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The Booms and Busts of Beta Arbitrage

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Abstract. Low-beta stocks deliver high average returns and low risk relative to high-beta stocks, an opportunity for professional investors to "arbitrage" away. We argue that beta-arbitrage activity generates booms and busts in the strategy's abnormal trading profits. In times of low arbitrage activity, the beta-arbitrage strategy exhibits delayed correction, taking up to three years for abnormal returns to be realized. In contrast, when arbitrage activity is high, prices overshoot and then revert in the long run. We document a novel positive-feedback channel operating through firm leverage that facilitates these boom-and-bust cycles.

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1. Introduction

The trade-off of risk and return is a key concept in modern finance. The simplest and most intuitive measure of risk is market beta—the slope in the regression of a security's return on the market return. In the capital asset pricing model (CAPM) of Sharpe (1964) and Lintner (1965), market beta is the only risk needed to explain expected returns. More specifically, the CAPM predicts that the relation between expected return and beta—the security market line (SML)—has an intercept equal to the risk-free rate and a slope equal to the equity premium.

However, empirical evidence indicates that the security market line is too flat on average (Black 1972, Frazzini and Pedersen 2014) and especially so during times of high expected inflation (Cohen et al. 2005), investor disagreement (Hong and Sraer 2016), and market sentiment (Antoniou et al. 2015). These patterns are not explained by other well-known asset pricing anomalies such as size, value, and price momentum.

We study the response of arbitrageurs to this failure of the Sharpe-Lintner CAPM in order to identify booms and busts of beta arbitrage.[1] In particular, we exploit the novel measure of arbitrage activity introduced by Lou and Polk (2022). They argue that traditional measures of such activity are flawed, poorly measuring a portion of the inputs to the arbitrage process for a subset of arbitrageurs. Lou and Polk's innovation is to measure the outcome of the arbitrage process, namely, the correlated price impacts that can result in excess return comovement in the spirit of Barberis and Shleifer (2003).[2]

We first confirm that our measure of the excess return comovement, relative to a benchmark asset pricing model of beta-arbitrage stocks (labeled *CoBAR*), is correlated with existing measures of arbitrage activity. In particular, we find that time variation in the level of institutional holdings in low-beta stocks (i.e., stocks in the long leg of the beta strategy), the assets under management (AUMs) of long-short equity hedge funds, aggregate liquidity, and the past performance of a typical beta-arbitrage strategy together forecast roughly 41% of the time-series variation in *CoBAR*. These findings suggest that not only is our measure consistent with existing proxies for arbitrage activity but also that no one single existing proxy is sufficient for capturing time-series variation in arbitrage activity. Indeed, one could argue that perhaps much of the unexplained variation in *CoBAR* represents variation in arbitrage activity missed by existing measures.

After validating our measure in this way, we then forecast the postformation abnormal returns to beta arbitrage. We first estimate time variation in the short-run and long-run security market lines, conditioning on *CoBAR*, the average pairwise partial weekly return correlation in the low-beta decile over the past 12 months. We find that in periods of high beta-arbitrage activity, the short-term security market line (e.g., in the six months after portfolio ranking) slopes downward, indicating profits to the low-beta strategy, consistent with arbitrageurs expediting the correction of market misvaluation. However, this correction is excessive, as the long-run security market line (e.g., in year 3 after ranking) dramatically slopes upward. In contrast, during periods of low beta-arbitrage activity, the security market line is weakly upward sloping in the short run and becomes downward sloping in the long term, consistent with delayed correction of the anomaly.

It is important to note that whereas our framework has unambiguous predictions about long-run beta-arbitrage returns, there is reversal after periods of crowded trading, and it does not have clear predictions for short-run beta-arbitrage returns. Beta-arbitrage returns right after portfolio ranking could move positively (if more arbitrageurs are closing the gap as we measure beta-arbitrage returns) or negatively (if arbitrageurs have already closed the gap before we measure the returns) with *CoBAR*. As a result, for the rest of the paper, our empirical focus is squarely on the time-varying *long-term reversal* pattern in the postholding period returns of stocks traded by the beta-arbitrage strategy (as opposed to prior studies in this literature that focus on the short-term profitability of beta-arbitrage strategies).

We next show, using both the security-market line approach and a calendar-time portfolio approach, that the long-term reversal pattern is robust to controlling for well-known results in the cross section. In particular, we classify all months into five groups based on *CoBAR*. We find, for example, that the difference in the year 3 postformation six-factor alpha (the Fama and French (2015) five-factor model augmented with a momentum factor) between high-*CoBAR* periods and low-*CoBAR* periods for the beta-arbitrage strategy is an impressive −1.50% per month with an associated *t*-statistic of −3.67. Further controlling for the lottery factor of Bali et al. (2017) reduces the alpha slightly to −1.29% per month (*t*-statistic = −3.14). In other words, the long-run reversal of beta-arbitrage returns varies predictably through time.[3]

In summary, our results reveal interesting patterns in the relationship between arbitrage strategy returns and the arbitrage crowd. When beta-arbitrage activity is low, the returns to beta-arbitrage strategies exhibit significant *delayed* correction. In contrast, when beta-arbitrage activity is high, the returns to beta-arbitrage activities reflect strong *overcorrection* because of crowded arbitrage trading. These results are consistent with time-varying arbitrage activity generating booms and busts in beta arbitrage.

We argue that these results are intuitive, as it is difficult to know how much arbitrage activity is pursuing beta arbitrage, and, moreover, the strategy is susceptible to positive-feedback trading. Specifically, bets on (against) low-beta (high-beta) stocks result in prices of those securities rising (falling). If the underlying firms are leveraged, this change in price will, all else equal, result in the security's beta falling (increasing) further, a key insight behind proposition II of Modigliani and Miller (1958). Thus, not only do arbitrageurs not know when to stop trading the low-beta strategy, their (collective) trades strengthen the signal based on realized beta. Consequently, beta arbitrageurs may increase their bets when trading is more crowded.

Consistent with our novel positive-feedback mechanism, we show that the cross-sectional spread in betas increases when beta-arbitrage activity is high and particularly so when beta-arbitrage stocks are relatively more levered. As a consequence, stocks remain in the extreme beta portfolios for a longer period of time. Our novel positive-feedback channel also has implications for cross-sectional heterogeneity in abnormal returns: we find that our boom-and-bust beta-arbitrage cycles are particularly strong among high-leverage stocks.

A variety of robustness tests confirm our main findings. In particular, we show that controlling for other factors either when measuring *CoBAR* or when predicting beta-arbitrage returns does not alter our primary conclusions that a) the excess comovement of beta-arbitrage stocks forecasts a time-varying security market line, b) the excess comovement of beta-arbitrage stocks forecasts time-varying reversal to beta-arbitrage bets, and c) the beta spread varies with this excess comovement.

Finally, Shleifer and Vishny (1997) link the extent of arbitrage activity to limits to arbitrage (LTAs). Based on their logic, trading strategies that bet on firms that are cheaper to arbitrage (e.g., larger stocks, more liquid stocks, or stocks with lower idiosyncratic risk) should have more arbitrage activity. This idea of limits to arbitrage motivates tests examining cross-sectional heterogeneity in our findings. We show that our results primarily occur in those stocks with the *least* limits to arbitrage: large stocks, liquid stocks, stocks with low idiosyncratic volatility, and stocks with low values of the maximum daily return signal (MAX) of Bali et al. (2011), known to identify stocks that retail investors favor. This cross-sectional heterogeneity in our return effect is again consistent with the interpretation that arbitrage activity causes much of the time-varying patterns we document. We emphasize that this finding is in contrast to most of the behavioral finance literature, which finds that anomalies are stronger in stocks with the highest limits to arbitrage.

The organization of our paper is as follows. Section 2 summarizes the related literature. Section 3 describes the data and empirical methodology. We detail our empirical findings regarding beta-arbitrage activity and predictable patterns in returns in Section 4 and present key tests of our economic mechanism in Section 5. Section 6 concludes.

2. Related Literature

There is mounting empirical evidence that contradicts the main prediction of the capital asset pricing model. Black et al. (1972) are the first to show that the security market line is too flat on average. Put differently, the risk-adjusted returns of high-beta stocks are too low relative to those of low-beta stocks. This finding is subsequently confirmed in an influential study by Fama and French (1992). Blitz and van Vliet (2007), Blitz et al. (2013), Baker et al. (2014), and Frazzini and Pedersen (2014) document that the low-beta anomaly is also present in both non-U.S. developed markets as well as emerging markets.

A variety of explanations have been proposed for the beta anomaly. Black (1972) and, more recently, Frazzini and Pedersen (2014) argue that leverage-constrained investors, such as mutual funds, tend to deviate from the capital market line and invest in high-beta stocks to pursue higher expected returns, thus causing these stocks to be overpriced relative to the CAPM benchmark.[4] Cohen et al. (2005) derive the cross-sectional implications of the CAPM in conjunction with the money illusion story of Modigliani and Cohn (1979). Hong and Sraer (2016) provide an alternative explanation drawing on Miller (1977)'s insights on investor disagreement at the market level. Kumar (2009) and Bali et al. (2011) show that high-risk stocks underperform low-risk stocks because some investors prefer volatile, skewed returns in the spirit of the cumulative prospect theory as modeled by Barberis and Huang (2008). Finally, Liu et al. (2018) attribute the low-beta anomaly to the positive correlation between market beta and idiosyncratic volatility.[5]

Regardless of the explanation (whether it is because of behavioral biases or market constraints), a natural question is why sophisticated investors, who can lever up and sell short securities at relatively low costs, do not fully take advantage of the low-beta anomaly and thus restore the theoretical relation between risk and returns. Our paper is aimed at addressing this exact question. Our premise is that professional investors indeed take advantage of this low-beta return pattern, often in dedicated strategies that buy low-beta stocks and/or sell high-beta stocks. However, the total amount of capital that is dedicated to this low-beta strategy is both time varying and unpredictable from a single arbitrageur's perspective, thus resulting in periods where the security market line remains too flat—that is, too little arbitrage capital—as well as periods where the security market line becomes overly steep, that is, too much arbitrage capital.

Not all arbitrage strategies have these issues. Indeed, some strategies have a natural fundamental anchor that is relatively easily observed (Stein 2009). For example, it is straightforward to observe the extent to which an American depositary receipt (ADR) is trading at a price premium (discount) relative to its local share. This ADR premium/discount is a clear signal of an opportunity, and, in fact, arbitrage activity keeps any price differential small, with deviations disappearing within minutes. Importantly, if an unexpectedly large number of ADR arbitrageurs pursue a particular trade, the price differential narrows. An individual ADR arbitrageur can then adjust demand accordingly.

There is, however, no easy anchor for beta arbitrage. Further, we argue that the difficulty in identifying the amount of beta-arbitrage capital is exacerbated by a novel, endogenous positive-feedback channel.[6] Namely, beta-arbitrage trading can lead to the cross-sectional beta spread increasing when firms are levered. As a consequence, stocks in the extreme-beta deciles are more likely to remain in these extreme groups with more extreme beta values when arbitrage trading becomes excessive. Given that beta arbitrageurs rely on realized betas as their trading signal, this beta expansion pattern resulting from firm leverage effectively causes a positive-feedback loop in the beta-arbitrage strategy.

In summary, in contrast to prior empirical work measuring the profitability of the beta strategy, our empirical focus is on the *time-varying long-term reversal* pattern in the postholding period returns of stocks traded by the beta-arbitrage strategy. Specifically, we show that the long-term return reversal to the beta-arbitrage strategy is stronger after periods of crowded trading. Moreover, whereas prior studies on limits to arbitrage argue and show that anomalies are generally weaker among stocks with low limits to arbitrage, our return pattern is, in fact, stronger among stocks with low limits to arbitrage (e.g., large cap, liquid stocks).

Our results, taken together, challenge the traditional view that an increase in the amount of arbitrage activity makes the market more informationally efficient (Friedman 1953). Put simply, whereas previous literature examines holding-period returns (anywhere from a month to a year post-portfolio formation, depending on the nature of the trading signal), our research focuses on postholding-period returns (in the years after the typical arbitrageur has exited their positions in those dynamic trading strategies) to trace out the long-run consequences of arbitrage activity.

