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Intradimensional Single-Peakedness and the Multidimensional Arrow Problem

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Abstract. Arrow's account (1951/1963) of the problem of social choice is based upon the assumption that the preferences of each individual in the relevant group are expressible by a single ordering. This paper lifts that assumption and develops a multidimensional generalization of Arrow's framework. I show that, like Arrow's original framework, the multidimensional generalization is affected by an impossibility theorem, highlighting not only the threat of dictatorship of a single individual, but also the threat of dominance of a single dimension. In particular, even if preferences are single-peaked across individuals within each dimension -- a situation called *intradimensional single-peakedness* -- any aggregation procedure satisfying Arrow-type conditions will make one dimension dominant. I introduce lexicographic hierarchies of dimensions as a class of possible aggregation procedures under intradimensional single-peakedness. The interpretation of the results is discussed.

1. Introduction

The problem of social choice discussed here is the problem of aggregating the preferences of the individual members of a group into the corresponding preferences of the group *as a whole*. The problem is illlustrated by a committee decision or an election, where individuals explicitly express their preferences over the alternatives by casting their votes, from which the collective outcome is then obtained. The votes could be either each individual's self-regarding preferences, or each individual's judgment of what is best for the group as a whole. For notational simplicity, I will use the language of 'preferences'.

Arrow's theorem (1951/1963) shows that there exists no procedure for aggregating individual preference orderings over a set of alternatives into collective, or social, ones where the procedure satisfies a set of arguably undemanding minimal conditions (transitivity of social orderings, universal domain, the weak Pareto principle, independence of irrelevant alternatives and non-dictatorship). Arrow's theorem is based on the assumption of one-dimensionality, the assumption that the preferences of each individual are expressible in terms of a single ordering. This assumption requires the existence of a single relevant 'respect' or 'dimension', like overall preference, in terms of which each individual can rank-order any set of alternatives.

The present paper lifts the assumption of one-dimensionality. We suppose that the preferences of each individual are expressed in terms of *multiple* orderings, one for each relevant dimension. I will show that, under this supposition, we are faced with a multidimensional Arrow problem.

There exists no procedure for aggregating individual multidimensional preference orderings into social orderings in accordance with a set of Arrow-type minimal conditions. Like Arrow's one-dimensional theorem, the multidimensional results highlight the threat of *dictatorship* of a single individual. But unlike Arrow's theorem, they also highlight the threat of *dominance* of a single dimension. Dominance and dictatorship are formally similar, yet their interpretation raises different issues, as discussed below.

I will also ask whether an escape-route from the impossibility result that is well known in the one-dimensional case is also available in the multidimensional case, namely the escape-route via single-peakedness. The main theorem of this paper shows that, even if preferences are single-peaked across individuals *within each dimension* -- a situation called intradimensional single-peakedness -- any aggregation procedure satisfying Arrow-type conditions will make one dimension *dominant*. Intradimensional single-peakedness is, however, sufficient for avoiding a dictatorship of one individual. I will introduce the class of *lexicographic hierarchies of dimensions* as a class of possible aggregation procedures under intradimensional single-peakedness.

There is not much related work in the literature; multidimensionality is usually not developed as an interpretation, and single-peakedness is not addressed in this context.[1]

In section 2, I will briefly discuss the political theory background of multidimensional social choice problems and the relevance of single-peakedness. In section 3, I will present the formal results, and in section 4, I will make some concluding remarks.

2. Single-Peakedness and Multidimensionality

The threat posed by Arrow's original one-dimensional theorem depends on how diverse the preferences across individuals are. If there is unanimity, aggregation is trivial. But although unanimity is sufficient for avoiding an impossibility result, it is not necessary. It is well known

[1] Formally related to the present results are Roberts's (1995) results concerning the aggregation of multiple opinions about the welfare of a group of individuals into a single social ordering and Khmelnitskaya's (1999) and Khmelnitskaya and Weymark's (2000) results concerning social welfare orderings for different scales of individual utility measurement in distinct population subgroups. Precursors are results by Fishburn (1971) and Batra and Pattanaik (1972) on multi-stage majority decisions. In multi-stage majority decisions, a given profile of preference orderings across individuals is first partitioned into several (possibly nested) subprofiles; the overall outcome is then determined by (possibly nested) aggregation over the aggregate outcomes corresponding to the appropriate subprofiles.

