AGGLOMERATION AND TRADE WITH INPUT-OUTPUT LINKAGES AND CAPITAL MOBILITY

Frédéric Robert-Nicoud[†]

LSE and CEPR

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Abstract. This paper put forth a nesting 'New trade-New Economic Geography' model in which agglomeration is driven by input-output linkages among firms, trade in goods and capital mobility. The NEG sub-model exhibits the same positive and dynamic properties as a wide class of models based on other agglomeration mechanisms. Its normative implications are nuanced: equity and efficiency do not necessarily conflict. When input-output linkages are strong, agglomeration might Pareto dominate dispersion because agglomeration lowers producer prices. When vertical linkages are weak, the market is biased in favour of agglomeration if the planer has a strong aversion to inequalities.

Keywords: New Economic Geography; Capital mobility; International trade; Welfare.

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[†] Correspondence: Department of Geography and Environment, London School of Economics, Houghton Street, London WC2A 2 AE. F.L.Robert-Nicoud@lse.ac.uk

1. Introduction

This paper works out in an integrated fashion a 'New Trade, New Economic Geography' (NT, NEG hereafter) model that nests, on the one hand, the NT model due to Lawrence and Spiller (1983), Flam and Helpman (1985) and extended to a geography framework by Martin and Rogers (1995) and, on the other hand, a NEG model that shares similarities with Venables' (1996) model. Let me refer to the former as the FHMR model after the initials of its authors and to the latter as the FCVL model for 'Footloose Capital, Vertical Linkages'.[1] Specifically, the all-encompassing model is a NT model in which countries are engaged in intra-industry trade à-la Dixit-Stiglitz-Krugman (DSK hereafter) as well as in inter-industry trade and in which capital is internationally mobile. In addition, firms buy each other's output as intermediate inputs, giving rise to so-called forward and backward linkages; as a result of these linkages, market sizes are endogenous—certainly *the* distinctive feature of NEG models. On the positive side, the paper describes the equilibrium properties of the model and shows how to generate the FCVL and the FHMR models as special cases. This is convenient because an attractive feature of the FHMR model is its full analytical tractability, whilst the FCVL model is useful because its dynamic properties are as rich as in typical NEG models, but it is (marginally) simpler than most of them. On the normative side, the main contribution of the paper is to study the welfare analysis of the FCVL model; as we will see, these properties are distinguishable from those of Krugman's (1991) model, as studied by Charlot et al. (2006), as well as from those of Krugman and Venables' (1995) model, as studied by Ottaviano and Robert-Nicoud (2006). The main result from the welfare section is perhaps that agglomeration, whereby all manufacturing activities are concentrated in a single region, can Pareto-dominate dispersion because strong input-output linkages are passed onto consumers everywhere in the form of low prices.

The remainder of the paper is organised as follows. In the remainder of the introduction, I first provide some statistics to show that the two key ingredients of the model—capital mobility and input-output linkages—are important current features of the world economy. I then briefly and discriminatorily review the relevant literature, explaining along the way where and how the current paper contributes. Section 2 sets up the structure of the model. Section 3 solves the equilibria, mostly in the special case of ex-ante symmetric regions. Sections 4 and 5 convey the welfare analysis in that case. Section 6 derives the FHMR model as a special case of the nesting model.. Section 7 concludes.

Two important features of 'globalisation'

Two of the main ingredients put forth in the current paper –to repeat: capital mobility and input-output linkages– are two noticeable features of the current international environment. Moreover, these features are closely related (in both the model and in the data). In the model, the form of international capital mobility that is closest to the real world is perhaps foreign direct investment (FDI), that is, long term investments in the real economy; in recent years, FDI flows have grown enormously, albeit from a low base. Using UNCTAD and World Bank data, Barba-Navaretti et al. (2005) report that worldwide real inflows of FDI have grown at an annual rate of 17.7% over 1985-1999 (that is, by a factor eleven), faster than worldwide export (annual rate 5.6%, a factor two), faster still than worldwide real GDP (annual rate 3.1%, a 50% increase). Almost as striking perhaps is the volume of employment or turnover of multinational companies. Since the FCVL model explicitly features firm-to-firm sales, let me provide some figures for export volumes and sales of multinational corporations. As an illustration, take the volume of manufacturing sales of US-owned EU affiliates in the EU economy: these about nearly 4 times as large as US exports of manufactures into the

EU. More globally, UNCTAD (various years) estimates that a third of world trade is conducted within the boundaries of multinational corporations (between foreign affiliates or between the parent company and affiliates). Of the remaining two thirds, a good share of world trade takes place between unrelated firms. A recent descriptive paper by Bernard and Jensen (2005) provides a fresh look at this. They report that the 'most globally engaged' (MGE) firms of their almost exhaustive sample of US enterprises are responsible for 80% of US trade. Those MGE firms are defined as the firms that both import and export, and at least some of those are to and from 'related parties' (by opposition to arms length). Thus, firm to firm international business represents perhaps the most important share of world trade. The model in this paper does not model explicitly the boundaries of the firm, thus the FCVL model is consistent with both forms of firm-to-firm trade (whether they are distinct entities or whether they belong to the same multinational corporation).[2]

Policies are also becoming ever friendlier towards FDI flows. UNCTAD (2004) reports that among the 70 countries that have changed their FDI regulations between 1992 and 2002, more than 94% of the regulatory changes are classified as being more favourable to FDI, that is, impediments to such capital movements have decreased over the period. Likewise, the number of bilateral investment treaties have increased 5-fold between 1990 and 2002.

In short, capital is increasingly footloose –certainly more than workers– and vertical linkages are ubiquitous. Thus, it is important to have a NEG model that incorporates those facts for its own sake.

