The Voting Power Approach: A Theory of Measurement
A Response to Max Albert

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Abstract. Max Albert (2003) has recently argued that the theory of power indices "should not ... be considered as part of political science" and that "[v]iewed as a scientific theory, it is a branch of probability theory and can safely be ignored by political scientists". Albert's argument rests on a particular claim concerning the theoretical status of power indices, namely that the theory of power indices is not a positive theory, i.e. not one that has falsifiable implications. I re-examine the theoretical status of power indices and argue that it would be unwise for political scientists to ignore such indices. Although I agree with Albert that the theory of power indices is not a positive theory, I suggest that it is a theory of measurement that can usefully supplement other positive and normative social-scientific theories.

1. Introduction

Power indices have received increasing attention in political science, especially in the field of European Union politics. They are frequently used for investigating, first, the present distribution of voting power among EU member states in the Council of Ministers and the European Parliament, and, second, the effect of proposed institutional changes or EU enlargement on that distribution (e.g. Felsenthal and Machover 1997; Nurmi 1997, 2000; Nurmi and Meskanen 1999; Dowding 2000; Aleskerov et al. 2002). In a recent article, however, Max Albert (2003) argues that the theory of power indices "should not ... be considered as part of political science" (p. 1), and further that "[v]iewed as a scientific theory, it ... can safely be ignored by political scientists" (p. 1). His argument rests on a particular diagnosis of the theoretical status of power indices. The theory of power indices, Albert argues, is not a positive theory, i.e. not one that has falsifiable implications. Rather, he suggests, depending on the interpretation, the theory is either an empirically vacuous branch of probability theory or an unconvincing branch of political philosophy. In either case, the theory "has no factual content and can therefore not be used for purposes of prediction or explanation" (p. 1).

I seek to re-examine the theoretical status of power indices and to explain why, in my view, it would be unwise for political scientists to ignore such indices. I agree with Albert on what the theory of power indices is *not*. It is not, by itself, a positive theory. But I disagree with him on what it *is*. I suggest that, in terms of its theoretical status, the theory of power indices is similar to the theory of inequality indices. The theory of inequality indices is not, by itself, a free-standing theory. Rather, it is a *theory of measurement* that supplements other social-scientific theories. An inequality index is a statistical measure for summarizing certain properties of a given income (or other) distribution across a population. Inequality indices can thus supplement any theory that refers to such distributions, whether that theory is positive or normative. Analogously, the theory of power indices is a theory of measurement that supplements other social-scientific theories. A power index is a statistical measure for summarizing certain properties of a given voting game, as defined below. Power indices can thus supplement any theory that refers to such voting games, particularly cooperative game theory and its applications to modelling political institutions.

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2. Power indices as statistical measures on the set of voting games

The general definition clarifies that power indices are statistical measures on the set of voting games (e.g. Laruelle and Valenciano 2001). A *voting game* is a pair $<N, v>$, where $N = \{1, 2, ..., n\}$ is a set of players and v a function mapping each subset of N (a *coalition*) to either 0 (non-winning) or 1 (winning), such that:

(i) $v(\emptyset) = 0$ (the empty coalition is non-winning) and $v(N) = 1$ (the coalition of all players is winning);[2]

(ii) there exists at least one subset $S \subseteq N$ such that $v(S) = 1$ (there is at least one winning coalition);

(iii) for all subsets S, $T \subseteq N$, $S \subseteq T$ implies $v(S) \leq v(T)$ (a superset of a winning coalition is also winning);

(iv) for all subsets $S \subseteq N$, $v(S) + v(N\backslash S) \leq 1$ (for any partition of the set of players into two disjoint coalitions, at most one is winning).[3]

Each n-player voting game represents a particular voting procedure in an n-member electorate. For example, simple majority voting or unanimity voting in a 100-member electorate each correspond to a particular 100-player voting game. Let V_n denote the set of all logically possible n-player voting games. Then V_n can be interpreted as the set of all logically possible (binary) voting procedures in an n-member electorate.[4]

Now a *power index* is a function Φ (with domain V_n and co-domain $\mathbf{R}^n$) that maps each n-player voting game to a vector of real numbers, $<p_1, p_2, ..., p_n>$, called a *power profile*. For each i, p_i is interpreted as the voting power of player i.

The Penrose-Banzhaf (PB) index and the Shapley-Shubik (SS) index, discussed by Albert, are instances of such functions:

- PB: $\Phi_{\mathrm{PB}}(<N, v>) := <p_1, p_2, ..., p_n>$, where

  for each i, $p_i := \frac{1}{2^{n-1}} \sum_{S \subseteq N \,:\, i \in S} (v(S) - v(S\backslash\{i\}))$.

- SS: $\Phi_{\mathrm{SS}}(<N, v>) := <p_1, p_2, ..., p_n>$, where

  for each i, $p_i := \sum_{S \subseteq N \,:\, i \in S} \frac{(s-1)!(n-s)!}{n!} (v(S) - v(S\backslash\{i\}))$.

  (For each $S \subseteq N$, $s := |S|$).