One of the few papers, and arguably the first to study these long-term reversal patterns and connect them to arbitrage activity, is Lou and Polk (2022). Relative to Lou and Polk (2022), we document a novel, endogenous feedback mechanism that occurs as a result of equity betas on levered firms changing because of the arbitrage activity. Specifically, as arbitrageurs buy low-beta stocks and sell high-beta stocks, this activity reduces the market leverage of the former and increases the market leverage of the latter, thereby widening the beta gap between the two groups. This can create an endogenous feedback loop and exacerbate price overshooting.

3. Data and Methodology

The main data set used in this study is the stock return data from the Center for Research in Security Prices (CRSP). Following prior studies on the beta-arbitrage strategy, we include, in our study, all common stocks on NYSE, Amex, and NASDAQ. We then augment the stock return data with institutional ownership in individual stocks provided by Thompson Financial. We further obtain information on assets under management of long-short equity hedge funds from Lipper's Trading Advisor Selection System (TASS). Because the assets managed by hedge funds grow substantially in our sample period, we detrend this variable. In addition, we use fund-level data on hedge fund returns and AUMs.

We also construct, as controls, a list of variables that have been shown to predict future beta-arbitrage strategy returns. Specifically, (a) following Cohen et al. (2005), we construct a proxy for expected inflation using an exponentially weighted moving average (with a half-life of 36 months) of past log growth rates of the producer-price index; (b) we also include, in our study, the sentiment index proposed by Baker and Wurgler (2006, 2007); (c) following Hong and Sraer (2016), we construct an aggregate disagreement proxy as the beta-weighted standard deviation of analysts' long-term growth rate forecasts; and (d) finally, following Frazzini and Pedersen (2014), we use the TED spread—the difference between the LIBOR rate and the U.S. Treasury bill rate—as a measure of financial intermediaries' funding constraints. In addition, we include both the volatility of the daily TED spread as well as financial sector leverage (Chen and Lu 2019) as proxies for funding liquidity constraints.

We begin our analysis in January 1970 (i.e., our first measure of beta-arbitrage crowdedness is computed as of December 1969), as that was when the low-beta anomaly was first recognized by academics.[7] At the end of each month, we sort all stocks into deciles (in some cases, vigintiles) based on their preranking market betas. Following prior literature, we calculate preranking betas using daily returns in the past 12 months (with at least 200 daily observations). Our results are similar if we use monthly returns or different preranking periods. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is simply the sum of the six coefficients from the ordinary least squares (OLS) regression.

We then compute pairwise partial correlations using 52 (nonmissing) weekly returns for all stocks in each decile in the portfolio ranking period. We control for the Fama-French three factors when computing these partial correlations to purge out any comovement in stocks induced by known risk factors. We measure the excess comovement of stocks involved in beta arbitrage (*CoBAR*) as the average pairwise partial correlation in the *lowest* market beta decile. We focus on the low-beta decile as these stocks tend to be larger, more liquid, and have lower idiosyncratic volatility compared with the highest-beta decile; thus, our measurement of excess comovement will be less susceptible to issues related to asynchronous trading and measurement noise.[8] We operationalize this calculation by computing the average correlation of the three-factor residual of every stock in the lowest-beta decile with the rest of the stocks in the same decile:

$$CoBAR = \frac{1}{N}\sum_{i=1}^{N} Corr(retrf_i^L, retrf_{-i}^L \mid mktrf, smb, hml), \tag{1}$$

where $retrf_i^L$ is the weekly return of stock i in the lowest-beta decile, $retrf_{-i}^L$ is the weekly return of the equal-weight lowest-beta decile excluding stock i, and N is the number of stocks in the lowest-beta decile. We have also measured *CoBAR* using returns that are orthogonalized not only to the Fama-French factors but also to each stock's industry return or to other risk factors, and our conclusions continue to hold.

In the following period, we then form a zero-cost portfolio that goes long the value-weight portfolio of stocks in the lowest market beta decile and short the value-weight portfolio of stocks in the highest market beta decile.[9] We track the cumulative abnormal returns of this zero-cost long-short portfolio in months 1 through 36 after portfolio formation. To summarize the timing of our empirical exercise, year 0 is our portfolio formation year (during which we also measure *CoBAR*), year 1 is the holding year, and years 2 and 3 are our postholding period to detect any (conditional) long-run reversal to the beta-arbitrage strategy.

4. Main Results

We first document simple characteristics of our arbitrage activity measure. Panel A of Table 1 indicates that there is significant excess correlation among low-beta stocks on average and that this pairwise correlation varies substantially through time; specifically, the mean of *CoBAR* is 0.10, varying from a low of 0.04 to a high of 0.20.

Panel B of Table 1 examines *CoBAR*'s correlation with existing measures linked to time variation in the expected abnormal returns to beta-arbitrage strategies. We find that *CoBAR* is high when disagreement is high, with a correlation of 0.27. *CoBAR* is also positively correlated with the Ted spread, consistent with a time-varying version of Black (1972), though the Ted spread does not forecast (or, in some cases, forecasts in the wrong direction) time variation in expected abnormal returns to beta-arbitrage strategies (Frazzini and Pederson 2014). *CoBAR* is negatively correlated with the expected inflation measure of Cohen et al. (2005). However, in results not shown, the correlation between expected inflation and *CoBAR* becomes positive for the subsample from 1990 to 2016, consistent with arbitrage activity eventually taking advantage of this particular

Table 1. Summary Statistics

Panel A: Summary statistics

Variable	*N*	Mean	Std. Dev.	Minimum	Maximum
CoBAR	564	0.104	0.026	0.037	0.203
Inflation	564	0.003	0.002	−0.001	0.008
Sentiment	564	0.015	0.939	−2.420	3.200
Disagreement	420	0.054	1.012	−1.277	3.593
TED Spread	372	0.588	0.428	0.118	3.353
TED Volatility	372	0.068	0.081	0.005	0.813
Financial Leverage	564	0.000	0.001	−0.005	0.009

Panel B: Correlation

	CoBAR	*Inflation*	*Sentiment*	*Disagreement*	*TED Spread*	*TED Volatility*	*Financial Leverage*
CoBAR	1						
Inflation	−0.272	1					
Sentiment	0.024	−0.361	1				
Disagreement	0.271	−0.242	0.132	1			
TED Spread	0.290	0.277	0.007	−0.211	1		
TED Volatility	0.202	0.266	0.077	−0.135	0.763	1	
Financial Leverage	−0.017	0.126	−0.068	0.011	0.198	0.143	1

Notes. This table provides characteristics of *CoBAR*, the *excess* comovement among low-beta stocks over the period from 1970 until 2016 (we then examine beta-arbitrage returns in the following three years, so the return sample ends in 2019). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. Pairwise partial return correlations (controlling for the Fama-French three factors) for all stocks in the bottom beta decile are computed based on weekly stock returns in the previous 12 months. *CoBAR* is the average pairwise correlation between any two stocks in the low-beta decile in the previous 12 months. *Inflation* is the smoothed inflation rate used by Cohen et al. (2005), who apply an exponentially weighted moving average (with a half-life of 36 months) to past log growth rates of the producer price index. *Sentiment* is the sentiment index proposed by Baker and Wurgler (2006, 2007). *Disagreement* is the beta-weighted standard deviation of analysts' long-term growth rate forecasts, as used in Hong and Sraer (2016). *TED Spread* is the difference between the LIBOR rate and the U.S. Treasury bill rate. *TED Volatility* is the standard deviation of daily *TED Spread*. *Financial Leverage* is the AR(2) residual of financial leverage, constructed following Chen and Lu (2019). Panel A reports the summary statistics of these variables. Panel B shows the time-series correlations among these variables for the entire sample period.

source of time variation in beta-arbitrage profits. There is little to no correlation between *CoBAR* and sentiment.

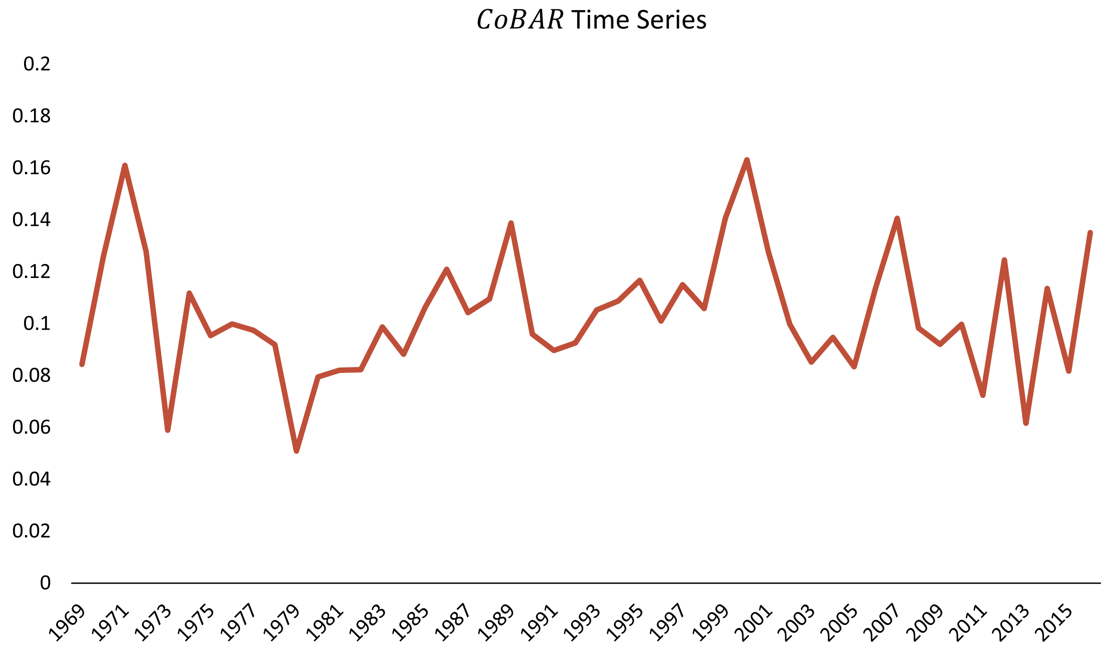
Figure 1 plots *CoBAR* as of the end of each December. Note that we do not necessarily expect a trend in this measure. Though there is clearly more capital invested in beta-arbitrage strategies, in general, markets are also deeper and more liquid. Nevertheless, after an initial spike in December 1971, *CoBAR* trends slightly upward for the rest of the sample. However, there are clear cycles around this trend. These cycles tend to peak before broad market declines. Also, note that *CoBAR* is essentially uncorrelated with market volatility. A regression of *CoBAR* on contemporaneous realized market volatility produces a loading of 0.01 with a t-statistic of −0.36.

Consistent with our measure tracking arbitrage activity, Web Appendix Table A1 shows that *CoBAR* is persistent in event time. Specifically, the correlation between *CoBAR* measured in year 0 and year 1 for the same set of stocks is 0.14. In fact, year 0 *CoBAR* remains highly correlated with subsequent values of *CoBAR* for the same stocks all the way out to year 3. The average value of *CoBAR* remains high as well. Recall that in year 0, the average excess correlation is 0.10. We find that in years 1, 2, and 3, the average excess correlation of these same stocks remains around 0.07.[10]

4.1. Determinants of *CoBAR*

To confirm that our measure of beta-arbitrage is sensible, we estimate regressions forecasting *CoBAR* with four variables that are often used to proxy for arbitrage activity. The first variable we use is the aggregate institutional ownership (*Inst Own*) of the low-beta decile—that is, stocks in the long leg of the beta strategy—based on 13F filings. We include institutional ownership, as these investors are typically considered smart money, at least relative to individuals, and we focus on their holdings in the low-beta decile, as we do not observe their short positions in the high-beta decile. We also include the *AUMs* of long-short equity hedge funds, the prototypical arbitrageur. We further include a measure of the past profitability of beta-arbitrage strategies, the realized four-factor alpha of Frazzini and Pedersen's betting-against-beta (BAB) factor. Intuitively, more arbitrageurs should be trading the low-beta strategy after the strategy has performed well in recent past. Finally, we include in the regressions a list of variables that have been shown to

Figure 1. (Color online) This Figure Shows the Time Series of the December Observations of the *CoBAR* Measure



Notes. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. *CoBAR* is the average pairwise partial return correlation in the low-beta decile measured in the ranking period. We begin our analysis in 1970, as it is the year when the low-beta anomaly was first recognized by academics. The time series average of *CoBAR* is 0.10.

predict future beta-arbitrage returns: the expected inflation in Cohen et al. (2005), the sentiment index of Baker and Wurgler (2006, 2007), aggregate disagreement about long-term growth following Hong and Sraer (2016), the Ted spread and its daily volatility in Frazzini and Pedersen (2014), as well the financial sector leverage of Chen and Lu (2019). We measure these variables contemporaneously with *CoBAR*, as we will be running horse races against these variables in our subsequent analysis.

