WHY IDEALIZED MODELS IN ECONOMICS HAVE LIMITED USE

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Introduction
This paper divides idealized models into two classes--causal and non-causal--according to whether the idealized model represents causes or not. Although the characterization of a causal idealized model may be incomplete, it is sufficiently well-defined to ensure that idealized models specified using restrictive antecedent clauses are non-causal. The contention of this paper is that whilst such idealized models are commonly used in economics, they are unsatisfactory; they do not predict reliably. Moreover, notions of causation that cut across such models are required to suggest when the idealized model will provide a sufficiently good approximation and when it will not.

Doubt is cast on the ability of simple causal idealized models to capture sufficiently the causal complexity of economics in such a way as to provide useful predictions.

The causalist philosophical standpoint of this paper is close to that of Nancy Cartwright.


**Causal and non-causal idealized models**

For the purposes of this paper a “causal idealized model” is an idealized model that rests on simple idealized causes. The idealized models represent causes, or the effects of causes, that operate in reality.

The inverse square law of gravitational attraction, whose idealized model consists of forces operating on point masses, is a causal idealized model by virtue of the fact that the forces of the model are causes. The standard idealized models of the operation of a spring, a pendulum or an ideal gas are causal by virtue of the fact that there are immediately identifiable causes that underpin these models. Cartwright shows, in *Nature’s Capacities and their Measurement*, how certain idealized models of the operation of lasers were shown to be causal (Cartwright, 1989, pp. 41-55).

Causal idealized models can on occasion tell us what happens in real, non-ideal situations when the causes identified in the idealized model are modelled sufficiently accurately and operate in a sufficiently undisturbed way in the real situation under consideration. It is the presence and operation of the identified causes in reality that, under the right circumstances, allow the idealized model to approximate the behaviour of that reality. Empirical evidence may allow us to calibrate the degree of approximation.

A “non-causal idealized model” is an idealized model that is not causal--it does not attempt to capture causes or the effect of causes that operate in reality. It is not apparent how a non-causal
idealized model approximates the behaviour of real, non-ideal situations.

On occasion a model may regularly and accurately predict the behaviour of a real system, although the causes that underpin such behaviour have not been identified. Such was the case with the early models of lasers--Cartwright’s account tells how much importance the early laser scientists attached to finding the causes. Such an idealized model may be causal even though the causes are unknown.

**Non-causal idealized models in economics**

Many idealized models used in economics are non-causal. Such idealized models are generally identified by restrictive antecedent clauses which are true of nothing actual.

An example of such a restrictive antecedent clause occurs in the assumption of perfect knowledge that is used most commonly in relation to consumers or producers to underpin the achievement of certain maximizing behaviour. The assumption is of course false of all actual consumers or producers; it is indeed true of nothing actual. It partially defines an idealized model--a model in which behaviour is more simple and more predictable than in reality.

Friedman’s 1953 essay still dominates the methodology of economics despite its apparent shortcomings (Friedman, 1953). Nagel suggests that the important sense in which Friedman defends “unrealistic assumptions” is that they are restrictive antecedent clauses (Nagel, 1963, pp. 217-18). Nagel’s analysis highlights an ambiguity underlying Friedman’s paper concerning the role of assumptions in idealized models:
1. The assumptions are merely a device for stating the conclusion so that it is only the conclusion which is descriptive of reality. In this case the assumptions and the idealized model are of little interest because they cannot support in any way the validity of the conclusion which must stand or fall on accordance with available empirical evidence.

   Nagel’s example of such an assumption is “firms behave as if they were seeking rationally to maximize returns” (ibid., p. 215). The assumption concerns the behaviour of firms but does not require that they do actually rationally seek to maximize returns—merely that they behave as if this were the case.

2. The assumptions are understood as dealing with “pure cases” and the model is then descriptive of the pure case. The assumptions are then restrictive antecedent clauses. Nagel comments: “laws of nature formulated with reference to pure cases are not useless. On the contrary, a law so formulated states how phenomena are related when they are unaffected by numerous factors whose influence may never be completely eliminable but whose effect generally vary in magnitude with differences in attendant circumstances under which the phenomena recur. Accordingly, discrepancies between what is asserted for pure cases and what actually happens can be attributed to factors not mentioned in the law” (ibid., pp. 215-16).