Literature review, terminology, and new results

The main insight of the new economic geography, or NEG, comes from its formalisation of agglomeration mechanisms based on endogenous market size. Various trade models predict that sectors characterized by increasing returns to scale, imperfect competition and transportation costs will be disproportionately active in locations with good market access (Krugman, 1980). In a simple two-country model, this 'Home market effect' implies that the country with the larger demand for the good produced by such sector will end up exporting that good (this result sharply contrasts with the predictions of the Ricardian and Heckscher-Ohlin-Viner models of trade; but see Behrens et al. 2005 and Ottaviano and Thisse, 2005). NEG models add cumulative causation to this effect: hosting a larger share of increasing returns activities increases local demand and profitability. If there is factor mobility of sorts, then more of these factors will in turn locate themselves in the already large market, and the cycle repeats under certain conditions. Endogenous agglomeration results from this mechanism, namely, the manufacturing sector clusters in a single region, usually called the 'core' and opposed to the 'periphery'. Therefore, initially symmetric regions might end up hosting very different sectors as increasing returns activities have a tendency to locate in few places (Fujita and Thisse, 2002). As for the transmission mechanism whereby agglomeration occurs, the NEG exploits in a spatial setting the kind of circularity causality recurrent in models of monopolistic competition (Matsuyama, 1995). Typically, these models predict that agglomeration (resp. dispersion, whereby the manufacturing sector is active in both regions or countries), is the unique stable equilibrium when international trade and transaction costs are low (resp. large) and that both agglomeration and dispersion are stable equilibria for intermediate values of trade costs. For syntheses of this literature, see Fujita et al. (1999), Baldwin et al. (2003) and Combes et al. (2005).

This brings us back to the first contribution of this paper. The model here emphasizes, as the agglomeration mechanism, the interplay between capital mobility and input-output linkages. Other mechanisms have been put forth in the literature, like skilled labour migration (Krugman 1991, Forslid and Ottaviano 2003) and human capital accumulation (Baldwin 1999). The mechanism put forth thus far that is closest to the current paper originally due to Venables (1996), extended in Krugman and Venables (1995) and simplified by Ottaviano and Robert-Nicoud (2005).[3] There, agglomeration stems jointly from inter-sector labour mobility and input-output linkages (a.k.a. vertical linkages) among firms. Namely, workers can move between sectors within the same spatial entity (a region or a country) and firms buy each other's output as intermediate inputs in the spirit of Ethier (1982).[4] As Robert-Nicoud (2005) and Ottaviano and Robert-Nicoud (2005) show, all these NEG models are isomorphic and thus exhibit the same dynamic properties (for a comprehensive survey and a synthesis, see also Baldwin et al. 2003).[5] As it turns out, this is also the case for the model presented in the current paper, thus the NEG model of section 3 belongs to the same family of models.

The paper also studies the normative properties of the model. Unlike most previous contributions, it does so mostly using the Pareto criterion and compensation criteria, for social welfare functions are problematic when people face different price indices (Wildasin, 1986)[6]. As we shall see, the market delivers the ‘desirable’ outcome when trade is quite free: this desirable outcome is agglomeration. (an equilibrium outcome is said to be ‘desirable’ if everyone can be made better off, using compensation schemes if necessary, relative to another equilibrium outcome). As it turns out, if the vertical linkages that bind firms with each others are large agglomeration might even Pareto-dominate dispersion, that is, agglomeration might be preferable to dispersion even for the population left behind in the periphery. To understand this seemingly counterintuitive result, remember that firms trade with each other. Thus, when all firms are clustered in a single location intermediate inputs are cheapest because firms do not have to pay for transportation or trade costs when they purchase those inputs. This cost-saving aspect of agglomeration, which benefits all firms, is passed onto mill prices at equilibrium. When trade costs are low and vertical linkages are strong enough, these lower mill prices might event translate into a lower consumer price index (which includes trade costs) for the residents in the periphery. The market also delivers a socially optimal outcome in the opposite case, that is, when vertical linkages are modest and when trade costs are near prohibitive.

A third contribution of this paper is to underline the distinct role of two parameters of the model that are taken to be the same for simplicity in the aforementioned papers on vertical linkages. As I shall see, the share of manufactured in consumers' expenditure exclusively influences the normative properties of the model, in an intuitive way that I shall explain in detail. By contrast, the share of intermediates in firms' cost function affects both the positive and the normative features of the model. When this share increases, it makes agglomeration both likely and more desirable. In addition, I show that the elasticity of substitution plays a modest role (in a well-defined sense) in the welfare analysis but is crucial for the positive properties of the model.

Finally, there is a good technical reason to integrate NEG forces into a model in which countries trade goods and capital alike, as the model does. To repeat, the model in this paper nests both a NEG model with all the usual properties described in details in Fujita et al. (1999) and Baldwin et al. (2003), among others, and the NT model due to Harry Flam, Elhanan Helpman, Philippe Martin and Carol Rogers as special cases. This is useful, because applications to, say, the analysis of the location effects of preferential trade agreements in such a setting emphasizes how and where the departure from the tractable FHMR trade model, by incorporating vertical linkages, changes the resulting picture. This is the route undertaken by Baldwin et al. (2003, chapter 14). Finally, it is worth stressing that these extensions are facilitated by the fact that the current model is easier to manipulate than the model due to Krugman and Venables (1995).

Minor results and results that appear elsewhere in the literature are cast as 'Lemmas'. All new results appear as 'Propositions' or 'Corollaries'; they appear in bold characters and there are six of them.