that if preferences across individuals satisfy the structure condition of single-peakedness, aggregation in accordance with Arrow's conditions (apart from universal domain) is possible.[2]

Single-peakedness may be interpreted as an *implication* of *agreement at a meta-level*, in the following sense. Agreement and disagreement are usually defined in terms of how *substantively* similar or dissimilar the preferences of different individuals are. But preferences may be different in *substance*, in that different individuals disagree on the rankings of options, and yet all individuals' preferences may be systematically aligned along the same common 'left'/'right' dimension. Each individual may have a most preferred position (possibly different for different individuals) on that dimension and rank options according to their distance from the most preferred position. Agreement on such a structuring dimension -- a form of *agreement at a meta-level* -- implies *single-peakedness* (see also List, 2002).

Miller (1992) argues that, when preferences are not single-peaked across individuals, this is caused by the combination of several normatively relevant issue-dimensions into a single preference ordering for each individual. The issue-dimensions which are collapsed into a single preference ordering, say on which industrial policy to pursue, could for instance be (1) economic growth, (2) employment, and (3) ecological sustainability. On Miller's account, a precondition for meaningful collective decisions is the identification of these multiple relevant issue-dimensions and the 'factorization' of overall preferences into dimension-specific preferences. Each individual would rank all options along each dimension separately, and these dimension-specific orderings would then form the basis of a collective choice. However, once dimension-specific individual preferences have been identified, we are still faced with a problem of aggregation, namely the problem of aggregating sets of dimension-specific orderings into corresponding collective orderings. This is precisely a multidimensional aggregation problem of the kind addressed in this paper. The main theorem stated below shows that, even in the best-case scenario from Miller's perspective, a multidimensional Arrow problem occurs, and I will discuss its implications.

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[2] See Black (1948) and Arrow (1951/1963). Other proposed structure conditions which are sufficient for aggregation in accordance with Arrow's conditions (apart from universal domain) are single-cavedness (Inada, 1964), separability into two groups (Inada, 1964), latin-square-lessness (Ward, 1965) and triplewise value-restriction (Sen, 1966). Triplewise value-restriction is the most general one of all these, being implied by, but not implying, each of the other conditions.

3. The Multidimensional Framework

Let $N = \{1, 2, ..., n\}$ be a set of individuals, and $X = \{x_1, x_2, ...\}$ a set of alternatives. Suppose, further, that there are k relevant issue-dimensions, contained in $K = \{1, 2, ..., k\}$. We will assume $n > 1$, $|X| > 2$, and, unless stated otherwise, $k > 1$.[3]

To each individual $i \in N$, there corresponds a k-tuple $R_i := \langle R_{ij} \rangle_{j \in K} = \langle R_{i1}, R_{i2}, ..., R_{ik} \rangle$ of personal preference orderings on X, one for each issue-dimension $j \in K$, where each R_{ij} is reflexive, transitive and connected. For example, X might be a set of industrial policy alternatives and the numerals 1, 2 and 3 might represent the dimensions of economic growth, employment and ecological sustainability, respectively. Then R_{i1}, R_{i2} and R_{i3} would represent individual i's preference orderings of the policy alternatives from the perspectives of economic growth, employment and ecological sustainability, respectively. For each $i \in N$ and each $j \in K$, the ordering R_{ij} induces a strong ordering P_{ij} and an indifference relation I_{ij}, defined as follows: for all x_1, $x_2 \in X$,

$x_1 P_{ij} x_2$ if and only if $x_1 R_{ij} x_2$ and not $x_2 R_{ij} x_1$;
$x_1 I_{ij} x_2$ if and only if $x_1 R_{ij} x_2$ and $x_2 R_{ij} x_1$.

A *profile of k-tuples of personal preference orderings* is an n-tuple $\{R_i\}_{i \in N} = \{R_1, R_2, ..., R_n\}$ of such k-tuples, one for each individual in N. As a notational convention, {}-brackets will be used to denote n-tuples across individuals and <>-brackets to denote k-tuples across dimensions.