   Nagel labels idealized models “pure cases,” and suggests that the restrictive antecedent clauses that define them succeed by eliminating extraneous causes not dealt with by the law (ibid., p. 216).
The simple idealized world is tractable to analysis. The antecedent clauses define idealized models that are clearly non-causal. The key question is how behaviour in such an idealized model relates to the behaviour of systems in the real world. Behaviour in the idealized model is derived using deductive analysis that appeals to the simplifying assumptions--the restrictive antecedent clauses. In practice, an implicit assumption is made but never stated in economics that the derived behaviour pictured in the idealized model does approximate the behaviour of systems in reality--this is here termed the “Approximate Inference Assumption” and is discussed below.

Consider, for instance, the idealized model of perfect competition. The assumptions are normally stated broadly as follows (McCormick et al., 1983, p. 348):

1. Producers aim to maximize their profits and consumers are interested in maximizing their utility.
2. There are a large number of actual and potential buyers and sellers.
3. All actual and potential buyers and sellers have perfect knowledge of all existing opportunities to buy and sell.
4. Although tastes differ, buyers are generally indifferent among all units of the commodity offered for sale.
5. Factors of production are perfectly mobile.
6. Productive processes are perfectly divisible; that is, constant returns to scale prevail.
7. Only pure private goods are bought and sold.
Conclusions are typically derived by economists concerning behaviour within such conditions over both the short run and the long run. The structure is causally complex. The short run/long run distinctions are not easy to explicate, so that it is far from clear, even in principle, that the behaviour pictured in the idealized model could be produced in reality. How close is the behaviour pictured in the model to the behaviour of a given real economy? It is not clear how a user of this non-causal idealized model could begin to answer this question. The idealized model of perfect competition is a causally complex structure which, if the economists are right, allows certain functional relationships (in (1913) sense) that are the effects of the behaviour of firms, to be exhibited within the ideal circumstances prescribed by the assumptions. It is the contention of this paper that such an idealized model has very limited use in predicting the behaviour of real economies.

**The “Approximate Inference Assumption”**

The Approximate Inference Assumption (AIA) states that if a situation is “close” in some sense to the one captured by an idealized model then it may be inferred that the behaviour in that situation is approximately that pictured in the idealized model. For a causal idealized model, the presence of the causes modelled in the real situation provides some basis for supposing that under the right circumstances such an approximation may hold. In the case of non-causal idealized models, the AIA is unsound.
Chaos theory leads us to expect that in many situations the AIA is incorrect. Small changes to the boundary conditions of a model can lead to major changes in the behaviour of the system. There are thus serious theoretic problems concerning the applicability of the AIA.

Nevertheless Gibbard and Varian comment:

It is almost always preposterous to suppose that the assumptions of the applied model are exactly true.... The prevailing view is that when an investigator applies a model to a situation he hypothesizes that the assumptions of the applied model are close enough to the truth for his purposes. (1978, pp. 668-9)

On this view of the behaviour of economists, which seems to me to be correct, economists do indeed rely upon the AIA even in the case of non-causal idealized models.

**Mathematical economics**

Mathematical economics requires highly idealized models of reality. Within the mathematical model, results are derived and these are deemed to be descriptive of reality. In order to apply the results of the mathematical model to reality, the AIA is usually required.

A particularly good example is Arrow and Debreu’s proof of the existence of a general equilibrium within an economy (Debreu, 1959). The proof rests on modelling the economy as a multi-dimensional vector space in which certain sets of points are assumed to be compact—a quality which only in the loosest sense could be deemed to be descriptive of reality. This idealization of an economy
is non-causal. Once the nature of the relationship between the mathematical model and reality is brought into question, and the problems with the AIA accepted, it becomes clear that the proof of the existence of an equilibrium within an economy is simply an artifact of the mathematical model, and does not demonstrate the existence of a corresponding counterpart in reality.