Each index can be interpreted in multiple ways. For the PB index, we say that player i is *pivotal* for a particular coalition if i's leaving that coalition turns it from a winning to a non-winning one. The PB index for each i can then be interpreted as the proportion among all logically possible

[2] The condition $v(N) = 1$ is not strictly necessary, as it is already implied by the conjunction of $v(\emptyset) = 0$ and (ii), (iii), (iv) below.

[3] Technically, a voting game is a *simple superadditive game*.

[4] Under this interpretation, conditions (i), (ii), (iii), (iv) are minimal consistency conditions on such voting procedures.

coalitions for which player i is pivotal. For the SS index, consider all (n!) logically possible sequences in which the n players can join a coalition one-by-one. We say that player i is *pivotal* for a particular sequence if i's joining the coalition of all players preceding i in the sequence turns that coalition from a non-winning to a winning one. The SS index for each i can then be interpreted as the proportion among all logically possible such sequences for which player i is pivotal. Other interpretations of the indices are possible, e.g. in terms of players' probabilities of being pivotal. But while such interpretations help our intuitive understanding of a given power index, they are not definitions of the index. A more precise way to characterize a particular index is to state a set of axioms – minimal conditions on summarizing voting power – such that the given index is the unique function Φ satisfying these axioms (Laruelle and Valenciano 2001).

A power index is thus a statistical measure for summarizing each logically possible voting game into a corresponding summary statistic, namely a power profile across players. As each possible voting procedure in the Council of Ministers or the European Parliament (including relevant weights) corresponds to a particular voting game, a power index can serve as a statistical measure for summarizing certain procedural features of such voting procedures *taken in isolation*.

3. The analogy with inequality indices

To illustrate the usefulness of such statistical measures, consider the example of an inequality index. An *inequality index* is a function that maps each logically possible income (or other) distribution across a population into a single quantity: the level of inequality. Prominent such indices are the Gini and Atkinson indices, but others have been discussed (Sen 1997). Just as a power index summarizes each voting game into a single summary statistic (the power profile), an inequality index summarizes each income (or other) distribution into a single summary statistic (the level of inequality). Power indices and inequality indices summarize different items, and thus the resulting summary statistics have different interpretations. But the *theoretical status* of both kinds of indices is similar. They are both functions aggregating relatively complex items into less complex summary statistics, and they can thus supplement any theory requiring such statistics.[5]

In the case of inequality indices, the resulting summary statistics are known to be useful from normative and positive perspectives. Normatively, ranking alternative socio-economic policies in an order of desirability may involve assessing the level of inequality under each policy, which requires using an inequality index. Positively, the level of inequality, measured by the Gini index, has been shown to be a predictor of several phenomena. For example, inequality of land distribution correlates negatively with the stability of democracy (e.g. Russett 1968), and income inequality correlates negatively with voting turnout (e.g. Goodin and Dryzek 1980).

In the case of power indices, the generated summary statistics may be relevant for normatively evaluating alternative voting procedures (or voting weights) in a given context. While the distribution of voting power is unlikely to be the only normatively relevant consideration here, it is plausibly one of several such considerations (others being the avoidance of stalemate or the consistency of voting outcomes). Most of the recent applications of power indices to EU politics

[5] Indeed, power indices and inequality indices can even be usefully combined to obtain a summary measure of *inequality of voting power*: using a power index we can assign to each voting game a corresponding power profile, and using an inequality index we can then assign to each such power profile a corresponding summary statistic capturing the level of inequality of voting power under the given voting game.

fall into this normative category. Power indices are used for evaluating alternative institutional arrangements in the EU and the effects of potential changes, and sometimes for making recommendations on how to equalize voting power across member states or across EU citizens. The potential of using power indices in positive research, by contrast, has been largely unexplored so far. So Albert's complaint that power indices are disconnected from positive research is correct *to the extent* that we have not yet seen much evidence of their usefulness in positive research. But there is no reason why power indices cannot in principle be used in such research too. Like inequality, voting power might plausibly serve as a regressor in models of certain empirical phenomena. For instance, it is conceivable (though still an untested hypothesis) that voting power might affect decision outcomes: policies preferred by agents with greater voting power might prevail more often than ones preferred by agents with less voting power. Similarly, the distribution of voting power might conceivably affect the dynamic of decision processes and perhaps the nature of deliberation in a collectivity: if there are significant inequalities in voting power, certain agents might frequently be agenda-setters while others might be marginalized. There are clearly avenues for positive research here. The results, to be sure, are open.

4. The informational poverty of power indices

Albert might grant that power and inequality indices are similar in that they are both statistical measures for summarizing certain items. But he might argue that their difference lies in the fact that inequality indices are *useful* such measures while power indices are not. Following the claims in his paper, he might argue that inequality indices are useful because they capture certain social-scientifically relevant properties of the items they summarize, whereas power indices are not useful because they capture only very abstract, and social-scientifically detached properties of the items in their domain: "the definition of voting power ... is disconnected from any positive theory and, therefore, useless for purposes of political science" (Albert 2003, p. 13).

