Corrigendum to “A Possibility Theorem on Aggregation over Multiple Interconnected Propositions” [Mathematical Social Sciences 45 (2003), 1-13]

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In this note, I correct an error in List (2003). I warmly thank Ron Holzman for drawing my attention to this error, and Franz Dietrich for giving me some key insights that have led to the present correction, particularly the formulation of assumption (a*) below.

Theorem 2 (specifically, the claim that (i) implies (ii) and the associated Proposition 2) in List (2003) requires an additional assumption on the set X of propositions under consideration (the *agenda*). Let me use the definitions and notation from List (2003). Also, define a set of propositions to be *satisfiable* if some truth-value assignment makes all its members true, and, for each $Y \subseteq X$, define $Y^{\neg} := \{\neg\phi : \phi \in Y\}$.

In List (2003), I assumed that (a) X contains at least two distinct atomic propositions P, Q and their conjunction $(P \wedge Q)$, and (b) X contains proposition-negation pairs (i.e. if $\phi \in X$, then $\neg\phi \in X$, where $\neg\neg\phi$ is identified with ϕ). While assumption (b) is correct as stated, Theorem 2 requires assumption (a*) instead of (a).

(a*) X has a satisfiable subset Y such that $Y^{\neg}$ is not satisfiable, and
X has pairwise disjoint subsets X_1, X_2, X_3, where $X_1 \cup X_2 \cup X_3$ contains a member from each proposition-negation pair in X, such that
$X_1 \cup X_2 \cup X_3$, $X_1^{\neg} \cup X_2 \cup X_3$, $X_1 \cup X_2^{\neg} \cup X_3$ are each satisfiable, and
$X_1^{\neg} \cup X_2^{\neg} \cup X_3$ is not satisfiable.

Many standard agendas satisfy (a*), including the examples in List (2003) such as $X = \{P, \neg P, Q, \neg Q, (P \wedge Q), \neg(P \wedge Q)\}$ and $X = \{P, \neg P, Q, \neg Q, R, \neg R, (R \leftrightarrow (P \wedge Q)), \neg(R \leftrightarrow (P \wedge Q))\}$.

Once assumption (a) is replaced with (a*), Theorem 2 is correct as stated. In the original example $X = \{P, \neg P, Q, \neg Q, (P \wedge Q), \neg(P \wedge Q)\}$, which satisfies (a*), the proof of Theorem 2 is also correct as stated. In the general case, the error lies in my assertion (made without argument in Claim 3 of the proof) that the partially specified profile in Table 3 can be extended to a full profile satisfying unidimensional alignment. While this assertion is correct for the agenda $X = \{P, \neg P, Q, \neg Q, (P \wedge Q), \neg(P \wedge Q)\}$, it is incorrect for some supersets of this agenda such as $\{P, \neg P, Q, \neg Q, (P \wedge Q), \neg(P \wedge Q), (\neg P \wedge Q), \neg(\neg P \wedge Q), (P \wedge \neg Q), \neg(P \wedge \neg Q), (\neg P \wedge \neg Q), \neg(\neg P \wedge \neg Q)\}$. For such agendas, a counterexample to Claim 3 and hence to Theorem 2 (“(i) implies (ii)”) can be constructed, as noted by Ron Holzman. Assumption (a*) rules out this problem.

To correct the original proof so as to establish Theorem 2 under assumption (a*), it suffices to replace Claim 3 with the following.

Claim 3 (corrected). For any $k, l \in \{0, 1, ..., n\}$ $(k, l \neq n/2)$, $k < l$ implies $g(k) \leq g(l)$.

Proof of Claim 3 (corrected). First note that $g(n) = 1$. If $g(n) \neq 1$ then, by Claim 2, $g(0) = 1$. Take a profile $\{\Phi_i\}_{i \in N} \in \mathbf{UAD} \subseteq \mathbf{D}$ such that $Y \subseteq \Phi_1 = ... = \Phi_n$,

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where $Y \subseteq X$ is satisfiable with $Y^{\neg}$ not satisfiable (Y exists by (a*)). By Claim 1, as $|N_{accept-\phi}| = 0$ for all $\phi \in Y^{\neg}$, we have $Y^{\neg} \subseteq F(\{\Phi_i\}_{i \in N})$; but then $F(\{\Phi_i\}_{i \in N})$ is not satisfiable, which contradicts the consistency and deductive closure of $F(\{\Phi_i\}_{i \in N})$.

Now assume, for a contradiction, that there exist $k, l \in \{0, 1, ..., n\}$ $(k, l \neq n/2)$ such that $k < l$ and $g(k) > g(l)$, i.e. $g(k) = 1$ and $g(l) = 0$; then $g(n - l) = 1$, by Claim 2 (as $l \neq n/2$). Take the profile $\{\Phi_i\}_{i \in N} \in \mathbf{UAD} \subseteq \mathbf{D}$ uniquely specified by Table 1 (by (a*), each Φ_i is complete, consistent and deductively closed, and $\{\Phi_i\}_{i \in N}$ is clearly unidimensionally aligned).

	Ind. 1	...	Ind. k	Ind. $k+1$	...	Ind. l	Ind. $l+1$	...	Ind. n
every $\phi \in X_1^{\neg}$	Yes	...	Yes	No	...	No	No	...	No
every $\phi \in X_2^{\neg}$	No	...	No	No	...	No	Yes	...	Yes
every $\phi \in X_3$	Yes	...	Yes	Yes	...	Yes	Yes	...	Yes

Table 1

By Claim 1, as $|N_{accept-\phi}| = k$ for all $\phi \in X_1^{\neg}$ and $g(k) = 1$, we have $X_1^{\neg} \subseteq F(\{\Phi_i\}_{i \in N})$; and as $|N_{accept-\phi}| = n - l$ for all $\phi \in X_2^{\neg}$ and $g(n - l) = 1$, we have $X_2^{\neg} \subseteq F(\{\Phi_i\}_{i \in N})$; finally, as $|N_{accept-\phi}| = n$ for all $\phi \in X_3$ and $g(n) = 1$, we have $X_3 \subseteq F(\{\Phi_i\}_{i \in N})$. But, by (a*), $F(\{\Phi_i\}_{i \in N}) \supseteq X_1^{\neg} \cup X_2^{\neg} \cup X_3$ is not satisfiable, which contradicts the consistency and deductive closure of $F(\{\Phi_i\}_{i \in N})$. ■

The corrected proof rehabilitates Theorem 2 (and Proposition 2), given the corrected assumption that X satisfies (a*) and (b). For an extension to general logics and other generalizations, see Dietrich and List (2005).

References

Dietrich, F., List, C., 2005. Judgment aggregation on restricted domains. Working paper, LSE.

List, C., 2003. A Possibility Theorem on Aggregation over Multiple Interconnected Propositions. *Mathematical Social Sciences* 45, 1-13.