**The Black and Scholes argument**
The Black and Scholes (hereafter ‘B&S’) paper on option pricing is a first-class example in its economic field--mathematical in style and almost universally accepted (Black and Scholes, 1973). For this reason it is also a good example of economics that, for its practical application and in order to claim empirical content, rests wholly upon the AIA to make inferences from a non-causal idealized model to reality. This paper uses the B&S argument to illustrate the use of the AIA and to give an example of its failure.

B&S make the following assumptions:

(a) The short term interest rate is known and is constant through time.
(b) The stock price follows a random walk in continuous time with a variance rate proportional to the square root of the stock price. Thus the distribution of the stock price at the end of any finite interval is lognormal. The variance rate of return on the stock is constant.
(c) The stock pays no dividends or other distributions.
(d) The option is “European,” that is, it can only be exercised at maturity.

(e) There are no transaction costs in buying or selling the stock or the option.

(f) It is possible to borrow any fraction of the price of a security to buy it or to hold it at the short term interest rate.

(g) There are no penalties for short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

(a), (e), (f) and (g) are false. (c) is generally false. (d) is false if the option is American. In the case of (b) a far weaker assumption, namely that the price of the stock is a continuous function of time, is false. These assumptions function as restrictive antecedent clauses that define a non-causal idealized model.

Using these assumptions B&S derive a solution to the option pricing problem. By the end of their paper it is clear that the authors believe the solution in the idealized model situation is applicable to real options.

It is rather surprising and worthy of note that many followers of B&S have not only employed, implicitly, the AIA but appear to have used a stronger assumption--a “precise inference assumption”--to the effect that the idealized solution is precise in real situations. Cox and Rubenstein for instance write a section of their leading
textbook on option pricing under the title “An Exact Option Pricing Formula” (1985, pp. 165-252).

B&S themselves conclude that:

the expected return on the stock does not appear in the equation. The option value as a function of the stock price is independent of the expected return on the stock. (1973, p. 644)

The failure of the expected return on the stock to appear as a parameter of the value in the option is a direct result of the powerful assumptions that B&S employ for defining their idealized model. They have not succeeded in showing that in real situations the expected return on the stock is not a parameter in the value of an option. This logically incorrect conclusion demonstrates their use of the AIA. (This is the equivalent of the conclusion that walking uphill is as quick as walking downhill in the Narrow Island analogy below.)

Many economists have sought to demonstrate the relevance of the B&S solution to real options by showing that similar results hold under different (usually claimed to be weaker) assumptions than those used for the B&S idealized model. Beenstock’s attempt in “The Robustness of the Black-Scholes Option Pricing Model” (1982) is typical. Unfortunately, these “relaxations” of the assumptions merely tell us what would happen in ideal circumstances and do little to bridge the gap to the real world. Beenstock’s first conclusion is that “[o]ption prices are sensitive to the stochastic processes that determine underlying stock prices.... Relaxation of these assumptions can produce large percentage changes in option prices”
The B&S solution is not necessarily a good approximation even in the carefully controlled idealized situations where all except one of their assumptions are held constant. The AIA simply does not work.

A more tangible practical example of the failure of the AIA arises when the stock price movement is discontinuous. Discontinuities arise in a wide range of practical situations, but perhaps most markedly in relation to bids. A situation arose recently where one morning a major stake in a top 100 company changed hands. The purchaser made known its intention to make a statement concerning its intentions at 1pm. Expert analysts considered there to be a significant possibility of a bid—probabilities in the 10% to 50% range were typical assessments. In the event of a bid, the stock price would rise by some 20% or more. In the event of no bid, the stock price would drop some 10%. For a slightly in-the-money, short-dated call option, cash receipt at expiry would almost certainly be zero if no bid were received, and close to 20%-odd of the stock price per underlying share if a bid were forthcoming. Under any practical method of valuation, the value of the option must be close to the probability of a bid times 20%-odd of the stock price. The B&S solution simply does not work in this situation. (This breakdown of the B&S solution is the equivalent of the breakdown of Professor White’s solution for villages on opposite coasts at broader parts of Narrow Island (see below). The assumption that stock price moves are continuous is a good one most of the time, but is totally wrong here, just as Professor White’s assumption that the island is a line breaks down for the corresponding case.)
Practicing options experts recognize the shortcomings of B&S and will commonly adjust their results for the non-lognormality of the stock price distribution since by an \textit{ad hoc }increase to the stock price volatility estimate, empirical evidence shows that most real stock price distributions have higher kurtosis than the lognormal distribution. But B&S provides no basis for such adjustments. The key question is “If a real situation is close to the B&S assumptions, how close is it to the B&S conclusion?” The major complaint about the B&S derivation is that it does not allow an answer to this question. Their precise solution to the idealized case tells us nothing about real situations. In the case of the imminent bid announcement, B&S breaks down entirely. What characteristics of the real situation will ensure the B&S solution “works” sufficiently well? The B&S approach provides no answer.

