Ec319
Mathematical Economics

Problem Set 1

Constrained Optimization

1. For each of the following sets of constraints either say, without proof, what the maximum of z^2 subject to the constraints is, or explain why there is no maximum.

$z \leq 0, z \geq 1$	$z \leq 2$
$0 \leq z \leq 1$	$z \geq 2$
$-1 \leq z \leq 1$	$1 \leq z < 2$
$1 \leq z \leq 1$	$1 < z \leq 2$

2. For each of the following sets say whether or not it is non-empty, closed and bounded. Give reasons for your explanations.

(a) The consumption set for a household defined by $x_j \geq 0$, $j=1,2,...,n$, and

$p_1x_1+p_2x_2+...+p_nx_n \leq y$, where x_j is the consumption of good j, p_j

its price (where $p_j>0$) and $y>0$ the household's income.

(b) The production set of a firm defined by $y \geq 0$, $x \geq 0$, and $y \leq 2x$, where

y is the output and x is the input, x and y are both scalars.

3. Suppose that the price of the firm's input in question 2b is w and the price of its output is p, and that neither of these prices is affected by how much it buys or sells. Its objective is to maximize profits $py - wx$. For what non-negative

values of p and w does the firm have an optimal production plan? What is this plan?

4. *XYZ* is a profit maximizing firm selling a good in a perfectly competitive market at price 4. It can produce any non-negative quantity of such good y at cost $c(y)=y^2$. However there is a transport bottle-neck which makes it impossible for the firm to sell more than k units of y, where $k \geq 0$.

(a) Write down *XYZ*'s profit maximization problem.

(b) Show on a graph the sets B and C for this problem, as defined in the lecture. Using the graph write down the solution to the problem for all non-negative values of k.

(c) What is the maximum value function for this problem?

(d) Can you use the Lagrangian Sufficiency Theorem to solve this problem? If your answer is yes use the theorem to confirm that your answer to part b of this question is correct. If your answer is no explain why.

5. *XYZ*'s production technology changes, so its cost function is now $c(y)=y^3-3y^2+4y$. Answer question 4 for this new cost function.