Putting the Price in Asset Pricing

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ABSTRACT

We propose a novel way to estimate a portfolio's *abnormal price*, the percentage gap between price and the present value of dividends computed with a chosen asset pricing model. Our method, based on a novel identity, resembles the time-series estimator of abnormal returns, avoids the issues in alternative approaches, and clarifies the role of risk and mispricing in long-horizon returns. We apply our techniques to study the cross-section of price levels relative to the CAPM, finding that a single characteristic dubbed *adjusted value* provides a parsimonious model of CAPM-implied abnormal price.

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How well does an asset pricing model explain the observed *price levels* of stocks? And which stock characteristics signal model-specific *abnormal price*, the deviation of price from the present value of future dividends?[1] These are central questions in finance, since stock price levels can drive the financing and investment decisions of firms as well as the portfolio decisions of long-term buy-and-hold investors. Understanding stock price levels can also reveal whether abnormal returns on a given expected return anomaly are earned on price convergence or divergence.

The techniques with which to study asset price levels at our disposal are extremely limited. Since discount rates vary over time, abnormal price cannot simply be inferred from short-horizon abnormal returns (alphas), the literature's traditional focus. Cohen et al. (2009) proposed an approximate estimator of abnormal price that nonetheless relies on strong assumptions about the data. An estimator based on directly discounting subsequent dividends and a terminal cash flow, as explored in van Binsbergen et al. (2023), can be significantly biased and suffers from a serious overlapping observations issue.

Our contribution is to develop a new estimator of abnormal price. The estimator resembles the time-series alpha estimator and avoids the issues in alternative approaches. Applying our techniques, we document empirical facts about the cross-section of stock price levels. (i) The CAPM explains some, but not all, of the cross-sectional variation in stock price levels. (ii) Net issuance, investment, and beta predict significant CAPM-implied abnormal price. (iii) The classic momentum strategy bets on overpriced stocks. (iv) A single characteristic that we dub adjusted value provides a parsimonious model of CAPM abnormal price.

We define a portfolio's average formation-period abnormal price—which we also call

[1] Abnormal price could signal either a misspecification of the model of risk or an actual distortion in asset price levels. That is, like abnormal returns, abnormal price is subject to the joint hypothesis problem emphasized in Fama (1970).

δ ("delta") or mispricing—as follows:

$$\delta \;\equiv\; E\left[\frac{P_t - V_t}{P_t}\right] \quad \text{with} \quad V_t = E_t\left[\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} D_{t+j}\right], \tag{1}$$

where t is the portfolio-formation period, P_t is the portfolio's market price at t, and V_t is the present value of post-formation dividends $\{D_{t+j}\}_{j=1}^{\infty}$ with respect to the candidate cumulative stochastic discount factor (SDF) $\widetilde{M}_{t,t+j}$ implied by a chosen asset pricing model of risk. Abnormal price (δ) is a price-level analogue of short-horizon abnormal return (α) in a return analysis and is specific to the asset pricing model used to specify $\widetilde{M}$, just like α.[2]

We estimate δ via a novel exact identity expressing today's abnormal price using post-formation buy-and-hold excess returns. For intuition, consider a portfolio that is currently overpriced relative to an asset pricing model. If that overpricing undergoes a subsequent "correction," the capital gain component of future returns must be low on a risk-adjusted basis. If, on the other hand, the price remains elevated—which our identity also allows for—the dividend yield component of future returns must be low on a risk-adjusted basis. In both cases, we expect initial overpricing to be reflected in lower subsequent risk-adjusted returns, an idea our identity formalizes.

Applying our novel identity, we derive a calendar-time estimator of δ, denoted $\widehat{\delta}$, based on the portfolio's post-formation buy-and-hold excess returns:

$$\widehat{\delta} = \frac{1}{T}\sum_{t=1}^{T} \widetilde{\delta}_t \quad \text{with} \quad \widetilde{\delta}_t = -\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \tfrac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}, \tag{2}$$

where t here indexes the month in which returns occur (i.e., calendar time) and the $(t-j)$ argument in the subscript indicates that the particular cumulative capital gain

[2] This perspective on $\widetilde{M}$ is similar to that of Hansen and Jagannathan (1991, 1997): δ_t measures the abnormal component of P_t relative to a potentially misspecified candidate model of risk, $\widetilde{M}$.

($\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$) or one-month excess return ($R^e_{(t-j),t}$) is earned on buying and holding a portfolio formed at time $t-j$. Using $J = 15$ years (180 months) turns equation (2) into an estimator of δ, whereas using $J = 1$ month reduces it to the conventional alpha estimator, $\widehat{\alpha} = \frac{1}{T}\sum_{t=1}^{T} \widetilde{M}_t R^e_t$.[3]

Our abnormal price estimator in equation (2) has three advantages relative to estimators in the literature. First, estimating $\widehat{\delta}$ with excess (rather than gross) returns prevents measurement errors in the candidate $\widetilde{M}$ from materially biasing $\widehat{\delta}$, since it forces $\widetilde{M}$ in equation (2) to act mainly as a risk adjustment rather than a time discount. This benefit is similar to the way using excess returns in the expected return framework allows researchers to focus on risk adjustment rather than time discount (p.9 of Cochrane (2009)). Second, equation (2) avoids using overlapping observations and minimizes serial correlations, since each $\widetilde{\delta}_t$ combines the contemporaneous time-t returns on portfolios formed in prior periods ($t-1$, $t-2$, $t-3$, ...). Third, our estimator implies a natural measure of long-horizon return that can be decomposed into "risk" and "abnormal price" (δ) components, facilitating the parallel between the expected return and price frameworks. In contrast, the aforementioned dividend-based approach of van Binsbergen et al. (2023) and others leads to a bias, has statistical inference that is challenging due to its reliance on monthly discounted sums of future dividends that generate overlapping 15-year windows, and does not imply a natural measure of long-horizon risk and return.

What is the statistical power of our price-level test? To reject the null of zero abnormal price, δ must be roughly 10 times larger in absolute value than the annualized α (abnormal return) required for significance in a traditional return test, consistent with a time-series delta observation $\widetilde{\delta}_t$ (equation (2)) being a discounted cross-sectional sum of 15 years of post-formation returns. For example, if return volatility were such that an annualized α of 2 to 4 percent achieves significance, we need a δ of roughly 20 to 40 percent to find

[3] This correspondence holds after a sign flip and an interest rate adjustment $1/\overline{\widetilde{M}_t}$.

significance.[4]

Applying equation (2), we study cross-sectional variation in stock prices with respect to the CAPM, providing a foundation for multifactor refinements in subsequent research. Following Korteweg and Nagel (2016), we specify a loglinear candidate SDF and choose the SDF parameters to make the market portfolio's in-sample abnormal price (δ) and abnormal return (α) zero, just as the time-series *return* regression restricts a model's factor portfolios to have zero in-sample α's. Our method does not assume the portfolio's factor betas to be constant and instead allows these betas—computed implicitly as part of estimation—to vary over different post-formation months. Based on Monte Carlo analysis, we recommend GMM standard errors with a Newey-West bandwidth of 24 months.

Our initial analysis focuses on two signals recently studied in the price-level context: book-to-market (B/M) and *quality* which, according to Golubov and Konstantinidi (2019) and Asness et al. (2019) respectively, predict α precisely because they proxy for δ. In isolation, however, we find that neither is a statistically significant signal of CAPM δ (see Cohen et al. (2009) for similar evidence on B/M). A quintile sort on *quality* generates an estimated δ variation of just 11.4 percentage points, whereas B/M generates a larger but statistically insignificant spread of 27.2 percentage points.

Next, we combine information about price, profitability, and CAPM risk to develop a simple novel characteristic signaling abnormal price.[5] This signal—dubbed *adjusted value*—generates variation in δ that is economically large at 51.9 percentage points and statistically significant. Hence, adjusted value generates the sort of price variation that should attract long-term buy-and-hold CAPM investors and challenge researchers devel-

[4] Interestingly, this finding provides conceptual and statistical grounds supporting Black (1986)'s conjecture that markets are efficient if prices are within a factor of two of intrinsic value.

[5] Our new signal draws from the ideas developed in Frankel and Lee (1998), Piotroski (2000), Vuolteenaho (2002), Piotroski and So (2012), Novy-Marx (2013), Asness et al. (2019), and several others. The primary advantage of our proposed measure is its simplicity.

oping asset pricing models of price levels.

Turning to portfolios sorted on seven other characteristics (net issuance, investment, accruals, beta, size, momentum, and profitability), we document four main findings. First, net issuance is a robust signal of abnormal price relative to the CAPM, consistent with firm managers timing the stock market based on the perceived mispricing of their stocks relative to the CAPM. Second, investment and beta also generate price-level variation not explained by the CAPM. Third, on average, momentum bets on overpriced stocks despite their short-term CAPM alpha being positive. Finally, adjusted value subsumes the ability of net issuance and investment to generate significant δ variation, indicating that adjusted value is a parsimonious signal of CAPM abnormal price.

Relation to the literature. This paper applies a novel identity linking abnormal price to subsequent returns to develop an estimator of abnormal price resembling the time-series regression for abnormal returns. Our novel identity mapping model-specific abnormal price to subsequent returns allows one to estimate a portfolio's unconditional abnormal price without a structural assumption on the evolution of abnormal returns, clearly departing from the existing identities in Cohen et al. (2009) or van Binsbergen and Opp (2019). Our identity achieves this benefit within a general SDF framework by putting market price in the denominator of our definition of mispricing, $\delta_t = 1 - V_t/P_t$, which allows subsequent returns—which are inversely related to initial price—to appear in the numerator of the identity in an additively separable manner. Furthermore, unlike the cumulative abnormal return (CAR) or buy-and-hold abnormal return (BHAR) constructs, our delta measure has an exact interpretation as the price deviation from the present value of cash flows, or, equivalently, abnormal price from the perspective of buy-and-hold investors.

On the empirical front, Cohen et al. (2009) study the cross-section of price levels, focusing on B/M-, size-, and beta-sorted portfolios, and apply the calendar-time reformulation to price-level analysis to avoid the overlapping observations issue. Cho (2020) documents that institutional trading of return anomalies can generate abnormal price, as trading by those intermediaries turns alphas into higher betas with intermediary asset pricing factors. Belo et al. (2013) use the relation between Tobin's q and investment under constant returns to scale to study the cross-section of price levels. Belo et al. (2022) introduce labor and heterogeneous capital inputs to Belo et al. (2013)'s framework in order to structurally decompose the sources of firm value.

Chernov et al. (2022) discipline popular linear factor models by requiring them to price their own factors at multiple horizons. Keloharju et al. (2021) document that discount rates on stocks tend to converge over time. van Binsbergen et al. (2023) estimate mispricing for 57 anomalies to classify them as either convergence or divergence bets and then correlate those estimates with investment as done in Polk and Sapienza (2009). They estimate δ based on post-formation dividends and a terminal value, which, as mentioned above, suffers from a potentially large bias from misspecifying the yield curve component of the candidate SDF and is exposed to serious autocorrelation issues. Chen and Kaniel (2021) develop a new methodology to study long-horizon expected returns.

Organization of the paper. After explaining the drawbacks of existing approaches (Section I), we develop a novel identity (Section II) and a new estimator of abnormal price (Section III). We then present data and our empirical results on B/M, *quality*, and *adjusted value* (Section IV), extend our analysis to other characteristic sorts (Section V), and conclude (Section VI).

I. Section

This section discusses the drawbacks of existing techniques that our novel estimator addresses. We begin by specifying the asset pricing environment.

A. Asset pricing environment

Consider a portfolio formed at time t with post-formation dividends $\{D_{t+j}\}_{j=1}^{\infty}$ and a *candidate* stochastic discount factor (SDF) $\left\{\widetilde{M}_{t+j}\right\}_{j=1}^{\infty}$ that may or may not be the true SDF. We want to compare the portfolio's market price P_t to the present value of post-formation dividends discounted using $\widetilde{M}$, which we denote by V_t:

$$V_t = \sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j} D_{t+j}\right], \tag{3}$$

where $\widetilde{M}_{t,t+j} = \Pi_{s=1}^{j} \widetilde{M}_{t+s-1,t+s}$ is the cumulative candidate SDF. There is a base asset whose return, denoted R_b, satisfies the fundamental asset pricing equation with respect to $\widetilde{M}$ in all periods:

$$E_{t+j-1}\left[\widetilde{M}_{t+j}\left(1 + R_{b,t+j}\right)\right] = 1 \quad \forall j. \tag{4}$$

In our particular CAPM implementation, a natural choice for the base asset b is the market portfolio itself.[6] Hence, we compute excess returns with respect to returns on the market portfolio rather than the Treasury bill.

We define (conditional) *abnormal price*, denoted δ_t, as the percentage deviation of price from present value:

$$\delta_t = \frac{P_t - V_t}{P_t}. \tag{5}$$

Hence, $\delta_t > 0$ if the portfolio is overpriced, and $\delta_t < 0$ if it is underpriced. The range of values δ_t can take is $(-\infty, 1]$, the opposite of the range of abnormal returns, $[-1, \infty)$. Define *log abnormal price* as

$$\delta_t^{log} = \log(P_t) - \log(V_t). \tag{6}$$

[6]For a multifactor implementation of our estimator, future research may need to think carefully about the most appropriate base asset, given that Chernov et al. (2022) show that popular unconditional multifactor models cannot simultaneously price the short- and long-horizon returns on their own factors.

We use $\delta \equiv E[\delta_t]$ and $\delta^{log} \equiv E[\delta_t^{log}]$ to denote the unconditional mean of abnormal price and log abnormal price, respectively.