\textbf{The Narrow Island analogy}

In the Southern Seas, some way to the east and south of Zanzibar is a thin strip of land that modern visitors call “Narrow Island.” An indigenous people inhabit the island whose customs derive from beyond the mists of time. At the northern end of the island is a hill cut at its midpoint by a high cliff which crashes vertically into the sea. On the headland above the cliff, which to local people is a sacred site, a shrine has been erected. It is the custom of the island that all able-bodied adults walk to the shrine to pay their respects every seventh day.

Despite its primitive condition the island possesses the secret of accurate time-keeping using instruments that visitors recognize as
simple clocks. In addition to a traditional name, each village has a “numeric name,” which is the length of time it takes to walk to the shrine, pay due respects and return to the village. The island being of a fair length, the numeric names stretch into the thousands.

Many years ago one of the first visitors to the island from Europe was a traveller the islanders called Professor White. Modern opinion has it that White was an economist. What is known is that at the time of his visit the local people were wrestling with the problem of establishing how long it took to walk between villages on the island.

The argument of Professor White is recorded as follows:

The problem as it stands is a little intractable. Let us make some assumptions. Suppose that the island is a straight line. Suppose the speed of walking is uniform. Then the time taken to walk between two villages is half the absolute difference between their numeric names.

The islanders were delighted with this solution and more delighted still when they found how well it worked in practice.

It was noted with considerable interest that the time taken to walk between two villages is independent of the height difference between them. As the island is quite hilly this astonishing result was considered an important discovery.

Stories tell that so great was the islanders’ enthusiasm for their new solution that they attempted to share it with some of their neighboring islands. To this day there is no confirmation of the dreadful fate which is said to have befallen an envoy to Broad Island,
inhabited by an altogether more fearsome people, who were apparently disappointed with the Narrow’s solution.

Although the Narrow Islanders have used their solution happily for many years, more recently doubts have begun to emerge. In some parts of Narrow, where the island is slightly broader, reports suggest that the time between villages on opposite coasts is greater than Professor White’s solution would suggest. Others who live near the hills quite frankly doubt that the time taken to walk from the bottom of the hill to the top is the same as the time taken to walk from the top to the bottom.

It is a tradition amongst the islanders that land is passed to sons upon their marriage. Upon the marriage of a grandson the grandfather moves to a sacred village near the middle of the island which is situated in the lee of a large stone known as the McCarthy. The McCarthy Stoners report that Professor White’s solution suggests it is quicker to walk to villages further south than it is to walk to villages that appear to be their immediate neighbors.

The island’s Establishment continues to point out that Professor White’s solution gives “the time to walk between two villages,” so that the time taken on any particular journey—resting as it does on a particular person in particular climactic conditions—is hardly adequate contrary evidence. Those who would talk of average times are considered to be wide of the mark. Whilst few islanders seriously doubt the veracity of the theory, it is reported that many have taken to making *ad hoc* adjustments to the official solution, a practice that is difficult to condone.
Last year a visitor to the island was impolite enough to question the logic of Professor White’s solution itself. His main points were as follows:

1. The “time taken to walk between two villages” is not well defined. A workable definition might be “the average time taken to walk between the two villages in daylight hours, by able-bodied men between the ages of eighteen and forty, traveling at a comfortable pace, without rest stops, during normal climatic conditions, e.g., excluding unduly wet or windy weather.”
2. Both of the assumptions are incorrect. The conclusion is not always approximately correct.
3. A more robust solution which should provide answers that are approximately correct for all pairs of villages is to regress the average times as proposed in (1) against the two principal causal factors underlying journey times—horizontal distance and height of climb, positive or negative. If a linear equation does not provide a sufficiently good fit, a higher order polynomial might be used.

