those essential elements: as we may say, "normalizing" the agenda.

A normalized agenda $G = \{\{p\} \cup W \cup \{q\}\}$ is a set $\{\{p\} \cup W \cup \{q\}\}$ of propositions such that:

- it contains a (possibly empty) set of propositions W, a conjunction of premises p, and a conclusion q;
- it contains the complementary of each proposition $v \in$, W of p and of q;
- each $v \in W$ is true for every person in the group;
- $\{p\} \cup W$ entails q.

It should be noted that a normalized agenda induces a domain restriction. If $\boldsymbol{Z^*}$ is the set of all generalized judgment relations definable on the set $\{\{p\} \cup W \cup \{q\}\}$, the "normalized" or "relevant" restriction induced by $G = \{\{p\} \cup W \cup \{q\}\}$ is the subset of all the $R \in \boldsymbol{Z^*}$ such that for every $v \in W$, $v \in \text{Max}(G \mid R)$.

It should be noted that for every individual $_i$ and every v, $r \in W$:
the only triples $s^1, s^2, s^3 \in G$ such that $s^1 P_j s^2$ and $s^2 P_j s^3$, are the following:
(1) $vP_i q$, $qP_i \sim q$, (2) $vP_i \sim q$, $\sim qP_i q$, (3) $vP_i p$, $pP_i \sim p$, (4) $vP_i \sim p$, $\sim pP_i p$,
(5) $vP_i q$, $qP_i p$, (6) $vP_i q$, $qP_i \sim p$, (7) $vP_i \sim q$, $\sim qP_i p$, (8) $vP_i \sim q$, $\sim qP_i \sim p$.

So, in all those cases, it cannot be that $s^3 P_j s^1$. Hence, *extremal restriction* holds for every triple $s^1, s^2, s^3 \in G$, and R^J is *transitive* (Sen, 1986: 1139).

Regarding logical constraints, R^J *transfers support* from the argument p to the conclusion q. By hypotheses $\{p\} \cup W$ entails q, and for every $v \in W$, $v \in \text{Max}(G \mid R_i)$ for every person i. Given that R_i is *complete* and meets *combined deductive transfer*, there is some $s \in \{p\} \cup W$, such that $qR_i s$. If $s \neq p$, then $s \in \text{Max}(G \mid R_i)$, and $sR_i p$. For transitivity, $qR_i p$. Hence, $qR_i p$ for every person I, and $qR^J p$ by unanimous agreement.

To sum up. If asking the relevant question and aggregating the relevant judgments, the discursive dilemma and related phenomena disappear. On the other hand, asking questions on judgments other than the relevant ones and aggregating correpondingly may lack a clear meaning.

8.- **SUMMARY AND FINAL REMARKS**

In this paper we have considered six scenarios depending on the available information on the points of view of the group's individuals. In the most simple one, individual points of view are represented by relations in set $\boldsymbol{B}$, that is, *segmented relations of categorical judgments*. In another, individual points of view are represented by relations in set $\boldsymbol{A}$, that is, *levelled relations of categorical judgments*. We have considered also the possibility that individual points of view are represented by *segmented relations of non-categorical* or *of partitioned judgments*, with $\boldsymbol{VP^n}$ and $\boldsymbol{VQ^n}$ respectively being the aggregation domain. Finally, we kept in mind the possibility that individual point of view are represented by *regular relations of non-categorical* or *categorical judgments*, with $\boldsymbol{Z^n}$ and $\boldsymbol{H^n}$ respectively being the aggregation domain.

In the first case, the majority method variants that can be properly applied are the *vertical restriction of the majority method* (VMD) and the *net majority method* (NMMD). R^V denotes the aggregate relations generated by the former, R^S those generated by the latter. If, on the contrary, individual relations are levelled, in addition to the former two methods we can also apply the *levelled variant of the majority method* (LMD) and the *majority method* itself (MMD). R^L denotes the aggregate relations generated by LMD, R^J those generated by MMD.

These methods and the aggregate relations induced by them hold many attractive properties.

For instance, if *levelled relations of categorical judgments* are aggregated, R^L, R^J and R^S are *complete* and *vertically balanced preorderings*, R^L is also *levelled*, and $R^J = R^S$, while R^V is *vertically restricted* and, instead of being *complete*, it is only *vertically complete*. R^V also satisfies *vertical balance* but only in a vacuous way. If, on the contrary, the individual relations that are being aggregated are *segmented relations of* *categorical* or *of non-categorical* or *of partitioned judgments*, applying LMD or MMD becomes meaningless, and R^V and R^S hold the same properties as when aggregating profiles in set $\boldsymbol{A^n}$.

Let now this same set $\boldsymbol{A^n}$ of individual profiles be the aggregation domain. Then, LMD, MMD and NMMD (and VMD) hold *anonimity*, *neutrality*, *independence of irrelevant alternatives*, *non-negative responsiveness*, *unanimity*, the *vertical version of the* (*strong*) *Pareto condition* (equivalent with *unanimity* in this scenario), *non-dictatorship*, *non-vertical-dictatorship*, *non-oligarchy* and *non-vertical-oligarchy*. MMD and NMMD (and VMD) satisfy, in addition, *positive responsiveness* and the (*strong*) *Pareto condition*.

