Preferences over equality in the presence of costly income sorting

Gilat Levy and Ronny Razin, LSE[1]

Abstract: We analyze preferences over redistribution in societies with costly (positive) income sorting. We identify a new motivation for redistribution, where individuals support more taxation in order to reduce the costs of sorting. We characterize a simple condition over income distributions which implies that even relatively rich voters -with income above the mean- will prefer full equality (and thus no sorting) to societies with costly sorting. In line with the empirical literature which illustrates that support for redistribution is stronger in equal societies, this condition is satisfied when income inequality is not too high. We show that the condition is satisfied by a large family of distribution functions and specifically for sufficiently equal Pareto and Lognormal distributions which have been used in estimations of income distributions.

1 Introduction

The presence of income sorting or stratification in society has received plenty of attention in the economics and sociology literature.[2] Relocating to a leafy suburb, sending your child to a private school, or conspicuous consumption of sports car, jewelry or designer clothes, have all been suggested as actions that individuals take to guarantee that they mix, interact, or match only with those with the same or higher income than theirs.[3]

In societies in which individuals engage in such costly sorting, what are their preferences over redistribution? Beyond being a traditional tool for creating equality, income redistribution will potentially decrease the incentive to sort; it might decrease the benefit of mixing with other rich individuals, and thus also reduce the cost of sorting. In this paper we explore how costly income sorting shapes individual and political preferences over policies of redistribution.

To analyze this question, we put forward a simple model in which individuals differ in their income. We assume that the utility of an individual exhibits complementarities in his disposable

[1]We thank Nageeb Ali, Tim Besley, Thomas Piketty, Andy Postlewaite, Tom Roemer, Omer Moav and Richard Van Weelden for helpful comments. We also thank seminar participants in the LSE/NYU Political Economy conference 2012, CEPR Public Policy conference 2012, and the Priorat workshop 2012.

[2]See for example Benabou (1996), Fernandez and Rogerson (2001), Kremer (1997) and Wilson (1987).

[3]The literature on conspicuous consumption includes contributions by Liebenstien (1950), Bagwell and Bernheim (1996), Pesendorfer (1995) and Heffetz (2011). Glazer and Konrad (1996) consider signalling of wealth via charitable donation which exhibits positive externalities. Moav and Neeman (2012) analyze the trade-off between conspicuous consumption and human capital as signals for unobserved income.

income and that of those he interacts with. A sorting equilibrium is the partition of society into "clubs", where all individuals who use the same costly signal insure that they interact only with each other. The tax rate along with the income distribution are the parameters affecting the benefit and cost of each club. In this environment we characterize preferences of individuals over linear taxes. This simple framework can be seen as a reduced form of several economic environments with costly income sorting:

The education market: The literature on sorting in children's education (see for example Epple and Romano 1998 and Fernandez and Rogerson 2003) typically assumes a signal crossing condition, i.e., that richer individuals care more about the education of their child. If there are peer effects, i.e., complementarities in the ability of pupils, or if education is financed locally with school quality determined by a majority vote in the community, then individuals will sort into schools or neighborhoods according to income. Our model can be viewed as a reduced form of such environments. The costly signals are the entry fees to private schools or house prices in a wealthy suburb (where children would attend the state school); in both cases these costs imply that the child mixes with the children of the relatively rich.[4]

In tertiary education, income might have a more direct complementarity when one considers networks and potential for future investment and work opportunities. A recent study by Cohen and Malloy (2010) on alumni relations finds that U.S. mutual fund portfolio managers placed larger concentrated bets on companies to which they were connected through an education network, and that the fund managers performed significantly better on those connected positions to the tune of around 8%. These college network effects imply complementarities in knowledge, human capital, and connections, all correlated with and enabled by income. Thus, individuals may sort into expensive business schools in order to enjoy these network effects.

The marriage market: Another example explored in the literature is that of the assortative matching in the marriage market. Becker (1973) has shown that positive assortative matching will arise on wealth when there are wealth complementarities. Lam (1998) formalizes conditions on both utilities and bargaining procedures in marriages to show how positive sorting arises on wage incomes as well. Pesendorfer (1995) describes a "dating" market where individuals of different types, be it their education, entertainment skills, or human capital are matched with one another. The utility from matching is supermodular, which induces high types to distinguish themselves by acquiring the newest fashion design. As human capital

[4]See Bradford and Kelejian (1973) who show empirically that the decision of the middle classes to live in the suburbs depend (negatively) on the share of the poor in the city.

and education attainment are correlated with income, our model is a reduced form for this matching environment as well; the different signals would be the different fashion labels that would allow individuals to identify one another.[5]

In environments such as the ones described above, would individuals prefer to live in an equal society without the need to sort, or in an unequal society where one can mix with the rich but has to pay a cost for doing so? Intuitively, it is the middle income groups who may be particularly affected by pressures to sort. The poor will be simply left out as others sort away from them; for them, the option of a private school is not viable. The wealthy groups who enjoy sorting will probably not be too sensitive to its cost. In contrast, for the middle income groups, the incentive to interact with richer individuals may put a large strain on their budgets.

The distribution of income in society might be an important element in determining the preferences of these middle income groups. Income inequality creates a strong desire to match with the rich as in this case sending your child to a state versus a private school, or missing out on a place in a business school with well-connected alumni, may have large consequences. One intuition would be that inequality would push the middle classes to advocate more redistribution as it will soften the pressures to engage in costly signalling. On the other hand, it might induce the middle classes to be more concerned about the incomes of the groups they wish to mingle with, and therefore support less redistribution.[6]

In our main result we identify a necessary and sufficient condition over income distributions which implies that all individuals up to the mean (and possibly some above) prefer full equality to all sorting equilibria and tax levels. We show that the condition is satisfied for relatively equal societies, and illustrate that the more equal is the society, the larger is the share of agents with income above the mean that would support redistribution. Specifically, we show that the condition is satisfied for sufficiently equal Gamma, Pareto and Lognormal distributions, often used to estimate real income distributions.

On the other hand, if a society is sufficiently unequal, there will be some individuals below the mean that would oppose redistribution. High inequality implies that the middle class can, by sorting, avoid a large mass of very poor individuals, while keeping the cost of sorting relatively low. To illustrate this, consider the symmetric Beta distributions on [0,1]:

[5] If society treats men and women equally, or if marriages can occur among any pair regardless of gender, it is also sufficient to consider a symmetric matching model as in Pesendorder (1995) and as we do here. Alternatively one can enrich the model to consider two different income distributions for each side of the market.

[6] Naturally, for individuals with income below the mean, there is also the standard motivation to support redistribution to simply increase their own income.

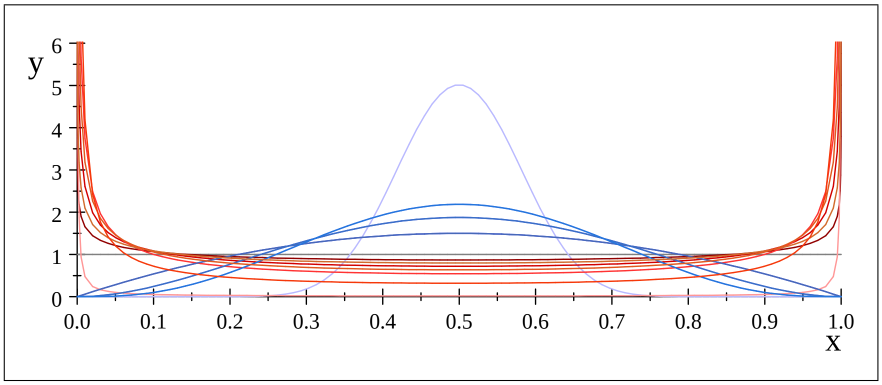

Figure 1: $f(x) = \frac{x^{\alpha-1}(1-x)^{\alpha-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\alpha-1}du}$. The inverted U-shaped densities have $\alpha > 1$, and the U-shaped ones have $\alpha < 1$.

For these distributions the Gini coefficient is monotonically decreasing in α. When $\alpha > 1$, the density is inverted U-shaped and the mean and some above prefer maximum taxation to any costly sorting environment. Moreover, the higher is α, the more there are individuals above the mean who prefer full equality. On the other hand, when $\alpha < 1$, the density is U-shaped, and it is possible to sustain sorting equilibria and low taxes with the support of individuals poorer than the mean or the median. Such bimodal distributions include a large mass of the very poor which the mean and some below are able to avoid with cheap enough signals. Indeed India is one example of a society with a large fraction of a very poor population, coupled with low income tax rates and a large degree of income sorting, as manifested for example in the marriage market.[7]

In the classical paper of Meltzer and Richards (1981), an individual favours taxation if (and only if) her income is below the mean income. While the empirical literature supports a positive relation between income and preferences over taxation, a familiar puzzle is the observation that many voters with income below the mean vote for parties on the right who traditionally oppose further taxation.[8] The opposite happens as well; De la O and Rodden (2008) use the Eurobarometers and World Values Survey data to show that on average well over 40% of the richest individuals vote for parties of the left in Europe. Moreover, some evidence also indicates that voters in more equal societies are more positive towards further taxation and transfers, while voters in relatively unequal societies have less positive attitudes towards taxation.[9]

[7] See Banerjee et al (2010) who measure the effects of castes (often correlated with income) as well as costly signals (such as education) on the marriage market.

[8] See for example Frank (2004). Gelman et al (2007) show that the positive relation between income and voting right is strong in poor American states and weak in rich states.

[9] See Perotti (1996) and Kerr (2011).