B. *Cohen, Polk, and Vuolteenaho (2009) (CPV)*

Using the decomposition of Campbell and Shiller (1988),

$$\delta_t^{log} \approx -\sum_{j=1}^{\infty} \rho^{j-1} E_t[r_{t+j}] - E_t[r_{V,t+j}], \tag{7}$$

where $r_t \equiv \log(P_t + D_t) - \log(P_{t-1})$ and $r_{V,t} \equiv \log(V_t + D_t) - \log(V_{t-1})$ denote log returns on price and value, respectively, and $\rho < 1$ is a parameter. Equation (7) and joint lognormality implies that long-horizon returns are related to mean log abnormal price as follows:

$$E\left[\sum_{j=1}^{\infty} \rho^{j-1} R_{t+j}\right] \approx E\left[\sum_{j=1}^{\infty} \rho^{j-1} R_{b,t+j}\right] + E\left[\sum_{j=1}^{\infty} \rho^{j-1} Cov_{t+j-1}(r^e_{V,t+j}, -\widetilde{M}_{t+j})\right] + \frac{1}{2} E\left[\sum_{j=1}^{\infty} \rho^{j-1} \{Var_{t+j-1}(r_{t+j}) - Var_{t+j-1}(r_{V,t+j})\}\right] - \delta^{log}. \tag{8}$$

We provide a detailed derivation of this equation in the Internet Appendix. Note that CPV estimate δ^{log} using a closely related equation (their Equation 9) in the cross-section of portfolios:

$$E\left[\sum_{j=1}^{J} \rho^{j-1} R_{k,t+j}\right] = \lambda_0 + \lambda_1\, \beta_k^{CF} + u_k, \tag{9}$$

where k indexes a portfolio, β_k^{CF} is measured by regressing the portfolio's long-horizon cash flows on that of the market, with the horizon capped at J.[7]

[7] CPV highlight that tests of the CAPM may be distorted when there is market-wide mispricing. Their use of a ROE CAPM, as motivated by the Vuolteenaho (2002) decomposition, nicely avoids this concern. Of course, we can similarly use a ROE-based SDF in our return-based identity approach. However, mispricing in *firm-level* returns does not hinder us when using the distorted covariance between returns and the candidate SDF to estimate δ based on our identity. The Internet Appendix provides more details

In addition to the potentially large measurement errors when estimating β_k^{CF}, justifying equation (9) under the null where $r_v = r$ requires strong intertemporal restrictions that guarantee

$$E\left[\sum_{j=1}^{\infty}\rho^{j-1}Cov_{t+j-1}(r^e_{t+j}, r^{mkt}_{t+j})\right] = Cov\left(\sum_{j=1}^{\infty}\rho^{j-1}r^e_{t+j}, \sum_{j=1}^{\infty}\rho^{j-1}r^{mkt}_{t+j}\right), \tag{10}$$

in which case $\lambda_1 = b_1 Var\left(\rho^{j-1}r^{mkt}_{t+j}\right)$ if the candidate SDF is given by $\widetilde{m}_t = b_0 - b_1 r^{mkt}_t$ for the log market return r^{mkt}. The simplest way to guarantee equation (10) is to assume that returns are independently and identically distributed (i.i.d.). However, assuming that returns are i.i.d. is not only inconsistent with the properties of return data but also obviates the need to study differences between abnormal price and short-horizon abnormal returns in the first place. Indeed, estimating both sides of equation (10) in our sample reveals a large discrepancy arising from (i) a portfolio's conditional market beta covarying with market volatility (which affects the left-hand side) and (ii) the long-run reversal effect generating negative cross-autocovariances between portfolio excess returns and market returns.

C. A direct discount of post-formation cash flows

A potential estimator of δ is to directly discount post-formation dividends and a terminal cash flow:

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \approx 1 - E\left[\sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{t+j}}{P_t}\right] - E\left[\widetilde{M}_{t,t+j}\frac{P_{t+J}}{P_t}\right]. \tag{11}$$

Suppose one takes equation (11) to the data using the method of moments estimator $\widehat{\delta}^{CF}_t$:

$$\widehat{\delta}^{CF} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t^{CF} \quad \text{with} \quad \widetilde{\delta}_t^{CF} = 1 - \sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{(t),t+j}}{P_{(t),t}} - \widetilde{M}_{t,t+j}\frac{P_{(t),t+J}}{P_{(t),t}}, \tag{12}$$

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and a simple example that illustrates this point.

where the notation $X_{(t),t+j}$ indicates a time $t+j$ realization of X from buying and holding a portfolio formed at time t. The resulting dividend-based estimator of δ in equation (12) is potentially biased and also subject to a serious overlapping observations problem.

To see the bias point, restate the dividend-based estimator in equation (12) using post-formation returns (see Section IA.C.6 in the Internet Appendix for the equivalence):

$$\widehat{\delta}^{CF} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t^{CF} \text{ with } \widetilde{\delta}_t^{CF} = -\widetilde{M}_{t,t+j-1}\frac{P_{(t),t+j-1}}{P_{(t),t}}(\widetilde{M}_{t+j}(1+R_{(t),t+j})-1). \tag{13}$$

Here, getting $\widetilde{M}_{t+j}(1+R_{(t),t+j})-1$ close to zero on average requires getting both the risk premium and the interest rate parts of $\widetilde{M}$ right, and measurement error in the interest rate part of $\widetilde{M}$ could bias $\widetilde{M}_{t+j}(1+R_{(t),t+j})-1$ in the same direction for all periods, leading to a large bias in $\hat{\delta}^{CF}$. Importantly, simply introducing a T-bill rate into $\widetilde{M}$ is not a remedy for this issue. Indeed, van Binsbergen et al. (2023) grant that applying their dividend-based estimator to a simple strategy of rolling over T-bills for 15 years results in a $\hat{\delta}^{CF}$ of around 0.50 (i.e., 50%), despite the strategy requiring relatively little risk adjustment, highlighting the challenge of dealing with time discounts in price-level analyses.[8] (They propose correcting for such a bias through bootstrap that requires a structural model of how returns are distributed.) Applying our excess-return-based estimator in equation (2) to the same 15-year roll-over T-bill strategy results in a $\widehat{\delta}$ of just 0.001 (i.e., 0.1%) without having to apply a separate bias adjustment. We return to this point about the bias in Section III after we formally introduce our excess-return-based estimator.

Second, to understand the overlapping samples issue, note that the covariance between

[8] In our sample, the event-time gross-return approach estimates the $\hat{\delta}^{ET,gross}$ of the T-bill roll-over strategy to be 0.497 (49.7%), similar to the number in van Binsbergen et al. However, for other portfolios, we find that the event-time gross-return approach typically finds estimated mispricing that is similar to our calendar-time excess-return approach.

$\widetilde{\delta}_t^{CF}$ observations s months apart is

$$Cov(\widetilde{\delta}_t^{CF}, \widetilde{\delta}_{t+s}^{CF}) = \sum_{j=s+1}^{J} Cov(\frac{\widetilde{M}_{t,t+j}}{P_{(t),t}} D_{(t),t+j}, \frac{\widetilde{M}_{t+s,t+j}}{P_{(t+s),t+s}} D_{(t+s),t+j}) + other\ terms. \tag{14}$$

Since both $\widetilde{\delta}_t^{CF}$ and $\widetilde{\delta}_{t+s}^{CF}$ depend on dividend realizations in periods $t+s+1$ through $t+J$, the cross-sectional covariance in dividend yields results in severe time-series covariances. For example, for the growth portfolio (lowest B/M portfolio in a quintile sort), the time-series of δ observations based on the cash-flow approach ($\widetilde{\delta}_t^{CF}$) have 1-, 5-, and 15-year sample autocorrelations of 0.78, 0.34, and −0.86 whereas the corresponding autocorrelations based on our approach are 0.15, −0.05, and 0.01 (Table IA.IV in the Internet Appendix). Furthermore, unlike returns, dividends on a stock are extremely serially correlated over time such that the event-time-to-calendar-time rearrangement does not sufficiently lower the serial correlation in the $\widetilde{\delta}_t^{CF}$ observations. This high serial correlation of a dividend-based approach makes standard errors imprecise and inference unreliable.[9] We also discuss this issue further in Section III.

D. *Simple long-horizon returns*

If short-horizon α cannot proxy for price-level δ, could we use simple long-horizon abnormal return measures such as the cumulative abnormal return (CAR) or buy-and-hold abnormal return (BHAR) instead? The issue with these measures is that they put equal weight on all future abnormal returns and do not differentiate among abnormal returns earned in different time periods or states of nature. As explained in the next section, our δ estimator appropriately discounts the abnormal returns earned in different periods and states, which can lead to a very different magnitude of estimated δ.

One may argue that the CAR or the BHAR could generate the direction of price distortion and associated statistical significance that tend to be correct under the *null*

[9]Panel D of Figure 2 makes this point using Monte Carlo analysis.

hypothesis. However, measuring the exact magnitude of price-level δ under the alternative hypothesis is important for firm managers and investors who wish to use our novel estimation approach and the magnitude of the resulting δ estimate to inform their investment/issuance decisions. For example, firm managers may wish to quantify the extent to which a particular characteristic is historically a signal of δ from the CAPM perspective. Similarly, long-term buy-and-hold investors would like to understand the magnitude of the δ associated with the stocks they bought. Indeed, equity analyst reports provide a price target and the magnitude of estimated δ on a stock. When those investors undertake that analysis, they would not want to rely on methods like CAR or BHAR, which do not have an interpretable magnitude under the alternative of non-zero δ and, as we emphasize and document in our analysis in Table A5 in the Internet Appendix, can have magnitudes that depart substantially from δ estimated using our approach.

II. The Mispricing Identity

Under the asset pricing environment specified in the previous section, we derive a novel identity that yields our new estimator of abnormal price in Section III.

A. *The law of motion for abnormal price*

The first step is to derive a simple law of motion for conditional abnormal price, δ_t. Equation (3) and the law of iterated expectations imply that the fundamental asset pricing equation holds for V_t with respect to $\widetilde{M}$:

$$1 = E_t\left[\widetilde{M}_{t+1}\frac{V_{t+1} + D_{t+1}}{V_t}\right]. \tag{15}$$

Next, using equation (5) to substitute the empirically unobserved quantities V_t and V_{t+1} with $V_t = (1 - \delta_t) P_t$ and $V_{t+1} = (1 - \delta_{t+1}) P_{t+1}$,

$$\begin{aligned}\delta_t &= 1 - E_t \left[\widetilde{M}_{t+1} (1 + R_{t+1})\right] + E_t \left[\widetilde{M}_{t+1} \tfrac{P_{t+1}}{P_t} \delta_{t+1}\right] \\ &= -E_t \left[\widetilde{M}_{t+1} R^e_{t+1}\right] + E_t \left[\widetilde{M}_{t+1} \tfrac{P_{t+1}}{P_t} \delta_{t+1}\right], \end{aligned} \tag{16}$$

where the last equality uses equation (4) to express mispricing δ_t at time t in terms of excess return $R^e_{t+1} = R_{t+1} - R_{b,t+1}$ at time $t+1$ and mispricing δ_{t+1} at time $t+1$.

Equation (16) is intuitive. It says that overpricing (underpricing) at time t is either "paid out" as a negative (positive) abnormal return, $E_t \left[\widetilde{M}_{t+1} R^e_{t+1}\right]$, or contributes to overpricing (underpricing) at time $t+1$. The correct discount factor on δ_{t+1} is the SDF times the capital gain because δ_{t+1} has been normalized by P_{t+1}. δ_{t+1} matters more at time t if it arises in a state in which P_{t+1} is high (hence the capital gain term) or has a higher present value (hence the SDF term).

B. *The identity: linking abnormal price to subsequent returns*

Our identity is derived under the relatively mild assumption of no explosive bubbles in prices.

ASSUMPTION 1 (**No explosive bubble**): *Price is not explosive with respect to the candidate* $\widetilde{M}$*:* $\lim_{j\to\infty} E_t \left[\widetilde{M}_{t,t+j} P_{t+j}\right] = 0$.

To understand this assumption, note that by definition, an analogous condition on value V also holds: $\lim_{j\to\infty} E_t \left[\widetilde{M}_{t,t+j} V_{t+j}\right] = 0$. Hence, Assumption 1 implies that the present value of the deviation in price from value at the limit $j \to \infty$ is zero:

$$\lim_{j\to\infty} E_t \left[\widetilde{M}_{t,t+j} (P_{t+j} - V_{t+j})\right] = 0. \tag{17}$$

This condition is not restrictive, as it allows for most types of price deviations from value, including a permanent gap (e.g., $\delta_{t+j} = \delta \neq 0 \, \forall j$), which our identity correctly detects.[10]

Iterating equation (16) forward and imposing Assumption 1 to set $\lim_{j\to\infty} E_t \left[\widetilde{M}_{t,t+j} \frac{P_{t+j}}{P_t} \delta_{t+j}\right] = 0$ expresses abnormal price as a discounted sum of subsequent excess returns.

LEMMA 1 (**Mispricing identity**): *Under Assumption 1, a portfolio's abnormal price δ_t is the negative of the sum of expected subsequent excess returns discounted by the price-weighted SDF:*

$$\delta_t \equiv \frac{P_t - V_t}{P_t} = -\sum_{j=1}^{\infty} E_t \left[\widetilde{M}_{t,t+j} \tfrac{P_{t+j-1}}{P_t} R^e_{t+j}\right], \tag{18}$$

where $\frac{P_{t+j-1}}{P_t}$ and R^e_{t+j} are, respectively, the portfolio's cumulative capital gains from time t to $t+j-1$ and excess returns at time $t+j$. Hence, mean (unconditional) abnormal price, denoted δ, equals

$$\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right] = -\sum_{j=1}^{\infty} E\left[\widetilde{M}_{t,t+j} \tfrac{P_{t+j-1}}{P_t} R^e_{t+j}\right]. \tag{19}$$

Equation (18) can also be stated with conditional abnormal returns. By the law of iterated expectations, $E_t\left[\widetilde{M}_{t,t+j} \frac{P_{t+j-1}}{P_t} R^e_{t+j}\right] = E_t\left[\widetilde{M}_{t,t+j-1} \frac{P_{t+j-1}}{P_t} E_{t+j-1}\left[\widetilde{M}_{t+j}\right] E_{t+j-1}\left[\frac{\widetilde{M}_{t+j}}{E_{t+j-1}\left[\widetilde{M}_{t+j}\right]} R^e_{t+j}\right]\right] = E_t\left[\widetilde{M}_{t,t+j} \frac{P_{t+j-1}}{P_t} \alpha_{t+j}\right]$, where α_{t+j} denotes the conditional abnormal return.