All else equal, we expect *CoBAR* to be lower if markets are more liquid. However, as arbitrage activity is endogenous, times when markets are more liquid may also be times when arbitrageurs are more active. Indeed, Cao et al. (2013) show that hedge funds increase their activity in response to increases in aggregate liquidity. Following Cao et al., we further include past market liquidity as proxied by the Pástor and Stambaugh (2003) liquidity factor (*PS liquidity*) in our regressions to measure which channel dominates. All regressions in Table 2 include a trend to ensure that our results are not spurious.

Regression (1) in Table 2 documents that *Inst Own*, *AUM*, and *PS liquidity* forecast *CoBAR*, with an R^2 of approximately 41%.[11] Regression (2) shows that three of the extant predictors of beta-arbitrage returns help explain *CoBAR*. The Ted spread adds some incremental explanatory power, with the sign of the coefficient consistent with arbitrageurs taking advantage of potential time variation in beta-arbitrage returns linked to this channel. Indeed, as we show later, the Ted spread does a poor job forecasting beta-arbitrage returns in practice, perhaps because arbitrageurs have compensated appropriately for this potential departure from Sharpe-Lintner pricing. The disagreement measure and inflation rate also help explain variation in *CoBAR*. In both specifications, past profitability of a prototypical beta-arbitrage strategy strongly forecasts relatively high arbitrage activity going forward. It seems reasonable that strong past performance of an investment strategy may result in the strategy becoming more popular. Regression (3) shows that proxies for market funding constraints are uncorrelated with *CoBAR*.

Overall, these findings make us comfortable in our interpretation that *CoBAR* is related to arbitrage activity and distinct from existing measures of opportunities in beta arbitrage. As a consequence, we turn to the main analysis of the paper, the subsequent, especially long-run, performance of beta-arbitrage returns conditional on *CoBAR*.

4.2. Predicting the Security Market Line

We first look for predictable returns linked to beta arbitrage by documenting time variation in the shape of the security market line as a function of lagged *CoBAR*. Such an approach ensures that the time variation we find is not restricted to a small subset of extreme-beta stocks but, instead, is a robust feature of the cross section. (We

Table 2. Determinants of *CoBAR*

Dependent variable	$CoBAR_t$ (1)	(2)	(3)
$Inst\ Own_{t-1}$	0.030***	0.015***	0.012***
	(0.003)	(0.003)	(0.004)
AUM_{t-1}	0.004**	0.004**	0.004**
	(0.002)	(0.002)	(0.002)
$BAB\ Alpha_{t-1}$	0.003*	0.006***	0.006***
	(0.001)	(0.001)	(0.002)
$Inflation_t$		−0.006**	−0.004
		(0.002)	(0.003)
$Sentiment_t$		0.001	0.001
		(0.003)	(0.003)
$Disagreement_t$		0.011***	0.011***
		(0.002)	(0.002)
$TED\ Spread_t$		0.013***	0.016***
		(0.002)	(0.004)
$TED\ Volatility_t$			−0.001
			(0.003)
$Financial\ Leverage_t$			−0.002
			(0.002)
$PS\ Liquidity_t$	0.007***	0.011***	0.011***
	(0.001)	(0.001)	(0.002)
Trend	Yes	Yes	Yes
Adjusted R^2	0.413	0.567	0.566
Number of observations	288	288	288

Notes. This table reports regressions of *CoBAR*, described in Table 3, on lagged variables plausibly linked to arbitrage activity in the post-1993 period (constrained by the availability of the hedge fund AUM data). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. The dependent variable in the regressions, *CoBAR*, is the average pairwise partial weekly return correlation in the low-beta decile over 12 months. *Inst Own* is the aggregate institutional ownership of the low-beta decile; *AUM* is the logarithm of the total assets under management of long-short equity hedge funds (detrended). *BAB Alpha* is the realized four-factor alpha of Frazzini and Pedersen's BAB factor. *Inflation* is the smoothed inflation rate used by Cohen et al. (2005), who apply an exponentially weighted moving average (with a half-life of 36 months) to past log growth rates of the producer price index. *Sentiment* is the sentiment index proposed by Baker and Wurgler (2006, 2007). *Disagreement* is the beta-weighted standard deviation of analysts' long-term growth rate forecasts, as used in Hong and Sraer (2016). *TED Spread* is the difference between the LIBOR rate and the U.S. Treasury bill rate. *TED Volatility* is the standard deviation of daily *TED Spread*. *Financial Leverage* is the AR(2) residual of financial leverage, constructed following Chen and Lu (2019). We also include, in the regression, the Pastor-Stambaugh liquidity factor (*PS Liquidity*). A trend dummy is included in all regression specifications. All independent variables are standardized to have a mean of zero and standard deviation of one so that the coefficient represents the effect of a one-standard-deviation change in the independent variable on *CoBAR*. Standard errors are shown in parentheses.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

note that beta-arbitrage activity can affect the entire cross section of stocks rather than just the extreme deciles because arbitrageurs may bet against the low-beta anomalies by selecting portfolio weights that are inversely proportional to the market beta.) At the end of each month, we sort all stocks into 20 value-weighted portfolios by their preranking betas. We track these 20 portfolios' returns in four distinct postranking periods—months 1–6, months 7–12, months 13–24 (year 2), and months 25–36 (year 3)—after portfolio formation, computing both postranking betas and the corresponding security market lines.

For example, for the months 1–6 portfolio returns, we compute the postranking betas by regressing each of the 20 portfolios' monthly value-weight returns on the market's excess returns. Following Fama and French (1992), we use the entire sample to compute postranking betas. That is, we pool together those six monthly returns across all calendar months to estimate a single beta for that portfolio for that postformation period. We estimate postranking betas for the other three groups in a similar fashion. The four sets of postranking betas are then labeled $\beta_1^{1-6},\ldots,\beta_{20}^{1-6};\beta_1^{7-12},\ldots,\beta_{20}^{7-12};\beta_1^{13-24},\ldots,\beta_{20}^{13-24}$; and $\beta_1^{25-36},\ldots,\beta_{20}^{25-36}$.

To calculate the intercept and slope of the short-term and long-term security market lines, we estimate the following cross-sectional regressions:

$$XRet_{i,t}^{1-6} = intercept_t^{1-6} + slope_t^{1-6}\beta_i^{1-6}, \quad (2)$$

$$XRet_{i,t}^{7-12} = intercept_t^{7-12} + slope_t^{7-12}\beta_i^{7-12}, \quad (3)$$

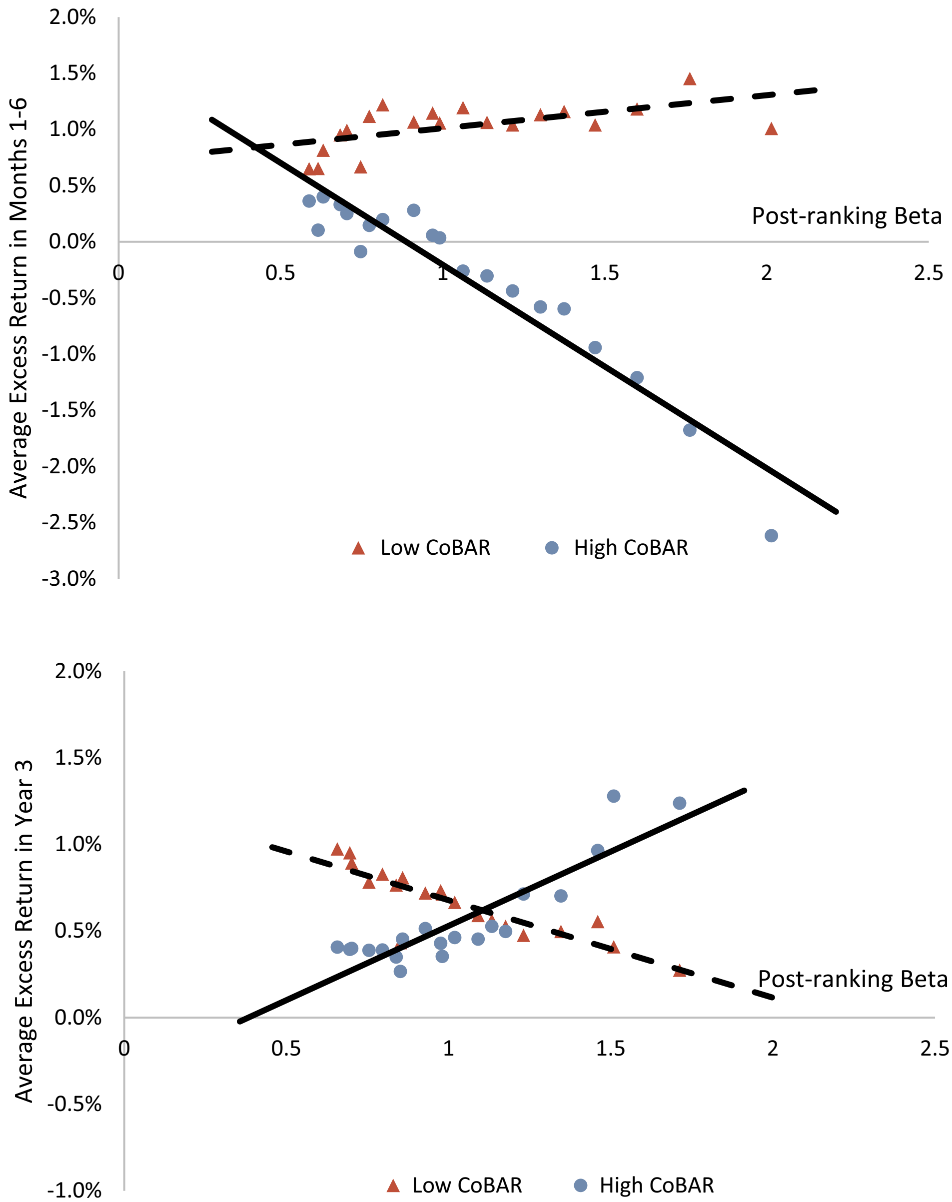
$$XRet_{i,t}^{13-24} = intercept_t^{13-24} + slope_t^{13-24}\beta_i^{13-24}, \quad (4)$$

$$XRet_{i,t}^{25-36} = intercept_t^{25-36} + slope_t^{25-36}\beta_i^{25-36}, \quad (5)$$

where $XRet_{i,t}^{1-6}$ is portfolio i's monthly excess returns in months 1 through 6, $XRet_{i,t}^{7-12}$ is portfolio i's monthly returns in months 7 through 12, $XRet_{i,t}^{13-24}$ is portfolio i's monthly returns in months 13 through 24, and $XRet_{i,t}^{25-36}$ is portfolio i's monthly returns in months 25 through 36. These four regressions then give us four time series of coefficient estimates of the intercept and slope for each particular security market line: $(intercept_t^{1-6}, slope_t^{1-6})$, $(intercept_t^{7-12}, slope_t^{7-12})$, $(intercept_t^{13-24}, slope_t^{13-24})$, and $(intercept_t^{25-36}, slope_t^{25-36})$. As the average excess returns and postranking betas are always measured at the same point in time, these pairs $(intercept_t, slope_t)$ fully describe the security market line over the postformation period in question.