Our paper ties together these two empirical observations, as in the model it is in sufficiently equal (unequal) societies where one might find rich (poor) agents voting to the left (right). There is a large literature in Political Economy explaining one or both of the above empirical observations, and we contribute to this literature by identifying an explanation that is based on the effects that redistribution has on the patterns of costly sorting in societies.

Individuals in our model care about their disposable income and -indirectly- about the distribution of income, as it affects the sorting equilibrium. This effect strengthens the usual distaste of the poor for inequality, as they are left behind when others sort away from them. Our model can then also shed light on recent empirical findings on inequality and happiness, which show that happiness can decrease even when everyone's income had increased, if inequality increases as well.[10] In the standard approach (e.g., Meltzer and Richards 1981) when utility is proportional to disposable income this cannot arise. One explanation that has been put forward in this literature is that agents have direct preferences over income inequality (e.g., preferences over status and relative standing). In our model such preferences arise endogenously, and indeed, it is easy to find examples in our model in which the income of all individual increases along with inequality, while the utility of a sizable fraction of the population decreases as a result of the changes in the cost and benefit of sorting.

We discuss the relation to the theoretical literature in the next section. Section 3 presents the model. In Section 4 we characterize indirect utilities over taxation, and in Section 5 we derive our main result showing when maximum taxation or full equality is supported by a substantial majority in the population. In Section 6 we discuss other policy tools such as non-linear taxes, the provision of efficient (costless) sorting and taxes on the revenues from sorting. We also consider preferences over the exclusiveness of sorting. We show how an "ends against the middle" coalition might arise in which the middle class faces opposition from the rich and the poor both preferring to increase the exclusiveness of sorting. An appendix contains all proofs not in the text.

2 Related literature

Previous literature in Political Economy explaining why the poor might vote right (or why the rich might vote left), and why preferences for redistribution are stronger in equal societies, can in general be split into dynamic and static models.[11]

Dynamic models take into account the possibility of mobility, different beliefs about luck

[10] A recent example is Oishi, Kesebir and Diener (2011). See also Alesina, Di Tella and MacCulloch (2004).

[11] For a good summary of this literature, see Alesina and Giuliano (2009).

vis a vis effort as determining income, or induced preferences over the income distribution through growth externalities. Piketty (1995) and Benabou and Tirole (2004) show how different beliefs, i.e., whether success is a function of luck or effort, could induce multiple equilibria, one with a large welfare state and low effort and one with a small government and high effort.[12] Another dynamic model that considers preferences over redistributions is Benabou and Ok (2002) who show how a future redistribution of a concave (convex) function of the current income distribution will, by Jensen's inequality, induce those below (above) the mean to vote against (in favour) redistribution. Benabou (2000) shows how in the presence of imperfect credit and insurance markets the relation between inequality and the welfare state is potentially U-shaped. Galor and Zeira (1993) show how credit constraints and education externalities imply that middle income voters prefer a more equal society, as this will allow the poor to gain higher education levels.

There are also static models, as is ours, that explain why agents vote against their standard economic interest. Some consider a multidimensional policy space (Roemer 1998, Levy 2004, Alesina and La Ferrara 2005). For example, poor agents who care about religion might vote for a religious right-wing party, whereas racist preferences may induce voters to vote for less redistribution if tax revenues will be spent on other groups. Shayo (2009) introduces social group identity and shows that when voters identify with their nation as opposed to their economic class only, the tax rate is lower. The effect of inequality on redistribution however is ambiguous. Our model has a unidimensional heterogeneity among voters and no additional elements in the utility function beyond utility from (matching) income. Within this literature of static explanations a related paper is Corneo and Gruner (2000) who assume that agents' consumption levels signal their (pre-tax) wealth, and therefore redistribution reduces the information value of signalling. Our analysis is complementary, as we consider the after-tax income as an important component for matching which implies that redistribution changes the market for signalling, e.g., the price of costly signals such as luxury goods.

Beyond the literature on the political economy of taxation, our paper is also related to the literature on sorting in the tradition of Tiebout models, where agents who have different preferences over the provision of public goods sort themselves into different communities.[13] Within the sorting literature several papers consider the effect of redistributive policies. Fernandez and Rogerson (2003) consider provision of quality of schooling and analyze different equalizing policies which target the finance of education. Epple and Romano (1998) model the supply

[12] See also Alesina and Angeletos (2005).

[13] For an example of this approach see Fernandez and Rogerson (2001).

side, i.e., the market for private schools, and show how richer and more able agents are screened into better quality schools. They consider the policy of school vouchers and show that it is mainly high ability and high income types who benefit from the introduction of vouchers to private schools.

Tournaments has been analyzed as another form of sorting; Fernandez and Gali (1997) show that with credit constraints, markets perform less well than tournaments at sorting individuals according to ability. Hopkins and Korneiko (2011) analyze a model in which they explore in the context of a tournament the effect of equality in the distribution of rewards vis a vis an equality in the distribution of income. They show that the latter induces effort whereas the former hampers it.

Finally, our model is related to recent literature on the cost of signalling. Hoppe, Moldovanu and Sela (2009) consider a model in which individuals signal their attributes. Their model is an incomplete information model with two-sided heterogeneity, finite types and perfect signalling. We discuss the relation of our results to theirs in more detail in Section 5. Several other papers focus on coarse matching, for example Hoppe, Moldovanu and Ozdenoren (2011) and McAfee (2002), and show the conditions under which coarse matching provides sufficiently high surplus compared with random or perfect matching.[14]

3 The model

The basic model includes an income distribution function and linear taxation. In addition, we assume income complementarities which give rise to an equilibrium of costly sorting. Preferences over taxation will be shaped by how tax rates affect the sorting equilibrium outcomes.

Income distribution and taxation: Suppose that agents differ in their income, x, which is distributed according to some $F(x)$ and density $f(x)$ (positive everywhere) on some $[\underline{v}, \bar{v}]$, $0 \leq \underline{v} < \bar{v} \leq \infty$. Let μ (m) denote the mean (median) of the distribution. We will consider income distributions with $m \leq \mu$. We will look at a simple linear taxation scheme in which the disposable income of an agent of type x is $x^t = x(1-t) + t\mu$ (we discuss tax distortions and other taxation schedules in Section 6).

Income complementarities: Assume that when an individual with (pre-tax) income x interacts with an individual with income y, as in the marriage market, or mixes in an

[14] See also Rege (2003).

environment where the average income is y, as in the case of peer or network effects in education, he receives a utility $x^t y^t$.[15] Our results qualitatively hold as long as the after-tax incomes plays some role in income complementarities.

We keep this part of the model deliberately simple in order to focus on general sorting structures and distribution functions. The results will qualitatively hold if for example we allow for more dimensions of heterogeneity (as long as some income complementarities remain) or if the complementarity is derived as a reduced form of a more complicated mechanism or interaction as discussed in the literature, e.g., a public good game with credit constraints, peer and network effects in education, or the bargaining process in the marriage market in the presence of transferable utilities.[16]

Costly signalling and sorting: Consider a set of costly signals such as private schools with different fees. We assume that when some individuals use a costly signal they will interact randomly with, and only with, other individuals agents who use the same signal, e.g., all those that had paid the entry fee to the school or the elite college.

Thus, when an agent with income x_i uses a signal that costs b, his utility (for a given tax level t) will be

$$x_i^t E[x_j^t | j \in X_b] - b$$

where X_b is the set of other agents who use the same signal and $E[x_j^t | j \in X_b] = (1-t)E[x_j | j \in X_b] + t\mu$.

The quasi-linear nature of the utility function is simple to use but is not necessary for our results; our main result can be extended to the case in which the utility of an agent with income x_i who mixes in the same "club" with the population whose average income is x_j is $(x_i^t - b)(E(x_j^t) - b)$ instead (see Remark 1). One might also think that the signal has intrinsic utility on top of the sorting value, e.g., that private schools provide, aside from peer effects, better education. This, with some monotonicity condition, can also be accommodated in the model.

By single crossing, if some agent with x_i prefers to use a signal with cost $b > b'$, all agents with $x > x_i$ will prefer b over b'. We will therefore focus on monotone sorting, i.e., with connected intervals. We will abstract away from the supply side, i.e., how the signals or their costs are being determined.[17] But as agents are assumed to choose optimally which signal to use, no matter how the supply side arises, the costs satisfy some incentive compatibility

[15] The analysis could be adjusted to allow for other supermodular functions, such as some $h(x^t)g(y^t)$.

[16] See Lam (1988).

[17] For such analysis see Damiano and Li (2007) and Rayo (2005).

constraints which will define the sorting equilibrium.

Definition 1: *A sorting equilibrium (SE) is a vector* $\mathbf{x} = (x_0, x_1, ...x_{n-1}, x_n)$ *with* $x_0 = \underline{v}$, $x_n = \bar{v}$ *and* $x_i < x_{i+1}$, *such that all agents with type* $x \in [x_i, x_{i+1})$ *for* $i = 0, 1, ..., n-1$ *pay* b_i *and interact with agents in* $[x_i, x_{i+1})$ *only,*[18] *with*

$$\begin{aligned} b_0 &= 0 \\ b_i - b_{i-1} &= x_i^t(E[x_j^t|x_j \in [x_i, x_{i+1})] - E[x_j^t|x_j \in [x_{i-1}, x_i)]). \end{aligned}$$

Note that it must be that $b_i \geq b_{i-1}$. Also note that we focus on coarse sorting, i.e., a finite number of signals (we discuss this below).[19] For simplicity of exposition and without loss of generality, we are restricting the price of joining the lowest club in the partition to zero.