COROLLARY 1 (**Identity in abnormal returns**): *Today's abnormal price δ_t is the expectation of a simple discounted sum of subsequent conditional abnormal returns:*

$$\delta_t = -\sum_{j=1}^{\infty} E_t\left[\widetilde{M}_{t,t+j} \tfrac{P_{t+j-1}}{P_t} \alpha_{t+j}\right], \tag{20}$$

where α_{t+j} is the time $t+j$ abnormal return conditional on information at time $t+j-1$.

Equation (20) is intuitive. The economic surplus, relative to a candidate SDF $\widetilde{M}$,

[10] Section IA.C.1 in the Internet Appendix gives an example illustrating this point.

from a buy-and-hold strategy on a portfolio is the net present value of all subsequent abnormal payoffs: $V_t - P_t = E_t\left[\sum_{j=1}^{\infty} \widetilde{M}_{t,t+j} X_{t+j}^{Abnormal}\right]$. The abnormal payoff at $t+j$ is $X_{t+j}^{Abnormal} = P_{t+j-1}\alpha_{t+j}$, the conditional abnormal return from $t+j-1$ to $t+j$, converted into monetary payoff through a multiplication by price at time $t+j-1$. Finally, divide both sides by P_t and change sign to arrive at equation (20).

We note in passing that equation (20) is fundamentally different from an identity van Binsbergen and Opp (2019) exploit in their quantitative analysis:

$$P_t = \sum_{j=1}^{\infty} E_t\left[\frac{\widetilde{M}_{t,t+j}}{\Pi_{k=1}^{j}\left(1+\alpha_{t+k}^{*}\right)} D_{t+j}\right], \tag{21}$$

where $1+\alpha_{t+k}^{*} = E_{t+k-1}\left[\widetilde{M}_{t+k}\left(1+R_{t+k}\right)\right]$.[11] Since equation (21) writes price in the numerator of the left-hand side, abnormal returns appear in the denominator of the identity. This choice is innocuous for the structural approach taken in their paper but does not render a simple expression for unconditional abnormal price as in equation (19).

C. *Theoretical implications of the abnormal price identity*

Equation (20) clarifies the exact relation between price-level δ and subsequent buy-and-hold short-horizon αs missing in the literature, summarized as Corollary 2.

COROLLARY 2 (**Implications of the identity**): *Ceteris paribus, ex-ante abnormal price δ is larger if subsequent abnormal returns*

1. *are larger*
2. *are more persistent*
3. *occur sooner*
4. *occur in more valuable states*
5. *occur after relatively large capital gains*

[11] This is a discrete-time version of their continuous-time expression. See the Internet Appendix Section IA.C.3 for further details.

While the first two points are relatively obvious (i.e., they are present in the Campbell-Shiller based approach of CPV, at least to some degree), the last three points emphasize that to recover ex-ante abnormal price, one must correctly discount the subsequent abnormal returns according to the time and state in which they occur. Put differently, the correct way to aggregate subsequent abnormal returns to arrive at ex-ante abnormal price is clearly distinct from existing long-run return measures such as CAR or BHAR, which do not distinguish between abnormal returns earned in the near vs. distant future or those earned in more valuable vs. less valuable cumulative states.

Mathematically, the time discount (#3) arises because the no-explosive-bubble condition implies $\lim_{j\to\infty} E_t\left[\widetilde{M}_{t,t+j}\frac{P_{t+j-1}}{P_t}\right] = 0$.[12] Intuitively, an abnormal return that arises in the distant future matters less for the stock's current price level, since such an abnormal return only affects the discounting of the dividends to be earned in the periods that follow the abnormal return, not the dividends to be earned prior to the timing of the abnormal return. In contrast, an abnormal return in the immediate future affects the discounting of all future dividends and therefore matters more for today's stock price level. The state discount (#4 and #5) arises because the conditional abnormal return, α_{t+j}, is earned on P_{t+j-1}, which has a large present value if either the cumulative capital gain has been large or the cumulative candidate SDF is high.[13] While we speculate that our abnormal price identity can be applied in other ways, the present paper applies the identity to derive a return-based estimator of abnormal price discussed next.

[12]CPV effectively incorporate a time discount through the Campbell-Shiller discount parameter although their approach does not account for differences in cash-flow duration across different stocks.

[13]Since the time $t+j$ component of the cumulative candidate SDF, $\widetilde{M}_{t+j}$, is orthogonal to α_{t+j} by definition, it only generates a time discount. The covariance between α_{t+j} and the past cumulative state, $\widetilde{M}_{t,t+j-1}$, could be nonzero.

III. A Return-Based Estimator of Abnormal Price

This section derives a return-based estimator of a portfolio's mean formation-period abnormal price, $\delta \equiv E[\delta_t] = E\left[(P_t - V_t)/P_t\right]$, and studies its statistical properties using Monte Carlo.

A. *The identity in calendar time*

To get to our abnormal price estimator, we first rearrange the terms of the identity in Equation (19) to obtain an equivalent *calendar-time* expression for unconditional abnormal price δ.[14]

LEMMA 2 (**Calendar-time δ expression**): *Consider a series of portfolios formed every period based on a predetermined rule (e.g., characteristic cutoffs). Then, the portfolio's unconditional formation-period abnormal price, $\delta \equiv E\left[\frac{P_t - V_t}{P_t}\right]$, equals the unconditional expectation of the sum of appropriately discounted time-t excess returns on all portfolios formed between periods $t-\infty$ and $t-1$:*

$$\delta = -E\left[\sum_{j=0}^{\infty} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right], \tag{22}$$

where $\frac{P_{(t-j),t}}{P_{(t-j),t-j}}$ and $R^e_{(t-j),t}$ denote, respectively, the time-t realizations of the cumulative capital gain and one-period excess return on the portfolio formed at $t-j$. The stochastic discount $\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$ tends to place a greater weight on portfolios formed in the recent past.

[14] Fama (1998) emphasizes the usefulness of calendar-time techniques in his discussion of the literature on post-event, long-horizon abnormal returns. Cohen et al. (2009) modify a calendar-time approach to have portfolio weights decline as a function of time-from-event so that the return on the resulting portfolio approximates a buy-and-hold investor's experience in price-level units. We thank Rob Rogers for suggesting a similar approach for our estimation of delta using our exact identity.

Proof. Applying equation (19) to a multi-asset portfolio implies that a portfolio's time-t_0 ex-ante conditional abnormal price is $\delta_{t_0} = -\sum_{j=0}^{\infty} E_{t_0}[\widetilde{M}_{t_0,t_0+j} \frac{P_{(t_0),t_0+j-1}}{P_{(t_0),t_0}} R^e_{(t_0),t_0+j}]$, where (t_0) in a subscript indicates that the quantity is from a buy-and-hold strategy on a portfolio formed at t_0. Take an unconditional expectation of both sides of the expression and use calendar time $t \equiv t_0 + j$ (the time when the excess returns are realized) to obtain the expression in equation (22). □

Figure 1 visualizes the difference between an event-time and a calendar-time approach. The original identity in equation (19) shows that unconditional δ is the expectation of the event-time δ_t that appropriately discounts the post-formation buy-and-hold monthly excess returns on a growth portfolio formed today, as illustrated in Panel A. In contrast, the equivalent calendar-time identity in equation (22) shows that unconditional δ also equals the expectation of the calendar-time δ_t that appropriately discounts today's realizations of monthly excess buy-and-hold returns on growth portfolios formed in the past, as illustrated in Panel B.

B. A new estimator of abnormal price

In practice, truncating the infinite-horizon sum in equation (22) at some large J provides a good approximation, since $E\left[\widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ converges to zero as j gets large. This convergence is because both the discount part ($\widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$) and the conditional abnormal return part ($E_{t-1}\left[\widetilde{M}_t R^e_{(t-j),t}\right]$) of the expression converge to zero as $j \to \infty$ due to the no-explosive-bubble condition and the long-term convergence in (abnormal) returns of stocks (Keloharju et al. (2021)), respectively. We find that $J = 15$ years works well both empirically and in simulations (see Figure IA.4 in the Internet Appendix).

Our estimator of unconditional δ is therefore the sample analogue of the true unconditional δ in equation (22) with the truncation of the infinite sum at a large finite J.

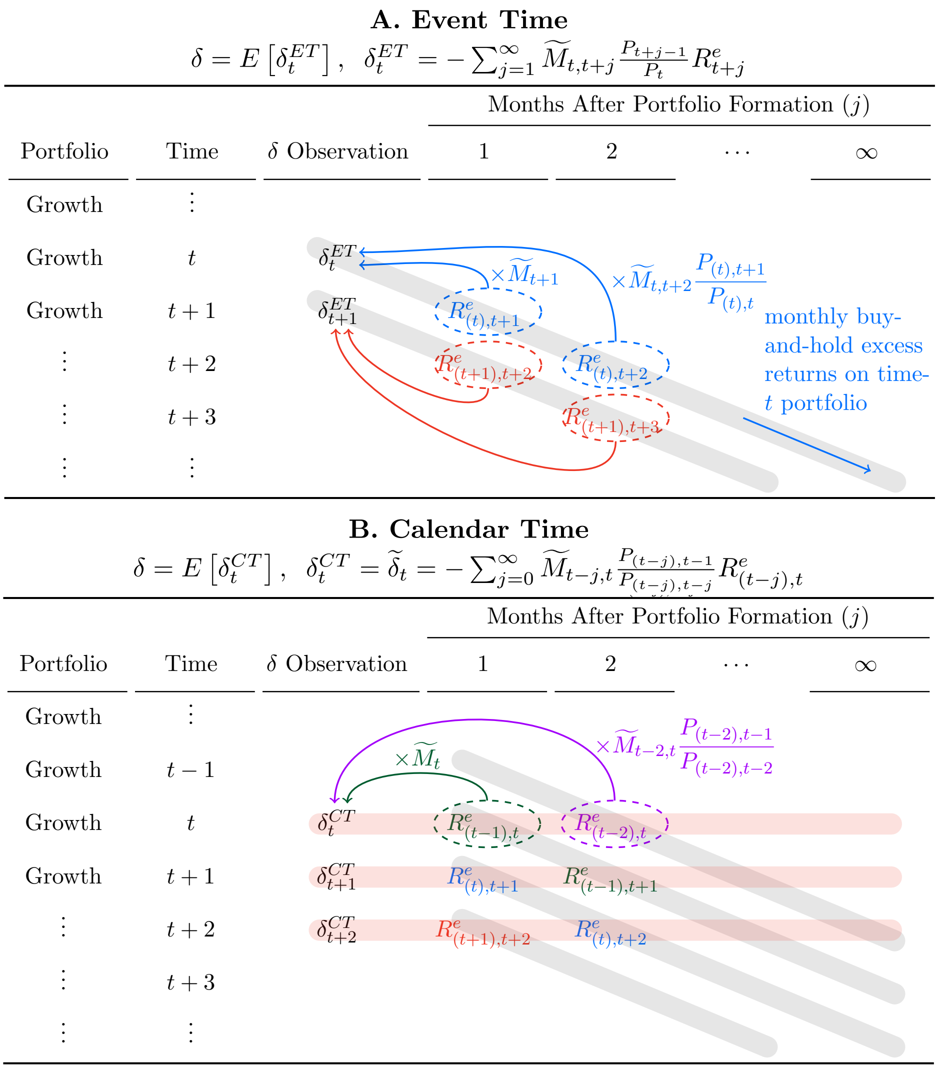

Figure 1. Event time vs. calendar time approach to δ estimation. Panel A visualizes the event-time approach to estimating δ. The equation there shows that unconditional δ is the expectation of the event-time δ that appropriately discounts the post-formation buy-and-hold monthly excess returns on a growth portfolio formed today. Hence, the event-time approach takes a sum of all discounted post-formation excess returns diagonally in the southeast direction. Panel B visualizes the equivalent calendar-time approach. The equation there shows that unconditional δ also equals the expectation of the calendar-time δ that appropriately discounts today's realizations of monthly excess buy-and-hold returns on growth portfolios formed in the past. Hence, the calendar-time approach takes a sum of all discounted excess returns in the concurrent period (on portfolios formed in the past) horizontally.

COROLLARY 3 (**A return-based estimator of abnormal price**): *The unconditional abnormal price δ of a portfolio formed on a predetermined rule (e.g., "value portfolio") can be estimated by*

$$\widehat{\delta} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t \quad \textit{with} \quad \widetilde{\delta}_t = -\sum_{j=1}^{J}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}, \tag{2}$$

for $J = 15$ years, where the time-t observation of δ is the sum of appropriately discounted time-t excess returns on all portfolios formed on the characteristic over the last J periods. The expression coincides with the true δ in equation (22) as $J, T \to \infty$.

An important advantage of our estimator in equation (2) is that using excess returns rather than gross returns for estimation makes our mispricing estimates less subject to a potential bias. This can be best explained by drawing an analogy with the expected return framework. Researchers typically estimate time-series abnormal returns using the excess-return formulation, $E\left[\frac{\widetilde{M}_t}{E[\widetilde{M}_t]}R^e_t\right]$, not the gross-return formulation, $E\left[\widetilde{M}_t(1+R_t)-1\right]$, since the former only requires the SDF to explain the risk premium component of excess returns, whereas the latter requires the SDF to explain both the risk premium and the interest rate components of gross returns.[15] Since our excess-return estimator of abnormal price uses $\widetilde{\delta}_t = -\sum_{j=1}^{J}\left(M_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}\right)\widetilde{M}_t R^e_{(t-j),t}$, getting $\widehat{\delta}$ close to zero mostly hinges on getting $\widetilde{M}_t R^e_{(t-j),t}$ close to zero on average through the risk premium component of $\widetilde{M}$, whereas the interest rate component of $\widetilde{M}$ only affects how $\widetilde{M}_t R^e_{(t-j),t}$ is time-discounted back to the portfolio formation period through $\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$. Thus, our excess return formulation ensures that measurement error in the interest rate component of $\widetilde{M}$ does not materially affect our $\widehat{\delta}$.