As is well-known since Fama and Macbeth (1973), the time-series of $intercept_t$ and $slope_t$ are excess returns. We then examine the way these returns vary as a function of our measure of beta-arbitrage capital. As can be seen from the top panel of Figure 2, the intercept of the months 1–6 security market line significantly increases in *CoBAR*, and its slope significantly decreases in *CoBAR*. When *CoBAR* is relatively high, that is, during periods when beta-arbitrage capital is relatively high, the short-term security market line strongly slopes downward,

Figure 2. (Color online) This Figure Shows the Security Market Line as a Function of Lagged *CoBAR*



Notes. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. We then estimate two security market lines based on these 20 portfolios formed in each period: one SML using portfolio returns in months 1–6 (the top panel) and the other using portfolio returns in year 3 after portfolio formation (the bottom panel); the betas used in these SML regressions are the corresponding postranking betas. The y axis reports the average monthly excess returns to these 20 portfolios, and the x axis reports the postranking betas of these portfolios. Beta portfolios formed in high-*CoBAR* periods are depicted with a blue circle and fitted with a solid line, and those formed in low-*CoBAR* periods are depicted with a red triangle and fitted with a dotted line.

indicating strong profits to the low-beta strategy, consistent with arbitrageurs expediting the correction of market misevaluation. In contrast, when *CoBAR* is low, that is, when beta-arbitrage capital is relatively low, the short-term security market line is weakly upward sloping, and the beta-arbitrage strategy, as a consequence, is unprofitable, consistent with delayed correction of the beta anomaly.

The pattern is completely reversed for the security market line during year 3 (i.e., months 25–36). The intercept of this security market line is significantly negatively related to *CoBAR*, whereas its slope is significantly positively related to *CoBAR*. As can be seen from the bottom panel of Figure 2, two years after high-*CoBAR* periods, the long-term security market line turns upward sloping; indeed, the slope is so steep (resulting in a negative intercept) that the beta strategy loses money, consistent with overcorrection of the low-beta anomaly by crowded arbitrage trading.[12] In contrast, after low-*CoBAR* periods, the months 25–36 security market line turns downward sloping, reflecting eventual profitability of the low-beta strategy in the long run.

Table 3 formally measures these effects using the methodology developed in Cohen et al. (2005). Specifically, we regress the returns represented by the time series of $intercept_t$ and $slope_t$ on a constant, the contemporaneous excess market return ($r^e_{M,t}$), and lagged *CoBAR* ($CoBAR_{t-1}$):

$$intercept_t = a_1 + b_1 r^e_{M,t} + c_1 CoBAR_{t-1} + u_{1,t}, \qquad (6)$$

$$slope_t = a_2 + b_2 r^e_{M,t} + c_2 CoBAR_{t-1} + u_{2,t}. \qquad (7)$$

Following Cohen et al. (2005), the excess slope is defined as $g_0 + g_1 CoBAR_{t-1}$, where $g_0 \equiv a_2/b_2$ and $g_1 \equiv c_2/b_2$. The excess intercept is computed as $h_0 + h_1 CoBAR_{t-1}$, where $h_0 \equiv a_1 - a_2 b_1/b_2$ and $h_1 \equiv c_1 - c_2 b_1/b_2$.[13] These formulas correct for the fact that the betas we use as inputs to the Fama-MacBeth stage are not perfect measures of betas; there is no guarantee that $b_1 = 0$ and $b_2 = 1$ exactly.

Table 3 reports the estimates of the excess slope and intercept for our four postformation periods of interest. In the first six months, the excess slope and intercept move with *CoBAR* in a manner consistent with Figure 2. Specifically, the excess slope of the security market line moves negatively ($g_1 = -0.185$ with a t-statistic of -2.26) with the amount of beta-arbitrage activity, whereas the excess intercept ($h_1 = 0.186$ with a t-statistic of 2.29) moves positively with the amount of beta-arbitrage activity.

In contrast, in year 3, the excess slope of the security market line moves positively ($g_1 = 0.379$ with a t-statistic of 3.78) with the amount of beta-arbitrage activity, whereas the excess intercept ($h_1 = -0.317$ with a t-statistic of -3.70) moves negatively with the amount of beta-arbitrage activity. These patterns are consistent with the nonparametric analysis in Figure 2.[14]

It is important to note that, whereas our framework has unambiguous predictions about long-run beta-arbitrage returns (that we should see reversal after periods of crowded trading), it does not have clear-cut predictions for short-run beta-arbitrage returns. Beta-arbitrage returns right after portfolio ranking could move positively (if more arbitrageurs are closing the gap as we measure beta-arbitrage returns) or negatively (if arbitrageurs have already closed the gap before we measure the returns) with *CoBAR*.[15] In the rest of the paper, we focus squarely on the time-varying long-term reversal pattern in the postholding period returns of stocks traded by the beta-arbitrage strategy.

4.3. Robustness of Key Results

Table 4 examines variations to our methodology to ensure that our main finding of time-varying reversal of beta-arbitrage profits is robust. For simplicity, we report the estimates of the excess slope and intercept in year 3. For reference, the first row of Panel A of Table 4 reports the baseline results from Table 3.

In Panel A, we consider different subsample results. In rows 2 and 3, we exclude the tech bubble crash and the recent financial crisis from our sample. Our estimates of the way the security market line varies with beta-

Table 3. Forecasting Security Market Lines with *CoBAR*

Time	g_0	g_1	h_0	h_1
Months 1–6	0.013	**−0.185**	−0.013	**0.186**
	(1.52)	(−2.26)	(−1.56)	(2.29)
Months 7–12	0.001	−0.079	0.000	0.080
	(0.07)	(−0.83)	(−0.03)	(0.83)
Year 2	**−0.016**	0.087	**0.016**	−0.082
	(−2.14)	(1.14)	(2.34)	(−1.17)
Year 3	**−0.045**	**0.379**	**0.040**	**−0.317**
	(−4.33)	(3.78)	(4.54)	(−3.70)

Notes. This table shows the estimated function that maps *CoBAR* into the excess slope and intercept of the security market line in different time windows. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. We then estimate four security market lines based on these 20 portfolios formed in each period: one SML using portfolio returns in months 1–6, months 7–12, year 2, and year 3 after portfolio formation; the betas used in these SML regressions are the corresponding postranking betas. We regress the intercepts ($intercept_t$) and the slopes ($slope_t$) on a constant, the contemporaneous excess market return ($r^e_{M,t}$), and lagged *CoBAR* ($CoBAR_{t-1}$):

$$intercept_t = a_1 + b_1 r^e_{M,t} + c_1 CoBAR_{t-1} + u_{1,t}$$
$$slope_t = a_2 + b_2 r^e_{M,t} + c_2 CoBAR_{t-1} + u_{2,t}$$

The excess slope is defined as $g_0 + g_1 CoBAR_{t-1}$, where $g_0 \equiv a_2/b_2$ and $g_1 \equiv c_2/b_2$. The excess intercept is computed as $h_0 + h_1 CoBAR_{t-1}$, where $h_0 \equiv a_1 - a_2 b_1/b_2$ and $h_1 \equiv c_1 - c_2 b_1/b_2$. The t-statistics computed using the delta method are in parentheses. Five percent statistical significance is indicated in bold.

Table 4. Forecasting Security Market Lines with *CoBAR*: Robustness

	g_0	g_1	h_0	h_1
Panel A: Subsamples				
(1) Full sample: 1970–2016	−0.045	0.379	0.040	−0.317
	(−4.33)	(3.78)	(4.54)	(−3.70)
(2) Excluding 2001	−0.045	0.375	0.040	−0.315
	(−4.33)	(3.87)	(4.65)	(−3.85)
(3) Excluding 2007–2009	−0.045	0.381	0.039	−0.310
	(−4.07)	(3.59)	(4.22)	(−3.47)
Panel B: Alternative definitions of *CoBAR*				
(1) Controlling for *UMD*	−0.047	0.415	0.042	−0.351
	(−3.91)	(3.42)	(4.14)	(−3.39)
(2) Controlling for *Large-/Small-Cap HML*	−0.041	0.345	0.037	−0.292
	(−4.02)	(3.46)	(4.45)	(−3.59)
(3) Controlling for *FF Five Factors*	−0.043	0.373	0.037	−0.303
	(−4.09)	(3.54)	(4.21)	(−3.35)
(4) Controlling for *FF Five Factors + UMD*	−0.045	0.408	0.039	−0.334
	(−3.83)	(3.31)	(3.91)	(−3.14)
(5) Controlling for *FF Five Factors + UMD + FMAX*	−0.043	0.391	0.038	−0.320
	(−4.00)	(3.43)	(4.02)	(−3.18)
(6) Controlling for *Industry Factors*	−0.045	0.400	0.040	−0.339
	(−3.53)	(2.97)	(4.04)	(−3.17)
(7) Correlation between *High-* and *Low-Beta Stocks*	−0.015	0.029	0.014	−0.022
	(−3.26)	(2.46)	(3.90)	(−2.21)
Panel C: Residual*CoBAR*				
(1) Controlling for *CoMOM*	−0.005	0.382	0.006	−0.320
	(−2.04)	(3.83)	(3.00)	(−3.73)
(2) Controlling for *CoValue*	−0.005	0.369	0.006	−0.310
	(−2.06)	(3.84)	(2.98)	(−3.67)
(3) Controlling for *Market Correlation*	−0.006	0.378	0.007	−0.318
	(−2.17)	(3.94)	(3.19)	(−3.85)
(4) Controlling for *BAB*	−0.006	0.365	0.007	−0.306
	(−2.14)	(3.66)	(3.13)	(−3.62)
(5) Controlling for *Volatility (BAB)*	−0.006	0.379	0.007	−0.317
	(−2.25)	(3.86)	(3.25)	(−3.73)
(6) Controlling for *Volatility (Market)*	−0.006	0.379	0.007	−0.317
	(−2.25)	(3.79)	(3.22)	(−3.71)
(7) Controlling for *Trend*	−0.006	0.380	0.007	−0.317
	(−2.22)	(3.67)	(3.24)	(−3.61)
(8) Controlling for *Preformation CoBAR*	−0.006	0.377	0.007	−0.315
	(−2.24)	(3.72)	(3.22)	(−3.64)
(9) Controlling for *Inflation*	−0.006	0.399	0.007	−0.333
	(−2.22)	(3.96)	(3.24)	(−3.98)
(10) Controlling for *Sentiment*	−0.006	0.379	0.007	−0.317
	(−2.25)	(3.86)	(3.25)	(−3.75)
(11) Controlling for *Disagreement*	−0.007	0.394	0.008	−0.328
	(−2.45)	(3.05)	(3.07)	(−2.69)
(12) Controlling for *TED Spread*	−0.005	0.319	0.006	−0.250
	(−1.75)	(2.29)	(2.11)	(−1.90)
(13) Controlling for *TED Volatility*	−0.005	0.318	0.006	−0.247
	(−1.81)	(2.45)	(2.13)	(−1.95)
(14) Controlling for *Financial Leverage*	−0.005	0.343	0.007	−0.260
	(−1.82)	(2.69)	(2.12)	(−1.91)

Notes. This table shows the estimated function that maps *CoBAR* into the excess slope and intercept of the security market line in year 3. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. We then estimate four security market lines based on these 20 portfolios formed in year 3 after portfolio formation; the betas used in these SML regressions are the corresponding postranking betas. Year 0 is the beta portfolio ranking period. We regress the intercepts ($intercept_t$) and the slopes ($slope_t$) on a constant, contemporaneous excess market return ($r^e_{M,t}$), and lagged *CoBAR* ($CoBAR_{t-1}$):

$$intercept_t = a_1 + b_1 r^e_{M,t} + c_1 CoBAR_{t-1} + u_{1,t}$$
$$slope_t = a_2 + b_2 r^e_{M,t} + c_2 CoBAR_{t-1} + u_{2,t}$$