To the Narrow Islanders such talk was incomprehensible.

**A robust solution to the option pricing problem**

The Narrow Island analogy shows how the use of a non-causal idealized model breaks down. It may represent a good approximation most of the time, but on occasion it is no
approximation at all. Moreover, the model itself provides no clues as to when it is, and when it is not, applicable. We have this knowledge (if at all) from a consideration of broader factors; it would seem these must include causal factors.

A causal idealized model always provides a good approximation whenever it captures enough of the causes sufficiently accurately, and the causes modelled operate in reality in a sufficiently undisturbed way. Often our knowledge of the situation will allow us to judge whether this is likely to be the case.

The visitor’s solution to the Narrow Island problem is correct; it is a robust causal solution. A similar solution exists for the option pricing problem.

In the absence of any generally accepted definition of investment value, we may choose as a robust measure the expected value of receipts using a discount rate appropriate to the net liabilities.

The causal parameters of value are those which affect the stock price at expiry. These may be categorized as known and unknown causes.

**Known causes:**

1. Expected growth of stock price. Unless the dividend yield unusually high, growth in stock price is part of the anticipated investment return.
2. Bifurcating event. To the extent that the market is efficient (all causes known to any market participant are reflected in the stock price, say), the probability density function (pdf)
that describes the stock price at expiry should be evenly distributed about the central expectation. A bifurcating event induces a double-humped pdf.

3. Market ineffectiveness. If we have insight into a stock which differs from that of other market participants then we may on occasion anticipate a movement in the market price of the stock during the period to expiry.

Unknown causes:
John Stuart Mill referred to “disturbing causes,” causes that do not appear explicitly within the model (1967, pp. 330-31). Such unknown causes may be allowed for by choosing a pdf that allows for a range of outcomes. It is usual to use a lognormal distribution for stock prices, but empirical evidence suggests that the chosen distribution should have higher kurtosis than this.

A pdf may be chosen that makes allowance for all the causes, both known and unknown, and the discounted expected value calculated to provide a robust approximate measure of value. Sensitivity to changing assumptions may be checked.

Causal idealized models in economics
In the physical sciences real working systems can sometimes be constructed that arrange for causes to operate in just the right way to sustain the prescribed behaviour of the system. Pendula, lasers and car engines are examples. Whilst the systems are working, aspects of
their behaviour may be described by functional relationships (in Russell’s (1913) sense) between variables. The design of the system ensures consistency, repeatability and reversibility.

Real economies are not so neat; they have a complex causal structure, and are more akin to machines with many handles, each of which is turned continually and independently. The state of the machine is continually changing and evolving as the handles are turned. The effect of turning a single handle is not in general consistent, repeatable or reversible. Simple functional relationships between variables are at best approximate. Simple mathematical models do not capture such a complicated process, but may on occasion approximate part of the process for some duration.

Economists do on occasion use causal idealized models. Examples are equilibrium models where causes are identified that tend to push the economic system back towards equilibrium. One such model equates money supply with money demand. Economic textbooks argue that behavioural patterns create causes that ensure equality holds “in equilibrium.” Such causal idealized models leave out the vast bulk of causes and simplify to an extreme extent. It is far from clear that such models can demonstrate any predictive success--their usefulness is at best very limited.

**Conclusion**
Economists’ use of non-causal idealized models is problematic. No account can be provided in terms of the idealized model as to when the model will provide a sufficient approximation to reality. Such models do not predict reliably.
Moreover, the causal complexity of economies suggests that they may be intractable to simple models, so that causal idealized models may also have very limited predictive success.

References