[15]Cochrane (2009): "In fact, much asset pricing focuses on excess returns. Our economic understanding of interest rate variation turns out to have little to do with our understanding of risk premia, so it is convenient to separate the two phenomena by looking at interest rates and excess returns separately" (p.9).

Table I. CAPM abnormal price (δ) of a T-bill rollover strategy by estimation approach. The table reports the estimated CAPM δ, computed based on four different methods, of a strategy that rolls over the 1-month Treasury bill return for 15 years. The calendar-time excess return method proposed in the present study (top left) ensures that the T-bill rollover strategy has a near-zero estimated CAPM δ. The event-time excess return method (top right) also leads to a small CAPM δ of the T-bill strategy but is subject to large serial correlations. The gross-return method based on excess returns (bottom left) or the direct discount of cash flows (bottom right, equivalent to the event-time gross return method) leads to large estimated CAPM δs of the T-bill rollover strategy of around 50%. For each method, the candidate SDF's two parameters are estimated by imposing the restriction that the estimated delta of the market portfolio is zero as well as the complementary restriction that either the market's gross one-month return (first two approaches) or its excess one-month return (last two approaches) is perfectly explained.

	Calendar Time	Event Time
Excess Return	$\widehat{\delta} = 0.001$ (This Paper)	$\widehat{\delta}^{excess,ET} = 0.028$
Gross Return	$\widehat{\delta}^{gross,CT} = 0.457$	$\widehat{\delta}^{CF} = 0.497$ (Discounted CF)

Indeed, the strategy that rolls over the one-month T-bill return for 15 years has drastically different CAPM delta estimates depending on whether or not the excess return method is employed. Table I shows that using the proposed calendar-time excess-return method (top left corner),

$$\widehat{\delta} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t \quad \text{with} \quad \widetilde{\delta}_t = -\sum_{j=1}^{J}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t},$$

leads to an estimated delta of 0.1%, consistent with how restrictions used to estimate the candidate SDF parameters include an implicit assumption on the T-bill return being correct. The delta remains relatively small at 2.8% even if we use the event-time excess-return method that does not take advantage of the calendar-time formulation (top right corner),[16]

$$\widehat{\delta}^{excess,ET} = \frac{1}{T}\sum_{t=1}^{T}\widetilde{\delta}_t^{excess,ET} \quad \text{with} \quad \widetilde{\delta}_t^{excess,ET} = -\sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{P_{(t),t+j-1}}{P_{(t),t}}R^e_{(t),t+j}$$

In contrast, the calendar-time gross-return method,

$$\widehat{\delta}^{gross,CT} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T}\widetilde{\delta}_t^{gross,CT} \quad \text{with} \quad \widetilde{\delta}_t^{gross,CT} = -\textstyle\sum_{j=1}^{J}\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}(\widetilde{M}_t(1+R_{(t-j),t})-1),$$

and the dividend-discount method (equivalent to the event-time gross-return method),

$$\widehat{\delta}^{CF} = \tfrac{1}{T}\textstyle\sum_{t=1}^{T}\widetilde{\delta}_t^{CF} \quad \text{with} \quad \widetilde{\delta}_t^{CF} = 1-\textstyle\sum_{j=1}^{J}\widetilde{M}_{t,t+j}\frac{D_{(t),t+j}}{P_{(t),t}} - \widetilde{M}_{t,t+j}\frac{P_{(t),t+J}}{P_{(t),t}},$$

lead to large estimated deltas of 45.7% and 49.7%, respectively (bottom left and right corners of Table I; the latter estimate is similar to the one found in van Binsbergen et al. (2023)). The biases remain just as large even if we use candidate SDFs estimated using the excess return method, implying that the problem arises with the delta estimator itself, not in the restrictions used to estimate the candidate SDF coefficients.

Another advantage of working with this return-based calendar-time estimator of δ is that its time-series observations, $\widetilde{\delta}_t$, have very little serial correlation, simplifying the inference problem. To see this, consider estimating a value portfolio's δ. Then, the time-t observation of the value portfolio's calendar-time δ is the discounted sum of time-t excess returns on all value portfolios formed over the last J periods: $\widetilde{\delta}_t = -\sum_{j=1}^{J}\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}$. Hence, even if value portfolios formed over the span

[16]The following restrictions are used to estimate the candidate SDF parameters. (1) Calendar-time excess-return method: $\frac{1}{T}\sum_{t=1}^{T}\widetilde{M}_t(1+R_t^{mkt}) = 1$ and $\widehat{\delta} = 0$ for the market portfolio. (2) Event-time excess-return method: $\frac{1}{T}\sum_{t=1}^{T}\widetilde{M}_t(1+R_t^{mkt}) = 1$ and $\widehat{\delta}^{excess,ET} = 0$. (3) Calendar-time gross-return method: $\frac{1}{T}\sum_{t=1}^{T}\widetilde{M}_t(R_t^{mkt} - R_{f,t}) = 0$ and $\widehat{\delta}^{gross,CT} = 0$. (4) Dividend discount method: $\frac{1}{T}\sum_{t=1}^{T}\widetilde{M}_t(R_t^{mkt} - R_{f,t}) = 0$ and $\widehat{\delta}^{CF} = 0$.

of past few years have cross-sectionally correlated buy-and-hold one-period returns at time t, by putting all these time-t returns into a single observation $\widetilde{\delta}_t$, the expression ensures that this cross-sectional return correlation increases the variance of $\widetilde{\delta}_t$ instead of generating serial correlation in $\widetilde{\delta}_t$. On the other hand, since the stochastic discount part of the formula, $\widetilde{M}_{t-j,t}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$, must always remain positive and multiplies an excess return $R^e_{(t-j),t}$ that fluctuates around zero, the overlapping nature of the stochastic discount expression does not generate an empirically discernible serial correlation in $\widetilde{\delta}_t$ (Figure IA.2 and Table IA.IV of the Internet Appendix).

C. Decomposing long-horizon return: risk vs. abnormal price

Similarly to how (short-horizon) expected return can be decomposed into an abnormal return and a risk premium, a measure of long-horizon return can be written in terms of abnormal price, long-horizon risk, and a cumulative state adjustment component. To see this, apply the covariance identity, $E[XY] = E[X]E[Y] + Cov(X,Y)$, to equation (2).

COROLLARY 4 (**Long-horizon return and long-horizon risk**): *Estimated abnormal price $\widehat{\delta}$ is a deviation of long-horizon return from long-horizon risk, adjusted by the cumulative state in which the risk-return distortion arises:*

$$\underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] E_T\left[\widetilde{M}_t\right] E_T\left[R^e_{(t-j),t}\right]}_{\text{“long-horizon return”}} = -\,\widehat{\delta} \qquad (23)$$

$$+\underbrace{\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(R^e_{(t-j),t}, -\widetilde{M}_t\right)}_{\text{“long-horizon risk”}}$$

$$\underbrace{-\sum_{j=1}^{J} E_T\left[\phi_{(t-j),t-1}\right] Cov_T\left(\frac{\phi_{(t-j),t-1}}{E_T\left[\phi_{(t-j),t-1}\right]}, \widetilde{M}_t R^e_{(t-j),t}\right)}_{\text{“cumulative state adjustment”}}$$

where subscript T denotes a sample moment over T periods and $\phi_{(t-j),t-1} \equiv$

$\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}$ *discounts post-formation returns in more distant future or less important states more heavily.*

Long-horizon return summarizes the term structure of discount rates on the portfolio's cash flows. It puts more (less) weight on returns in the more imminent (distant) future, since in a dividend discount model, return in an imminent future (e.g., R_{t+1}) discounts all future cash flows, whereas return in a more distant future (e.g., R_{t+j}) discounts a smaller subset of cash flows that arise subsequently: $P_t = E_t\left[\frac{D_{t+1}}{1+R_{t+1}} + \frac{D_{t+2}}{(1+R_{t+1})(1+R_{t+2})} + ... + \frac{D_{t+j}}{(1+R_{t+1})(1+R_{t+2})...(1+R_{t+j})} + ...\right]$. By the same logic, *long-horizon risk* applies different time discounts to the term structure of risk premia. *Cumulative state adjustment* accounts for the way the cumulative *state* in which returns deviate from risk premia—not just the *time* at which those deviations happen—matters for abnormal price.

To obtain a measure of *risk-neutral abnormal price*, simply set $\widetilde{M}_t$ in equation (23) to be one.[17]

DEFINITION 1 (**Risk-neutral abnormal price**): *Estimated risk-neutral abnormal price, denoted* $\widehat{\delta}^{RN}$*, equals*

$$\widehat{\delta}^{RN} = -\sum_{j=1}^{J} E_T\left[\widetilde{M}_{t-j,t-1}\frac{P_{(t-j),t-1}}{P_{(t-j),t-j}}R^e_{(t-j),t}\right]. \tag{24}$$

For instance, when excess returns are taken with respect to the market portfolio, risk-neutral abnormal price measures the extent to which the term structure of portfolio returns differs from the term structure of market portfolio returns, where the weight on each excess return depends on the time and the cumulative state in which it occurs.

[17] We thank an anonymous referee for this suggestion. One could alternatively set $\widetilde{M}_t$ to be the inverse of the average monthly risk-free rate, but the difference is very small at our monthly frequency.

D. Implementation

To take our estimator in equation (2) to data, we use the CAPM to specify the candidate SDF.[18] While analyzing short-horizon returns requires choosing risk factors, analyzing price levels requires choosing both risk factors and the functional form of the candidate SDF. We follow Korteweg and Nagel (2016) and use a loglinear SDF specification, a natural choice for price-level analysis given its ability to explain short-horizon returns and prices simultaneously:

$$\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \textstyle\sum_{s=0}^{j-1} r_{t-s}^{mkt}\right), \tag{25}$$

where $r_t^{mkt} = \log\left(1 + R_t^{mkt}\right)$ is the log market return and (b_0, b_1) are constant parameters, and $\widetilde{M}_t \equiv \widetilde{M}_{t-1,t} = \exp\left(b_0 - b_1 r_t^{mkt}\right)$. We choose b_0 and b_1 to explain the market portfolio's prices and returns perfectly in sample, which makes our approach analogous to the conventional time-series approach to estimating abnormal returns. Specifically, the following two moment conditions pin down b_0 and b_1:

$$\begin{aligned} 0 &= E\left[\widetilde{M}_t(1 + R_t^{mkt})\right] - 1 \\ 0 &= \sum_{j=1}^{J} E[\widetilde{M}_{t-j,t} \frac{P_{t-1}^{mkt}}{P_{t-j}^{mkt}} \left(R_{(t-j),t}^{mkt} - R_{f,t}\right)], \end{aligned} \tag{26}$$

where $R_{(t-j),t}^{mkt}$ is the time-t return on market portfolio formed at $t - j$. The Internet Appendix shows that the estimated values of b_0 and b_1 vary depending on the choice of the number of post-formation years J that we include in our estimate of δ, but not

[18] Our analysis in subsequent sections will provide a direction for future research on multifactor models of prices. In addition, it would be natural to check whether the intertemporal CAPM specification of Campbell et al. (2018), which incorporates stochastic volatility into the ICAPM framework of Campbell and Vuolteenaho (2004), significantly reduces the pricing errors relative to the CAPM in standard SDF return tests. We leave a multifactor analysis of price levels to future work.

dramatically so, with confidence intervals at each horizon covering the point estimates of other horizons. These results are consistent with the CAPM evidence from Chernov et al. (2022).

We compute excess returns on a characteristic-based portfolio with respect to returns on the market portfolio, exploiting the CAPM implication that the market should be correctly priced and the fact that the market has a zero in-sample δ with respect to the model SDF. Benchmarking test assets against the market reduces the sensitivity of the time-series of δ observations to market return shocks, reducing the volatility of the resulting series and improving the precision of our estimates (as in Campbell et al. (2013), Campbell et al. (2018), and Korteweg and Nagel (2022)). Simulation shows that taking excess returns with respect to the market portfolio reduces the estimator's volatility by seven percent. An added benefit of this approach could be that the resulting estimates of δ may be less affected by the near-money feature of the T-bill (Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2016)).[19]

We use the generalized method of moments (GMM) standard errors that account for the time-series and cross-sectional covariances in the data as well as the uncertainty in estimating the SDF parameters. Our Monte Carlo analysis in the next subsection shows that the Bartlett kernel of Newey and West (1987) with a bandwidth of 24 months works well, given the low level of autocorrelation observed in the time series of $\tilde{\delta}_t$.

[19]Note also that any convenience yield properties of the short-term US government debt could still affect our results through its impact on candidate SDF parameter estimates, since the market portfolio's δ in equation (26) is estimated using its excess return with respect to the Treasury-bill rate. In practice, computing excess returns with respect to the T-bill return makes only a small difference to the point estimates.

E. Monte Carlo analysis

To study the performance of our delta estimator, we simulate a numerical model that resembles the one in Korteweg and Nagel (2016). The model generates realistic moments that resemble those of the high adjusted value quintile portfolio and those of the market portfolio.

We first analyze bias. Since the choice of candidate SDF parameters b_0 and b_1 affect $\widehat{\delta}$ in a highly nonlinear fashion, the uncertainty arising from having to estimate b_0 and b_1 using the base asset (market portfolio) can lead our estimated $\widehat{\delta}$ to deviate from the true δ. Panel A of Figure 2 shows that such a bias, if any, leads to a small attenuation in the estimated $\widehat{\delta}$ and a more conservative rule in rejecting an asset pricing model.

Next, turning to size and power, we find that our estimator under the null tends to under-reject the null relative to the 5% significance level and that our price-level test has statistical power similar to a conventional return test (Table IA.I in the Internet Appendix). As equation (2) implies, abnormal price δ is a discounted sum of 15 years of post-formation returns. This means that the magnitude of δ required for statistical significance is roughly 10-to-12 times that for annualized α, consistent with δ being a discounted sum of future αs over roughly 15 years. Panel B of Figure 2 visualizes this rule of thumb.

Panel C and D highlight the main issue with doing inference on δ using cash flows, which is that the resulting large serial correlation makes the number of independent time-series observations in the cash-flow approach small and the range of standard errors wide (Panel D). Our return-identity-based approach addresses this problem and tends to estimate standard errors accurately.