In particular, Albert criticizes the "assumption of simple random voting" underlying power indices.[6] In terms of the informal interpretation of the PB and SS indices offered above, Albert's point is a critique of the method of 'brute counting' across all logically possible coalitions (in the PB case) or across all logically possible sequences (in the SS case), without considering any potentially relevant facts on how likely each such coalition or sequence is to arise. For instance, if a player's voting power stems solely from his or her being pivotal for coalitions that are unlikely to arise (e.g. ones between libertarian and Marxist players), then his or her alleged voting power seems a vacuous quantity. In short, the PB and SS indices are informationally poor. By focusing solely on the *formal structure* of the voting game and *not* on the players' behaviour, they screen out potentially relevant information.

This point is forceful, but we should be clear about what follows from it. First, the fact that standard power indices are sensitive exclusively to the formal structure of a voting game may sometimes be a virtue rather than a vice. For some *normative* purposes, certain behavioural facts about the players, such as their preferences, might be deemed normatively irrelevant. Veil of ignorance arguments are based on this view. Albert criticizes such arguments, but I think that the best response here is to point out that there *exist* several influential normative theories that make

[6] While some standard power indices can be *interpreted* in terms of random voting, note that this is an *interpretation* and *not* part of their *definition*. Other non-probabilistic interpretations can be given (like the ones in section 2 above). Thus these power indices are *not* strictly speaking based on an *assumption* of random voting.

use of veil of ignorance arguments – whether or not one endorses them (e.g. Rawls's, Harsanyi's and Buchanan's theories) – and such theories can thus employ power indices as methodological tools. On the other hand, whether or not the informational restrictions of standard power indices impair their usefulness in *positive* research remains to be seen.

Second, it is conceivable that, for at least some purposes (whether normative or positive), the informational restrictions *do* pose significant limitations. We might be interested, for instance, not in the proportion of *logically possible* coalitions or sequences for which a given player is pivotal, but rather in the proportion of *realistically feasible* such coalitions or sequences. Once we recognize this point, as Albert does, one response might be to pursue Albert's route and to abandon power indices for the purposes of political science. But there exists a more constructive route: namely not to *abandon*, but rather to *extend* the theory of power indices. Nurmi (2000) explains how this can be done. If we assume that not *all* logically possible coalitions, but only *some* specific ones are likely to arise, we can easily accommodate this behavioural assumption in the construction of a power index. In the definition of the PB and SS indices, we simply need to replace summation over *all* logically possible coalitions $S \subseteq N$ (such that $i \in S$) with summation over all coalitions $S \in \mathbf{C}$ (such that $i \in S$), where **C** is the set of those coalitions that are assumed to be feasible. As an illustration, Nurmi (2000, p. 368, Table 3) computes the modified SS index for the Council of Ministers under the assumption that only 4 particular coalitions between member states are feasible (e.g. Franco-German, Mediterranean, Benelux, Neutral-plus-Nordic). Nurmi concludes that "… the criticism of the power index studies that is based on the equiprobability of coalitions assumption misses the point in so far as various kinds of player groupings can be modelled using the same apparatus". Formally, all that such an extension requires is defining power indices on a domain that is richer than the one traditionally used. Such a richer domain might for instance be the Cartesian product of [the set of all logically possible n-player voting games] and [the set of all logically possible sets **C**, as just defined].

Again the analogy with inequality indices is instructive. Standard methods of inequality measurement are often criticized for their narrow focus on income. Just as power indices screen out certain information, so inequality indices, applied to just one attribute such as income, screen out potentially relevant information, for instance about each person's capacity to convert income into welfare. Someone with a medical condition might require more income to attain a particular welfare level than someone without that condition, and therefore what superficially seems like an equal distribution (in terms of income) might actually be an unequal one (in terms of welfare) (for a famous discussion, see Sen 1980). But it would be unwise, as a consequence, to abandon inequality indices for social-scientific purposes. Rather, a more promising route (and one pursued by many welfare economists) is to *extend* the theory of inequality indices, and to construct indices that are sensitive to a richer information set. For example, multi-attribute inequality indices have been developed to meet this demand (e.g. Koshevoy and Mosler 1997; Tsui 1999).

So power indices and inequality indices can each be defined on informationally poor domains as well as on informationally rich ones, depending only on the required social-scientific application and on the amount of information that is available.

5. Conclusion

I have invoked the analogy with inequality indices to illustrate why Albert's conclusion – that political scientists can safely ignore power indices – does not follow from his diagnosis of the *theoretical status* of these indices. The premises concerning *theoretical status* that Albert uses to support his conclusion seem to be met equally by the theory of inequality indices, and yet (I think) we would not conclude that social scientists can afford to ignore inequality indices. The fact that something is not a free-standing (positive or normative) theory, but 'merely' a statistical measure does not undermine its usefulness (for positive or normative purposes, respectively). Something may be useful *precisely because* it is a statistical measure.

Just as inequality indices usefully supplement theories that refer to income (or other) distributions, so power indices can play a potentially useful role in theories that refer to voting games. There is no doubt that our methodological toolbox would be poorer without inequality indices. Power indices are a more recent addition to that toolbox and have had less time to prove their value. But throwing them out at this point seems premature.

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