2. The basic model

The model developed in this section is an extension of the FHMR trade and geography model and builds on Dixit and Stiglitz's (1977) framework of monopolistic competition. The novelty here is to add agglomeration forces in the form of vertical linkages, as in Venables (1996a). As I proceed, I make choices of units and of the numéraire that are standard in the NEG literature.

Tastes and production

Consider a country consisting of two regions or countries, j=1,2 (I use the two terms interchangeably). The typical individual is assumed to supply one unit of labour L (the reward of which is w) and k units of capital K (the reward of which is π) inelastically. There is a measure L of workers and a unit measure of capital in this economy, so the typical worker owns k=1/L units of capital and, as a consequence, her income is w+kπ. Tastes for a typical individual in j take a Cobb-Douglas form in which j spends a share μ of her income y_j on a composite good M (for 'manufacturing' or 'monopolistically competitive') and a share 1-μ on the homogenous good A (for 'agriculture', say). The composite good M comes in N different varieties. Tastes over the different varieties are captured by a CES, 'love-for-variety', functional form, with an elasticity of substitution σ>1 between any pair of varieties. The dual of this, the indirect utility function of region j's representative consumers, can therefore be written as:

$$\textbf{(1)} \qquad V_j = \frac{y_j}{p_A^{1-\mu} G_j^{\mu}}; \; G_j \equiv \left(\int_{i=0}^{N} p(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}; \; 0 < \mu < 1, \quad \sigma > 1$$

where p(i) is the consumer price of variety i, G_j is the true CES price index over the N varieties of the manufacturing good, and p_A is the price of good A (the reason p_A and π are not indexed by j will become clear shortly). Each firm i∈ [0,N] produces a different variety, the buyer price of which is p(i); this brings us to production.

Each producer enjoys monopoly power over his own variety. No producer has any incentive to produce a variety already being produced by another producer, for she would then directly compete for the market of that variety with the incumbent producer. As a result her profits be lower. Hence, N is also the number (mass) of firms operating in sector M.

The manufacturing sector M is the usual monopolistic competition sector a-la Dixit and Stiglitz (1977). It produces a CES aggregate under increasing returns. Specifically, each firm needs a fixed amount a_{KX} of capital K to start producing and a constant amount β of a Cobb-Douglas composite input made out of labour (with share 1-α) and intermediates produced by sector M itself (with share α) for each unit of output it produces. Mathematically, the cost function for the typical firm located in j is given by:

$$\textbf{(2)} \qquad C_j(x_j) = a_{KX}\pi_j + \beta x_j w_j^{1-\alpha} G_j^{\alpha}; \quad 0 < \alpha < 1$$

where G_j is the same CES price index as in (1), π_j is the cost of one unit of K prevailing in j, and x_j is a typical firm output. Observe that the same G_j enters (2) and (1); this means that the elasticity of substitution among varieties of manufacture is the same for consumers and for firms. Qualitatively, this is an innocuous assumption, but this is required to keep the analysis manageable. See also Fujita et al. (1999, chapter 14).

In FHMR, α is equal to zero. Shutting down vertical linkages, as this assumption does, makes the analysis entirely tractable, but this is at the cost of no-longer being a NEG model (when $\alpha=0$ market sizes are exogenous; see below).

The background sector A produces a homogenous good under constant returns using labour only: by choice of units, the per-unit output labour requirement is set to unity. A is assumed to be freely traded (hence p_A is the same in both regions) and parameter values are chosen so that no region ever specializes in M (I make the 'no-specialisation' condition more precise below); further, choose A as the numéraire. These imply $w_j=p_A=1$, $j\in\{1,2\}$. As a consequence, we can rewrite (1) as $V_j=y_jG_j^{-\mu}$.

By contrast to A, interregional trade in M is subject to Samuelson-type iceberg transportation costs $\tau\geq1$. That is, in order to sell one unit abroad a firm has to ship τ units. The difference τ-1 melts in transit (hence the name). Monopolistic pricing yields the usual relation $p_j(1-1/\sigma)= \beta w_j^{1-\alpha}G_j^{\alpha}$ for the producer price of a typical firm in j. The term in the right-hand side is the marginal cost, and σ is the perceived elasticity of demand; this requires us to impose $\sigma>1$ as a regularity condition. We choose units so that $\beta=1-1/\sigma$ (this is without loss of generality, see Baldwin et al. 2003), hence $p_j=G_j^{\alpha}$. In Dixit-Stiglitz monopolistic competition transportation costs are fully passed onto consumers so the producer price p_j holds irrespective of the market served.

Would be entrepreneurs bid for units of capital. Free entry and exit in M ensures that these entrepreneurs make no pure profits, so the operating profits (which are a share $1/\sigma$ of sales in Dixit-Stiglitz) of a typical firm active in j just cover the capital reward π_j:

(3) $$\pi_j \equiv \frac{p_j x_j}{\sigma}, \quad p_j = G_j^{\alpha}$$

Now normalize a_{KX} to 1. By symmetry among all varieties and full-employment of capital this implies N=1; define n as the share of N operating in j=1. As a result of these, we can rewrite the price indices in (1) as

(4) $$\Delta_1 = n\Delta_1^{\alpha} + \phi(1-n)\Delta_2^{\alpha}; \quad 0 \le \Delta_j \equiv G_j^{1-\sigma} \le 1, \quad 0 \le \phi \equiv \tau^{1-\sigma} \le 1$$

and Δ_2 is defined analogously. Note that the definitions of Δ_1 and Δ_2 are implicit and simultaneous. The variable G_j and the primary parameter τ usually come raised at the power 1-σ, so it is more convenient to use Δ_j and φ instead. The parameter φ measures the degree of free-ness of inter-regional trade in manufactures M (it is zero when trade is prohibited and equals unity when trade is perfectly free). Like φ, Δ_j lies in the unit interval because $n\in[0,1]$ and α<1. (This claim is easily demonstrated by contradiction.)