A *multidimensional social welfare function* (*MSWF*) is a function F which maps each profile of k-tuples of personal preference orderings (in a given domain), $\{R_i\}_{i \in N}$, to a corresponding social ordering $R = F(\{R_i\}_{i \in N})$ on X, where R is required to be reflexive, transitive and connected. R also induces a strong ordering P and an indifference relation I, defined as above. In this framework, a one-dimensional *social welfare function* (*SWF*) is a MSWF for the case k=1.

To derive a simple multidimensional Arrow theorem, we use the conditions of universal domain, the weak Pareto principle and independence of irrelevant alternatives, and we require that an

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[3] While the question of how the relevant dimensions are to be identified and demarcated (i.e. the question of how the set K is to be interpreted) is a very important philosophical and political matter, it will here be taken to lie outside the scope of social choice theory and will therefore be bracketed out. For a discussion of this question, see Dryzek and List (2002).

acceptable MSWF be -- in a relevant sense -- non-dictatorial. The multidimensional generalization of the first three conditions is straightforward.

Given any profile of k-tuples of personal preference orderings, $\{R_i\}_{n\in N}$, we define $R := F(\{R_i\}_{n\in N})$.

UNIVERSAL DOMAIN (U). The domain of F is the set of all logically possible profiles of k-tuples of personal preference orderings.

WEAK PARETO PRINCIPLE (P). Let $\{R_i\}_{i\in N}$ be any profile in the domain of F. For any $x_1, x_2 \in X$, we have $x_1 P x_2$ whenever, for all $i\in N$ and all $j\in K$, $x_1 P_{ij} x_2$.

INDEPENDENCE OF IRRELEVANT ALTERNATIVES (I). Let $\{R_i\}_{i\in N}$ and $\{R^*_i\}_{i\in N}$ be any profiles in the domain of F. Suppose $x_1, x_2 \in X$ such that, for all $i\in N$ and all $j\in K$, $x_1 R_{ij} x_2$ if and only if $x_1 R^*_{ij} x_2$. Then $x_1 R x_2$ if and only if $x_1 R^* x_2$.

The multidimensional generalization of Arrow's non-dictatorship condition is less straightforward. In the multidimensional case, there are two issues to be addressed: the issue of *dictatorship of an individual* and the issue of *dominance of a dimension*. Non-dictatorship requires the non-existence of a fixed single individual such that whenever *this individual* strictly prefers option x_1 to option x_2 *in all issue-dimensions*, then the social ordering also ranks option x_1 strictly above option x_2. Non-dominance requires the non-existence of a fixed single dimension such that whenever *all individuals* strictly prefer option x_1 to option x_2 *in that issue-dimension*, then the social ordering also ranks option x_1 strictly above option x_2. As we will see below, however, there are plausible situations in which social choice procedures violating non-dominance are attractive.

NON-DICTATORSHIP (D). There does not exist an individual $i\in N$ (a *dictator* for F) such that, for all profiles $\{R_i\}_{i\in N}$ in the domain of F and all $x_1, x_2 \in X$, $[\forall j\in K\ x_1 P_{ij} x_2]$ implies $x_1 P x_2$.

NON-DOMINANCE (DOM). There does not exist a dimension $j\in K$ (a *dominant dimension* for F) such that, for all profiles $\{R_i\}_{i\in N}$ in the domain of F and all $x_1, x_2 \in X$, $[\forall i\in N\ x_1 P_{ij} x_2]$ implies $x_1 P x_2$.

A double dictatorship is the situation in which the personal preference ordering of a *fixed single individual* in a *fixed single dimension* always determines the social ordering.

NON-DOUBLE-DICTATORSHIP (DD). There does not exist an individual $i \in N$ and a dimension $j \in K$ such that, for all profiles $\{R_i\}_{i \in N}$ in the domain of F and all $x_1, x_2 \in X$, $x_1 P_{ij} x_2$ implies $x_1 P x_2$.

Arrow's original theorem immediately implies the following result:

Theorem 1. There exists no MSWF satisfying (U), (P), (I) and (DD).

This result is no surprise. If we extend Arrow's original framework by asking n individuals to submit k separate personal preference orderings (rather than one), allowing as much diversity as logically possible both within and across individuals, then effectively we are faced with a one-dimensional Arrow problem for a group of nk individuals.