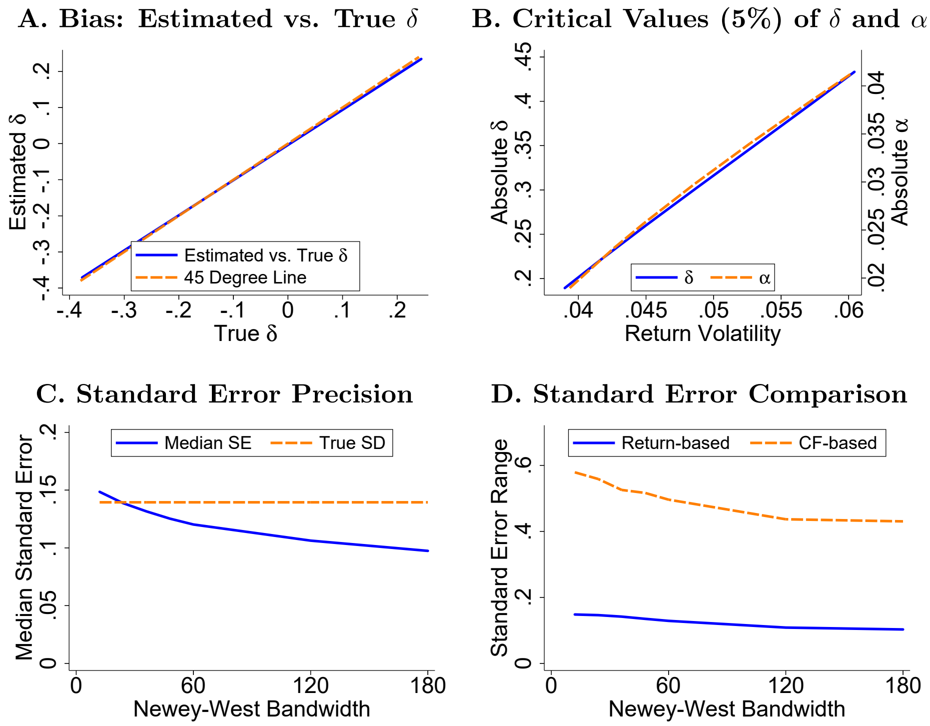

Figure 2. Bias, power, and standard errors of the return-identity-based δ estimator. Panel A analyzes the bias of our estimator. The solid blue line relating the mean estimated $\hat{\delta}$ to the true δ from a Monte Carlo simulation almost coincides with the 45-degree line in dotted line apart from a small attenuation (less than 1% point) in large values of δ. Panel B plots, for different levels of return volatility, the smallest absolute value of δ (left vertical axis) and α (right vertical axis) needed to reject the null at the 5% significance level when there is a true CAPM mispricing of $\delta = 36.6\%$ and $\alpha = 0.26\%$. This shows that the magnitude of δ required for statistical significance is roughly 10-to-12 times that for annualized α, consistent with δ being a discounted sum of future αs over roughly 15 years. Panel C reports results from a Monte Carlo simulation analyzing whether a Newey-West standard error ("SE") with a bandwidth of around 2 years accurately estimates the true standard deviation ("SD") of $\hat{\delta}$. The Monte Carlo simulation uses the parameter values reported in Table IA.II except that the simulation for Panel B varies the volatilities of cash flow shocks and conditional δ_t shocks to generate variation in return volatility.

IV. Asset Pricing Tests on Prices

A. Data

We combine monthly stock price data from the Center for Research in Security Prices (CRSP), annual accounting data from CRSP/Compustat Merged (CCM), and the pre-Compustat book equity data from Davis et al. (2000) to create our basic dataset. We compute the gross market portfolio return, the factor in our candidate SDF, as the sum of the market excess return and the one-month Treasury bill rate from Kenneth French's data library.

Our tests estimate the δs of diversified portfolios to minimize the impact of idiosyncratic returns. We typically form value-weight quintile portfolios by sorting stocks on single characteristic and applying NYSE cutoffs. When double-sorting, we form nine value-weight portfolios by sorting stocks independently on each characteristic and applying 30% and 70% NYSE cutoffs.

Estimating a portfolio's δ requires data on post-formation returns and capital gains over 180 months (15 years). Hence, for a portfolio formed at t, we track its monthly buy-and-hold returns and capital gains over $t+1, \ldots, t+180$.[20] That is, as illustrated in Figure 1, the post-formation returns on a portfolio formed in t are $R_{(t),t+1}$, $R_{(t),t+2}$, . . . , $R_{(t),t+3}$, following the diagonal arrow pointing southeast, where $R_{(t),t+j}$ denotes the time-$t+j$ return on buying and holding a portfolio formed at time t.

In summary, we construct three-dimensional data for each sorting characteristic: buy-and-hold monthly returns (and capital gains) over J post-formation months for T different months in which the post-formation return data are available for the full J holding periods, all together on N different portfolios. In contrast, a conventional short-horizon

[20]Post-formation returns at $t+j$ for the portfolio formed at t are from a buy-and-hold strategy that does not reinvest dividends into the same or different stocks. Section IA.C.4 in the Internet Appendix explains how to apply our identity correctly to a portfolio of stocks in an empirical analysis.

Table II. Descriptive statistics on stock characteristics. The table describes the ten characteristics we use to study CAPM-implied abnormal price. Column 2 reports the sample period over which post-formation returns for $j = 1$ through 180 months are available. Columns 3–5 report the CAPM alphas of the lowest and highest portfolio quintiles as well as the difference in the alphas between the two portfolios. Here, excess return is taken with respect to market returns. We report t-statistics based on heteroskedasticity-robust standard errors in parentheses. Column 6 reports Persistence, the value-weighted probability that the characteristic decile of a stock in the portfolio does not change after a year.

	Sample Period	α_{Low}^{CAPM}	α_{High}^{CAPM}	α_{H-L}^{CAPM}	Persistence
Book-to-Market	Jun48-Dec22	-0.04	0.25	0.29	0.67
		(-0.99)	(2.87)	(2.46)	
Quality	Jun48-Dec22	-0.28	0.17	0.46	0.56
		(-3.84)	(4.56)	(4.57)	
Adjusted Value	Jun48-Dec22	-0.29	0.47	0.76	0.59
		(-5.58)	(6.39)	(7.15)	
Size	Jun48-Dec22	0.03	0.01	-0.02	0.89
		(0.23)	(0.29)	(-0.15)	
Momentum	Jun48-Dec22	-0.64	0.33	0.97	0.23
		(-5.74)	(4.39)	(5.81)	
Net Issuance	Jun48-Dec22	0.24	-0.28	-0.51	0.49
		(5.39)	(-5.24)	(-6.23)	
Beta	Jun48-Dec22	0.24	-0.38	-0.62	0.63
		(3.58)	(-4.67)	(-4.94)	
Investment	Jun72-Dec22	0.25	-0.18	-0.43	0.35
		(3.30)	(-2.88)	(-3.75)	
Profitability	Jun72-Dec22	-0.08	0.09	0.17	0.77
		(-0.90)	(1.31)	(1.29)	
Accruals	Jun72-Dec22	0.04	-0.20	-0.24	0.37
		(0.57)	(-3.08)	(-2.14)	

return test uses two dimensional data, since it tracks just $J = 1$ month post-formation returns for T different months on N different portfolios.

Our baseline analysis uses post-formation returns over 1948m6–2022m12, where 1933m6 is the first month in which most of the characteristics other than accruals can be computed and 1948m6 is the first month in which the full 15 years of calendar-time observations of post-formation returns and capital gains are available (the horizontal arrows in Figure 1). Our modern subsample analysis uses 1972m6–2022m12. Table II provides descriptive statistics for the portfolios formed from a univariate sort on each of the nine characteristics we consider in the rest of the paper. The Internet Appendix provides further details on data construction.

B. *Initial analysis on book-to-market and quality*

Our initial analysis studies the book-to-market equity ratio (B/M) and *quality*, which the recent literature argues proxies for abnormal price. Golubov and Konstantinidi (2019) decompose B/M into the market-to-value ratio (abnormal price) and the value-to-book ratio using within-industry cross-sectional regressions of equity values on firm fundamentals, finding that the return predictability of B/M stems from the abnormal price component. Asness et al. (2019) measure *quality*, a z-score measure based on sixteen characteristics that rewards profitable, fast-growing, safe, and high-payout stocks and find that quality predicts price-level distortions, measured by cumulative abnormal returns over five years.

Looking first at returns, Rows 1 and 2 of Table II show that the CAPM does a poor job explaining the cross-section of returns on quintile portfolios sorted on B/M or *quality*. The long-short portfolio based on B/M generate an annualized CAPM alpha of 3.5% and the long-short portfolio based on *quality* generates an annualized alpha of 5.5%. In conjunction with the fact that both B/M and *quality* are relatively persistent characteristics, these two anomalies are natural candidates to generate significant price-

Table III. Pricing B/M- or $Quality$-sorted portfolios: returns vs. prices. The table shows that the book-to-market equity ratio (B/M) and *quality* (from Asness et al. (2019)) are not statistically significant univariate signals of abnormal price relative to the CAPM (the last row), although they are significant signals of CAPM alpha (the first row). We form value-weight quintile portfolios based on NYSE breakpoints and track post-formation returns for 15 years (180 months). In the first "return" row, δ measures -1 times the average one-month abnormal return:

$$\delta(1) \;=\; -E\left[\widetilde{M}_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of abnormal price defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \;\approx\; \delta(180) = -E\left[\sum_{j=1}^{180} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where V_t is the portfolio's buy-and-hold value implied by the CAPM, $(t-j)$ is the portfolio formation month, t is the month in which returns are realized, and j is the number of post-formation months. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. The last two columns report $\delta_{H-L} \equiv \delta_{High} - \delta_{Low}$ and its risk-neutral counterpart (Definition 1). We use the candidate SDF implied by the CAPM, $\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r_{t-s}^{mkt}\right)$, where r_t^{mkt} is log market returns and b_0 and b_1 are chosen to make the market portfolio's prices ($\delta = 0$) and returns ($\delta(1) = 0$) correct in sample. We report t-statistics (in parentheses) based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12.

A. Book-to-Market

J	δ_{Low}	δ_2	δ_3	δ_4	δ_{High}	δ_{H-L}	δ_{H-L}^{RN}
1mo	0.04	0.02	-0.14	-0.16	-0.24	-0.28	-0.25
("return")	(0.97)	(0.44)	(-2.65)	(-2.30)	(-2.77)	(-2.38)	(-2.21)
1yr	0.52	-0.23	-1.22	-2.38	-2.81	-3.33	-2.97
	(0.78)	(-0.43)	(-1.40)	(-2.19)	(-2.34)	(-1.92)	(-1.88)
3yrs	0.70	-0.79	-2.07	-6.43	-7.89	-8.58	-7.38
	(0.30)	(-0.56)	(-0.77)	(-2.00)	(-2.22)	(-1.54)	(-1.54)
5yrs	1.90	-0.59	-3.53	-9.68	-12.64	-14.53	-12.87
	(0.44)	(-0.25)	(-0.82)	(-1.77)	(-1.97)	(-1.39)	(-0.08)
10yrs	4.58	1.30	-1.42	-14.42	-17.61	-22.19	-23.99
	(0.55)	(0.28)	(-0.15)	(-1.41)	(-1.41)	(-1.09)	(-1.17)
15yrs	5.98	1.59	-2.14	-15.31	-21.19	-27.18	-29.94
("price")	(0.49)	(0.23)	(-0.16)	(-1.08)	(-1.26)	(-0.96)	(-1.54)

B. Quality

J	δ_{Low}	δ_2	δ_3	δ_4	δ_{High}	δ_{H-L}	δ_{H-L}^{RN}
1mo	0.29	0.04	0.06	-0.03	-0.17	-0.46	-0.26
(“return”)	(3.85)	(0.65)	(1.51)	(-0.86)	(-4.55)	(-4.58)	(-2.66)
1yr	2.49	0.61	0.71	-0.65	-1.88	-4.37	-2.27
	(2.78)	(0.71)	(1.19)	(-1.52)	(-3.31)	(-3.24)	(-1.78)
3yrs	4.69	1.31	1.44	-1.17	-3.79	-8.48	-2.81
	(1.90)	(0.53)	(0.99)	(-0.84)	(-2.05)	(-2.12)	(-0.81)
5yrs	4.19	1.37	1.39	-1.28	-3.74	-7.93	1.53
	(1.02)	(0.31)	(0.49)	(-0.63)	(-1.09)	(-1.12)	(0.03)
10yrs	7.83	4.70	3.03	-1.78	-5.27	-13.10	6.61
	(0.86)	(0.57)	(0.43)	(-0.69)	(-0.74)	(-0.83)	(0.58)
15yrs	6.09	8.25	4.49	-4.13	-5.32	-11.41	19.58
(“price”)	(0.48)	(0.71)	(0.41)	(-1.14)	(-0.52)	(-0.51)	(1.32)

level errors as well.

Table III provides a formal analysis of prices, estimating quintile δs and their difference between the “high” and “low” quintile portfolios ($\delta_{H-L} \equiv \delta_{High} - \delta_{Low}$), along with t-statistics (in parentheses) for $J \in \{1\text{mo}, 1\text{yr}, 3\text{yrs}, 5\text{yrs}, 10\text{yrs}, 15\text{yrs}\}$. The $J = 1$ month estimates (the first row of each panel) recovers results close to the conventional time-series return regression results in Table II after the appropriate sign change, but with a loglinear model of the SDF, whereas $J = 15$ years (180 months; the last row of each panel) proxies for price-level results given by $J \to \infty$.[21] The intermediate values of J allow us to see how the performance of the asset pricing model changes as the return horizon increases gradually from 1 month to 15 years.