The excess slope is defined as $g_0 + g_1 CoBAR_{t-1}$, where $g_0 \equiv a_2/b_2$ and $g_1 \equiv c_2/b_2$. The excess intercept is computed as $h_0 + h_1 CoBAR_{t-1}$, where $h_0 \equiv a_1 - a_2 b_1/b_2$ and $h_1 \equiv c_1 - c_2 b_1/b_2$. In Panel A, we consider different subsample results. Row 1 shows the baseline results from the full sample. In rows 2 and 3, we exclude the tech bubble crash and the recent financial crisis from our sample. In Panel B, we explore alternative definitions of *CoBAR*. In row 1, we control for the UMD factor in computing *CoBAR*. In row 2, we control for both large- and small-cap HML in computing *CoBAR*. In row 3, we control for the Fama-French (FF) five-factor model that adds profitability and investment to their three-factor model. In row 4, we control for the Fama-French five factors and Cahart's momentum factor. In row 5, we control for the Fama-French five factors, momentum factor, and lottery factor from Bali et al. (2017) (denoted as FMAX). In row 6, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 7, we instead measure the correlation between the high- and low-beta portfolios, with a low correlation indicating high arbitrage activity. In Panel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to, respectively, *CoMOM* and *CoValue* (Lou and Polk 2022, rows 1–2), the average pairwise correlation in the market (row 3), the BAB factor (Frazzini and Pedersen 2014; row 4), the lagged 36-month volatility of the BAB factor (row 5), market volatility over the past 24 months (row 6), a trend (row 7), lagged *CoBAR* (where we hold the stocks in the low-beta decile constant but calculate *CoBAR* using returns from the previous year; row 8), smoothed past inflation (Cohen et al. 2005, row 9), a sentiment index (Baker and Wurgler 2006, row 10), aggregate analyst forecast dispersion (Hong and Sraer 2016, row 11), the TED spread (Frazzini and Pedersen 2014, row 12), the TED volatility (row 13), and the AR(2) residual of financial leverage (Chen and Lu 2019, row 14). The *t*-statistics computed using the delta method are in parentheses.

arbitrage activity barely change and remain both economically and statistically significant.

In Panel B, we explore alternative definitions of *CoBAR*. In row 1, we control for the up-minus-down (UMD) factor (Carhart 1997) when computing *CoBAR*. In row 2, we control for both large- and small-cap high minus low (HML) (Cremers et al. 2013) when computing *CoBAR*. In row 3, we control for the Fama and French (2015) five-factor model that adds profitability and investment to their three-factor model. In row 4, we control for the Fama-French five factors and Carhart's momentum factor. In row 5, we control for the Fama-French five factors, momentum factor, and the lottery factor from Bali et al. (2017). In row 6, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 7, we instead measure the correlation between the high- and low-beta portfolios, with a low correlation indicating high arbitrage activity. Throughout these robustness tests, we continue to find that the security market line moves with our various measures of beta-arbitrage capital.

In Panel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to, respectively, *CoMOM* and *CoValue* (Lou and Polk 2022, rows 1–2), the average pairwise correlation in the market (row 3), the return on the BAB factor (Frazzini and Pedersen 2014, row 4), the lagged 36-month volatility of the BAB factor (row 5), market volatility over the past 24 months (row 6), a trend (row 7), lagged *CoBAR* (where we hold the stocks in the low-beta decile constant but calculate *CoBAR* using returns from the previous year; row 8), smoothed past inflation (Cohen et al. 2005, row 9), a sentiment index (Baker and Wurgler 2006, row 10), aggregate analyst forecast dispersion (Hong and Sraer 2016, row 11), the TED spread (Frazzini and Pedersen 2014, row 12), the volatility of the TED spread (row 13), and the AR(2) residual of financial leverage (Chen and Lu 2019, row 14). Our finding of a time-varying security market line linked to beta-arbitrage capital remains economically and statistically significant in every case.

4.4. Forecasting Beta-Arbitrage Portfolio Returns

The results in Figure 2 and Tables 1 and 5 document that the security market line moves with our proxy for beta-arbitrage activity. Though our findings are consistent with prices overshooting after periods of crowded arbitrage trading, that analysis does not control for well-known patterns in the cross section of average returns. In this section, we measure the abnormal returns on traditional long-short portfolios relative to popular factor models.

To this end, we sort all stocks into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial weekly return correlation in the low-beta decile over the past 12 months.

We report, in Table 5, the difference in six-factor alpha (the Fama and French (2015) five-factor model augmented with a momentum factor) and seven-factor alpha (adding the lottery factor of Bali et al. (2017) to the six-factor Fama-French-Carhart model) in postformation year 3 returns on the beta-arbitrage strategy between high-*CoBAR* periods and low-*CoBAR* periods. In this table, we study the same variation in methodology from Table 4 to ensure that our finding of time-varying reversal of beta-arbitrage profits is robust.

In Panel A, we consider different subsample results. Row 1 shows the baseline results from the full sample. The six-factor alpha is an impressive −1.50% per month with an associated *t*-statistic of −3.67. Adding the lottery factor of Bali et al. (2017) results in a still quite large estimate of −1.29% per month (*t*-statistic of −3.14). Rows 2 and 3 confirm that dropping either the tech bust or the global financial crisis does not have much of an effect on either the economic or statistical significance.

In Panel B, we explore the same alternative definitions of *CoBAR* studied in Panel B of Table 4. Again, we find

Table 5. Long-Term Portfolio Returns from Beta Arbitrage

	Six-factor alpha		Seven-factor alpha	
	Estimate (%)	*t*-statistic	Estimate (%)	*t*-statistic
Panel A: Subsamples				
(1) Full Sample: 1970–2016	−1.50	(−3.67)	−1.29	(−3.14)
(2) Excluding 2001	−1.42	(−3.59)	−1.21	(−3.04)
(3) Excluding 2007–2009	−1.44	(−3.32)	−1.21	(−2.76)
Panel B: Alternative definitions of *CoBAR*				
(1) Controlling for *UMD*	−1.65	(−4.29)	−1.46	(−3.77)
(2) Controlling for *Large-/Small-Cap HML*	−1.48	(−3.69)	−1.27	(−3.19)
(3) Controlling for *FF Five Factors*	−1.41	(−3.22)	−1.18	(−2.78)
(4) Controlling for *FF Five Factors + UMD*	−1.50	(−3.74)	−1.32	(−3.33)
(5) Controlling for *FF Five Factors + UMD + FMAX*	−1.35	(−3.70)	−1.20	(−3.27)
(6) Controlling for *Industry Factors*	−1.23	(−2.78)	−1.10	(−2.48)
(7) Correlation between *High-* and *Low-Beta Stocks*	−1.27	(−3.47)	−0.98	(−3.05)
Panel C: Residual*CoBAR*				
(1) Controlling for *CoMOM*	−1.54	(−3.88)	−1.35	(−3.33)
(2) Controlling for *CoValue*	−1.66	(−4.03)	−1.44	(−3.47)
(3) Controlling for *Market Correlation*	−1.63	(−4.13)	−1.41	(−3.60)
(4) Controlling for *BAB*	−1.49	(−3.38)	−1.28	(−2.91)
(5) Controlling for *Volatility (BAB)*	−1.48	(−3.86)	−1.31	(−3.41)
(6) Controlling for *Volatility (Market)*	−1.50	(−3.67)	−1.29	(−3.14)
(7) Controlling for *Trend*	−1.51	(−3.49)	−1.30	(−2.98)
(8) Controlling for *PreformationCoBAR*	−1.61	(−3.93)	−1.41	(−3.50)
(9) Controlling for *Inflation*	−1.62	(−4.14)	−1.44	(−3.87)
(10) Controlling for *Sentiment*	−1.52	(−3.72)	−1.31	(−3.18)
(11) Controlling for *Disagreement*	−2.21	(−6.12)	−2.09	(−6.16)
(12) Controlling for *TED Spread*	−1.27	(−2.83)	−1.27	(−2.98)
(13) Controlling for *TED Volatility*	−1.58	(−3.44)	−1.50	(−3.49)
(14) Controlling for *Financial Leverage*	−1.41	(−2.63)	−1.38	(−2.73)

Notes. This table reports year 3 returns to the beta-arbitrage strategy as a function of lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial weekly return correlation in the low-beta decile over the past 12 months. Reported below is the difference in six-factor alpha (FF five factors and momentum factor) and seven-factor alpha (FF five factors, momentum factor, and lottery factor) to the beta-arbitrage strategy between high-*CoBAR* periods and low-*CoBAR* periods. Year 0 is the beta portfolio ranking period. In Panel A, we consider different subsample results. Row 1 shows the baseline results from the full sample. In rows 2 and 3, we exclude the tech bubble crash and the recent financial crisis from our sample. In Panel B, we explore alternative definitions of *CoBAR*. In row 1, we control for the UMD factor in computing *CoBAR*. In row 2, we control for both large- and small-cap HML in computing *CoBAR*. In row 3, we control for the Fama-French five-factor model that adds profitability and investment to their three-factor model. In row 4, we control for the Fama-French five factors and Cahart's momentum factor. In row 5, we control for the Fama-French five factors, momentum factor, and lottery factor from Bali et al. (2017) (denoted as FMAX). In row 6, we perform the entire analysis on an industry-adjusted basis by sorting stocks into beta deciles within industries. In row 7, we instead measure the correlation between the high- and low-beta portfolios, with a low correlation indicating high arbitrage activity. In Panel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to, respectively, *CoMOM* and *CoValue* (Lou and Polk 2022, rows 1–2), the average pairwise correlation in the market (row 3), the BAB factor (Frazzini and Pedersen 2014, row 4), the lagged 36-month volatility of the BAB factor (row 5), market volatility over the past 24 months (row 6), a trend (row 7), lagged *CoBAR* (where we hold the stocks in the low-beta decile constant but calculate *CoBAR* using returns from the previous year; row 8), smoothed past inflation (Cohen et al. 2005, row 9), a sentiment index (Baker and Wurgler 2006, row 10), aggregate analyst forecast dispersion (Hong and Sraer 2016, row 11), the TED spread (Frazzini and Pedersen 2014, row 12), the TED volatility (row 13), and the AR(2) residual of financial leverage (Chen and Lu 2019, row 14). We compute *t*-statistics, shown in parentheses, based on standard errors corrected for serial dependence of 12 lags.

that the large negative abnormal returns that we find across high- and low-*CoBAR* periods are robust to these methodological changes. The weakest abnormal return differential is still −0.98% per month with a *t*-statistic of −3.05.

In Panel C, we replace *CoBAR* with residual *CoBAR* from a time-series regression where we purge from *CoBAR* variation linked to the same variables studied in Panel C of Table 4. In all 14 rows of Panel C, we continue to find a reversal in the year 3 returns on beta-arbitrage stocks following times when beta-arbitrage capital is relatively high in year 0. If anything, controlling for these variables typically increases the magnitude of the effect we find, with alphas exceeding 2% per month and *t*-statistics exceeding six in one case.

In every row of Table 5, *CoBAR* predicts time variation in year 3 returns. The estimates are always economically significant, with most point estimates larger than 1% per month. Statistical significance is always strong as well, with most *t*-statistics larger than 3.[16] Taken together, these results confirm that our measure of crowded beta arbitrage *robustly* forecasts times of strong reversal to beta-arbitrage strategies.

4.5. Smarter Beta-Arbitrage Strategies

One way to measure the economic importance of these boom-and-bust cycles is through an *out-of-sample* calendar-time trading strategy. We combine these time-varying overreaction and subsequent reversal patterns as follows. We first time the standard beta-arbitrage strategy using current *CoBAR*. If *CoBAR* is above the 80th percentile (of its distribution up to that point), we invest in the long-short beta-arbitrage strategy studied in Table 5 for the next six months. Otherwise, we short that portfolio over that time period. (Because we go short the beta-arbitrage strategy 80% of the time, our smarter beta strategy has a negative loading on the original beta-arbitrage strategy.)