However, the situation that Miller and others have in mind when arguing for solving social choice problems by identifying multiple relevant issue-dimensions is not the one captured by theorem 1 and its underlying condition (U). The (empirical) claim, though not explicitly spelled out by Miller, is the following. Once multiple relevant issue-dimensions have been 'factored out', preferences along each separate issue-dimension are more likely to be single-peaked across individuals than the unstructured, non-singlepeaked, preferences which are the result of collapsing several issue-dimensions into one. Independently, Mueller (1989, pp. 89 - 90) argues that "[g]iven that we have a single-dimensional issue, single-peakedness does not seem to be that strong an assumption. What is implausible is the assumption that the issue space is one dimensional". On this account, introducing several *dimension-specific* orderings, each of which concerns only a single issue-dimension, would make it more likely that, for each separate dimension, the *dimension-specific* orderings are single-peaked across individuals. This situation is captured by the condition of *intradimensional single-peakedness*.

First, define a profile $\{R_i\}_{i \in N}$ of *one-dimensional* personal preference orderings to be *single-peaked* if there exists at least one strict linear ordering Ω of the alternatives in X (a *structuring dimension*) such that, for all $i \in N$ and all $x_1, x_2, x_3 \in X$, *if* ($x_1 \Omega x_2$ and $x_2 \Omega x_3$) or ($x_3 \Omega x_2$ and $x_2 \Omega x_1$) (i.e. "x_2 is between x_1 and x_3 on the structuring dimension defined by Ω"), *then* $x_1 R_i x_2$ implies $x_1 P_i x_3$ ("each individual's preference ordering has only one peak with respect to Ω") (Black, 1948; Arrow, 1951/1963). Let D_S be the set of all single-peaked profiles of one-dimensional personal preference orderings.[4]

[4] For simplicity, I will not address issues of indifference and unconcerned individuals here. However, the present results can be stated in a more general form, by extending D_S along the lines discussed in Sen (1966).

INTRADIMENSIONAL SINGLE-PEAKEDNESS (ISP). The domain of F is the set of all profiles of k-tuples of personal preference orderings, $\{R_i\}_{i\in N}$, such that, for each $j\in K$, $\{R_{ij}\}_{i\in N}\in D_S$.

Given (ISP), the impossibility result of theorem 1 can be circumvented.

Proposition 2. There exist MSWFs satisfying (ISP), (P), (I) and (D).

A MSWF F is a *lexicographic hierarchy of dimensions* if there exist k one-dimensional SWFs, $G_1, G_2, ..., G_k$ (possibly different), defined on the domain D_S and satisfying (P), (I) and (D), and a permutation σ of K such that, for any $\{R_i\}_{i\in N}$ in the domain of F and any $x_1, x_2\in X$,

$$
\begin{array}{lll}
x_1 P x_2 \text{ if and only if} & & \\
 & x_1 P^*_{\sigma(j)} x_2 & \text{for some } j\in K \\
\text{and} & x_1 I^*_{\sigma(h)} x_2 & \text{for all } h<j,
\end{array}
$$

where, for each $j\in K$, $R^*_j = G_j(\{R_{ij}\}_{i\in N})$.

Suitably defined lexicographic hierarchies of dimensions satisfy the conditions of proposition 2, for instance, lexicographic hierarchies with all G_j defined as pairwise majority voting (i.e. for each j, G_j is defined as follows: for any $x_1, x_2\in X$, $x_1R^*_jx_2$ if and only if the number of individuals with $x_1R_{ij}x_2$ is at least as large as the number of individuals with $x_2R_{ij}x_1$). While lexicographic hierarchies of dimensions do not make a fixed single individual dictatorial, they violate condition (DOM) in that they install a fixed dominance hierarchy of dimensions. The overall social ordering is determined, first, exclusively on the basis of the dimension-specific profile $\{R_{ij}\}_{i\in N}$ for the highest ranked dimension; if there are ties, the dimension-specific profile for the second highest ranked dimension acts as a tie-breaker; if there are still ties, the dimension-specific profile for the third highest ranked dimension acts as a tie-breaker, and so on.

Are there any MSWFs satisfying the conditions of proposition 2 and condition (DOM)? The main theorem of this paper shows that the answer to this question is negative, even if we do not insist on condition (D).