Political economy of taxation. The key premise that is built into the analysis is that when income inequality is reduced, so are the incentives to sort or the willingness to pay for sorting. For example, in the extreme, with full equality, in any SE, it has to be that $b_i = 0$ for all i. Note that as long as the absolute after-tax income has some effect on the quality of the match, the incentive to sort will decline with redistribution and our results will qualitatively hold.

Below we conduct both local and global analysis. The local analysis will allow us to understand the shape of the indirect utility as a function of t, while fixing some SE. Changes in the tax rate imply that the prices of the signals will change, along with the income of the average agent that uses this signal. Note that more generally, when taxes change, it might be that the SE (i.e., the partition) will change as well. This possibility will be considered in our global analysis. In the global analysis we derive simple conditions on $F(x)$ which, if satisfied, imply that all agents up to the mean (and possibly some above) prefer full equality ($t = 1$) to *any* SE and any tax rate t. Such an analysis will therefore accommodate possible changes in the SE. Moreover, it will allow us to provide predictions without any information on the structure of the market.[20]

From a political point of view, we will show in the local and the global analysis that the median is decisive. Thus the preferences we characterize will be manifested in a political

[18] For completeness, when $i = n - 1$, the last interval is closed from above as well, i.e., $[x_{n-1}, x_n]$.

[19] McAfee (2002) and Hoppe, Moldovanu and Ozdenoren (2011) consider environments with one signal.

[20] Our approach, i.e., considering a general family of sorting equilibria rather than focusing a particular one, allows us to pursue general results in environments in which we, the modelers, do not have a precise grasp of the supply side of the sorting market. A different approach is taken by Moav and Neeman (2012) who focus on a refinement of equilibria.

outcome of any two-candidate competition or political model that supports the median voter result (we discuss other political alternatives in Section 6). Note that we do not consider the preferences of the firms or organizations that provide signals; to maintain the political model, one can assume that they compose a negligible part of the population.

Finally, we consider only preferences over the tax rate t although potentially one could also levy a tax on b; we discuss this in Section 6. Generally, any more complicated tax or redistributive regime will demand a particular knowledge of the SE equilibrium whereas we are concerned with environments in which no such knowledge is available for the modeler or public policy practitioner.

Benchmarks: no sorting and perfect sorting. Note that in the absence of costly sorting, preferences over redistribution in the model are "standard": all agents up to the mean will prefer redistribution and higher taxes, and all those above will be against redistribution and higher taxes.

In the presence of perfect sorting on the other hand, all agents up to some cutoff -higher than the mean- are against sorting (we show this formally in the appendix). With perfect sorting, each individual mixes with those with the same income as himself; thus, even the mean in the population would rather equalize income in society, in which case he at least saves on the sorting cost.

When sorting is coarse however, which is likely to be the case in reality, individuals will enjoy information rents. Thus, if there is any SE that will be politically viable for the middle income groups, it is more likely to be coarse which is why we focus on such structures.

4 Preferences over taxation: a local analysis

We now analyze the indirect utility from taxation for a fixed SE. Suppose first that the SE is such that there is only one signal, or one sorting class, where all agents above some cutoff $\hat{x}$ pay $b(t, \hat{x})$ and all below pay nothing. The type at the cutoff $\hat{x}$ will be indifferent between paying the cost of sorting and achieving $\hat{x}^t \bar{E}_{\hat{x}^t}$, vs. not paying and gaining $\hat{x}^t \underline{E}_{\hat{x}^t}$, where

$$\begin{aligned}
\underline{E}_{\hat{x}^t} &= E[x^t | x \leq \hat{x}] = \frac{\int_{\underline{v}}^{\hat{x}} x^t f(x)dx}{F(\hat{x})}, \\
\bar{E}_{\hat{x}^t} &= E[x^t | x \geq \hat{x}] = \frac{\int_{\hat{x}}^{\bar{v}} x^t f(x)dx}{1 - F(\hat{x})}.
\end{aligned}$$

Thus the price of the signal must satisfy:

$$b(t, \hat{x}) = \hat{x}^t (\bar{E}_{\hat{x}^t} - \underline{E}_{\hat{x}^t})$$

The expected utility of an individual $x < \hat{x}$ is therefore $x^t \underline{E}_{\hat{x}^t}$ and the expected utility of an individual $x > \hat{x}$ can be written as $x^t \bar{E}_{\hat{x}^t} - b(t, \hat{x}) = x^t \bar{E}_{\hat{x}^t} - \hat{x}^t(\bar{E}_{\hat{x}^t} - \underline{E}_{\hat{x}^t})$ or:

$$(x^t - \hat{x}^t)\bar{E}_{\hat{x}^t} + \hat{x}^t \underline{E}_{\hat{x}^t} \tag{1}$$

Expected utility from using the signal can be interpreted as the utility of the cutoff type (the second expression in (1)), plus an information rent component that depends on the distance from the cutoff (the first expression). This utility is increasing and convex in the income x; the slope for $x < \hat{x}$ is $(1-t)\underline{E}_{\hat{x}^t}$ and the slope for $x > \hat{x}$, is $(1-t)\bar{E}_{\hat{x}^t}$.

We now consider how the indirect utility over t changes with t for some type x. Whenever $\hat{x} < \mu$, for all agents with income $x < \hat{x}$, increasing taxation is always beneficial. It increases both their own income and the average income in the "club", and thus the optimum tax rate for all below $\hat{x}$ is $t = 1$.

The same argument implies that for agents with income $x > \hat{x}$, an increase in t increases the second expression in (1), $\hat{x}^t \underline{E}_{\hat{x}^t}$. However, taxation decreases the gain from the first expression, the information rent. First, it decreases the income of the average agent one mixes with. Second, it decreases the effective distance from the cutoff, as $(x^t - \hat{x}^t) = (1-t)(x - \hat{x})$. For $x > \hat{x}$, the first order condition of $(x^t - \hat{x}^t)\bar{E}_{\hat{x}^t} + \hat{x}^t \underline{E}_{\hat{x}^t}$ w.r.t. t is:

$$(x - \hat{x})(-\bar{E}_{\hat{x}^t} + (1-t)(\mu - \bar{E}_{\hat{x}})) + (\mu - \hat{x})\underline{E}_{\hat{x}^t} + \hat{x}^t(\mu - \underline{E}_{\hat{x}}),$$

where the first element is negative and the others are positive (when $\hat{x} < \mu$). Plugging $t = 1$ in the first order condition, it is easy to see that it is positive for all $x < 2\mu - \underline{E}_{\hat{x}}$ (note that $2\mu - \underline{E}_{\hat{x}} > \mu$ and thus it is positive at least for $x \leq \mu$). Intuitively, when society is equal enough, the loss in the information rent component is sufficiently small. More generally, we can show:

Proposition 1: *(i) For any SE, there exists t' such that for all $t > t'$, all agents below some $x' > \mu$ prefer to increase taxation. (ii) There exist $F(x)$, an SE and $t'' > 0$, such that for $t < t''$, all agents above (and including) the median prefer to decrease taxation.*

To see the second result, consider for example $\underline{v} = 0$ and a low enough cutoff $\hat{x} \to 0$. In this case, by income complementarities, the increase in the utility of the type at the cutoff from taxation -when this is already sufficiently low- is small. On the other hand, the loss in information rent of an agent who is sufficiently far from the cutoff is large. Specifically, for any $x > \hat{x}$, when $\hat{x} \to 0$ and $t \to 0$, the first order condition converges to $x(-\mu) < 0$.

An implication of Proposition 1 is that for some SE's and low enough taxation, the incentive for taxation can be lower under sorting than in its absence, as in the absence of sorting the median always favours taxation. For all SE on the other hand, when taxation is already high enough, or society relatively equal, the incentive to tax is greater in the presence of sorting than in its absence. Thus, equality will lead to more equality, and inequality may lead to more inequality. We will further illustrate the positive relation between preferences for redistribution and equality in the income distribution in Section 5.

To understand the shape of the indirect utility function, consider the second derivative of (1) with respect to t:

$$2(x - \hat{x})(\bar{E}_{\hat{x}} - \mu) + 2(\mu - \hat{x})(\mu - \underline{E}_{\hat{x}}) > 0$$

The utility function (for a simple SE with one signal and $\hat{x} < \mu$) is therefore convex in t and the optimal tax will be set for all $x > \hat{x}$ at the extremes, either $t = 0$ or $t = 1$. By the linearity of taxation, it can be either convex or concave; convexity arises because of income complementarities. That is, the negative effect of taxation on the information rent is less pronounced once one's income is already reduced, whereas the positive effect of taxation on the utility of the type at the cutoff becomes more beneficial once this utility is high. For a general SE the analysis is slightly more subtle but we can show the convexity of the utility function for all types with $x \leq \mu$.

Lemma 1: *Fix an SE. For all agents $x \leq \mu$, the indirect utility over taxes is convex, i.e., the optimal tax level is either $t = 0$ or $t = 1$.*

The next result can be easily seen from the first order condition for the case of one signal which decreases in $(x - \hat{x})$:

Lemma 2: *Fix an SE. Suppose that at some t, an agent x' prefers to reduce taxation. Thus all $x > x'$ want to reduce taxation as well.*

Together the above two imply:

Proposition 2: *Fix an SE. In any two-candidate political competition between different levels of $t \in [0, 1]$, the median is the decisive voter and the Condorcet winner is either $t = 0$ or $t = 1$.*

Proof of Proposition 2: Lemma 1 implies that if the median's optimal tax level is $t = 1$, then this will hold for all $x < m$. To see why, note that for all agents the utility from $t = 1$ is the same whereas the utility from $t = 0$ is lower for lower x.