Endowments and factor mobility

Potentially, the two regions differ in size: region 1 is endowed with a share s of world labour and world capital stock alike; assume s≥½ without loss of generality.[7] Labour is embodied and immobile; capital is disembodied and perfectly mobile in the long run (consequently, n≠s is possible).[8] Both workers and capital owners are themselves immobile. We further assume that the capitalists own a perfectly diversified portfolio, namely, each of them own the same share of each firm.[9] Hence, their portfolio return is $\pi \equiv n\pi_1+(1-n)\pi_2$. Also, remember that $w_1=w_2=1$ holds by free trade in A and by the choice of numéraire.

All these imply that aggregate income Y in region 1, say, is equal to $Y_1=s(L+\pi)$. Location 1 expenditure on M is given by $E_1=\mu Y_1+\alpha n p_1 \beta x_1$. The term μY_1 in the previous expression is the share of final demand (and, since there are no savings, income) spent on M; it follows by applying Roy's identity on (1). The second term in the expression for E_1, $\alpha n p_1 \beta x_1$, is the share of intermediate demand spent on M that emanates from other manufacturing firms. It can be inferred from (2) using Shepard's lemma. By analogy, location 2 expenditure on M is given by $E_2=\mu Y_2+\alpha(1-n)p_2\beta x_2$.

To close the model, note that the value of total output in sector M at producer prices must equal the value of global M-sector private expenditures, viz. $np_1x_1+(1-n)p_2x_2=\alpha\beta[np_1x_1+(1-n)p_2x_2]+\mu[L+\pi]$. Making use of the pricing rules and the free-entry condition (3), we get $\pi=\mu L/[(1-\alpha)\sigma+\alpha-\mu]$.[10] Observe that the equilibrium π is function of parameters and exogenous endowments only; in particular, this expression holds for any n. Importantly, it does not depend upon ϕ or τ.

As an aside, we now have everything at hand to make the no-specialisation condition more precise: if all firms cluster in a single location, we require the labour supply of this region to be larger that the labour these firms demand so that sector A is active in both regions and nominal wages are the same worldwide. Mathematically, this requires $\min\{L_1,L_2\}>(1-\alpha)\beta\sigma\pi$. Using the equilibrium expression for π, the closed-form condition is $(1-s)>\mu(1-\alpha)(\sigma-1)/(\sigma(1-\alpha)+\alpha-\mu)$. We assume it holds throughout.

Short run and long run equilibria

In the short run capital is immobile while it becomes mobile only in the long run. Thus, in a short run equilibrium consumers maximize utility, firms maximize profits and all markets clear.

Define q_j as the ratio of the actual operating profit in region j to the equilibrium value of π, that is, $q\equiv\pi_j/\pi$, and e_j as the share of expenditure that emanates from region j, viz. $e_j\equiv E_j/\sigma\pi$.[11] Together with the expressions for p and E_j above, we obtain the following closed form solutions for e_j:

(5) $e_1 = s + \alpha\beta(n-s) + \alpha\beta n(q_1 - 1);\ e_2 = (1-s) - \alpha\beta(n-s) + \alpha\beta(1-n)(q_2 - 1)$

This expression makes clear that a region's share of expenditure is larger, the more populated it is, the larger its number of firms per capita, and the more profitable are the firms that have located there.

We then use Sheppard's lemma and Roy's identity to get the demand for a typical variety, $p_j^{-\sigma}\mu E_j/\Delta_j$. The equilibrium operating q-ratios can be derived as follows:

(6)
$$q_1 \equiv \frac{\pi_1}{\pi} = \frac{p_1 x_1}{\pi\sigma} = \underbrace{\frac{E_1}{\pi\sigma}}_{=E_1/E}\frac{p_1^{1-\sigma}}{\Delta_1} + \underbrace{\frac{E_2}{\pi\sigma}}_{=E_2/E}\frac{(p_1\tau)_1^{1-\sigma}}{\Delta_2}$$
$$= \left(\frac{e_1}{\Delta_1} + \phi\frac{e_2}{\Delta_2}\right)\Delta_1^{\alpha}$$

The equilibrium expression for q_2 is symmetric. The first equality after the identity derives from (3); the second one derives from Cobb-Douglas preferences and costs; the final step follows from the facts that equilibrium profits are proportional to sales and expenditures (π=E/σ), the equilibrium price $p_j=\Delta_j^{\alpha}$, the equilibrium value for Δ_j in (4) and from the definition of e_j in (5). It is obvious from the definition of π and the q's that $q_1+q_2=1$ always holds. Thus, $n\in(0,1)$ implies, first, $q_1<1$ if, an only if, $q_2>1$ (and conversely) and, second, $q_1=1$ if, and only if, $q_2=1$. In words, the firms located in a given region make above normal operating profits whenever the firms located in the other one make below normal operating profits.

The system (3)-(6) completely characterizes the so-called *instantaneous equilibrium*. (In an instantaneous equilibrium, n is an exogenous variable and q_1 and q_2 are functions of n.)