The results for B/M in Panel A of Table III show that value stocks are undervalued relative to growth stocks only from the perspective of CAPM investors with a short investment horizon of $J = 1$ month. Beyond an investment horizon of 1 month, B/M is a weak signal of CAPM δ. In particular, the price-level result with $J = 15$ years in

[21]Figure IA.4 in the Internet Appendix shows that $J = 15$ years captures most, if not all, of the consequences of the post-formation abnormal returns associated with characteristics we study.

the last row shows that value stocks are 27.2 percentage points underpriced relative to growth stocks but with a t-statistic of 0.96.

Turning to *quality*, Panel B of Table III shows that high-quality stocks are undervalued and low-quality stocks are overvalued from the perspective of CAPM investors with an investment horizon of $J = 3$ year or less. However, for $J = 5$ or more years, the estimated δs are statistically indistinguishable from zero for all quality-sorted portfolios and imply that the market price correctly accounts for the quality difference. For example, for $J = 15$, we find that high-quality stocks are only 11.4 percentage points underpriced relative to low-quality stocks with an associated t-statistic of 0.51. Our finding based on an exact definition of price distortion δ is contrary to the conclusion drawn by Asness et al. (2019), whose analysis instead studies either cumulative five-year abnormal returns or a cross-sectional regression of the M/B ratio on quality.

Part of the reason that we find insignificant δ is that accounting for market risk reduces the CAPM-implied δ. Comparing the last two columns for $J = 1$ month shows that accounting for market risk leads to a larger spread in risk-adjusted returns than that in simple returns. In contrast, at $J = 15$ years, CAPM δs are smaller in magnitude than their corresponding risk-neutral δ for both B/M and *quality*, albeit to a small extent for B/M. Figure 3 makes this point graphically: whereas CAPM-implied risk and average excess returns tend to have an anomalous negative relation over a one-month post-formation horizon, they have a flat (for *quality*) or positive (for B/M) relation over a long horizon.[22]

Figure 4 plots the post-formation behavior of excess returns and market betas to analyze why risk and return tend to have a less anomalous relation over a long horizon.

[22]Figure 3 plots the long-horizon components of risk and return in equation (23) for direct comparison with the corresponding short-horizon measures. We thank an anonymous referee for this suggestion. Figure IA.8 shows that the cumulative state adjustment is not a negligible component of δ, although this component typically has an absolute magnitude below 10% and does not have a strong univariate cross-sectional relation to δ.

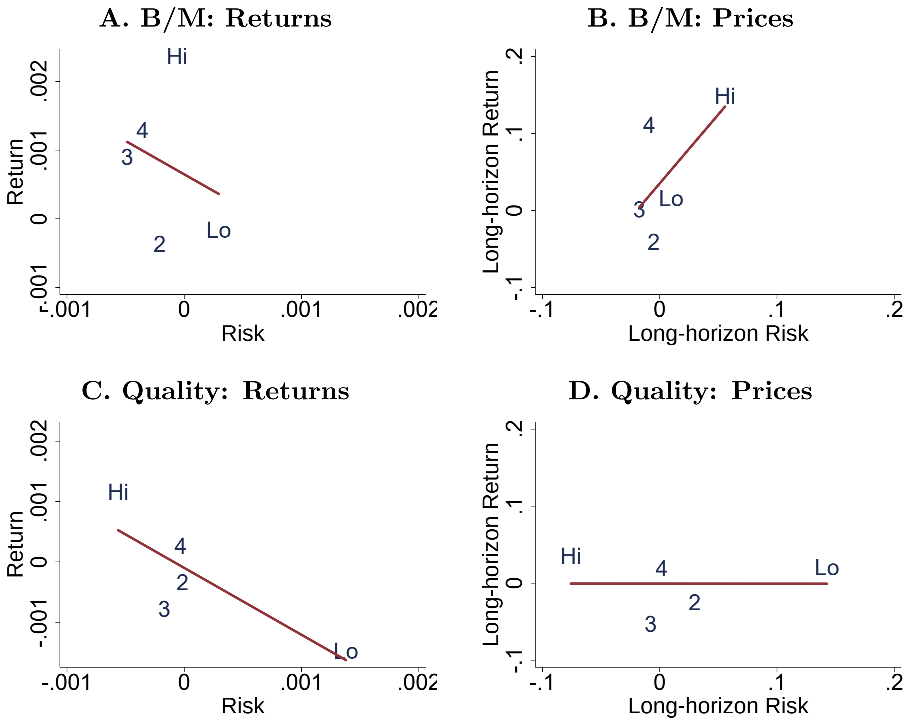

Figure 3. The risk-return relations in returns and price levels: book-to-market and quality. The plots show that, for portfolios sorted on book-to-market or quality, the relation between long-horizon risk and long-horizon return (Panel B and D) tends to be less anomalous than that between short-horizon risk and return (Panel A and C). This improvement contributes to the statistically insignificant CAPM abnormal price associated with book-to-market and quality sorts. Long-horizon return and long-horizon risk summarize the term structure of post-formation average excess returns and of risk premia, respectively, as in equation (23). The sample period is 1948m6–2022m12.

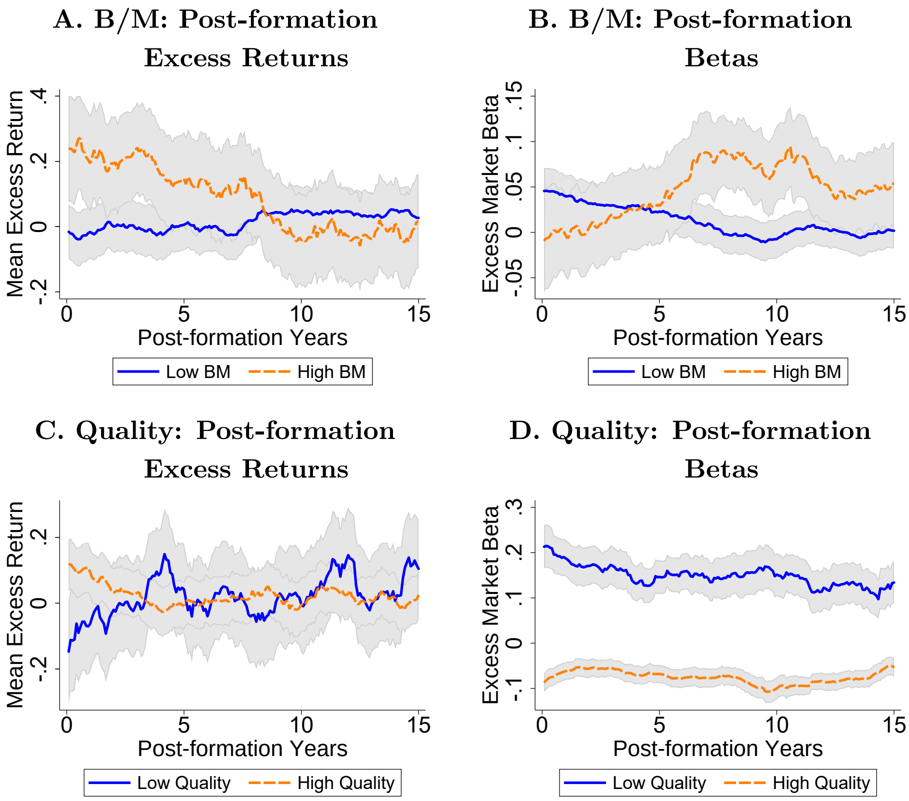

Figure 4. Post-formation behavior of return and risk: book-to-market and quality. The plots study the post-formation behavior of returns and CAPM betas of extreme quintile B/M or *quality* portfolios. The top plots (Panels A and B) show that the post-formation beta of value (high BM) stocks exceeds that of growth (low BM) stocks from around year 5, consistent with value stocks having higher post-formation mean returns than growth stocks until around year 8. The bottom plots (Panels C and D) show that junk (low *quality*) stocks have higher mean returns than quality stocks from around year 3, consistent with junk stocks having higher betas than quality stocks post formation. Excess returns used in the left panel are taken relative to post-formation returns on the market portfolio. The sample period is 1948m6–2022m12.

These plots show that a contributing factor for the smaller spread in δs for B/M-sorted portfolios is that value stocks have slightly higher betas than growth stocks from year five, allowing the higher long-horizon risk of value stocks to exceed that of growth stocks.[23] As for *quality*-sorted portfolios, junk stocks have persistently higher betas than quality stocks, which lines up with the fact that junk stocks have higher returns than quality stocks for most of their 15-year post-formation horizon.

C. Primary sorting characteristic: adjusted value

A more powerful test on price levels requires test assets that a priori are likely to exhibit large variation in δ. One way to generate more spread in δ is by adjusting the traditional value signal (B/M) for the effect of profitability and risk.

Intuitively, the present-value logic says that a stock could be cheap (i.e., B/M high) because it is (i) expected to have low dividend growth, (ii) risky, or (iii) truly undervalued. And for (i), profitable stocks with high expected future returns on equity will tend to have faster dividend growth as more earnings per book equity gets plowed back. Therefore, cheap stocks that are nonetheless (i) profitable and (ii) safe are likely underpriced, an idea we capture in *adjusted value*:

$$\textit{adjusted value} \equiv \underbrace{z(B/M)}_{\text{value (cheap)}} + \underbrace{z(\text{Prof})}_{\text{profitable}} - \underbrace{z(\text{Beta})}_{\text{risky}}, \tag{27}$$

where z denotes the z-score of the characteristic's cross-sectional rank.

Formally, the loglinear present-value model of Vuolteenaho (2002) implies that a stock's CAPM-implied log value ($v_t = \log V_t$) is the log book value (b_t) plus expected log clean-surplus returns on equity ($E_t roe^{cs}_{t+j+1}$) minus CAPM-implied discount rates

[23] That is, the insignificant delta associated with B/M is not merely an outcome of post-formation alpha decay over time but of a small reversion in post-formation alphas.

($E_t r_{t+j+1}$):

$$v_t \approx b_t + \sum_{j=0}^{\infty} \rho^j E_t roe^{cs}{}_{t+j+1} - \sum_{j=0}^{\infty} \rho^j E_t r_{t+j+1} \tag{28}$$

Since CAPM underpricing is the deviation of CAPM-implied value from price, it follows that a characteristic that adds the z-scores of B/M and profitability and subtracts the z-score of beta should proxy for CAPM underpricing:

$$\begin{aligned} v_t - p_t \quad &\approx \quad \underbrace{b_t - p_t}_{\text{book-to-market}} + \underbrace{\sum_{j=0}^{\infty} \rho^j E_t roe^{cs}{}_{t+j+1}}_{\text{profitability}} - \underbrace{\sum_{j=0}^{\infty} \rho^j E_t r_{t+j+1}}_{\text{beta}} \\ &\propto \quad z(B/M) \; + \; z(\text{Prof}) \; - \; z(\text{Beta}), \end{aligned} \tag{29}$$

which equals *adjusted value* in equation (27).[24,25]

Our three-characteristic model of CAPM underpricing is similar in spirit to 'quality at a reasonable price' in Asness et al. (2019) but addresses the potential concern that a composite measure based on sixteen characteristics plus B/M can be difficult to interpret and subject to overfitting. Adjusted value is also related to the idea in Piotroski and So (2012) that one can isolate the underpricing component of B/M through its interaction with proxies for the stock's future fundamentals.[26] Although our baseline analysis uses the z-score of *current* gross profitability of Novy-Marx (2013) to measure expected *future* profitability, we explore an alternative approach that more directly proxies for future profitability as well.

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[24] We use the z-score of current gross profitability to proxy for future profitability. When gross profitability data are unavailable, we use the z-score of return on equity.

[25] Cho et al. (forthcoming) derive a loglinear present-value identity that links today's market-to-book equity ratio to future investment ("scale"), profitability ("yield"), and discount rates. Their more granular decomposition suggests that separately controlling for firms' scale and yield characteristics may provide a better adjustment of B/M than our current adjusted value measure.

[26] See also Frankel and Lee (1998), Piotroski (2000), Cohen et al. (2003), Polk et al. (2006), Novy-Marx (2013), and Gonçalves and Leonard (2023).

D. *Adjusted value and the cross-section of price levels*

Table IV shows that portfolios sorted on *adjusted value* are economically and statistically mispriced at every horizon we consider. The first row shows that high-*adjusted-value* stocks outperform low-*adjusted-value* stocks by 75 basis points a month with an associated t-statistic of 6.88. At the 15-year horizon, low-*adjusted-value* stocks are 51.9 percentage points more overpriced than their high-*adjusted-value* counterparts. The large CAPM δ difference seems to arise from the CAPM risk adjustment to long-horizon returns on high vs. low adjusted-value portfolios being too large (i.e., the ex-post security market line being too flat), since the difference in risk-neutral δ is much smaller at 8.5 percentage points. Indeed, consistent with this interpretation, Figure IA.7 Panel P (Internet Appendix) visually confirms the strong negative relation between long-horizon risk and long-horizon return for portfolios sorted on adjusted value. Thus, Table IV documents exactly the sort of variation a buy-and-hold CAPM investor should be interested in exploiting and represents a challenge for future asset pricing models that make understanding price-level risk a priority.[27]

We find that the exact way in which information in the three characteristics is used matters less. For instance, defining *adjusted value* in a way that puts more weight on the "price" component of adjusted value rather than putting an equal weight on all three z scores generates similar results. We also find that using VAR-implied expected future profitability z-score instead of current profitability z-score slightly improves the performance of the *adjusted value* characteristic (columns three and four in the table).[28]

[27]Table IA.VI uses double sorts to generate variation in adjusted value.

[28]The improvement is relatively small, since profitability is one of the most persistent among the characteristics we consider, making current profitability a strong predictor of its future values (Table II). Hence, for parsimony as well as to facilitate replication of our work, we present the *adjusted value* characteristic based on current profitability as the baseline. The Internet Appendix details how we modify the baseline approach through a VAR model of characteristic z scores to use information about expected future profitability.