If the aggregation domain is $\boldsymbol{B^n}$ or $\boldsymbol{VP^n}$ or $\boldsymbol{VQ^n}$, (that is, if individual relations are *segmented relations of categorical*, or *of non-categorical* or *of partitioned judgments*) NMMD conveys the same information as when its domain is $\boldsymbol{A^n}$ and it also meets all the former properties, as does VMD as well.

These facts notwithstanding, those aggregation methods are habitually disqualified because the tertio excluso principle and the deductive closure restriction are imposed on the aggregation R^F. A simple example of a tie between the people who accept a proposition and the people who reject it suffices to show that these four method do not lead to the fulfilment of the *tertio excluso principle*. Analogously, the discursive dilemma illustrates how these methods can lead to a violation of *set-wise deductive closure*.

In this paper it has been suggested that such restrictions may be excessive as far as they cannot be required from any individual who must base his judgments on beliefs. In particular, the *tertio excluso principle* should not be postulated at all, and *set-wise deductive closure* may be substitued by some weakened logical restrictions, like *proposition-wise*, *combined* and *doubly combined deductive closure* and *deductive transfer*.

Specifically, we have shown that if VMD or NMMD is applied with the domain $\boldsymbol{B^n}$, that is, when individual relations are *segmented relations of categorical judgments*, (or if either of the two former methods or LMD or MMD is employed with the set $\boldsymbol{A^n}$, when individual relations are *levelled relations of categorical judgments*), the aggregation domain, then R^F alwayssatisfies *collective combined deductive closure* (*collective combined deductive transfer*). In these cases, then, individual relations satisfy *set-wise deductive closure* (*transfer*), and aggregate relations hold *collective combined deductive closure* (*transfer*).

Suppose now that individual relations are *segmented relations of non-categorical judgments* (or *regular relations of non-categorical judgments*), that the aggregation domain is $\boldsymbol{VP^n}$ (or $\boldsymbol{W^n}$) and that VMD or NMMD (or LMD or NMMD or MMD) is applied. In these cases, individual and aggregate relations meet *proposition-wise deductive closure* (*transfer*).

Thirdly, imagine that individual relations are *segmented relations of partitioned judgments* (or *regular relations of partitioned judgments*), that the aggregation domain is $\boldsymbol{VQ^n}$ (or $\boldsymbol{Z^n}$) and that VMD or NMMD (or LMD or NMMD or MMD) is applied. In these cases, *individual combined deductive closure* (*transfer*) and aggregate relations meet *doubly combined deductive closure* (*transfer*).

We have defended that *proposition-wise deductive closure*, *proposition-wise deductive transfer*, *collective combined deductive closure*, *collective combined deductive transfer*, *doubly combined deductive closure* and *doubly combined deductive transfer* are logical constraints that may be required from groups more properly than the *tertio excluso principle* and *deductive closure,* save that the group's point of view is unanimous. In this latter case, the former conditions and *deductive closure* are logically equivalent. Therefore, were my suggestion accepted, the majority method and its examined variants would be viable aggregation methods, unlike the habitual position defended in the literature. Specifically, it would be accepted (1) that VMD can be properly applied when information about individual points of view is given by individual relations of any of the kinds we have considered, (2) that NMMD can be properly applied and conveys richer information at the aggregate level than VMD (and LMD) when information about individual points of view is given by *segmented relations of categorical or of non categorical judgments*, or by *levelled relations of categorical judgments*, and (3) that LMD and MMD can be properly applied when information about individual points of view is given by *levelled relations of categorical judgments*, when MMD conveys just the same aggregate information as NMMD, being in addition more informative than LMD at the aggregate level.

When individual relations are *regular relations of categorical* (domain $\boldsymbol{H^n}$), *non-categorical* (domain $\boldsymbol{W^n}$) *or partitioned judgments* (domain $\boldsymbol{Z^n}$), LMD and VMD convey the same information as when the aggregation domain is $\boldsymbol{A^n}$ and each of them meets the same properties. Hence, they may be properly applied also in this case, though it would be preferable to increase the information that they supply. But with regard to MMD and NMMD, things are somewhat different in this case. Specifically, R^J is not more *transitive*, nor even *acyclical*, and does not hold *vertical balance*. In turn, while conveying the same information as in former cases, with respect to the comparisons of non-complementary propositions NMMD may fail to meet properties like *neutrality*, *independence of irrelevant alternatives* and *positive responsiveness*.

These last difficulties can be avoided if individual degrees of support are known and interpersonally comparable, and if the method analogous to the utilitarian criterion method is applied. However, it should be taken into account that this method may give different outcomes than the majority method and its variants. These latter methods, on the other hand, may be applied when information about the individual points of view is merely qualitative. As pointed out before, these and earlier considerations suggest how heavily choice of the appropriate judgment aggregation method and the outcome reached in each case depend on the information available about the judgments that individuals hold and the firmness that they attach to them.

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