In the long run, capital owner seek the higher nominal returns, so that n adjusts so that $\pi_j=\pi=\mu L/[(1-\alpha)\sigma+\alpha-\mu]$ (and hence $q_j=1$) for any active firm. Following standard practice, assume that capital owners allocate their capital according to current nominal differences in rewards according to the following ad-hoc law of motion for n: [12]

(7) $$\dot{n} = \gamma n(1-n)(\pi_1 - \pi_2) = \gamma n(1-n)(q_1 - q_2)\pi$$

where γ is a strictly positive parameter and the second equality follows from the definition of π. The long run equilibrium is attained whenever $\dot{n}$ is zero. Three cases can occur: n=0 (in which case q_2=1), n=1 (in which case q_1=1), and 0<n<1 (and hence q_1=q_2=1). The first two cases are usually referred to as 'core-periphery' equilibria and the third as interior or 'dispersed' equilibria. By the symmetry of the model, the symmetric equilibrium n=½ always exists. More generally denote an interior long run equilibrium as n^0.

To assess the stability of these equilibria the NEG typically resorts on the following informal methods.[13] Consider that, staring from any long run equilibrium, the spatial allocation of capital n is hit by an exogenous, epsilon-small, perturbation. For the interior equilibria (in particular the symmetric equilibrium in the symmetric model n=s=½), one evaluates the sign of the change in the nominal profit gap, viz. π_1-π_2. If the displaced unit of capital increases the profit in the receiving region, then the symmetric equilibrium is unstable.

For the agglomerated (or core-periphery) equilibrium, one checks whether the perturbation creates a nominal profit in the periphery that is higher than the nominal profit in the core. If this is the case then this equilibrium is unstable.

Mathematically, these two tests can be written as:

(8) $$\left.\frac{d(\pi_1 - \pi_2)}{dn}\right|_{n=n^0} < 0, \quad \left.(\pi_1 - \pi_2)\right|_{n=1} > 0$$

The equilibrium under consideration is stable when the relevant inequality holds. It is unstable otherwise.

The description of the model is now complete: the long run equilibria consist of the values of n in the interval [0,1] that solve (3)-(7) for $\dot{n} = 0$.

3. Symmetric regions and vertical linkages: The FCVL model

Following enduring tradition of the NEG, my primary interest here is to discuss how regions that share identical tastes, endowments, and technology might endogenously diverge in terms of production structure and real incomes.

Sustainability of the concentrated equilibrium

Here the question is, is a core-periphery pattern sustainable in region 1? To answer this question, we check under which conditions n=1 and $q_2 \leq 1$ hold simultaneously. No firm wants to leave 1 if the shadow profit in 2 is inferior to π. Substituting n=1 into (4)-(6), we find that this condition holds whenever

(9) $$S_1(\phi) \equiv 2\phi^{1-\alpha} - [(1+\alpha\beta)+(1-\alpha\beta)(2s-1)]\phi^2 - [(1-\alpha\beta)-(1-\alpha\beta)(2s-1)] \geq 0$$

As explained in various papers and monographs mentioned earlier, this inequality holds for all ϕ large enough, i.e. for $\phi \in [\phi_1^{sust},1]$, where ϕ_1^{sust} is implicitly defined as the smallest root of $S_1(\phi)$ (1 is the unique other root). This root is always strictly in the (0,1) interval because the so-called no-black-hole condition $1>\alpha\beta$ always holds, which I assume throughout. In words, whenever trade costs are low enough, if it already happens that all firms have agglomerated in either region, then none has any incentive to leave the core and start producing in the periphery. It turns out that the clustering of all manufactures in the south can also be sustainable if ϕ is large enough; the resulting condition can be denoted by $S_2(\phi)>0$, with $S_2(\phi)$ being the perfect symmetric to $S_1(\phi)$. Intuitively, one would expect ϕ_2^{sust} to be larger than ϕ_1^{sust}, that is, if agglomeration is sustainable in the small country then it must also be sustainable in the large market.

This intuition turns out to be correct. To see this, note that $S_1(\phi)$ is always larger than $S_2(\phi)$ since s>½:

(10) $$\forall \phi \in [0,1): \quad S_1(\phi) - S_2(\phi) = (1-\alpha\beta)(2s-1)](1-\phi^2) > 0$$

It turns out that working with the general, asymmetric case would be a substantial task that is beyond the scope of this paper (one would have to use numerical simulations, something that I aim at avoiding to the maximum in this paper). Therefore, I impose s=½ in the remainder of the paper, unless otherwise specified. The resulting model is referred to as the FCVL model in Baldwin et al. (2003).[14] Denote the sustain point in the symmetric case by ϕ^{sust}; from (9), ϕ^{sust} satisfies:

(11) $$\phi^{sust} < 1: \quad 2(\phi^{sust})^{1-\alpha} - [1+\alpha\beta](\phi^{sust})^2 - [1-\alpha\beta] = 0$$

To sum-up, we have:

> Lemma 1. The Core-periphery equilibrium is said to be sustainable if ϕ is larger that ϕ^{sust}.

This result is reminiscent from Krugman (1991) and his followers. In addition, an increase in input-output linkages reinforces the agglomeration forces and thus makes agglomeration more likely: $\partial\phi^{sust} < \partial\alpha < 0$. Note that $\phi^{sust}>0$ if, and only if, $\alpha<1$ (more on this below).