To see how the median is decisive for $t = 0$, note that if the utility of types above the median is convex, then as the utility at $t = 0$ increases in x, while the utility from $t = 1$ is the same, this implies that for such types $t = 0$ is optimal. If their utility is concave, once it decreases at $t = 0$, it must decrease throughout. By Lemma 2 whenever the utility of the median decreases in t (which has to be the case for small t when his optimal policy is $t = 0$), so does the utility of all others above him which implies the result.||

In the next Section we conduct global analysis and, motivated by the local analysis, analyze the conditions under which $t = 1$ is the global maximizer for large coalitions of voters, i.e., they would support full equality over SE with tax rates that are smaller than one. Note that the median is the decisive voter also in the global analysis, as the utility from any SE increases with the income x whereas the utility from full equality -μ^2- is the same for all.

5 Preferences for full equality

In this Section we analyze the preferences for full equality. We start by looking at the preferences of the individual with the mean income; as this individual doesn't enjoy redistribution per se, this will allow us to identify a motivation for taxation and redistribution which arises due to sorting only. We can then characterize environments in which the relatively rich vote for redistribution.

We will then examine the incentives of the individual with the median income, which are a mix of both sorting and standard income motivations. The results about the median will provide additional conditions for full equality to be politically viable.

5.1 A sorting motive for redistribution

We now analyze the preferences of the individual with the mean income. Consider again a simple SE with one signal, given by a cutoff $\hat{x}$. If $\hat{x} \geq \mu$, then the mean prefers full equality which will grant him, at no cost, mixing with a higher average income. We therefore have to focus on SE's that satisfy $\hat{x} < \mu$.

Consider first the case when $t = 0$. The mean prefers full redistribution to any signal $\hat{x}$ (and $t = 0$) iff

$$(\mu - \hat{x})\bar{E}_{\hat{x}} + \hat{x}\underline{E}_{\hat{x}} \leq \mu^2$$

Divide by μ to get

$$(1 - \frac{\hat{x}}{\mu})\bar{E}_{\hat{x}} + \frac{\hat{x}}{\mu}\underline{E}_{\hat{x}} \leq \mu$$

As

$$\mu = (1 - F(\hat{x}))\bar{E}_{\hat{x}} + F(\hat{x})\underline{E}_{\hat{x}}, \tag{2}$$

then full equality is preferred to coarse sorting for any cutoff $\hat{x}$ iff:

$$\frac{x}{\mu} \geq F(x) \text{ for all } x < \mu \qquad \text{(Condition 1)}$$

Generalizing Condition 1 to any t is trivial. Generalizing it to any SE with more than one signal does not follow immediately however. In particular, whenever $\frac{x}{F(x)}$ is increasing, adding more signals below some cutoff $\hat{x} < \mu$ reduces the signalling cost for all types $x > \hat{x}$ and thus improves the utility from sorting. Still, we are able to show that Condition 1 is necessary and sufficient for all SE. The intuition is that Condition 1 insures that the mean prefers full equality both to a partition $[\underline{v}, x_1, \bar{\nu}]$ and to a partition $[\underline{v}, x_2, \bar{\nu}]$, for $\mu > x_2 > x_1$, which together imply that the mean would also prefer it to a partition $[\underline{v}, x_1, x_2, \bar{\nu}]$.

Proposition 3: *The mean prefers full equality to any SE and any tax level t iff $F(x)$ satisfies Condition 1.*

While Condition 1 is necessary for the mean as shown for the case of one signal, it is sufficient for all below the mean. Moreover, if it is satisfied with strict inequality for some $x's$, then there exists an interval of types above the mean for whom full equality is better than sorting, rationalizing the possibility that relatively rich agents -i.e., with income above the mean- can vote left.

We first provide the proof and in the next subsection explain the intuition behind the condition, its relation to income inequality, and for which income distributions it is satisfied.

Proof of Proposition 3: We provide a direct proof for all SE and t (i.e., we ignore the local analysis which tells us to focus on $t = 0$ and $t = 1$ for any SE). We have already considered the case of one signal or an SE with $n = 2$. Note that if we start from some positive level of taxation t, the condition becomes $\frac{x^t}{\mu} \geq F(x)$ for all $x < \mu$, for which Condition 1 is sufficient. The necessary part of the Proposition follows then from this case for $t = 0$.

We now show sufficiency using an induction on the number of signals. Suppose that the Proposition is true for any SE with $n = k - 1$. Consider all SE with $n = k$.

Note that if $\mu < x_1$, then the utility of the mean is like in an SE with $n = 2$ and the same x_1, and so Condition 1 applies. If $x_1 < \mu < x_2$, consider his utility from an SE with $n = 3$ and the same x_1, x_2, which is the same again. Thus if $x_{i-3} < \mu < x_{i-2}$ for $i \leq k$, his utility from the SE is the same as the utility from an SE with $n = i$ and the same $x_0, x_1, ..., x_{i-2}$ which by the induction hypothesis proves the result. Now assume that $x_{k-2} < \mu < x_{k-1}$. The mean's expected utility can be written as:

$$\begin{aligned} x_1^t E(x_j^t|x_j &\in [\underline{v}, x_1]) + (x_2 - x_1)(1-t)E(x_j^t|x_j \in [x_1, x_2]) + ... \\ +(x_{k-2} - x_{k-3})(1-t)E(x_j^t|x_j &\in [x_{k-3}, x_{k-2}]) + (\mu - x_{k-2})(1-t)E(x_j^t|x_j \in [x_{k-2}, x_{k-1}]) \end{aligned}$$

which is strictly lower than the utility from an SE with $n = k-1$ and the same $x_0, x_1, ..., x_{k-2}$ in which case the last expectations are replaced by $E(x_j^t|x_j \in [x_{k-2}, \bar{\nu}])$ and the rest is the same.

Finally consider the case of $\mu > x_{k-1}$. We first divide both sides by μ and then use Condition 1 repetitively:

$$\begin{aligned} \frac{x_1^t}{\mu} E(x_j^t|x_j &\in [\underline{v}, x_1] + \frac{x_2^t - x_1^t}{\mu} E(x_j^t|x_j \in [x_1, x_2]) + ... \\ +\frac{x_{k-1}^t - x_{k-2}^t}{\mu} E(x_j^t|x_j &\in [x_{k-2}, x_{k-1}]) + (1 - \frac{x_{k-1}^t}{\mu}) E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}]) = \\ \frac{x_1^t}{\mu}(E(x_j^t|x_j &\in [\underline{v}, x_1] - E(x_j^t|x_j \in [x_1, x_2])) + ... \\ +\frac{x_{k-1}^t}{\mu}(E(x_j^t|x_j &\in [x_{k-2}, x_{k-1}]) - E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}])) + E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}] \leq \\ F(x_1)(E(x_j^t|x_j &\in [\underline{v}, x_1]) - E(x_j^t|x_j \in [x_1, x_2])) + ... \\ +F(x_{k-1})(E(x_j^t|x_j &\in [x_{k-2}, x_{k-1}]) - E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}])) + E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}] = \\ F(x_1)(E(x_j^t|x_j &\in [\underline{v}, x_1]) + (F(x_2) - F(x_1))E(x_j^t|x_j \in [x_1, x_2]) + ... \\ (F(x_{k-1}) - F(x_{k-2}))E(x_j^t|x_j &\in [x_{k-2}, x_{k-1}]) + (1 - F(x_{k-1}))E(x_j^t|x_j \in [x_{k-1}, \bar{\nu}]) = \mu \end{aligned}$$

where the inequalities follow from Condition 1 as the difference in the expectations terms is negative. This completes the proof.||

Remark 1: As an alternative utility specification we can assume that the utility from sorting and interacting is $(x_i^t - b)(x_j^t - b)$. We show in the appendix that Condition 1 is sufficient in this case for the mean to prefer full equality. For other more general utility functions, for example if an agent enjoys his income on top of the utility from the interaction with others, such as in $u(x_i^t) - b + h(x_i^t)g(x_j^t)$, it is possible to construct an adjusted similar condition.

5.2 Condition 1, inequality, and real income distributions

We will now illustrate how Condition 1 is satisfied in more equal societies. To see first a graphical illustration, we consider below distributions which are symmetric around the (same) mean; all with $F(x)$ which is not too concave satisfy the condition (a necessary and sufficient condition for being "not too concave" is that $f(0) < \frac{1}{\mu}$) :

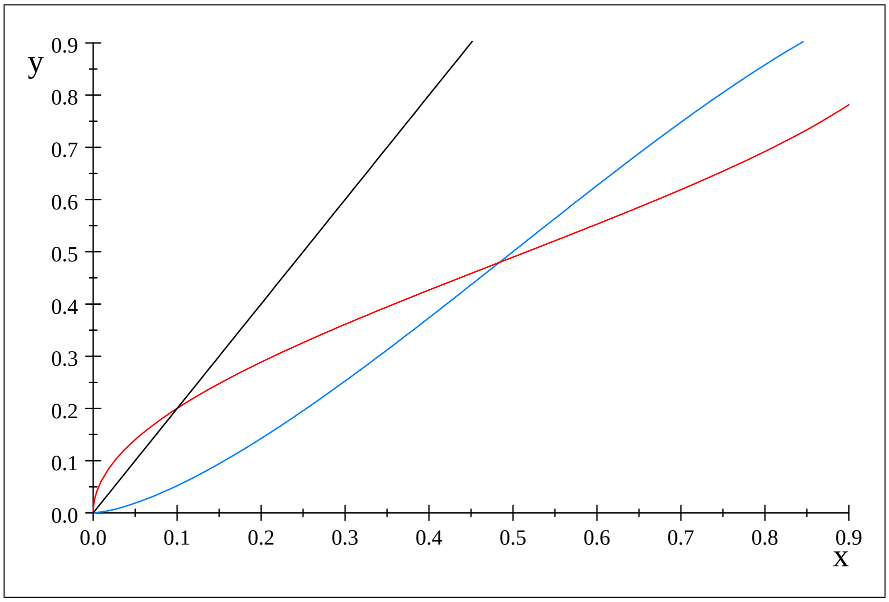

Figure 2: Illustration of Condition 1, drawn for the symmetric Beta distribution with $\alpha = \beta = 1.5$ and 0.5; the line coresponds to $\frac{x}{\mu}$ where $\mu = 0.5$. The condition is satisfied for the Beta distribution with $\alpha = 1.5$ ($F(x)$ is completely below the line) but not for $\alpha = 0.5$ (where $F(x)$ is above the line for small values of x).