Table IV. Pricing *Adjusted-value*-sorted Portfolios: Returns vs. Prices. The table shows that *adjusted value*, our proxy for the value-to-price ratio (V/P), generates cross-sectional variation in price levels (the last rows) and returns (the first row) not explained by the CAPM. *Adjusted value* combines B/M, profitability, and beta by taking the sum of their z scores: *Adjusted value* $= z(B/M) + z(\text{Prof}) - z(\text{Beta})$. We form value-weight quintile portfolios based on NYSE breakpoints for *adjusted value* and track post-formation returns for 15 years (180 months). In the first "return" row, δ measures -1 times the average one-month abnormal return:

$$\delta(1) \;\; = \;\; -E\left[\widetilde{M}_t R_t^e\right].$$

Positive (negative) δ here means negative (positive) one-month abnormal return. The reported δs in the last row are estimated values of abnormal price defined as

$$\delta = E\left[\frac{P_t - V_t}{P_t}\right] \;\; \approx \;\; \delta(180) = -E\left[\sum_{j=1}^{180} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right],$$

where V_t is the portfolio's buy-and-hold value implied by the CAPM, $(t-j)$ is the portfolio formation month, t is the month in which returns are realized, and j is the number of post-formation months. In the remaining rows, the reported δ estimates illustrate how the estimated $\delta(J) = -E\left[\sum_{j=1}^{J} \widetilde{M}_{t-j,t} \frac{P_{(t-j),t-1}}{P_{(t-j),t-j}} R^e_{(t-j),t}\right]$ changes as J takes values less than 180. The last two columns report $\delta_{H-L} \equiv \delta_{High} - \delta_{Low}$ and its risk-neutral counterpart (Definition 1). We use the candidate SDF implied by the CAPM, $\widetilde{M}_{t-j,t} = \exp\left(b_0 j - b_1 \sum_{s=0}^{j-1} r_{t-s}^{mkt}\right)$, where r_t^{mkt} is log market returns and b_0 and b_1 are chosen to make the market portfolio's prices ($\delta = 0$) and returns ($\delta(1) = 0$) correct in sample. We report t-statistics (in parentheses) based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12.

J	δ_{Low}	δ_2	δ_3	δ_4	δ_{High}	δ_{H-L}	δ_{H-L}^{RN}
1mo	0.29	-0.00	-0.17	-0.23	-0.46	-0.75	-0.39
("return")	(5.46)	(-0.01)	(-3.43)	(-4.01)	(-6.11)	(-6.88)	(-3.82)
1yr	3.24	0.10	-1.96	-3.04	-5.02	-8.26	-4.12
	(4.39)	(0.16)	(-3.01)	(-3.51)	(-5.24)	(-5.75)	(-3.03)
3yrs	7.08	0.76	-4.91	-8.06	-12.28	-19.37	-6.98
	(3.54)	(0.44)	(-2.24)	(-3.35)	(-4.34)	(-4.69)	(-1.74)
5yrs	9.34	2.45	-6.90	-12.17	-18.92	-28.26	-8.95
	(3.59)	(1.05)	(-2.03)	(-3.66)	(-3.57)	(-4.15)	(-0.05)
10yrs	14.72	0.44	-9.04	-20.74	-26.88	-41.60	-11.17
	(3.79)	(0.13)	(-2.07)	(-3.49)	(-3.05)	(-3.63)	(-0.62)
15yrs	18.45	2.32	-13.15	-29.92	-33.41	-51.86	-8.54
("price")	(3.33)	(0.47)	(-2.80)	(-2.46)	(-2.69)	(-3.14)	(-0.47)

At the same time, accounting for all three characteristics—B/M, profitability, and beta—is critical in isolating the large price-level variations unexplained by the CAPM. Interacting the z-scores of only two of the three characteristics—i.e., profitability and beta, B/M and beta, and B/M and profitability—fail to generate a statistically significant CAPM δ.

V. Are Return Anomalies Price Anomalies?

This second and final empirical section studies the extent to which the CAPM explains price-level variation associated with nine additional characteristics known to be associated with cross-sectional variation in average returns: net issuance, investment, accruals, beta, size, momentum, and profitability. The first four are chosen for their potential conceptual link to price-level distortions, while the next three are chosen for being prominent return anomalies (in conjunction with value, investment, and the market factor, they make up the widely-used Fama-French-Carhart six-factor model).

A. Characteristics conceptually related to abnormal price

Certain characteristics are interesting to analyze using our abnormal price measure either due to their conceptual association with abnormal price (net issuance, investment, and accruals) vis-à-vis the endogenous choices of managers or their mechanical link to the long-horizon risk component of abnormal price in Corollary 4 (beta). We explain the conceptual link that each characteristic has to abnormal price δ and study the extent to which the characteristic is associated with price-level variation that cannot be explained by exposure to market risk that the CAPM captures.

A.1. Net share issuance

A large literature in behavioral corporate finance views securities market mispricing as a primary factor in managerial financing and investment decisions.[29] In particular, several papers document evidence that share repurchase (issuance) indicate undervaluation (overvaluation) as perceived by firm managers (e.g., Loughran and Ritter (1995); Ikenberry et al. (1995)). Nevertheless, this hypothesis has not been tested using our definition of stock mispricing that explicitly accounts for the asset pricing model of risk (in our case, the CAPM): $\delta = E\left[(P_t - V_t)/P_t\right]$.

Table V shows that the spread in mispricing δs associated with net issuance is indeed large.[30] Share repurchases (low net issuance) are especially strong signals of CAPM underpricing, consistent with CFOs identifying market timing as the number one reason for stock repurchases (Brav et al. (2005)). In contrast, share issuances (high net issuance) are not strongly indicative of CAPM overpricing, and this weaker result may reflect that firm CFOs use stock issuance primarily to finance investment projects (Graham and Harvey (2001)).[31] The difference in δ's across the two extreme net issuance quintiles is 23.7 percentage points with a large t-statistic of 2.72. Similarly to the *adjusted value* sort, the risk-neutral delta difference is small and statistically insignificant, indicating that the implicit CAPM long-horizon risk adjustment is an important contributor to the difference in deltas that we find. A similar finding holds for the other two significant delta characteristics discussed below, investment and beta, highlighting that the flat security line continues to play a role in our price-level analysis.[32]

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[29] Baker et al. (2007) review this literature.

[30] Figure IA.7 in the Internet Appendix visualizes the cross-sectional relations between long-horizon risk and long-horizon return for the return anomalies studied in this section.

[31] The CFOs surveyed in the study identify market timing as the number two reason for stock issuance.

[32] Since the focus of our paper is to explain our novel method and document the corresponding empirical facts, we leave a more complete analysis of the flat price-level security market line to future work.

Table V. Pricing Anomaly-sorted Portfolios. The table reports estimated abnormal price with respect to the CAPM for portfolios sorted on characteristics conceptually linked to abnormal price or prominent return anomaly characteristics. For each characteristic, we form value-weight quintile portfolios based on NYSE breakpoints and track post-formation returns for 15 years. High (Low) denotes stocks with the highest (lowest) value of the characteristic. The reported δs are estimated values of CAPM abnormal price defined as $\delta = E\left[\frac{P_t - V_t}{P_t}\right]$, where V_t is the portfolio's buy-and-hold value implied by the CAPM. The last two columns report $\delta_{H-L} \equiv \delta_{High} - \delta_{Low}$ and its risk-neutral counterpart (Definition 1). We report t-statistics (in parentheses) based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12 except for investment, accruals, and profitability, which have a sample period of 1972m6–2022m12.

Sort	δ_{Low}	δ_2	δ_3	δ_4	δ_{High}	δ_{H-L}	δ_{H-L}^{RN}
Net issuance	-16.4	-4.2	2.0	-0.2	7.2	23.7	8.6
	(-2.67)	(-0.40)	(0.52)	(-0.03)	(1.08)	(2.72)	(0.93)
Investment	-17.6	-17.8	-3.0	9.0	11.8	29.4	16.7
	(-2.33)	(-2.50)	(-0.68)	(1.67)	(1.43)	(2.11)	(1.08)
Accruals	0.2	-11.7	0.2	4.9	20.9	20.7	9.4
	(0.02)	(-1.97)	(0.03)	(0.93)	(1.98)	(1.22)	(0.59)
Beta	-22.6	-15.9	-4.9	5.3	18.2	40.8	-26.9
	(-1.88)	(-2.19)	(-1.11)	(1.00)	(2.05)	(2.18)	(-1.21)
Size	-13.4	-16.9	-20.6	-13.5	3.5	17.0	54.6
	(-0.53)	(-0.92)	(-1.19)	(-1.14)	(1.04)	(0.60)	(1.91)
Momentum	-16.5	-7.7	-3.7	2.6	4.3	20.8	22.5
	(-1.15)	(-1.89)	(-1.10)	(0.53)	(0.74)	(1.49)	(1.85)
Profitability	13.3	-9.3	-14.4	-5.0	4.4	-9.0	2.3
	(0.63)	(-0.66)	(-1.27)	(-0.41)	(0.24)	(-0.25)	(0.08)

A.2. Investment

Arguably the most important anomaly to study in this context is investment, given the potential link between misvaluation and the allocation of capital by firms to real investment projects. That link may occur indirectly, through the equity issuance decision (Stein (1996), Baker and Wurgler (2002), Baker et al. (2003)), or directly, through catering by the firm to investor sentiment (Polk and Sapienza (2009)). Thus, it is naturally interesting to measure whether price-levels are also anomalous for portfolios sorted on investment, as measured by asset growth. The high investment and the low investment quintiles have a statistically significant difference in δ's of 29.4 percentage points, confirming the link between investment and price-level mispricing.

A.3. Accruals

Earnings management proxied by accruals (Sloan (1996)) is an interesting phenomenon to revisit with our explicit mispricing definition, as its typical interpretation is that companies with adverse operating results manage earnings to inflate the firm's market value. Thus, if the firms are successful in managing earnings, high accruals may proxy for overpricing perceived by firm managers. The results in Table V do not support this interpretation of accruals, as it is a statistically weak predictor of delta.

A.4. Beta

Equation (23) shows that long-horizon risk, defined as a discounted sum of contemporaneous covariances between excess returns and the candidate SDF $\widetilde{M}$, helps determine abnormal price δ. Hence, the persistence of market beta implies that market beta sorts have the potential to generate large variation in $\delta = E\left[\left(P_t - V_t\right)/P_t\right]$. In particular, we would find that sorts on beta generate spread in δ if the resulting spread in long-horizon risk that must generate spread in CAPM-implied V is not compensated by a corresponding spread in price (P).

Table V shows an estimated difference in δs of 40.8 percentage points across the high- and low-beta portfolios, consistent with the above prediction. Furthermore, this estimate is statistically significant with a t-statistic of 2.18. As discussed above, the flat security market line plays a particularly important role in our analysis, and the results for beta underscore this interpretation as risk-neutral δ is of the opposite sign as risk-adjusted δ.

B. Prominent return anomalies

Next, we turn to three remaining prominent return anomalies. Fama and French (2015) argue that profitability and size are characteristics that are important in summarizing the cross-section of returns, and price momentum has been a prominent return anomaly since Jegadeesh and Titman (1993). To what extent are these prominent return characteristics associated with variation in price levels unrelated to CAPM price-level risk?

B.1. Size and momentum

Size and momentum are interesting to study from the price-level perspective, given that momentum strongly predicts the cross-section of average returns but is a rather transitory firm characteristic while size weakly predicts the cross-section of average returns but is a rather persistent firm characteristic. In particular, Cohen et al. (2009) highlight that signal persistence is an important consideration when moving from the conventional return perspective to the price-level perspective, a point that Cochrane (2011) subsequently emphasizes.

> "For example, since momentum amounts to a very small time-series correlation and lasts less than a year, I suspect it has little effect on long-run expected returns and hence the level of stock prices. Long-lasting characteristics are likely to be more important. Conversely, small transient price errors can have a large impact on return measures" (p.1064).

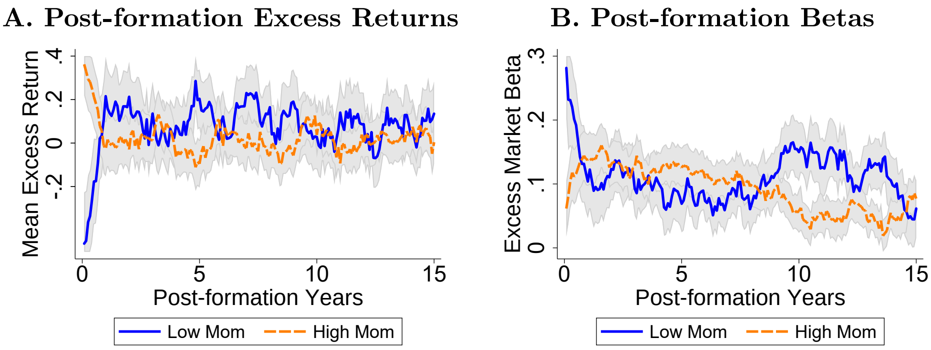

Figure 5. Post-formation behavior of return and risk: momentum. The plots study the post-formation behavior of returns and CAPM betas of extreme quintile momentum portfolios. They show that the anomalous pattern of high momentum stocks having higher excess returns but lower market betas than low momentum stocks quickly reverses such that low momentum stocks have higher excess returns and lower betas on those excess returns than high momentum stocks from around year 2 post portfolio formation. Excess return is measured relative to the post-formation returns on the market portfolio. The sample period is 1948m6–2022m12.

Consistent with Cochrane's conjecture, Table V shows that momentum is not a statistically significant predictor of CAPM abnormal price: the difference in δ between the high momentum and low momentum portfolios is 20.8 percentage points with a t-statistic of 1.49. Moreover, the point estimates suggest that high momentum stocks with large positive abnormal returns are overpriced, whereas low momentum stocks with negative abnormal returns are underpriced.[33] This finding is consistent with momentum profits often coming from continued overreaction.[34]

Figure 5 visualizes how momentum's initial positive α quickly turns into negative α

[33]The point estimates in van Binsbergen et al. (2023) are consistent with momentum stocks being overpriced as well. In contrast to our results, their analysis finds statistical significance, suggesting the importance of our method's more reliable statistical inference.