(Un)stability of the dispersed equilibrium

Define a dispersed equilibrium as the configuration in which n=½. With symmetric regions such an equilibrium always exists as can be seen from (4)-(7). It might not always be stable in the sense of (8), though. Here the thought experiment is, if a firm moves from region 2 into region 1, in which case would the gap π_1-π_2 thus created be positive and hence, by (7), widening? In which case would that gap be negative and hence the perturbation be self-correcting? Formally, answering this question is equivalent to signing dq_1/dn evaluated at $n=½$ (see Puga 1999).[15]

As we know from the symmetry of the model, $n=½$ implies $e_1=e_2$, $\Delta_1=\Delta_2$, and $q_1=q_2$. We therefore denote common variables with the nought subscript. From (4)-(6), we find that $q_0=1$, $e_0=½$, and $\Delta_0^{1-\alpha}=(1+\phi)/2$. A small perturbation of the model (in the form of $dn>0$) around the symmetric equilibrium has an impact on the Δ's, the q's and the e's. As an illustration, totally differentiate Δ_1 in (4) to get:

(12)
$$\begin{aligned} d\Delta_1\big|_{n=1/2} &= \left(\Delta_1^\alpha - \phi\Delta_2^\alpha\right)dn + \left(\alpha\Delta_1^{\alpha-1}d\Delta_1 + \phi\alpha\Delta_1^{\alpha-1}d\Delta_2\right) \\ &= (1-\phi)\Delta_0^\alpha dn + \alpha\Delta_0^\alpha(1-\phi)d\Delta \end{aligned}$$

where the second line derives from the fact that the effect of dn on the variables pertaining to region 2 is symmetric to the effect of dn on those pertaining to region 1 around the symmetric equilibrium; thus $de_0 \equiv de_1=-de_2$, $d\Delta_0 \equiv d\Delta_1=-d\Delta_2$ and $dq_0 \equiv dq_1=-dq_2$. Working out the derivatives of (5) and (6) in the same way, we obtain:

(13)
$$\begin{bmatrix} 0 & 1-\alpha(1-\phi)/(1+\phi) & 0 \\ 2 & 0 & -\alpha\beta \\ -2(1-\phi)/(1+\phi) & (1-\phi)/(1+\phi)-\alpha & 1 \end{bmatrix} \begin{bmatrix} de_0 \\ d\Delta_0/\Delta_0 \\ dq_0 \end{bmatrix} = 2\begin{bmatrix} (1-\phi)/(1+\phi) \\ \alpha\beta \\ 0 \end{bmatrix} dn$$

Using Cramer's rule, it is easy to see that $dq_0/dn \geq 0$ if, and only if,

(14)
$$B(\phi) \equiv \phi - \phi^{break} > 0, \quad \phi^{break} \equiv \frac{(1-\alpha)(1-\alpha\beta)}{(1+\alpha)(1+\alpha\beta)}$$

where ϕ^{break} is the so-called 'break point', which is strictly smaller than unity by inspection. There always exists a non-empty combination of parameters such that the symmetric equilibrium is stable provided that $\alpha<1$, that is, provided that firms use at least some labour. I assume that this condition holds throughout (this is the so-called no-black hole condition).

To sum-up we can write:

Lemma 2. The symmetric equilibrium is stable for all ϕ below ϕ^{break}.

This result is familiar to readers of Krugman (1991) and the following literature (see especially Puga 1999 who was the first to derive an analytical solution to the break point). It can be shown that $\phi^{sust}<\phi^{break}$ (for details see Robert-Nicoud 2005) thus there is a range of parameters for which both the symmetric and core-periphery long run equilibria are stable.

The symmetric equilibrium is unstable for lower values of ϕ if agglomeration forces are stronger. Ceteris paribus, agglomeration forces are increasing in α and β. α captures the strength of vertical linkages among firms (when α is large, firms buy a lot of each others' output as intermediate inputs). To study the effect of an increase of the elasticity of substitution on the break point, it is important to recall that β is equal to 1-1/σ and that ϕ is defined as τ at the power (1-σ). Thus, let $\tau^{break}\equiv(\phi^{break})^{1-\sigma}$. Standard algebra reveals that $\partial\tau^{break}/\partial\sigma<0$, that is, when goods become less close substitutes, agglomeration is the unique stable equilibrium over a larger range of trade costs. The intuition for this result is the following: when σ decreases, firms enjoy a higher mark-up which reinforces the strength of pecuniary externalities or, in other words, of backward and forward linkages.

Other equilibria and global stability

Thus far we have only shown the existence of long run equilibria for which $n\in\{0,½,1\}$ exist and the conditions under which they are stable. Now the following natural question may arise. Do there exist other equilibria, and what are their dynamic properties if they do?

Simulations undertaken by several researchers and a formal method derived in Robert-Nicoud (2005) provide the following answer to this question. Generically, there are five equilibria: the dispersed equilibrium (n=½), the concentrated equilibria (n=0,1), and two asymmetric, interior equilibria n' and n'' with n'+'n''=1. Moreover, whenever they exist, the latter equilibria are unstable in the sense that $dq_1/dn>0$ and $dq_2/dn<0$ at n=n',n''.

Regarding global stability, Baldwin (2002) has shown that the informal methods developed in Krugman (1991) and Puga (1999), designed to assess the local stability of the model, give the same answer as formal methods, that is, the break and sustain points describe the global stability of the model, too.

Comparison with the break and sustain points of the CPVL model

The reader familiar with the NEG literature already knows that break and sustain points in Krugman's (1991) Core-Periphery (CP) model and Fujita et al.'s (1999, p 245) Core-Periphery-Vertical-Linkage (CPVL) model are isomorphic, as are the corresponding point of the FCVL model of this section.[16] In particular, the break and sustain points of the CPVL model solve:

(15)
$$\phi_{CPVL}^{break}=\frac{(1-\alpha)(\beta-\alpha)}{(1+\alpha)(\beta+\alpha)},\quad 2(\phi_{CPVL}^{sust})^{1-\frac{\alpha}{\sigma-1}}-[1+\alpha](\phi_{CPVL}^{sust})^{2}-[1-\alpha]=0$$

The model requires the so-called 'no black hole conditions' $\beta>\alpha$ to hold. Without these, the break points would be negative, implying that the symmetric equilibrium is never stable. The similarity between the CPVL and the FCVL models is striking as both the functional forms for the cost functions and the mechanism driving agglomeration are different. In particular, agglomeration stems from labour mobility between the manufacturing and the background sectors within each region in the CPVL model. By contrast, international capital mobility is the driving mechanism in the FCVL model.