As a specific example consider the family of the symmetric Beta distributions $f(x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1}du}$ on $[0,1]$ with $\alpha = \beta$ as discussed in the introduction (see Figure 1 and Figure 2). For this family, a lower α represents a mean-preserving spread and the Gini coefficient is monotonically decreasing in α. When $\alpha > 1$ the density is inverted U-shaped, and Condition 1 holds; this is shown in Figure 2 for the case in which $\alpha = 1.5$ where the distribution function is completely below the line which corresponds to $\frac{x}{\mu}$. Moreover, with a higher α, Condition 1 is satisfied with a strict inequality with a greater wedge implying that more agents above the mean prefer taxation. On the other hand, when $\alpha < 1$, the density function is bimodal with the modes close to zero and one, and Condition 1 is not satisfied for an interval of small values of x. This can be seen in Figure 2 where we depict the cumulative distribution function for $\alpha = 0.5$. This distribution is above the line $\frac{x}{\mu}$ for values of x below the mean.

Why is Condition 1 not satisfied when $F(x)$ is sufficiently concave or sufficiently unequal? To see why sufficient inequality will convince the mean to opt away from full equality, note that a low cutoff $\hat{x}$ (for the case of one signal) guarantees a sufficiently low price for sorting. But high inequality also implies that this low cutoff $\hat{x}$ allows the mean to stay away from a relatively large mass of the very poor. This implies that sorting is with relatively high income types, at a low cost, and is thus beneficial.

We now show that Condition 1 is satisfied by a large family of distributions, some used to approximate real income distributions, that are characterized by a lower Gini coefficient. Recall that $F(x)$ has increasing (decreasing) failure or hazard rate, denoted IFR (DFR) if $\frac{f(x)}{1-F(x)}$ is increasing (decreasing). We then have:

Proposition 4: *Condition 1 is satisfied by (i) all IFR distribution functions, e.g., Gamma and Weibull with shape parameters greater than one; (ii) all Pareto distributions on [1,∞) with sufficiently high shape parameter* α ($\alpha \geq 1.5$)*; (iii) all Lognormal distributions with sufficiently low shape parameter* σ ($\sigma \leq 1.1$).

Proof: (i) Assume that $\bar{E}_x - x \leq \mu$ for any x. Using (2) we have that

$$\begin{aligned} \mu &= F(x)\underline{E}_x + (1-F(x))\bar{E}_x \leq F(x)\underline{E}_x + (1-F(x))(x+\mu) \Leftrightarrow \\ F(x)\mu &\leq F(x)\underline{E}_x + (1-F(x))x \Leftrightarrow \mu \leq \frac{x}{F(x)} + (\underline{E}_x - x) \Rightarrow \\ \mu &< \frac{x}{F(x)} \text{ for any } x \text{ as } \underline{E}_x < x. \end{aligned}$$

Finally, Barlow and Proschan (1966) have shown that any IFR function satisfies the property that $\bar{E}_x - x \leq \mu$ (called also *new better than used in expectations,* or NBUE).[21] (ii) Condition 1 is satisfied for the Pareto distribution iff $\frac{x}{1-(\frac{1}{x})^{\alpha}} > \frac{\alpha}{\alpha-1}$ (for all $x < \frac{\alpha}{\alpha-1} = \mu$, for the support $[1,\infty]$). This is satisfied for all $\alpha \geq 1.5$, but not otherwise. (iii) See the appendix.||

Note that by the proof, Condition 1 is satisfied with a strict inequality for any IFR distribution; this means that for this family of distributions there are agents above the mean who prefer full equality to sorting. Intuitively, these distributions do not provide sufficient benefits from matching with the rich as the tail on high income falls too quickly.

The Weibull and Gamma distributions -for shape parameters greater than 1- have IFR (as well as the Normal, Uniform and Exponential distributions). For the US in the 1960's, Salem and Mount (1974) have advocated a version of the Gamma distribution which is IFR, i.e., with a shape parameter estimated to be around two.[22] For these distributions the higher is the shape parameter, the lower is the Gini coefficient and hence Condition 1 is satisfied for the sufficiently equal Gamma and Weibull distributions.

Other distributions which are typically considered in the literature are Pareto (which is DFR) and the Lognormal (which is first IFR and then DFR). Singh and Maddala (1976) claim that

[21] Such distributions, in short NBUE, imply that the life span of a machine is less reliable with time. The opposite condition is NWUE (new worse than used in expectations).

[22] The distribution is $f(x) = \frac{\lambda^{\alpha}}{A(\alpha)}x^{\alpha-1}e^{-\lambda x}$ on $[0,\infty]$ for $A(\alpha) = \int_0^{\infty} e^{-u}u^{\alpha-1}du.$ For this distribution the median is $\frac{3\alpha-1}{3\lambda}$, $\frac{1}{\sqrt{\alpha}}$ is the parameter of skewness, and the mean is $\frac{\alpha}{\lambda}$. For the decades of the 60's, their estimate of α is around 2 and λ is around $\frac{3}{10^4}$.

income distributions should be DFR at least for high enough income, as the ability to make more money should increase with one's income, once some threshold is reached.[23] Atkinson, Piketty and Saez (2011) and Diamond and Saez (2011) provide evidence that the top tail of income distributions follows a Pareto distribution and the latter estimate a shape parameter for the upper tail of around $\alpha = 1.5$. In general both in the US and and in the UK in the 20'th century estimates of α are in the range $[1, 2.5]$.[24] For the Pareto, the higher is the shape parameter α, the lower is the Gini coefficient (which equals $\frac{1}{2\alpha-1}$), and thus Condition 1 is satisfied for the more equal Pareto distributions.

Finally, the Lognormal distribution is characterized by two parameters, $\tilde{\mu}$ (log-scale) and σ (the shape). The Gini coefficient is $2\Phi(\sigma/\sqrt{2}) - 1$ where $\Phi(x)$ is the standard normal distribution, and thus a lower σ is associated with a lower Gini. In this family of distributions we show that Condition 1 is satisfied as long as $\sigma < 1.1$ and irrespective of $\tilde{\mu}$. One example for the estimation of σ is the estimation of the distribution of earnings of UK full time male manual workers (see Cowell 2011) with an estimated $\sigma^2 = 0.13$ well below the cutoff above.[25]

Remark 2: Proposition 4(i) is related to the (utilitarian) efficiency of sorting. We can show that in our model, any SE leads to a higher (lower) average utility to society as a whole compared with full equality if and only if the income distribution satisfies $\bar{E}_x - x \leq (\geq)\mu$ for any x (denoted NBUE (NWUE), satisfied by any IFR (DFR) function). As the utility from sorting is convex in the income x, this also implies that whenever sorting is inefficient, the mean is against it.

To see the intuition for the efficiency result, note that positive assortative matching is beneficial and worth paying the cost when variability is sufficiently high, where random matching results in significant losses. Hall and Wellner (1984) showed that any NBUE function has a coefficient of variation $CV(x) = \frac{\sqrt{Var(x)}}{E(x)} \leq 1$, whereas for any NWUE, $CV(x) \geq 1$. Thus, under NBUE, the variability of the income distribution is too small and sorting is inefficient.[26]

[23] Singh and Maddala (1976) fit the data to some mixture of Pareto and Weibull, with an increasing *proportional* hazard rate ($x\frac{f(x)}{1-F(x)}$) which then converges to become constant. We note that Cramer (1978) advocates caution with respect to interpreting failure rates properties with regard to static distributions of income (where such properties should relate to time or age).

[24] See Cowell (2011).

[25] See also Pinkovskiy and Sala-i-martin (2009).

[26] For the case of perfect continuous signalling, Hoppe, Moldovanu and Sela (2009) show that $CV(x) \geq (\leq)1$ is a sufficient and necessary condition for sorting to be efficient (not efficient) compared with random matching. For their discrete model which has incomplete information on a discrete set of types but perfect signalling, a necessary and sufficient condition for efficiency (inefficiency) of signalling is for the function to have decreasing (increasing) failure rate.

5.3 Sorting and income motives for redistribution

Above we have focused on the agent with the mean income to identify a new motivation for taxation and redistribution which arises due to sorting motives only. In fact, for each individual voter we can derive an analogous condition which takes into consideration only sorting motives. That is, we can compare the utility from the SE and the utility from random interaction. For the median voter for example, the utility from random interaction is $m\mu$, and as in the proof of Proposition 3, we can show that he prefers this (which is dominated by full equality) to any SE and t iff $F(x)$ satisfies $\frac{x}{F(x)} > m$ for all $x < m$.[27] But as above, any sufficiently concave $F(x)$ for example will violate this condition.