In addition, if *CoBAR* from two years ago is below the 20th percentile (of its prior distribution), we long for the next 12 months the long-short beta-arbitrage strategy based on beta estimates from two years ago. Otherwise, we short that portfolio, again for the next 12 months. In other words, our smarter beta-arbitrage strategy has two components: exploiting variation in both holding and postholding period returns. We skip the first three years of our sample to compute the initial distribution as well as show in-sample results in Panel A of Table A4 for the sake of comparison.

This "smarter" beta-arbitrage strategy harvests beta-arbitrage profits much more wisely than unconditional bets against beta. As can be seen from Panel B of Table A4, the four-factor alpha is 43 basis points per month with a *t*-statistic of 2.35. The six-factor alpha (where we add the investment and profitability factors of Fama and French 2015) remains high at 45 basis points per month (*t*-statistic of 2.32). If we then include the lottery factor of Bali et al. (2017), the resulting seven-factor alpha is 57 basis points per month (*t*-statistic of 2.97). Finally, if we also include the BAB factor of Frazzini and Pedersen (2014) as an eighth factor, the abnormal return increases to 63 basis points per month with a *t*-statistic of 3.31.[17] In comparison, the standard value-weight beta-arbitrage strategy yields a four-factor alpha of 0.02% per month (*t*-statistic = 0.08) in our sample period.

We have also estimated conditional regressions where we interact each factor with *CoBAR* to control for conditional risk exposures. The alpha from this regression is significantly larger at 0.77% per month (*t*-statistic of 2.48).

5. Testing the Economic Mechanism

The previous section documents rich cross-sectional and time-series variation in expected returns linked to our proxy for arbitrage activity and the low-beta anomaly. In this section, we delve deeper to test specific aspects of the economic mechanism behind these patterns. Our interpretation of these patterns makes specific novel predictions in terms of the role of firm leverage, the limits to arbitrage, and the reaction of sophisticated investors to these patterns.

5.1. Beta Expansion

Beta arbitrage can be susceptible to positive-feedback trading. Successful bets on (against) low-beta (high-beta) stocks result in prices of those securities rising (falling). If the underlying firms are leveraged, this change in price will, all else equal, result in the security's beta falling (increasing) further.[18] Thus, not only do arbitrageurs not know when to stop trading the low-beta strategy, their (collective) trades also affect the strength of the signal. Consequently, beta arbitrageurs may increase their bets when trading becomes crowded and the expected profitability of the strategy has decreased.

We test this prediction in Panel A of Table 6. The dependent variable is the spread in betas across the high- and low value-weight beta decile portfolios, denoted *BetaSpread*, as of the end of year 1. The independent variables include lagged *CoBAR*, the beta-formation-period value of *BetaSpread* (computed from the same set of low- and high-beta stocks as the dependent variable), the average book leverage quintile (*Leverage*) across the high- and low-beta decile portfolios, and an interaction between *CoBAR* and *Leverage*. Note that because we estimate beta using 52 weeks of stock returns, the two periods of beta estimation that determine the change in *BetaSpread* do not overlap. (Our results are robust to including a time trend in the regression.)

Regression (1) in Panel A of Table 6 shows that when *CoBAR* is relatively high, future *BetaSpread* is also high, controlling for lagged *BetaSpread*. A one-standard-deviation increase in *CoBAR* forecasts an increase in *BetaSpread* of roughly 6% (of the average beta spread). Regression (2) shows that this is particularly true when *Leverage* is also high. If beta-arbitrage bets were to contain the highest book-leverage quintile stocks, a one-standard-deviation increase in *CoBAR* would increase *BetaSpread* by nearly 10%.

These results are consistent with a positive-feedback channel for the beta-arbitrage strategy that works through firm-level leverage. In terms of the economic magnitude of this positive-feedback loop, we draw a comparison with the price momentum strategy. The formation-period spread for a standard price momentum bet in the post-1963 period is around 115%, whereas the momentum profit in the subsequent year is close to 12% (e.g., Lou and Polk 2022). Put differently, if we attribute price momentum entirely to positive-feedback trading, such trading increases the initial return spread by about 10% (12% divided by 115%) in the subsequent year, which is similar

Table 6. Beta Expansion

Panel A: Time-series analysis

Dependent variable	$BetaSpread_{t+1}$	
	(1)	(2)
BetaSpread	0.244***	0.246***
	(0.058)	(0.057)
CoBAR	1.314**	0.320
	(0.545)	(0.645)
Leverage		−0.033**
		(0.015)
CoBAR × *Leverage*		0.433***
		(0.117)
Adjusted R^2	0.090	0.113
Number of observations	564	564

Panel B: Cross-sectional analysis

	$Post\ Ranking\ Beta_{t+1}$			
Dependent variable	(1)	(2)	(3)	(4)
CoBAR	−0.943***	−0.924***		
	(0.144)	(0.139)		
Distance	0.269***	0.253***	0.253***	0.226***
	(0.029)	(0.032)	(0.026)	(0.029)
CoBAR × *Distance*	0.842***	0.640**	0.640***	0.629**
	(0.264)	(0.291)	(0.242)	(0.264)
Leverage		−0.005***		−0.003**
		(0.001)		(0.001)
CoBAR × *Leverage*		0.025*		0.023*
		(0.014)		(0.013)
Leverage × *Distance*		−0.006		0.005
		(0.005)		(0.005)
CoBAR × *Leverage* × *Distance*		0.234***		0.096**
		(0.048)		(0.042)
Time fixed effects	No	No	Yes	Yes
Adjusted R^2	1,265,762	1,265,762	1,265,762	1,265,762
Number of observations	0.258	0.263	0.318	0.320

Notes. This table examines time-series beta expansion associated with arbitrage trading (Panel A) and cross-sectional regressions of postranking stock beta on lagged *CoBAR* (Panel B). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. In Panel A, the dependent variable is the beta spread between the high-beta and low-beta deciles (ranked in year t) in year $t + 1$. *CoBAR* is the average pairwise partial weekly three-factor residual correlation within the low-beta decile over the past 12 months. *Leverage* is a quintile dummy based on the average value-weight book leverage of the bottom and top beta deciles. We also include, in the regression, an interaction term between *CoBAR* and *Leverage*. Standard errors are shown in parentheses. In Panel B, the dependent variable is the postranking stock beta from year t to $t + 1$ (nonoverlapping periods). The main independent variable is lagged *CoBAR*, the average pairwise excess weekly return correlation in the low-beta decile over the past 12 months. *Distance* is the difference between a stock's preranking beta and the average preranking beta in year t. *Leverage* is the book leverage of the firm, measured in year t. We also include all double- and triple-interaction terms of *CoBAR*, *Distance*, and *Leverage*. Other (unreported) control variables include *lagged firm size*, *book-to-market ratio*, *past one-year return*, *idiosyncratic volatility* (over the prior year), and *past one-month return*. Time fixed effects are included in columns 3 and 4 (because *CoBAR* is a time-series variable, it is subsumed by the time dummies). Standard errors, shown in parentheses, are double clustered at both the firm and year-month levels.

*, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

in magnitude to the positive-feedback channel we document for beta arbitrage. Web Appendix Table A5 confirms that these results are robust to the same methodological variations as in Tables 1 and 2.

Panel B of Table 6 turns to firm-level regressions to document the beta expansion our story predicts. In particular, we estimate panel regressions forecasting beta with lagged *CoBAR*. At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. The dependent variable is *Post Ranking Beta*, the stock beta in year $t + 1$ (again, we use nonoverlapping periods). In addition to *CoBAR*, we also include *Distance*, the difference between a stock's preformation beta and the average preformation beta in year *t*. *Leverage* is the book leverage of the firm, measured in year *t*. We also include all double and triple interaction terms of *CoBAR*, *Distance*, and *Leverage*. Other control variables include the *lagged firm size*, *book-to-market ratio*, *lagged one-month and one-year stock return*, and the *prior-year idiosyncratic volatility*. Time fixed effects are included in columns 3 and 4. Note that because *CoBAR* is a time-series variable, it is subsumed by the time dummies in those regressions.

In all four regressions, stocks with higher *Distance* have a higher *Post Ranking Beta*, consistent with betas being persistent. This persistence is higher when *CoBAR* is relatively high. Our main focus is on the triple interaction among *CoBAR*, *Distance*, and *Leverage*. The persistence in a firm's beta is significantly stronger when *CoBAR* and *Leverage* are high. Taken together, these results are consistent with beta-arbitrage activity causing the cross-sectional spread in betas to expand.

As a natural extension, our positive-feedback channel suggests that booms and busts of beta arbitrage should be especially strong among more highly levered stocks. Web Appendix Figure A1 reports results where the sample is split based on leverage. Specifically, at the beginning of the holding period, we sort stocks into four equal groups using book leverage. For each leverage quartile, we compute the *CoBAR* return spread, that is, the difference in four-factor alpha to the beta-arbitrage strategy between high- and low-*CoBAR* periods. Reported in the figure is the cumulative difference in the *CoBAR* return spread between the highest and lowest leverage quartiles over the five years after portfolio formation.

As can be seen from the figure, the difference in the *CoBAR* return spread rises substantially in the first 12 months, by about 20% (1.67% × 12). It then reverses in the subsequent years. For example, the cumulative *CoBAR* return spread in year 4 is roughly −6% (−0.52% × 12). Both are significant at 10%. This finding is consistent with our novel positive-feedback channel facilitating excessive arbitrage trading activity that can potentially destabilize prices.

In Web Appendix Table A6, we confirm that leverage splits enhance the profitability of the calendar-time strategies studied in Table 5. Specifically, we go long a version of the beta-arbitrage strategy restricted to the top quartile of firms based on leverage and go short the corresponding low-leverage (bottom-quartile) version. The resulting in-sample alpha is 49 basis points per month with a *t*-statistic of 2.85 after controlling for the Fama and French (2015) five factors, momentum factor, the lottery factor, the Frazzini and Pedersen (2014) BAB factor, as well as our "smarter" beta-arbitrage portfolio studied in Table A4. The corresponding out-of-sample estimate still generates a statistically significant 37 basis points per month (*t*-statistic = 2.16).

5.1.1. Conditional Attribution. Give that beta is moving with *CoBAR*, we also estimate conditional performance attribution regressions (that is, we allow for the possibility that portfolio betas and expected market and factor returns comove in the time series). Figure 3 plots the conditional security market line in the short and long runs as a function of lagged *CoBAR*. It is easy to see, from the figure, our result that beta expansion and destabilization go hand in hand: the range of average beta across the 20 beta-portfolios is much larger during high-*CoBAR* periods than in low-*CoBAR* periods.

5.2. Low Limits to Arbitrage

We interpret our findings as consistent with arbitrage activity facilitating the correction of the slope of the security market line in the short run. However, in periods of crowded trading, arbitrageurs can cause price overshooting. In Table 7, we exploit cross-sectional heterogeneity to provide additional support for our interpretation. All else equal, arbitrageurs prefer to trade stocks with low idiosyncratic volatility (to reduce tracking error), high liquidity (to facilitate opening/closing of the position), and large capitalization (to increase strategy capacity). Finally, the maximum daily return signal of Bali et al. (2011) helps identify stocks that retail investors, rather than institutional investors, prefer. As a consequence, we split our sample along each of these dimensions. In particular, we rank stocks into quartiles based on the variable in question (as of the beginning of the holding period); we label the quartile with the weakest limits to arbitrage as "low LTA" and the quartile with the strongest limits to arbitrage as "high LTA." Our focus is on the long-run reversal associated with periods of high *CoBAR*.