4. Equity and efficiency in the FCVL model

This section studies the normative properties of the FCVL model. Since this model produces two radically different kind of equilibria, the goal here is to establish a welfare comparison between the two market outcomes.[17] From an equity-perspective one may ask: Who are the gainers and the losers from agglomeration? Which factor owner benefit and which suffer? Which regions are advantaged and which are disadvantaged? From an efficiency-perspective one may wonder: Can the gainers compensate the losers? Does the free working of market forces deliver too much or too little agglomeration? This section follows the methodology developed by Charlot et al. (2006) and adopted by Ottaviano and Robert-Nicoud (2006). Thus, I will also point to the similarities and differences between the models these papers study (the Krugman 1991 model and a cousin of the Krugman-Venables 1995 model, respectively) and the current one.

Pareto welfare analysis

The first thing to note is that nominal rewards are invariant in the FCVL model (thus the conflict between factors owners is unaffected by the equilibrium spatial configuration). Thus we write:

> Lemma 3. In all cases, capital owners and workers of region 1 (region 2) alike are best off when all firms are clustered in region 1 (region 2).

This implies that all the welfare effects occur via the price index. Specifically, welfare in region j, as a function of n and ϕ, reads $V_j(n,\phi)= [\Delta(n,\phi)]_j^{\mu}$. As is clear from (4), Δ_1 is maximised when n=1.

However, in the current setting it is not necessarily the case that increasing the share of industry n necessarily hurts residents of region 2. This is because and increase in n lowers the production costs of firms located in region 1, a reduction that is fully passed onto consumers. Therefore, from region 2's residents point of view, two competing effects arise when n increases. On the one hand, they have to import more varieties, that is, they have to pay transportation costs on a wider range of varieties. This clearly hurts them. On the other hand, the producer price of these varieties is lower and hence the consumer price net of transportation costs. This is good news to them. The net effect is thus a-priori ambiguous, as we shall see. This has an important consequence: the market might select an outcome that is dominated by another possible equilibrium.

Go back to Lemma 3; this lemma says that the preferred outcome of north's residents is n=1 but is silent on which outcome is their second best: are those people better off if they are in the periphery (n=0) or if firms are evenly spread between the two regions (n=½)? As it turns out, this is not a trivial question.

Take the case α=0 as a benchmark, that is, there are no forward or backward linkages – and hence no agglomeration economies. In such a case (4) reveals that Δ_1 is strictly increasing in n. This implies:

> Lemma 4. When the magnitude of the vertical linkages is small ($\alpha \approx 0$), then residents in 1 rank the possible equilibria as follows: there are best off under the core-periphery pattern n=1; their second best is the dispersed equilibrium n=½; they are least well off under the core-periphery pattern n=0.

Now turn to the case α>0. To a larger α corresponds greater agglomeration economies. Agglomeration economies ensure that producer prices are lower in the core-periphery outcome than in the dispersed outcome. Low transportation costs ensure that consumer prices are closer to producer prices. Thus, consumer prices are in the periphery can be lower under agglomeration than under dispersion if transportation costs are low enough and if vertical linkages are substantial.

To see this formally, assume that if firms are fully agglomerated in region 1. Then manipulating (4) reveals that in the dispersed outcome n=½ we have:

$$\textbf{(16)} \qquad \Delta_1 = \Delta_2 = \left(\frac{1+\phi}{2}\right)^{1/(1-\alpha)} < 1$$

By contrast Δ_1=1 and Δ_2=ϕ if firms are clustered in the north. Therefore residents in 1 always prefer the core-periphery pattern and residents in 2 agree with this ranking if, and only if,

$$\textbf{(17)} \qquad P(\phi) \equiv V_2(\tfrac{1}{2},\phi) - V_2(1,\phi) = \phi^{\mu} - (\frac{1+\phi}{2})^{\mu/(1\text{-}\alpha)}$$

is positive (P stands for 'Pareto'). This expression is negative for low values of ϕ and nil if ϕ=1 (in the latter case location is irrelevant). However, standard algebra reveals that this expression is increasing at ϕ=0 and everywhere concave. Moreover, it is increasing in ϕ at the limit ϕ=1, and thus everywhere on [0,1], if and only if α<½. In this case, (17) is negative for all admissible value of ϕ. If α>½ then the expression in (17) is decreasing in ϕ at the limit ϕ=1 and thus (17) is positive for any ϕ in (ϕ_P,1), where ϕ_P is the unique real root of this polynomial in (0,1). Hence, this analysis has shown:

> **Proposition** 5. If α>½ then there exists a ϕ_P in (0,1) such that residents in the periphery are better off under the core-periphery outcome than under the dispersed outcome n=½ if, and only if, transportation costs are low enough ($\phi>\phi_P$). Agglomeration is said to be 'efficient' in this case. If α≤½ then residents in the periphery are worse off than under n=½ for all ϕ. Since residents in the core are always better-off under

agglomeration than under dispersion in this model, dispersion is not Pareto-dominated by agglomeration only if $\phi<\phi_P$.