Theorem 3. There exists no MSWF satisfying (ISP), (P), (I) and (DOM).

A proof is given in the appendix. Theorem 3 implies that, if the individuals rank different industrial policies separately on the three dimensions of economic growth, employment and ecological sustainability and these *dimension-specific* rankings are single-peaked across individuals, any MSWF defined on the resulting domain and satisfying (P) and (I) will make one of the three dimensions dominant and use the other dimensions at most as tie-breakers. However, in some cases decision-makers might reach agreement on the relative importance of the three dimensions, thereby making the use of a lexicographic hierarchy of dimensions defensible.

3. Concluding Remarks

A full escape-route from the multidimensional Arrow problem via intradimensional single-peakedness is unavailable.[5] Miller's proposal on solving impossibility problems of social choice by identifying multiple relevant issue-dimensions therefore requires some elaboration. Even if we grant the (empirical) claim that 'factoring out' multiple relevant issue-dimensions helps to generate a situation of intradimensional single-peakedness, the problem of aggregating across these dimensions is not easily soluble. We have seen that (lexicographic) hierarchies of dimensions are the only formal solutions to that problem. Sometimes decision-makers can agree not only on what the relevant issue-dimensions are, but also on how to rank them in an order of importance. In such cases, lexicographic hierarchies of dimensions may be attractive solutions to multidimensional social choice problems. Often, however, there is as much, or even more, disagreement about issue-priority than about the actual ranking of options within each issue-dimension. A study by Budge, Robertson and Hearl (1987), for instance, suggests that electoral competition is often concerned more with matters of issue-priority than with the question of what the best option on a given issue-dimension is. If so, no lexicographic hierarchy of dimensions will command the assent of all decision-makers. Sometimes it is possible to subdivide the decision into several dimension-specific sub-decisions, for instance one on economic aspects, one on employment aspects, and one on ecological aspects, and, given intradimensional single-peakedness, separate dimension-specific aggregation raises no Arrow problem. This route in effect 'defines away' the multidimensional Arrow problem, by not aggregating preferences across multiple dimensions. But whenever decisions come in 'packages' and cannot be subdivided into separate dimension-specific subdivisions, this route is unavailable, and we are faced with a

[5] Of course, a stronger condition of *overall single-peakedness*, or a suitably defined condition of *interdimensional single-peakedness* in addition to *intradimensional single-peakedness*, would provide a full escape-route from the multidimensional Arrow problem. However, it is unclear whether such a stronger condition would be as empirically plausible as the condition of intradimensional single-peakedness. Suppose that the motivation for 'factoring out' multiple relevant issue-dimensions is the lack of single-peakedness in the *overall* preferences of the individuals. Then the reason for expecting dimension-specific preferences to be more likely to be single-peaked across individuals than overall preferences is precisely that overall preferences are determined by more than one formal structuring dimension (corresponding to different substantive issue-dimensions).

problem to which there is no straightforward solution (for a discussion of these issues in the context of a deliberative democracy, see Dryzek and List, 2002).

This paper can only hint at some of the implications of the multidimensional Arrow problem.[6] But, as in the famous one-dimensional case, there is one conclusion that should *not* be drawn, namely that meaningful collective decision making is impossible. Rather, the multidimensional Arrow problem should once again challenge us to explore the question of how collective decision making is affected by the constraints highlighted by Arrow's theorem.

Appendix. Proof of Theorem 3

Theorem 3 follows immediately from the following result concerning one-dimensional SWFs (i.e. MSWFs for k=1).

Let (U), (P), (I), (D) be Arrow's one-dimensional conditions of universal domain, the weak Pareto principle, independence of irrelevant alternatives, non-dictatorship, respectively (i.e. the conditions stated in section 3 for k=1). Given any $\{R_i\}_{i\in N}$, define $R := F(\{R_i\}_{i\in N})$.

STRUCTURED DOMAIN ASSUMPTION (SDA). $N = N_1 \cup N_2 \cup ... \cup N_k$ (k>1) such that, for each i, $N_i \neq \emptyset$ and, for each i, j, $i \neq j$ implies $N_i \cap N_j = \emptyset$. The domain of F contains all profiles of personal preference orderings, $\{R_i\}_{i\in N}$, with the property that, for each j, $\{R_i\}_{i\in N_j} \in D_S$.