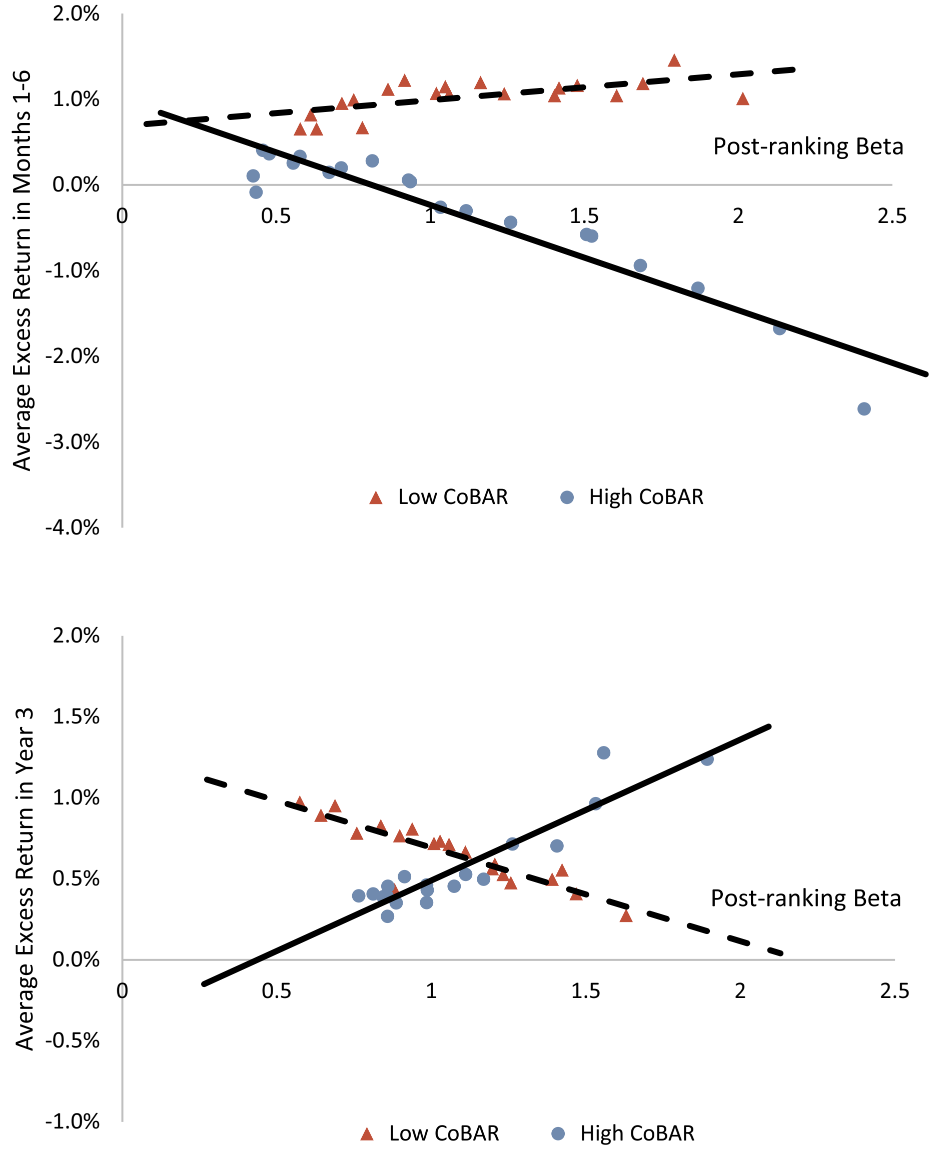
The first two columns report results based on market capitalization, the third and fourth based on idiosyncratic volatility, the fifth and sixth based on illiquidity, and the final two on MAX. The first column of each pair shows the difference in six-factor alpha to the beta-arbitrage strategy between high-*CoBAR* periods and low-*CoBAR* periods in year 3, whereas the second column shows the difference in seven-factor alpha in year 3.

For each of the four proxies for low limits to arbitrage, we find economically and statistically significant differences in the predictability of year 3 returns. In summary, Table 7 confirms that our effect is stronger among stocks with weaker limits of arbitrage, exactly where one expects arbitrageurs to play a more important role.[19]

5.3. Time-Series and Cross-Sectional Variation in Fund Exposures

We next use our novel measure of beta-arbitrage activity to understand time-series and cross-sectional variation in the performance of long-short/market-neutral hedge funds, typically considered to be the classic example of smart money, as well as active mutual funds, which are subject to more stringent leverage and short-sale constraints. Web Appendix Table A7 reports estimates of

Figure 3. (Color online) This Figure Shows the *Conditional* Security Market Line as a Function of Lagged *CoBAR* (i.e., Where Betas Are Allowed to Vary with *CoBAR*)



Notes. At the end of each month, all stocks are sorted into vigintiles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. We then estimate two conditional security market lines based on these 20 portfolios: one SML using portfolio returns in months 1–6 (top panel) and the other using portfolio returns in year 3 after portfolio formation (bottom panel); the betas used in these SML regressions are the corresponding postranking betas. The *y* axis reports the average monthly excess returns to these 20 portfolios, and the *x* axis reports the postranking beta of these portfolios. Beta portfolios formed in high-*CoBAR* periods are depicted with a blue circle and fitted with a solid line, and those formed in low-*CoBAR* periods are depicted with a red triangle and fitted with a dotted line.

panel regressions of monthly fund returns on the Fama-French-Carhart four-factor model augmented with the beta-arbitrage factor of Frazzini and Pedersen (2014). In particular, we allow the coefficient on the Frazzini-Pedersen BAB factor to vary as a function of *CoBAR*, a fund's AUM, and the interaction between these two variables. To capture variation in a fund's AUM, we create a dummy variable, *SizeRank*, that takes the value of zero if the fund is in the smallest-AUM tercile (within the active mutual fund or long-short equity hedge fund industry, depending on the returns being analyzed) in the previous month, one if it is in the middle tercile, and two otherwise. The first two columns analyze hedge fund returns, whereas the last two columns analyze active mutual fund returns.

We find that the typical long-short equity hedge fund increases its exposure to the BAB factor when *CoBAR* is relatively high. For the 20% of the sample period that

Table 7. Limits to Arbitrage

	Market cap		Idiosyncratic volatility		Illiquidity		Maximum	
	6F Alpha	7F Alpha	6F Alpha	7F Alpha	6F Alpha	7F Alpha	6F Alpha	7F Alpha
Low LTA	**−1.53%**	**−1.30%**	**−1.56%**	**−1.32%**	**−1.44%**	**−1.21%**	**−1.54%**	**−1.33%**
	(−3.55)	(−3.01)	(−4.24)	(−3.70)	(−3.11)	(−2.57)	(−3.99)	(−3.57)
High LTA	0.08%	0.13%	−0.34%	−0.27%	−0.39%	−0.36%	−0.24%	−0.17%
	(0.15)	(0.27)	(−0.72)	(−0.58)	(−0.80)	(−0.75)	(−0.57)	(−0.41)
Low-high	**−1.60%**	**−1.43%**	**−1.23%**	**−1.05%**	**−1.06%**	−0.85%	**−1.30%**	**−1.16%**
	(−2.74)	(−2.47)	(−2.48)	(−2.11)	(−2.11)	(−1.61)	(−3.06)	(−2.78)

Notes. This table reports year 3 returns to the beta-arbitrage strategy as a function of lagged *CoBAR* in various subsamples ranked by proxies for limits to arbitrage (LTAs) (as of the beginning of the holding period). At the end of each month, all stocks are sorted into deciles based on their market beta calculated using daily returns in the past 12 months. To account for illiquidity and nonsynchronous trading, on the right-hand side of the regression equation, we include five lags of the excess market return in addition to the contemporaneous excess market return. The preranking beta is the sum of the six coefficients from the OLS regression. All months are then classified into five groups based on *CoBAR*, the average pairwise partial return correlation in the low-beta decile over the past 12 months. Reported below is the difference in six-factor and seven-factor alphas to the beta-arbitrage strategy between high-*CoBAR* periods and low-*CoBAR* periods in year 3. Year zero is the beta portfolio ranking period. "Low LTA" corresponds to the subsample of stocks with low limits to arbitrage, and "high LTA" corresponds to the subsample with high limits to arbitrage. "Low-high" is the difference in monthly portfolio alpha between the two subsamples. We measure limits to arbitrage using four common proxies. In columns 1 and 2, we rank stocks into quartiles based on market capitalization; we label the top quartile as "low LTA" and the bottom quartile as "high LTA." In columns 3 and 4, we rank stocks into quartiles based on idiosyncratic volatility with regard to the Carhart four-factor model; we label the bottom quartile as "low LTA" and the top quartile as "high LTA." In columns 5 and 6, we rank stocks into quartiles based on the illiquidity measure of Amihud (2022); we label the bottom quartile as "low LTA" and the top quartile as "high LTA." In columns 7 and 8, we rank stocks into quartiles based on the maximum daily return of Bali et al. (2011); we label the bottom quartile as "low LTA" and the top quartile as "high LTA." We compute *t*-statistics, shown in parentheses, based on standard errors corrected for serial dependence of 12 lags. Five percent statistical significance is indicated in bold.

is associated with the lowest values of *CoBAR*, the typical hedge fund's BAB loading is −0.063. This loading increases by 0.017 for each increment in *CoBAR* rank. (It is noteworthy that the average long-short hedge fund is loading negatively on the BAB factor—i.e., on average, funds are tilting toward high beta stocks.)

Adding the interaction with AUM reveals that the ability of hedge funds to time beta-arbitrage strategies is decreasing in the size of the fund's assets under management. These findings seem reasonable, as we would expect large funds to be unable to time a beta-arbitrage strategy as easily as smaller (and presumably nimbler) funds.

The typical small fund's exposure increases by 0.030 for each increase in *CoBAR* rank. Thus, when *CoBAR* is in the top quintile, the typical small hedge fund's BAB loading is 0.047. In contrast, large hedge funds' BAB loading moves by 0.016 from the bottom to the top *CoBAR* quintiles, a much smaller increase in exposure to beta arbitrage. Indeed, when *CoBAR* is high, small hedge funds have loadings on BAB that are nearly twice as large.

As can be seen from columns 3 and 4, there is a vastly different pattern in the market exposures of mutual funds. To start, mutual funds have an average market beta that is larger than one. Second, none of the interactions are statistically significant. In particular, mutual funds' loadings on the beta-arbitrage strategy do not vary with *CoBAR*, our proxy for the strategy's crowdedness.

5.4. Fresh vs. Stale Beta

Though beta-arbitrage activity may cause the beta spread to vary through time, for a feedback loop to occur, beta arbitrageurs must base their strategies on fresh estimates of beta rather than on stale estimates. (Note that the autocorrelation in a stock's market beta is far less than one.) Consistent with this claim, we show that our predictability results decay as a function of beta staleness.

We repeat the previous analysis of Section 4.4 but replace our fresh beta estimates (measured over the most recent year) with progressively staler ones. In particular, we estimate betas in each of the five years prior to the formation year. As a consequence, both the resulting beta strategy and the associated *CoBAR* are different for each degree of beta staleness. For each of these six beta strategies, we plot CAPM alpha of the strategy in months 1–6. Web Appendix Figure A2 plots the CAPM alpha as a function of the degree of staleness of beta. We find that the return predictability decays as the beta signal becomes more and more stale.

6. Conclusion

We study the response of arbitrageurs to a flat security market line. Using an approach to measuring arbitrage activity first introduced by Lou and Polk (2022), we document booms and busts in beta arbitrage. Specifically, we find that when arbitrage activity is relatively low, abnormal returns on beta-arbitrage strategies take much longer to materialize, appearing three years after putting on the trade. In contrast, when arbitrage activity is relatively high, abnormal returns on beta-arbitrage strategies occur relatively quickly and then revert over the next three years. Thus, our findings are consistent with arbitrageurs exacerbating the time variation in the expected return to beta arbitrage we document.

We provide evidence on a novel positive-feedback channel for beta-arbitrage activity. Because the typical firm is levered and given the mechanical effects of leverage on equity beta (Modigliani and Miller 1958), buying low-beta stocks and selling high-beta stocks may cause the cross-sectional spread in betas to increase. We show that this beta expansion occurs when beta-arbitrage activity is high and particularly so when stocks typically traded by beta arbitrageurs are highly levered. Thus, beta arbitrageurs may actually increase their bets when the profitability of the strategy has decreased.