However, for the median voter (or all below the mean), there are also income incentives for redistribution which Condition 1 or its modifications as above do not take into account. We now focus on the median voter to combine sorting and standard income motivations for redistribution. We show that these additional income incentives imply that whenever the income distribution is sufficiently *unequal*, the median (and all those below) would favour full equality:

Proposition 5 *The median (and all those below) prefers full equality to any SE and any t, if $m\bar{E}_m \leq \mu^2$. This condition is satisfied by (i) All distributions with $m < \frac{1}{2}\mu$; (ii) Pareto distributions on [1,∞) with low enough α, specifically $\alpha \in (1, 2]$, (ii) Lognormal distributions with high enough σ, specifically $\sigma > 1.174$.*

Proof of Proposition 5: By Proposition 2, it is sufficient to show that $t = 1$ is preferred to $t = 0$ given any SE. For $t = 0$, $m\bar{E}_m$ is the highest utility the median can get in all SE's as $\bar{E}_m$ is the highest expected type he could match with in a club he belongs to, and we are excluding the cost of the match. Thus $m\bar{E}_m < \mu^2$ is a sufficient condition. We now show when this condition is satisfied. For (i), note that when $m < \frac{1}{2}\mu$, then also

$$m\bar{E}_m = m\frac{\int_m^\infty xf(x)dx}{1 - F(m)} = 2m\int_m^\infty xf(x)dx < \mu^2.$$

as $\mu > \int_m^\infty xf(x)dx$. Thus, the condition holds for sufficiently unequal distributions with half the population concentrated on relatively low incomes compared with the mean. For (ii), note that for the Pareto distribution, the condition is $\frac{(\frac{\alpha}{\alpha-1})^2}{\sqrt[\alpha]{2}} - 2\int_{\sqrt[\alpha]{2}}^\infty \frac{\alpha}{x^\alpha}dx \geq 0$, which, after standard

[27]This condition is consistent with Proposition 7 in Hoppe, Moldovanu and Sela (2009), which shows that if the distribution function is uniform or stochastically dominates the uniform, then all in society prefer random matching to (perfect) costly sorting.

manipulation, holds iff $\frac{\alpha}{\alpha-1} - 2^{\frac{2}{\alpha}} > 0$ which is positive for all $\alpha \in (1, 2]$ (note that given the support on $[1, \infty)$, it has to be that $\alpha > 1$). For (iii) see the Appendix.||

The Proposition adds a counterpart to Condition 1. As the sorting benefits arise through income complementarities, if the distribution is too unequal and the income of the median is simply too low compared to the mean, no sorting benefits will allow the median to prefer sorting to full equality. Specifically, not only the median's income is low enough, but also in this case $\bar{E}_m$ - although higher than μ - will not be sufficiently high.

Together, Propositions 4 and 5 imply that from the point of view of the median, either relatively equal or relatively unequal distributions would yield preferences for full equality (as Proposition 4 provides a sufficient condition for the median). From the point of view of the mean though and richer voters around him, it is only relatively equal distributions which will yield such preferences.

Proposition 5 shows that a political majority will support full equality if $m < \frac{1}{2}\mu$; note that for all DFR distributions, it is the case that $m \leq \mu \ln 2 \approx 0.69\mu$, and thus redistribution will be favoured in a large family of DFR distributions and in particular those that are relatively more concave or more unequal. This arises as all DFR's with the same mean as some Exponential, are more variable - i.e., stochastically dominated in a second-order sense - than the Exponential one, which satisfies $m = \mu \ln 2$.[28] Thus, together with Proposition 4, both IFR functions and a large family of DFR functions will imply support for full equality. Specifically, support for full equality according to the median voter's preferences should arise for all Pareto distributions on $[1, \infty)$.

Finally, note that we show in the appendix that the condition in Proposition 5 holds for all $\sigma > 1.1774$, the family of such functions which are characterized with more inequality, but that if in addition $\tilde{\mu} > 0$ (as in typical income distributions), then the condition in Proposition 5 holds also for all $\sigma \in [1.1, 1.1774]$. Thus, together with Proposition 4, also for all lognormal distributions society should be in favour of full equality.

6 Extensions and discussion

Our analysis implies that when preferences for sorting are taken into account, they can affect preferences over taxation. We have shown how in some cases agents with income around and above the mean will strictly favour redistribution -when Condition 1 is (strictly) satisfied- and

[28] Specifically, by Theorems 4.4 and 4.7 in Barlow and Proschan (1965), $F(x) \geq 1 - e^{\frac{-x}{\mu}}$ for all $x < \mu$ if $F(x)$ is DFR which implies that the median is lower in the DFR distribution.

that this is likely to be the case for some of the commonly estimated income distributions. This is consistent with the observation that relatively rich voters often vote left and that society is more likely to favour redistribution if it is sufficiently equal.

We have made a few simplifying assumptions to facilitate our analysis. We analyze a simple political process, while decision making in society does not always follow that of the median voter. We analyze a simple signalling environment, which is one-dimensional and characterized by a quasi-linear utility function; our results can be extended to consider other utilities and more dimensions. Also, we have provided general conditions for all forms of sorting, and a modelling of the supply side may provide more specific results. We now consider some possible extensions.

6.1 More general taxation or intervention schemes

We have focused on one simple tool of linear tax on income. We now discuss other possible tools potentially available for the government. One possibility is to impose taxes on the revenues from sorting accumulated by the owners of land, private school, sport cars or fashion design industries. Imposing taxes on luxury goods increases its signalling value while some of the tax proceeds are gained by the government. In particular, an efficient as well as politically viable course of action would be to fully tax these, and to return the proceeds to voters in a progressive manner, i.e., poor voters will have to be compensated more.

To implement such a scheme though, the government will have to acquire information about the particular sorting equilibrium. Such programs might also be hindered by tax distortions (i.e., beyond administration cost, the need to provide landowners with incentives to develop land, or sport car industries with the incentive to innovate, which will reduce the ability to fully tax). Another possibility for the government, in the context of our model, will be simply to publish all information on salaries or wealth. This might create efficient and costless sorting; still, to make this politically viable, the benefit of sorting will have to be transferred to the poor in a progressive manner.

In terms of taxes on income, we have ignored both the possibility of non-linear taxes and tax distortions. As long as these are not too non-linear, our analysis will remain robust.

6.2 Preferences over the sorting equilibrium

As discussed above, one possible intervention for the government is to introduce taxes or subsidies in the housing or education markets; these will not only generate revenues from sorting, but will also affect the price and composition of sorting. For example, a tax on luxury goods or private schools might increase the exclusiveness of sorting.

To shed some light on this, we ask whether agents will prefer their club to be more or less inclusive for the case of a simple sorting equilibrium with just club characterized by the cutoff $\hat{x}$. For the poor who are not in the club, the higher is $\hat{x}$ the higher is the average income of those left to interact with them. For those in the club, the derivative of the utility from sorting is (for some type x) is:

$$(\bar{E}_{\hat{x}^t} - \hat{x}^t)((x^t - \hat{x}^t)\frac{f(\hat{x})}{1-F(\hat{x})} - 1) + (\hat{x}^t - \underline{E}_{\hat{x}^t})(\hat{x}^t \frac{f(\hat{x})}{F(\hat{x})} - 1) \tag{3}$$

An increase in $\hat{x}$ directly increases $\bar{E}_{\hat{x}^t}$, $\underline{E}_{\hat{x}^t}$, and the price. What is clear from (3) is that once some x prefers an increase in $\hat{x}$, then all those above prefer an increase in $\hat{x}$ as well. This reveals a possible "ends against the middle" coalition for small local changes.

Proposition 6: *A coalition to increase $\hat{x}$ will always consist of agents below $\hat{x}$ and sometimes consists of all agents from some $x > \hat{x}$ and above. Moreover, there exist income distributions for which an "ends against the middle" coalition can arise to successfully increase the exclusiveness of the club.*

Policies that can increase or decrease x are subsidies or taxes imposed on the signalling devices; a fuller analysis will naturally include the cost of such tools. Still, the result above indicates who may gain and who may lose from such a policy. Moreover, it is possible to construct examples of such successful "ends against the middle" coalitions (consisting of more than 50% of the population but excluding the median). For example, consider the Gamma distribution as in Salem and Mount (1974) with the parameters they estimate for the income distribution in the US in the 1960's, $\alpha \approx 2$ and $\lambda \approx \frac{3}{10^4}$. For these parameters, $m \approx 5555$. When $\hat{x} = 5000$, all types with income above $11,308$ prefer to increase $\hat{x}$, and together with all types below 5000, they comprise a share of 58%.

6.3 The political process

We have used a simple majority rule to assess the political viability of different policies. Clearly in some environments the median voter's preferences or those of the majority more generally are not sufficient to determine the political outcome. Organized lobbies which are more likely to represent organized high income voters or the suppliers of private schools or luxury goods may bias the political outcome in their favour; these may imply that even if there are pressures for redistributions due to sorting, these are not necessarily successful. It would be interesting to combine in future research the sorting environment with a more sophisticated political model, taking into account the interests and possibly pressure exerted by private providers of sorting signals.