$M \subseteq N$ is *almost decisive* over some pair of alternatives $x, y \in X$ if and only if, for all $\{R_i\}_{i\in N}$ in the domain of F,

$$[[\forall i \in M\; xP_iy] \;\&\; [\forall i \in N \setminus M\; yP_ix]] \Rightarrow xPy.$$

$M \subseteq N$ is *decisive* over some pair of alternatives $x, y \in X$ if and only if, for all $\{R_i\}_{i\in N}$ in the domain of F,

$$[\forall i \in M\; xP_iy] \Rightarrow xPy.$$

[6] List (2001) shows that, like Arrow's original result, the multidimensional results depend crucially on certain informational assumptions, specifically, assumptions on measurability, interpersonal comparability and interdimensional commensurability of individual preferences (or welfare). While each of interpersonal comparability and interdimensional commensurability alone is not sufficient for avoiding an impossibility result, the conjunction of suitable interpersonal comparability and suitable interdimensional commensurability is sufficient for the existence of aggregation procedures satisfying non-dictatorship and non-dominance as well as other Arrow-type conditions.

Theorem. Suppose F is a SWF satisfying (SDA), (P) and (I). Then there exists a fixed $j \in \{1, 2, ..., k\}$ such that N_j is *decisive* over *all* pairs $x, y \in X$.

Proof. Suppose F satisfies (SDA), (P) and (I).

Step (1). We will show that there exists a fixed $j \in \{1, 2, ..., k\}$ such that N_j is *almost decisive* over *all* pairs $x, y \in X$. Construct a SWF G for a k-individual society as follows. For each logically possible $\{R_j\}_{j\in\{1, 2, ..., k\}}$, define a corresponding $\{R^*_i\}_{i\in N}$ such that, for each $i \in N$, $R^*_i := R_j$ where j is chosen from $\{1, 2, ..., k\}$ such that $i \in N_j$. Note that the mapping from $\{R_j\}_{j\in\{1, 2, ..., k\}}$ to $\{R^*_i\}_{i\in N}$ is an injection. Note further that, for each j, $\{R^*_i\}_{i\in N_j}$ is a profile of identical orderings, and so $\{R^*_i\}_{i\in N}$ is contained in the domain of F (as defined by (SDA)). Now define $G(\{R_j\}_{j\in\{1, 2, ..., k\}}) := F(\{R^*_i\}_{i\in N})$. By construction, G satisfies (U). Since F satisfies (P) and (I), so does G. So G satisfies the conditions of Arrow's (one-dimensional) theorem, and there exists $j \in \{1, 2, ..., k\}$ such that, for all $\{R_j\}_{j\in\{1, 2, ..., k\}}$ and all $x, y \in X$, xP_jy implies xPy, where $R = G(\{R_j\}_{j\in\{1, 2, ..., k\}})$. Now let $\{R^*_i\}_{i\in N}$ be a profile such that $[\forall i \in N_j\ xP^*_iy]$ and $[\forall i \in N \setminus N_j\ yP^*_ix]$. Since F satisfies (I), the rankings of any alternatives other than x and y are irrelevant to the relative ranking of x and y, and we may assume, without loss of generality, that, for each j, $\{R^*_i\}_{i\in N_j}$ is a profile of identical orderings. Consider the profile $\{R_j\}_{j\in\{1, 2, ..., k\}}$ to which $\{R^*_i\}_{i\in N_j}$ corresponds. Since xP_jy and j is a dictator for G, we must have xPy, where $R = G(\{R_j\}_{j\in\{1, 2, ..., k\}}) = F(\{R^*_i\}_{i\in N})$. Hence N_j is *almost decisive* over all pairs $x, y \in X$.

Step (2). Suppose N_j is *almost decisive* over *all* pairs $x, y \in X$. Given any pair $x, y \in X$, we will show that N_j is *decisive* over x, y. Let $\{R_i\}_{i\in N}$ be a profile such that, for all $i \in N_j$, xP_iy. Consider a new profile $\{R^*_i\}_{i\in N}$ with the following properties:

(i) for all $i \in N$, $xR^*_iy \Leftrightarrow xR_iy$ and $yR^*_ix \Leftrightarrow yR_ix$;

(ii) z is a third alternative such that, for all $i \in N_j$, xP^*_iz and zP^*_iy, and, for all $i \in N \setminus N_j$, zP^*_ix and zP^*_iy;

(iii) all other alternatives are ranked relative to x, y, z in such a way that $\{R^*_i\}_{i\in N}$ is single-peaked.