[34]Lou and Polk (2022) provide extensive analysis arguing that momentum can transition from an underreaction to an overreaction phenomenon in the presence of destabilizing activity by momentum traders. Blank et al. (2023) document similar findings in the cross-section for those stocks that are held relatively more by investors who react excessively to salient public news. Our price-level measure of mispricing δ can facilitate those sorts of empirical refinements of under and overreaction phenomena.

post formation. Low momentum stocks start having higher average returns and lower CAPM betas compared to high momentum stocks from around year 2, contributing to momentum being associated with overpricing. While size is a persistent characteristic, it generates small and statistically insignificant price-level variation unaccounted by the CAPM.

B.2. Profitability

Table V documents that profitability-sorted portfolios are associated with price-level errors that are statistically insignificant with an estimated δ spread of only 9.0 percentage points. The result is consistent with the observation that cross-sectional variation in the marginal product of capital, which our profitability measure could proxy for, does not necessarily imply a misallocation of capital (Joel et al. (2022)).

C. Why are (some) return anomalies not also price anomalies?

Having studied ten characteristics individually from the price-level perspective and estimated their CAPM deltas, we now study them together to understand the factors that determine whether a return anomaly is also a price anomaly.

Taking two extreme quintile portfolios from each of the ten characteristics, we run a cross-sectional regression of CAPM delta on variables motivated by our identity:

$$\delta_i = b_0 + b_1\,\alpha_i + b_2\left[\alpha_i \times \mathbf{1}(Reversal_i)\right] + b_3\,\beta_i + b_4\,CumStateAdj_i + \epsilon_i, \tag{30}$$

where α is the short-horizon (1-month) alpha, $\mathbf{1}(Reversal)$ is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month, β is the CAPM beta of the anomaly's excess monthly return in the first post-formation month, and $CumStateAdj$ is the cumulative state adjustment from the decomposition in

Corollary 4.[35] We include β as an incremental predictor of delta because a large initial β is likely to predict more persistent post-formation alphas. That is, since a portfolio's CAPM β is persistent, a flat security market line could cause a portfolio starting out with a large initial β to have persistent post-formation alphas. Table IA.VIII and Figure IA.9 of the Internet Appendix show that these four variables explain 94% of the cross-sectional variation in the deltas of the twenty portfolios in question.

Figure 6 plots the component of fitted delta associated with each of these four explanatory variables for all ten long-short portfolios; for example, $\widehat{b}_1 \times (\alpha_L - \alpha_S)$ is the component of long-short delta explained by short-horizon alpha and is plotted with a black bar for each anomaly. Given the high R^2 in column (4) of Table IA.VIII, the four components together explain almost all of the cross-sectional variation in delta.

Figure 6 highlights four findings. First, short-horizon alphas explain only a small fraction of the variation in abnormal price. Second, the largest price anomalies—*adjusted value* and beta—are those with a large initial beta, highlighting the importance of a flat security line in our price-level analysis. Third, the other two significant price anomalies—net issuance and investment—have delta contributions coming from a large initial alpha, a large initial beta, and (for investment) a large cumulative state adjustment. Although a sort on investment leads to an eventual reversal in returns, the other components of delta are enough to offset this effect, making investment a significant price anomaly. Finally, the return anomalies that do not make the cut for a significant delta either have weak contributions from short-horizon alpha and beta (e.g., B/M, profitability, size, accruals) and/or an offsetting effect from return reversal (B/M, quality, profitability, and momentum).

[35]Note that excess returns are measured in excess of the market return.

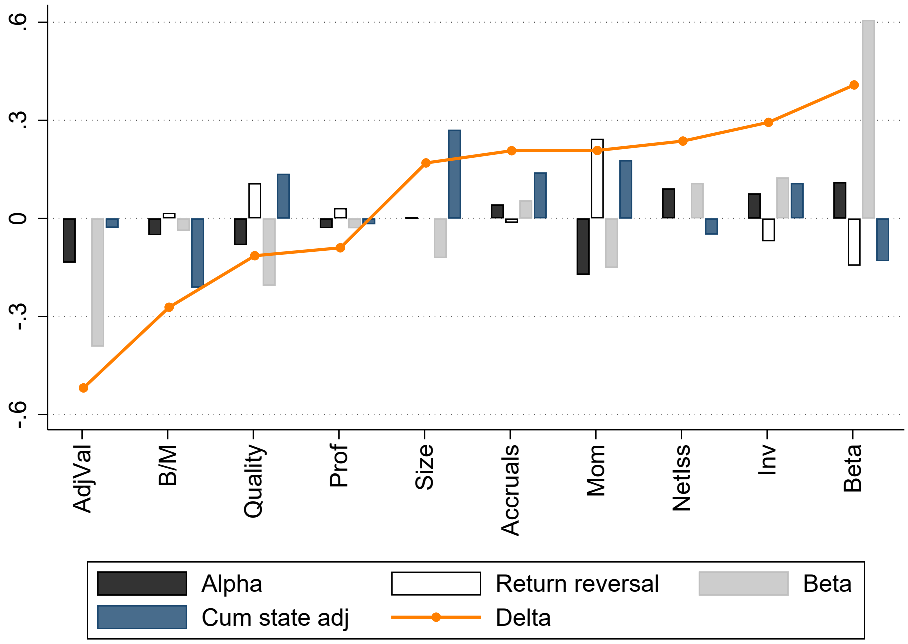

Figure 6. Why are (some) return anomalies not also price anomalies? The figure plots the extent to which short-horizon alpha, post-formation return reversal, short-horizon beta, and cumulative state adjustment explain portfolio long-short delta. The fitted values are based on a cross-sectional regression of delta on these four explanatory variables (Table IA.VIII; see also Figure IA.9), and each vertical bar measures the component of fitted value due to a particular explanatory variable. The sum across the four vertical bars for each characteristic portfolio is the fitted delta, with any remaining difference representing the regression residual. Overall, the figure shows that short-horizon beta contributes the most to delta for the two largest price anomalies (adjusted value and beta), whereas the other two statistically significant price anomalies (net issuance and investment) are driven by multiple factors. The other characteristics are not associated with a significant abnormal price typically because different components offset each other's effect (e.g., short-horizon alpha is offset by a subsequent reversal in returns). Return reversal is a dummy variable that takes the value of one if the average excess return in years three-to-15 following portfolio formation is opposite in sign to the average excess return in the first post-formation month. Short-horizon beta is the portfolio's one-month market beta immediately following portfolio formation and matters because beta tends to be persistent and large betas tend not to be accompanied by large excess returns (i.e., the security market line is flat). Cumulative state adjustment is defined in Section III.

D. Double sorts on characteristics and adjusted value

Our proposed *adjusted value* characteristic proxies for abnormal price by combining information in price, profitability, and beta. How well, then, does it explain variation in δ generated by other characteristics: net issuance, investment, accruals, size, and momentum?

Table VI synthesizes our analysis by examining double sorts on *adjusted value* and each of the five characteristics. Specifically, sorting stocks into three-by-three portfolios based on independent NYSE breakpoints, we report the δ and associated t-statistic for each of the nine portfolios on the left-hand side of the table. The right-hand-side of the table reports the δ's associated with the combination of the nine portfolios that results in either a characteristic-neutral portfolio that bets on *adjusted value* or a *adjusted-value*-neutral portfolio that bets on the second characteristic.

Table VI. Incremental Information About Prices: *Adjusted Value* vs. Others. The table shows that controlling for *adjusted value*, our proxy for the value-to-price ratio (V/P), subsumes the ability of other characteristics to predict CAPM abnormal price. In contrast, *adjusted value* retains its ability to predict CAPM abnormal price when controlling the other characteristic in question. To draw this conclusion, we form nine value-weight portfolios based on 30% and 70% NYSE breakpoints for *adjusted value* and, independently, the 30% and 70% NYSE breakpoints for the second sorting characteristic specified in column one. We study the five characteristics that do not comprise *adjusted value*. *Adjusted value* combines B/M, profitability, and beta by taking the sum of their z scores: *Adjusted value* $= z(B/M) + z(\text{Prof}) - z(\text{Beta})$. The left-hand side of the table reports the estimated δ and associated t-statistic for each portfolio. The right-hand-side of the table reports the δs associated with the combination of the portfolios that results in either a characteristic-neutral portfolio that bets on *adjusted value* or a *adjusted-value*-neutral portfolio that bets on the second characteristic. We report t-statistics (in parentheses) and p-values based on GMM standard errors that account for time-series and cross-sectional covariances in the data and uncertainty in estimating the parameters of the candidate SDF. The sample period is 1948m6–2022m12 except for investment and accruals, which has a sample period of 1972m6–2022m12.

	Adjusted value sort									*Adj val* sort (Second sort neutral)	Second sort (*Adj val* neutral)
	Low			2			High			$\frac{1}{3} * ((H1 + H2 + H3)$	$\frac{1}{3} * ((L3 + 23 + H3)$
Second sort →	1	2	3	1	2	3	1	2	3	$-(L1 + L2 + L3))$	$-(L1 + 21 + H1))$
Net issuance	10.7	16.6	11.6	-18.6	-8.7	-6.8	-37.7	-33.0	-29.0	-46.2	7.2
	(1.26)	(3.40)	(1.45)	(-2.53)	(-1.39)	(-1.12)	(-3.16)	(-2.63)	(-1.80)	(-2.80), [0.005]	(0.83), [0.408]
Investment	0.6	17.0	19.9	-25.1	-10.0	-3.9	-44.1	-41.4	-35.4	-52.8	16.4
	(0.05)	(2.17)	(2.29)	(-2.95)	(-1.42)	(-0.44)	(-2.80)	(-2.67)	(-2.16)	(-2.95), [0.003]	(1.62), [0.106]
Accruals	8.3	12.1	27.0	-14.5	-11.1	-1.0	-43.4	-33.9	-46.7	-57.2	9.6
	(0.79)	(1.90)	(2.46)	(-1.86)	(-1.23)	(-0.10)	(-1.94)	(-2.58)	(-2.16)	(-2.80), [0.005]	(0.81), [0.416]
Size	4.7	-3.8	15.7	-8.5	-24.6	-9.4	-41.4	-36.8	-33.7	-42.8	6.0
	(0.18)	(-0.25)	(3.25)	(-0.36)	(-1.49)	(-1.32)	(-1.49)	(-1.99)	(-2.34)	(-2.66), [0.008]	(0.25), [0.804]
Momentum	2.6	15.1	16.4	-25.5	-12.6	-4.4	-48.7	-33.6	-29.4	-48.6	18.1
	(0.34)	(2.26)	(2.54)	(-2.35)	(-2.09)	(-0.63)	(-2.06)	(-2.76)	(-2.28)	(-2.78), [0.005]	(1.61), [0.107]

Table VI has two important takeaways. First, across all of the rows, *adjusted value* repeatedly generates economically and statistically significant variation in CAPM δ. Hence, *adjusted value* appears to contain information about prices that is neither explained by CAPM-implied risk nor captured by another single characteristic. Second, after controlling for *adjusted value*, there is little incremental information about prices in the characteristics we study, in terms of the economic magnitude or the statistical significance. This finding is true even for net issuance and investment which both showed significant spread in δ in a univariate sort but are subsumed by adjusted value.[36]

E. Modern subsample

Our results and conclusions in Sections IV and V continue to hold in the modern subsample, 1972m6–2022m12. Combining information in price, profitability, and CAPM risk through our composite variable *adjusted value* describes economically and statistically significant variation in CAPM abnormal price (Tables IA.XIII and IA.XII in the Internet Appendix) while the other characteristics we study generally do not, with the exception of net issuance and beta (Tables IA.XI and IA.XIV).[37] Furthermore, in a horse race, all other characteristics are subsumed by *adjusted value* (Table IA.XV).

F. Alternative approaches

How do results change if we employ alternative approaches to estimating abnormal price? First, in terms of the point estimates, we show in Figure IA.6 that the cash-flow

[36]Our findings based on these double sorts suggest that adjusted value could be the natural second factor to put in a multifactor model of price levels. However, a proper treatment of all potential multifactor refinements would require a substantial expansion to the analysis. For this reason, we believe our single factor analysis based on the CAPM is one that most lucidly illustrates our novel return-based price-level analysis.

[37]Investment, accruals, and profitability are defined only over the modern subsample, so their results remain the same as before.

approach that directly discounts the cash flows and a terminal value generates essentially the same delta estimates as the version of our return-based approach that uses gross returns (rather than excess returns) and event-time observations (rather than calendar-time observations). This equivalence confirms that the cash-flow approach does not have an inherent advantage over the return-based approach but is less desirable in finite-sample inference. (See the discussion in Subsections I.C, III.B, and III.E.)

Second, Table IA.VII shows that cumulative abnormal return (CAR) generates different results, both in terms of magnitude and statistical significance at the 5% level. Third, Figure IA.9 shows that short-horizon CAPM alpha, its persistence (proxied by the persistence of the characteristic associated with that alpha), and the resulting interaction explain around half of the cross-sectional variation in delta across the twenty extreme quintile portfolios we study, implying that a simple measure based on short-horizon alpha and its persistence would miss important variation in abnormal price.

VI. Conclusion

Our novel identity precisely links ex-ante price distortion to subsequent returns and provides a new natural framework for studying the cross-section of stock price levels. Our primary tests reveal that portfolios formed on *adjusted value*, a composite signal that extracts the mispricing component from the market-to-book ratio, generate large variation in abnormal price, our measure of price distortion. Among all other prominent return anomalies, net equity issuance, investment, and beta sorts produce significant price-level distortions relative to the CAPM, and these distortions are subsumed by adjusted value.

As a consequence, our method identifies the stocks that a buy-and-hold investor who measures risk using a particular asset-pricing model—in our case, the monthly CAPM—should find the most attractive. Moreover, our approach highlights where new models that aim to explain both short- and long-run patterns in markets should focus. Indeed, by providing an exact metric of the extent to which a candidate asset-pricing model explains variation in prices, we aim to advance future research in both asset pricing and corporate finance. For the former, estimates of ex-ante price distortion could provide a useful lens through which to distinguish among risk-based, behavioral-based, and institutional-friction-based explanations for well-known empirical patterns in short-horizon returns. For the latter, our measure of mispricing with respect to a risk model may refine the results of a large literature (e.g. Baker and Wurgler (2002) and Shleifer and Vishny (2003)) that aims to link a firm's investment and financing decisions to price distortions.

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