7 Appendix

A. Additional results

Preferences over redistribution for the case of perfect sorting:

Note that when a type x^t pays $b(t,y)$, he will be matched with type y, gaining utility

$$x^t y^t - b(t,y)$$

taking the first order condition w.r.t. y and setting $y = x$ for incentive compatibility, we have:

$$\begin{aligned} b'(t,x) &= x^t(1-t) \rightarrow \\ b(t,x) &= (1-t)((1-t)\frac{x^2}{2} + t\mu x) \end{aligned}$$

First, the derivative of $(x^t)^2 - b(t,x)$ is strictly positive for all $x \leq \mu$ and so agents prefer higher t. Moreover, for all types that satisfy:

$$(x^t)^2 - b(t,x) \leq \mu^2$$

would prefer full equality, which amounts to all types $x \leq x'$ where x' is:

$$x' = \frac{\mu(\sqrt{2-t^2} - t)}{1-t} > \mu \text{ for all } t < 1.$$

Sufficiency of Condition 1 for other utility functions:

We now show that condition 1 is also sufficient for the utility function $(x-b)(y-b)$. Suppose there is just one signal. Then sorting is better than full equality for the median/mean if

$$\begin{aligned} (\mu - b)(\bar{E}_{x^t} - b) &\leq \mu^2 \Leftrightarrow \\ F(x)(\bar{E}_{x^t} - \underline{E}_{x^t}) + \frac{b}{\mu}(b - \mu - \bar{E}_x) &\leq 0 \end{aligned}$$

Note that at the cutoff x, we have that $(x^t - b)(\bar{E}_{x^t} - b) = x^t\underline{E}_{x^t}$, so that $x^t(\bar{E}_{x^t} - \underline{E}_{x^t}) = b(-b + x + \bar{E}_{x^t})$. This implies that the above holds if

$$F(x)(\bar{E}_{x^t} - \underline{E}_{x^t}) \leq \frac{x^t(\bar{E}_{x^t} - \underline{E}_{x^t})}{\mu}\frac{-b + \mu + \bar{E}_{x^t}}{-b + x^t + \bar{E}_{x^t}}$$

For which, as $\frac{-b+\mu+\bar{E}_{x^t}}{-b+x^t+\bar{E}_{x^t}} > 1$, a sufficient condition is Condition 1.$\blacksquare$

B. Omitted proofs:

Proofs of Proposition 1, Lemma 1 and Lemma 2: Consider an agent with income z, so that the largest element of the partition with $x_i < z$ is x_k. Let E_i^t (E_i) denote the expectations over the after tax (pre-tax) income of randomly drawn individual with income in $[x_i, x_{i+1}]$. The utility from of z from the SE is:

$$(z^t - x_k^t)E_k^t + (x_k^t - x_{k-1}^t)E_{k-1}^t + ... + (x_2^t - x_1^t)E_1^t + x_1^t E_0^t$$

Take the f.o.c w.r.t. t. We get:

$$(z - x_k)(-(1-t)2E_k + (1-2t)\mu) + (x_k - x_{k-1})((1-2t)\mu - 2E_{k-1}(1-t)) + ...$$
$$+(x_2 - x_1)((1-2t)\mu - 2E_1(1-t)) + (\mu - x_1)E_0^t + x_1^t(\mu - E_0)$$

Evaluated at $t = 1$, this is positive for all $z - \mu < \mu - E_0$ and negative otherwise. Similarly evaluating at $t = 0$, this is negative for $[0, x_1, \nu]$ when $x_1 \to 0$. This proves Proposition 1.

Suppose the first order condition is negative. In this case it has to be that $-(1-t)2E_k + (1-2t)\mu) < 0$ (even if $\mu < x_1$), so increasing z will make it more negative, which proves Lemma 2.

Take now the second derivative w.r.t. t. We have:

$$2(z - x_k)(E_k - \mu) + 2(x_k - x_{k-1})(E_{k-1} - \mu) +$$
$$+... + 2(x_2 - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0)$$

Note that this does not depend on t and thus will be either negative or positive.

Suppose $x_k \leq z \leq \mu$ and that $E_k < \mu$.

$2(z - x_k)(E_k - \mu) + 2(x_k - x_{k-1})(E_{k-1} - \mu)+$

$+... + 2(x_2 - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0) >$

$2(z - x_k)(E_1 - \mu) + 2(x_k - x_{k-1})(E_1 - \mu)+$

$+... + 2(x_2 - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0) =$

$2(z - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0) > 0$ as

$\mu - E_0 > \mu - E_1$ and $\mu > z$.

Suppose now that $x_k \leq z \leq \mu$ and that $E_k > \mu$. We know that $E_{k-1} < \mu$.

$2(z - x_k)(E_k - \mu) + 2(x_k - x_{k-1})(E_{k-1} - \mu)+$

$+... + 2(x_2 - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0) >$

$2(x_k - x_1)(E_1 - \mu) + 2(\mu - x_1)(\mu - E_0)$

as $\mu > x_k$.

We now need to show that this is true for types below x_k, which will hold by continuity. Look now at some one in x_{k-1}, x_k. For the mean it is positive, so also for the type at x_k as above. so if $(1-2t)\mu - 2E_{k-1}(1-t) < 0$ the expression will increase when we decrease z. If it is positive, then all expressions are positive, which implies that the derivative is positive. And so on. If $\mu < x_1$ then all below the mean are in the worst club so always want to increase taxation for all t. This completes the proof of Lemma 1.||

Proofs of Propositions 4 and 5: For the Lognormal distribution, with parameters $\tilde{\mu}$ and σ, Condition 1 becomes:

$$\begin{aligned} e^{\tilde{\mu}+\frac{\sigma^2}{2}} &< \frac{x}{(\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln x-\tilde{\mu}}{\sqrt{2\sigma^2}}))} \text{ for all } x \in [0, e^{\tilde{\mu}+\frac{\sigma^2}{2}}) \Leftrightarrow \\ \frac{e^{\tilde{\mu}+\frac{\sigma^2}{2}}(\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln x-\tilde{\mu}}{\sqrt{2\sigma^2}}))}{x} &< 1 \Leftrightarrow \\ e^{\frac{\sigma^2}{2}} e^{\tilde{\mu}-\ln x}(\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{\ln x-\tilde{\mu}}{\sqrt{2\sigma^2}})) &< 1 \Leftrightarrow \end{aligned}$$

A change of variable to $y = \tilde{\mu} - \ln x \in [-\frac{\sigma^2}{2}, \infty)$ implies,

$$e^{\frac{\sigma^2}{2}} e^{y}(\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{-y}{\sqrt{2\sigma^2}})) < 1 \text{ for all } y \in [-\frac{\sigma^2}{2}, \infty)$$

Note that this expression doesn't depend on $\tilde{\mu}$.

Suppose that the condition holds for some $\bar{\sigma} > 0$. We now show that it holds for all $\sigma < \bar{\sigma}$. Taking the derivative of the LHS above w.r.t to σ we get,

$$\frac{\partial(e^{\frac{\sigma^2}{2}} e^{y}(\frac{1}{2}+\frac{1}{2}\operatorname{erf}(\frac{-y}{\sqrt{2\sigma^2}}))}{\partial\sigma} = \frac{1}{2(\sigma)^{\frac{3}{2}}}(e^{y}e^{\frac{1}{2}\sigma^2}(\sigma)^{\frac{3}{2}}(1-\operatorname{erf}(\frac{y}{\sqrt{2\sigma^2}})) + \frac{\sqrt{2}}{\sqrt{\pi}} y e^{-\frac{1}{2}\frac{y^2}{\sigma^2}} e^{y} e^{\frac{1}{2}\sigma^2}) > 0$$

and so if the condition holds for $\bar{\sigma}$ it holds for all $\sigma < \bar{\sigma}$. Finally we show that $\bar{\sigma}$ exists. Assume that $\bar{\sigma} = 1.1$. We now find the maximal value of the LHS in $[-\frac{\sigma^2}{2}, \infty)$. The first order condition is, $\frac{\sqrt{2}}{\sqrt{\pi}\sqrt{\sigma^2}} e^{-\frac{1}{2}\frac{y^2}{(1.1)^2}} = (1-\operatorname{erf}(\frac{y}{\sqrt{2\sigma^2}}))$. Plugging $\bar{\sigma} = 1.1$ and solving we find that $y = 0.48784$ yielding a maximal value of $0.98044 < 1$, and the second order condition evaluated at $y = 0.48784$ is negative. Finally note that the expression decreases to zero when $y \to \infty$ and that when $y = -\frac{1}{2}\sigma^2$ the value of the LHS is $0.708\,84 < 0.98044$. Therefore, $y = 0.48784$ is a global maximizer and so the LHS is smaller than 1 in the whole region.

We now show the values of σ for for which the condition in Proposition 5 is satisfied. For the Lognormal this condition becomes,

$$\begin{aligned} 2e^{\tilde{\mu}} \int_{e^{\tilde{\mu}}}^{\infty} \frac{e^{-\frac{(\ln x-\tilde{\mu})^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dx &< e^{2\tilde{\mu}+\sigma^2} \Leftrightarrow \\ 2\int_{e^{\tilde{\mu}}}^{\infty} \frac{e^{-\frac{(\ln x-\tilde{\mu})^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}} dx &< e^{\tilde{\mu}+\sigma^2} \end{aligned}$$

Here we change variables to

$$z = \frac{(\ln x - \tilde{\mu})}{\sqrt{2\sigma^2}}$$

implying that the condition becomes,

$$\begin{aligned} 2\int_{e^{\mu}}^{\infty} \frac{e^{-z^2+z\sqrt{2\sigma^2}}}{\sqrt{\pi}} dz &< e^{\sigma^2} \Leftrightarrow \\ e^{\frac{1}{2}\sigma^2}(1 - \operatorname{erf}\left(e^{\tilde{\mu}} - \frac{1}{2}\sqrt{2\sigma^2}\right)) &< e^{\sigma^2} \end{aligned}$$

If $\sigma > 1.1774$ this is satisfied Suppose that $\sigma < 1.1774$, if $e^{\tilde{\mu}} > \frac{1}{2}\sqrt{2(1.1774)^2} = 0.83255$ then we have, $(1 - \operatorname{erf}\left(e^{\tilde{\mu}} - \frac{1}{2}\sqrt{2\sigma^2}\right)) < 1$ and the condition is satisfied.$\blacksquare$

References

[1] Alesina, A. and G. Angeletos (2005), "Fairness and Redistribution", American Economic Review, 95, 960-980.