The following table illustrates that a profile $\{R^*_i\}_{i\in N}$ with properties (i), (ii) and (iii) exists.

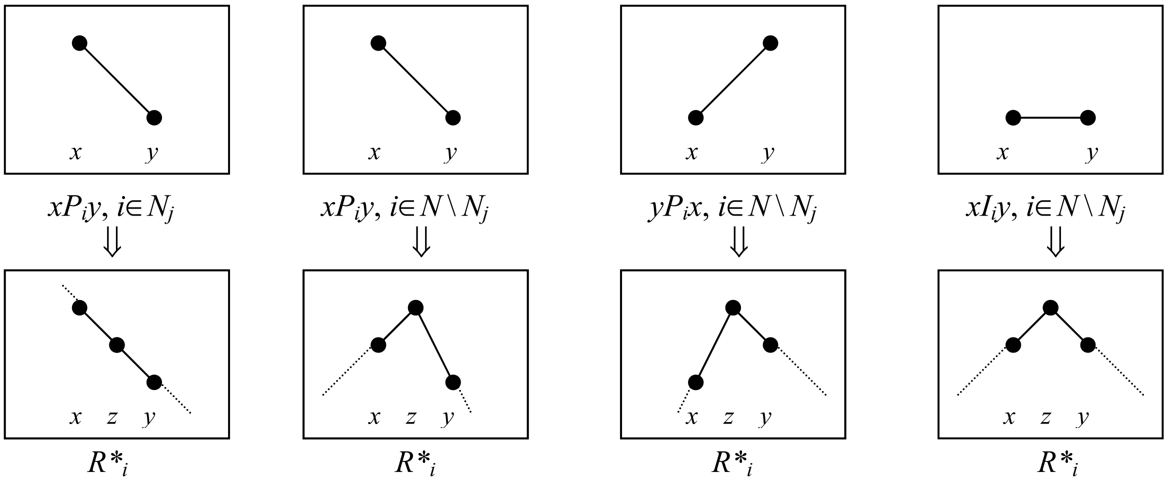

Table 1

Property (iii) implies that, for each j, $\{R^*_i\}_{i \in N_j}$ is single-peaked, so that $\{R^*_i\}_{i \in N}$ is contained in the domain of F (as defined by (SDA)). Let $R = F(\{R_i\}_{i \in N})$ and $R^* = F(\{R^*_i\}_{i \in N})$. Since F satisfies (I), property (i) implies that $xP^*y \Leftrightarrow xPy$. Since N_j is *almost decisive* over all pairs in X, N_j is almost decisive over x, z, and property (ii) implies that xP^*z. Since F satisfies condition (P), property (ii) further implies that zP^*y. By transitivity of P^*, it follows that xP^*y, and thus xPy, as required. **Q.E.D.**

The present proof strategy is related to the proof strategy of a result by Khmelnitskaya and Weymark (2000, theorem 2, hereafter K&W) concerning social welfare orderings for different scales of individual utility measurement in distinct population subgroups. In both proofs, the set of individuals is first partitioned into k subsets. In the present proof, each subset is a set of individuals such that the corresponding subprofile of personal preference orderings is single-peaked. In K&W, each subset is a set of individuals with a common scale of individual utility measurement. The first step of both proofs considers pairwise rankings where preferences (here) or utilities (K&W) over the relevant pair of options are identical within each subset of the partition. A standard Arrow-type theorem is then applied to show that, for the considered special pairwise rankings, one subset of individuals is decisive. In a following step, this decisiveness result for special pairwise rankings is generalized to any pairwise ranking by constructing suitable auxiliary options (here) or auxiliary utility vectors across individuals (K&W) in such a way that (i) the decisiveness result of the first step applies to pairwise rankings involving the constructed auxiliary options or utility vectors and (ii) these pairwise rankings, combined with Pareto dominance considerations, induce the require pairwise rankings in the given more general case.

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