From the Laplace, P. S. [1816]: Annales de Chimie et de physique*, **3**, pp. 238-241, Gay-Lussac and Arago, editors, Paris: Feugueray. Translated by Bryan Roberts.*

On the Speed of Sound in Air and Water

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(Read before the Academy of Sciences the 23rd of December, 1816.)

Newton, in the second book of *Mathematical Principles of Natural Philosophy*, gave the expression for the speed of sound. The manner in which he derived it is one of the most remarkable traits of his genius. The speed according to this expression is a little less than one sixth of the speed determined by careful experiment by members of the Academy in 1738, in experiments done with great care. Newton had already recognized this discrepancy in the experiments of his time, and tried to explain it. But modern discoveries about the nature of atmospheric air have destroyed this explanation, along with those proposed by various other mathematicians. Fortunately, these discoveries present us with a phenomenon which appears to me to be the true cause of the discrepancy between the observed and the calculated speed of sound, which most mathematical physicists have now accepted. This phenomenon is the heat that air develops when it is compressed. When one raises the temperature of air while maintaining constant pressure, only one part of the caloric received actually produces this effect; the other part, which becomes latent, produces an increase in volume. It is this part which manifests [*se developpe*] when one compresses the expanded air and reduces it to its original volume. The heat released by the approach of two neighboring molecules of a vibrating arial fiber elevates their temperature, and gradually diffuses into the air and the surrounding bodies. But this diffusion and propagation occurs extremely slowly relative to the speed of the vibrations, and one can assume without noticeable error that, in the period of a single vibration, the quanitity of heat stays the same between two neighboring molecules. Thus the molecules, in coming together, are repelled all the more: first, because as their temperature is assumed to be constant, their mutual repulsion increases in inverse proportion to their distance; and second, because the latent caloric which manifests [*se developpe*] increases their temperature. Newton had only considered the first of these two causes of repulsion, but it is clear that the second cause must increase the speed of sound, since it increases the elasticity of air. In entering this into the equation, I arrived at the following theorem:

> The actual speed of sound is equal to the product of the speed given by the Newtonian formula and the square root of the ratio of the specific heat of air under constant atmospheric pressure and varying temperature to its specific heat at a constant volume.
> [*Note: See the* Collected Works of Laplace, *vol. 5, pp. 105, 134, 157.*]

If we now assume, like many physicists, that the heat contained in a quanitity of air under constant pressure and varying temperature is proportional to its volume (which can't be far from the truth), the square root above becomes the square root of the ratio of the difference of two pressures to the difference of the quantities of heat which develop two equal volumes of atmospheric air with respect to these pressures, in passing from a given temperature to a lesser one, with the smallest of these quantities of heat and the smallest of these pressures being taken as units.

Wanting to compare this theorem to experimental fact, I was fortunate enough to find the observational conditions [*données d'observation*] assumed by my theorem among the numerous results of the interesting work that Mr. La Roche and Mr. Berard have done on the specific heat of gas. These able physicists have measured the quantities of heat which are released from two equal volumes of atmospheric air when the temperature is lowered by about 80 degrees; one quantity is compressed by the weight of the atmosphere, the other compressed the same weight increased by thirty-six hundredths. They found that the heat released relative to the greater pressure was 1.24, with the heat relative to the smaller pressure being taken as a unit. In order to calculate the actual speed of sound it is thus necessary, by the previous theorem, to multiply the speed deduced by Newton's formula by the square root of the ratio of 36 hundredths to 24 hundredths, or by the square root of 3/2. At a temperature of six degrees, this formula gives 282.42 meters per second. In multiplying by $\sqrt{(3/2)}$, we get 345.45 meters per second. Members of the French Academy found it to be 337.18. The difference between these two results might amount to experimental uncertainty. But the smallness of this discrepancy unarguably establishes that the difference between the observed speed of sound and the speed calculated by the Newtonian formula is due to the latent heat produced by the compression of air.

It is the result of all this that, given constant pressure, if one increases a given volume of air by raising its temperature, and then reduces it by compression to its original volume, it will release in this compression one third of the heat employed in its expansion. It remains for physicists to determine, by direct experiment, the ratio of the specific heats of air under constant pressure and at a constant volume; a ratio which we have just found to be 1.5. The speed of sound, as observed by members of the French Academy, gives 1.4254 for this ratio; perhaps, in light of the difficulty of direct experiments, this speed is the most accurate means of obtaining it.

I have concluded (page 166, October Workbook) that the speeds of sound in rainwater and sea-water are equal to 2642.8 and 2807.4 meters per second. These numbers were calculated on the basis of the experiments of Canton on the compression of these liquids, and with regard only to the linear decrease of the dimensions of the compressed volume. I recognized that it is necessary to consider the total decrease of this volume, and thus that the preceding numbers must be divided by $\sqrt{3}$, reducing them to 1525.8 and 1620.9. Hence, the speed of sound in fresh water is four and a half times larger than in air.