[2] Alesina, A. and P. Giuliano (2009), "Preferences for Redistribution", mimeo, Harvard and UCLA.

[3] Alesina, A. and E. La Ferrara, (2005), "Ethnic Diversity and Economic Performance", *Journal of Economic Literature*, vol. 43(3), 762-800.

[4] Alesina, A., Di Tella, R. and R. MacCulloch (2004), "Inequality and happiness: are Europeans and Americans different?", *Journal of Public Economics*, Volume 88, Issues 9–10, August, Pages 2009–2042.

[5] Atkinson, A., T. Piketty, and E. Saez (2011), "Top Incomes in the Long Run of History", Journal of Economic Literature 49:1, 3–71.

[6] Bagwell, L. S. and B.D. Bernheim (1996), "Veblen Effects in a Theory of Conspicuous Consumption," American Economic Review, vol. 86(3), pp 349-73.

[7] Banerjee, A., Duflo, E., Ghatak, M. and J. Lafortuney (2010), "Marry for What? Caste and Mate Selection in Modern India", mimeo.

[8] Barlow, R. E. and F. Proschan (1965), Mathematical Theory of Reliability (New York: Wiley).

[9] Barlow, R. E. and F. Proschan (1966), "Inequalities for Linear Combinations of Order Statistics from Restricted Families", Annals of Mathematical Statistics, vol. 37, pp. 1593–1601.

[10] Becker, G. "A Theory of Marriage: Part I" (1973), Journal of Political Economy 81:813–846.

[11] Benabou, R. (1996), "Heterogeneity, Stratification and Growth", American Economic Review, 584-609.

[12] Benabou, R. (2000), "Unequal Societies: Income Distribution and the Social Contract", American Economic Review, 90, 96-129.

[13] Benabou, R. and E. Ok (2001), "Social Mobility and the Demand for Redistribution: The POUM Hypothesis", Quarterly Journal of Economics 447-487.

[14] Benabou, R. and J. Tirole (2006), "Belief in a Just World and Redistributive Politics", Quarterly Journal of Economics, vol. 121(2), pp. 699-746.

[15] Bradford, D. H. Kelejian (1973), "An Econometric Model of the Flight to the Suburbs", Journal of Political Economy Vol. 81, No. 3, pp. 566-589.

[16] Cohen, L. and C. Malloy (2010), "The Power of Alumni Networks", Harvard Business Review.

[17] Corneo, G. and O. Jeanne (2000), "Social Limits to Redistribution", American Economic Review, 90, pp. 1491-

[18] Cowell, F. (2011), Measuring Inequality, Oxford University Press.

[19] Cramer, J. S. (1978), "Function for Size Distribution of Incomes: Comment", Econometrica Vol. 46, pp. 459-460.

[20] Damiano, E. and Li, H. (2007): "Price discrimination and efficient matching", Economic Theory, vol. 30, 243-263.

[21] De La O, A.L and J.A. Rodden (2008), "Does Religion Distract the Poor? Income and Issue Voting Around the World", Comparative Political Studies, vol. 41 pp. 437-476.

[22] Diamond, P. and E. Saez (2011), "The case for a Progressive Tax: from Basic Research to Policy Recommendations", Journal of Economic Perspectives, 25(4), 165–190.

[23] Epple, D. and R. Romano (1998), "Competition between Private and Public Schools, Vouchers, and Peer-Group Effects", American Economic Review, vol. 88, pp. 33-62.

[24] Fernandez, R. and R. Richardson (2003), “Equity and Resources: An Analysis of Education Finance Systems” , Journal of Political Economy, pp.

[25] Fernandez, R. and R. Richardson (2001), “Sorting and Long-Run Inequality” Quarterly Journal of Economics,

[26] Fernandez. R. and J. Gali, J. (1997), “To Each According to...? Markets, Tournaments and the Matching Problem with Borrowing Constraints”, Review of Economic Studies, 66, 799–824.

[27] Frank, T. (2004), *What’s the matter with Kansas*, Henry Holt.

[28] Galor, O. and J. Zeira (1993),"Income Distribution and Macroeconomics," Review of Economic Studies, Wiley Blackwell, vol. 60(1), 35-52.

[29] Gelman, A., Shor, B., Bafumi J. and Park, D. (2007), “Rich State, Poor State, Red State, Blue State: What’s the Matter with Connecticut?”, Quarterly Journal of Political Science: Vol. 2:No 4, pp 345-367.

[30] Glazer, A. and K.A. Konrad (1996), "A Signaling Explanation for Charity," American Economic Review, vol. 86(4), pp. 1019-28

[31] Hall, W. J. and Wellner, J. A. (1984), Mean residual life, In Proceedings of the international symposium in Statistics and Related Topics, pp. 169-184. Amsterdam: North-Holland

[32] Hassler J., J. Rodriguez-Mora, K. Storesletten and F. Zillibotti (2003), “The Survival of the Welfare State,” American Economic Review, XCIII, 87-112.

[33] Heffetz, O. (2011), “A Test of Conspicuous Consumption: Visibility and Income Elasticities”, Review of Economics and Statistics, Vol. 93, pp. 1101-1117.

[34] Hopkins, E. and T. Kornienko (2010), “Which Inequality? The Inquality of Endowments versus the Inequality of Rewards”, American Economic Journal: Micoreconomics 2, 106-137.

[35] Hoppe, H., B. Moldovanu, and A. Sela (2009), "The Theory of Assortative Matching Based on Costly Signals", Review of Economic Studies, vol. 76, p. 253-281

[36] Hoppe, H., B. Moldovanu, and E. Ozdenoren (2011), “Coarse Matching with Incomplete Information", Economic Theory, Vol. 47, 73-104.

[37] Kerr, W. (2011), "Income Inequality and Social Preferences for Redistribution and Compensation Differentials", mimeo, Harvard Business School.

[38] Kremer, M. (1997), "How Much Does Sorting Increase Inequality?" Quarterly Journal of Economics, CXII (1997), 115-139.

[39] Lam, D. (1988), "Marriage Markets and Assortative Mating with Household Public Goods: Theoretical Results and Empirical Implications" The Journal of Human Resources, Vol. 23, No. 4 , pp. 462-487.

[40] Leibenstein, H. (1950), "Bandwagon, snob and Veblen effects in the theory of consumer demand", Quarterly Journal of Economics 64, 183-207.

[41] Levy, G. (2004), "A Model of Political Parties", Journal of Economic Theory, Vol. 115(2), pp. 250-277.

[42] McAfee, R. P. (2002), "Coarse matching", Econometrica 70, pp. 2025-2034.

[43] Meltzer, A. and S. Richards (1981), "A rational theory of the size of government", Journal of Political Economy vol. 89, pp. 914-927.

[44] Moav, O. and Z. Neeman (2012) "Saving Rates and Poverty: The Role of Conspicuous Consumption and Human Capital", The Economic Journal, 122, 933-956.

[45] Oishi, S., Kesebir, S. and E. Diener (2011), "Income inequality and happiness", *Psychological science*, September, vol. 22 no. 9, 1095-1100

[46] Perotti, R. (1996), "Growth, Income Distribution and Democracy: What Does the Data Say", Journal of Economic Growth, 1(2), 149-187.

[47] Pesendorfer, W. (1995), "Design Innovations and Fashion Cycles", American Economic Review, vol. 85, pp. 771–792.

[48] Piketty, T. (1995), "Social Mobility and Redistributive Politics", Quarterly Journal of Economics CX, 551-583.

[49] Pinkovskiy, M. and X. Sala-i-martin (2009), "Parametric Estimations of the World Distribution of Income", mimeo, MIT and Columbia.

[50] Rayo, L. (2005), "Monopolistic Signal Provision", B.E. Journal in Theoretical Economics.

[51] Rege, M. (2003), “Why Do People Care About Social Status”, Journal of Economic Behavior & Organization, 66, pp. 233–242.

[52] Roemer, J. (1998), “Why the poor do not expropriate the rich”, Journal of Public Economics, 70, pp. 399-424.

[53] Salem, A. B. Z. and T. D. Mount (1974), “A Convenient Descriptive Model of Income Distribution: The Gamma Density”, Econometrica, Vol. 42, pp. 1115-1127.

[54] Shayo, M. (2009), “A Model of Social Identity with an Application to Political Economy: Nation, Class and Redistribution”, American Political Science Review 103(2), 147-174.

[55] Singh S. K. and G. S. Maddala (1976), “A Function for Size Distribution of Incomes”, Econometrica, Vol. 44, pp. 963-970.

[56] Wilson, W. (1987), *The Truely Disadvantaged: The Inner City, the Underclass*, and Public Policy, Chicago University Press.