Interestingly, the *unconditional* four-factor alpha of a value-weight beta-arbitrage strategy over our 1970–2019 sample is close to zero, much lower than the positive value one finds for earlier samples (also see Novy-Marx and Velikov 2022). Thus, it seems that arbitrageurs' response to Black et al. (1972)'s famous finding has been right *on average*. However, our conditional analysis reveals rich time-series variation that is consistent with the general message of Stein (2009): arbitrage activity faces a significant coordination problem for unanchored strategies that have positive-feedback characteristics.

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Endnotes

[1] The term "booms and busts" refers to two related phenomena. First, we use "booms and busts" to describe the significant time variation in beta-arbitrage activity. Second, we use this term to refer to our novel observation that when there is too much arbitrage activity, prices of stocks in the beta-arbitrage portfolio overshoot initially (the boom phase), which then reverse in the longer term (the bust phase).

[2] See, for example, Barberis et al. (2005), Greenwood and Thesmar (2011), Lou (2012), and Anton and Polk (2014).

[3] We use various methods to adjust the standard errors in our return analysis to adjust for the fact that returns are measured over overlapping horizons. The t-statistics reported in the paper are based on Newey-West adjustments with appropriate lags. The results are robust to other methods of assessing statistical significance. For example, if we bootstrap the standard errors in the aforementioned analysis, the resulting t-statistic is -3.43. If, instead, we measure the joint significance of the *nonoverlapping monthly* return spreads in year 3 (of -1.29%/month), we are unable to reject the null at the 1% level.

[4] See also Karceski (2002), Baker et al. (2011), Buffa et al. (2022), and Jylhä (2018).

[5] In addition, Campbell et al. (2018) document that high-beta stocks hedge time variation in the aggregate market's return volatility, offering a potential neoclassical explanation for the low-beta anomaly.

[6] The idea that positive-feedback strategies are prone to destabilizing behavior goes back to at least DeLong et al. (1990). In contrast, negative-feedback strategies like ADR arbitrage or value investing are less susceptible to destabilizing behavior by arbitrageurs, as the price mechanism mediates any potential congestion. See Stein (2009) for a discussion of these issues.

[7] Though eventually published in 1972, Black et al. (1972) had been presented as early as August of 1969. Mehrling (2005)'s biography of Fischer Black details the early history of the low-beta anomaly.

[8] Our results are robust to measuring *CoBAR* as the (minus) cross-correlation between high- and low-beta deciles.

[9] We focus on value-weight portfolios to ensure that our findings are robust and not driven by small-cap firms. Indeed, the use of equal-weight (or beta-weight) portfolios has recently been criticized by Novy-Marx and Velikov (2022).

[10] *CoBAR* is essentially uncorrelated with a similar measure of excess comovement based on the fifth and sixth beta deciles.

[11] We choose to forecast *CoBAR* in a predictive regression rather than explain *CoBAR* in a contemporaneous regression simply to reduce the chance of a spurious fit. However, our results are robust to estimating contemporaneous versions of these regressions.

[12] Whereas our focus in the paper is the subset of arbitrageurs that trade the low-beta anomaly, other arbitrageurs who trade other signals may play a role in facilitating the ultimate correction of these booms and busts in beta arbitrage. For example, suppose that low-beta arbitrageurs push up the price of low-beta stocks and push down the price of high-beta stocks, traditional value investors will start trading in the opposite direction, helping to bring prices back to their fundamental value.

[13] The original coefficients of a_1, b_1, c_1, a_2, b_2, and c_2 are reported in Web Appendix Table A2.

[14] Table A3 examines persistence in arbitrage activity and its impact on beta-arbitrage returns. Conditional on high *CoBAR*, in times when future *CoBAR* is also high, we see both larger short-term run-up as well as stronger long-run reversal to the BAB strategy, compared with times when future *CoBAR* is low.

[15] Lou and Polk (2022) document that for the classic 12-month momentum strategy, during periods of high *CoMOM* (that is, periods of relatively high momentum arbitrage activity), though the resulting overreaction occurs primarily in the formation period, the subsequent reversal of that overreaction occurs much sooner, starting in month 7 in the period following portfolio formation. Lou and Polk's results, viewed in conjunction with ours, suggest that the extent to which a trading strategy's signal is transitory might also be a factor in the timing of the overreaction and the eventual reversal. Indeed, the postformation returns to our high-versus-low leverage refinement of the classic low-beta strategy do exhibit stronger overreaction during the holding period.

[16] In untabulated results, we show that our documented variation in the long-run reversal to the beta-arbitrage strategy is significant among both overpriced and underpriced stocks classified by the mispricing score and is robust to controlling for the mispricing factor in Stambaugh et al. (2015).

[17] In the next section, we construct an "even-smarter" beta-arbitrage strategy by further exploiting differences between high-leverage and low-leverage firms. In particular, we divide all stocks into four quartiles based on their lagged leverage ratios. We then go long the smart-beta strategy formed solely with high-leverage stocks and short the smart-beta strategy solely with low-leverage stocks. This "even-smarter" beta strategy yields a monthly out-of-sample alpha of 37 basis points (bp) (t-statistic = 2.16) after controlling for the Fama and French (2015) five factors, momentum factor, the lottery factor, the Frazzini and Pedersen (2014) BAB factor, as well as our "smarter" beta-arbitrage portfolio studied in Table A4. See Web Appendix Table A6 for these results.

[18] The idea that, all else equal, changes in leverage drive changes in equity beta is, of course, the key insight behind proposition II of Modigliani and Miller (1958).

[19] Combined with the time-series result in Table 2 that *CoBAR* is strongly correlated with both the institutional ownership of low-beta stocks and the total AUM of hedge funds, these cross-sectional results in Table 7 mitigate the concern that variation in *CoBAR* is driven by correlated trades of traditional noise traders (i.e., retail investors). Throughout the paper, we follow the convention of labelling professional investors that exploit mispricing patterns as arbitrageurs. Destabilizing arbitrage activity could be viewed as a form of noise trading. As a result, one could instead summarize our work as studying a novel type of noise trading arising from a coordination failure among professional investors.

References

Amihud Y (2002) Illiquidity and stock returns: Cross-section and time-series effects. *J. Financial Markets* 5(1):31–56.
Anton M, Polk C (2014) Connected stocks. *J. Finance* 69:1099–1127.
Antoniou C, Doukas JA, Subrahmanyam A (2015) Investor sentiment, beta, and the cost of equity capital. *Management Sci.* 62:347–367.
Baker M, Wurgler J (2006) Investor sentiment and the cross-section of stock returns. *J. Finance* 61:1645–1680.
Baker M, Wurgler J (2007) Investor sentiment in the stock market. *J. Econom. Perspect.* 21:129–152.
Baker M, Bradley B, Taliaferro R (2014) The low-risk anomaly: A decomposition into micro and macro effects. *Financial Analysts J.* 70:43–58.
Baker M, Bradley B, Wurgler J (2011) Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly. *Financial Analysts J.* 67:40–54.
Bali TG, Cakici N, Whitelaw RF (2011) Maxing out: Stocks as lotteries and the cross-section of expected returns. *J. Financial Econom.* 99:427–446.
Bali TG, Brown SJ, Murray S, Tang Y (2017) A lottery-demand-based explanation of the beta anomaly. *J. Financial Quant. Anal.* 52:2369–2397.
Barberis N, Huang M (2008) Stocks as lotteries: The implications of probability weighting for security prices. *Amer. Econom. Rev.* 98:2066–2100.
Barberis N, Shleifer A (2003) Style investing. *J. Financial Econom.* 68:161–199.
Barberis N, Shleifer A, Wurgler J (2005) Comovement. *J. Financial Econom.* 75:283–317.
Black F (1972) Capital market equilibrium with restricted borrowing. *J. Bus.* 45:444–454.
Black F, Jensen MC, Scholes M (1972) The capital asset pricing model: Some empirical tests. Jensen MC, ed. *Studies in the Theory of Capital Markets* (Praeger, New York), 79–121.
Blitz D, van Vliet P (2007) The volatility effect. *J. Portfolio Management* 34:102–113.
Blitz D, Pang J, van Vliet P (2013) The volatility effect in emerging markets. *Emerging Markets Rev.* 16:31–45.
Buffa A, Vayanos D, Woolley P (2022) Asset management contracts and equilibrium prices. *J. Political Econom.* 130:3146–3201.
Campbell J, Giglio S, Polk C, Turley R (2018) An intertemporal CAPM with stochastic volatility. *J. Financial Econom.* 128:207–233.
Cao C, Chen Y, Liang B, Lo A (2013) Can hedge funds time market liquidity? *J. Financial Econom.* 109:493–516.
Carhart MM (1997) On persistence in mutual fund performance. *J. Finance* 52:57–82.
Chen Z, Lu A (2019) A market-based funding liquidity measure. *Rev. Asset Pricing Stud.* 9:356–393.0
Cohen RB, Polk C, Vuolteenaho T (2005) Money illusion in the stock market: The Modigliani-Cohn hypothesis. *Quart. J. Econom.* 70: 639–668.
Cremers M, Petajisto A, Zitzewitz E (2013) Should benchmark indices have alpha? Revisiting performance evaluation. *Critical Finance Rev.* 2:1–48.
DeLong JB, Shleifer A, Summers LH, Waldmann R (1990) Positive feedback investment strategies and destabilizing rational speculation. *J. Finance* 45:379–395.
Fama EF, French KR (1992) The cross-section of expected stock returns. *J. Finance* 47:427–465.
Fama EF, French KR (2015) A five-factor asset pricing model. *J. Financial Econom.* 116:1–22.
Fama EF, MacBeth JD (1973) Risk, return, and equilibrium: Empirical tests. *J. Political Econom.* 81(3):607–636.
Frazzini A, Pedersen LH (2014) Betting against beta. *J. Financial Econom.* 111:1–25.
Friedman M (1953) The methodology of positive economics. *Essays in Positive Economics* (Chicago University Press, Chicago), 3–46.
Greenwood R, Thesmar D (2011) Stock price fragility. *J. Financial Econom.* 102:471–490.
Hong H, Sraer D (2016) Speculative betas. *J. Finance* 71:2095–2144.
Jylhä P (2018) Margin requirements and the security market line. *J. Finance* 73:1281–1321.
Karceski J (2002) Returns-chasing behavior, mutual funds, and beta's death. *J. Financial Quant. Anal.* 37:559–594.
Kumar A (2009) Who gambles in the stock market? *J. Finance* 64: 1889–1933.
Lintner J (1965) The valuation of risky assets and the selection of risky investments in stock portfolios and capital budgets. *Rev. Econom. Statist.* 47:13–37.
Liu J, Stambaugh RF, Yuan Y (2018) Absolving beta of volatility's effects. *J. Financial Econom.* 128:1–15.
Lou D (2012) A flow-based explanation for return predictability. *Rev. Financ. Stud.* 25:3457–3489.
Lou D, Polk C (2022) Comomentum: Inferring arbitrage activity from return correlations. *Rev. Financ. Stud.* 35:3272–3302.
Mehrling P (2005) *Fischer Black and the Revolutionary Idea of Finance* (John Wiley and Sons, Hoboken, NJ).
Miller EM (1977) Risk, uncertainty, and divergence of opinion. *J. Finance* 32:1151–1168.
Modigliani F, Miller MH (1958) The cost of capital, corporation finance, and the theory of investment. *Amer. Econom. Rev.* 48: 655–669.
Modigliani F, Cohn R (1979) Inflation, rational valuation, and the market. *Financial Analysts J.* 35:24–44.
Novy-Marx R, Velikov M (2022) Betting against betting against beta. *J. Financial Econom.* 143:80–106.
Pástor L, Stambaugh RF (2003) Liquidity risk and expected stock returns. *J. Political Econom.* 111:642–685.
Sharpe W (1964) Capital asset prices: A theory of market equilibrium under conditions of risk. *J. Finance* 19:425–442.
Shleifer A, Vishny RW (1997) The limits of arbitrage. *J. Finance* 52: 35–55.
Stambaugh RF, Yu J, Yuan Y (2015) Arbitrage asymmetry and the idiosyncratic volatility puzzle. *J. Finance* 70:1903–1948.
Stein J (2009) Sophisticated investors and market efficiency. *J. Finance* 64:1517–1548.