Moreover, it is easy to see that people in the periphery are more likely to benefit from agglomeration, the larger the agglomeration economies. Indeed, at the limit α=1, $\Delta_2 \rightarrow \Delta_1=1$, so consumer prices in the periphery are the same as in the core, for all any value of ϕ. More formally, $P(\phi)>0$ if and only if $2\phi^{1-\alpha}-(1+\phi)>0$ and

(18) $$\frac{\partial}{\partial\alpha}\left(2\phi^{1-\alpha}-(1+\phi)\right)=-2\phi^{1-\alpha}\ln(\phi)>0$$

which implies that ϕ_P is decreasing in α. In other words,

Corollary 6. The range of ϕ over which everybody benefits from the clustering of industry in either region vis-à-vis the dispersed equilibrium is increasing in the magnitude of agglomeration economies.

This too is rather intuitive.

We can finally address the question of whether the market provides too much or too little agglomeration from the periphery's resident's point of view. To answer this question we rank ϕ_P, ϕ^{break}, and ϕ^{sust}. We already know that the sustain point comes before the break point, viz. $\phi^{sust}<\phi^{break}$, which implies that for all $\phi\in(\phi^{sust},\phi^{break})$ both the core-periphery and the dispersed outcomes are stable long run equilibria. Plug (14) into (17) to get:

(19) $$\text{sign}\{P(\phi^{break})\}=\text{sign}\{2(\phi^{break})^{1-\alpha}-[1+(\phi^{break})]\}$$

The sign of this expression is ambiguous and depends upon the magnitude of α and σ. As is to be expected, the this expression is most likely to be positive (and hence agglomeration Pareto dominated dispersion) at the break point when the magnitude of the vertical linkages α is large. Also, numerical comparisons show that the sign of ϕ_P-ϕ^{break} is ambiguous but suggest that $\phi^{sust}<\phi_P$ holds for all parameter values. The latter inequality means that dispersion is not Pareto dominated by agglomeration at the sustain point. Together, these facts imply first, $\phi^{sust}<\phi^{break}$, $\phi^{sust}<\phi_P$. As a consequence, no less than five cases can occur (See appendix). Therefore, we can write:

> Lemma 7. If $\phi<\phi^{sust}$ then the market delivers dispersion (n=½) and this outcome is Pareto efficient. If $\phi>\max\{\phi^{break},\phi_P\}$ then the market delivers agglomeration and this outcome is Pareto efficient. If $\phi^{break}<\phi_P$, then the market delivers too much agglomeration if $\phi\in(\phi^{break},\phi_P)$. In the remaining instances in which both agglomeration and dispersion may arise at the decentralized equilibrium, the actual outcome might or might not Pareto dominate the other one, and which does dominate the other depends on the parameters values of the model.

The really interesting question is how likely are those cases for different parameter configurations. I do this at the end of the next section, when I summarise all positive and normative analysis in Table 1 and Figure 1.

5. Potential Pareto improvements

When assessing the global welfare properties of the new economic geography model, Baldwin et al. (2003) use the utilitarian welfare function and, therefore, make interpersonal comparisons.[18] This is notoriously problematic, especially when different people face different price indices as in the present model (Wildasin, 1986).[19] For this reason, here I use compensation criteria based on the prevailing equilibrium prices and wages because they do not suffer from this caveat. Specifically, let us ask under which conditions the winners from agglomeration could compensate the losers (in the form of a transfer) from it and still be better off; answering this question involves making use of the Kaldor's (1939) compensation criterion. Alternatively, we could follow Hicks (1940) and evaluate the maximum transfer from periphery to core that under dispersion would let the core reach the same welfare level as under agglomeration. In both cases, we check the feasibility of transfers at the corresponding equilibrium prices, that is, we ask whether the location pattern with transfers is still part of an equilibrium, as this is not necessarily granted (Little 1949).

Kaldor's approach

Let us start with assessing the conditions under which agglomeration corresponds to a potential Pareto improvement with respect to dispersion. Let T_K be the per-capita transfer from region 1 residents to region 2's. People in the periphery are exactly compensated if they get additional income T_K such that their real income in (1) is the same in both instances:

(20) $$\left(1+T_K\right)V_2\,(1,\phi)=V_2\,(\tfrac{1}{2},\phi) \;\Leftrightarrow\; \left(1+T_K\right)\phi^{\mu}=\left(\frac{1+\phi}{2}\right)^{\frac{\mu}{1-\alpha}}$$

Since core and periphery host the same number of people, each resident in the core pays T_K. Clearly, T_K is a positive number unless agglomeration Pareto dominates dispersion (in which case no compensation is required).

It is straightforward to check that Kaldor's compensation scheme is feasible in the sense that the material balance conditions still hold at the consumption pattern corresponding to the compensated incomes, that is, that market clearing conditions still hold after T_K has been transferred.

The last step requires us to determine under which conditions region 1 residents prefer agglomeration with compensation to dispersion. Given preferences in (1), that is the case if and only if:

(21) $$\left(1-T_K\right)V_1(1,\phi) > V_1(\tfrac{1}{2},\phi) \;\Leftrightarrow\; \left(1-T_K\right) > \left(\frac{1+\phi}{2}\right)^{\frac{\mu}{1-\alpha}}$$

Using the value of T_K from (20), the condition above can be rewritten as:

(22) $$K(\phi) \equiv 2-(1+\phi^{-\mu})\left(\frac{1+\phi}{2}\right)^{\frac{\mu}{1-\alpha}} > 0$$

Using a similar reasoning as for P(ϕ), one can show that there exists a threshold value ϕ^{Kaldor} such that (22) holds for $\phi>\phi^{Kaldor}$ and is violated otherwise. Clearly, if agglomeration Pareto-dominates dispersion, then Kaldor's criterion favours agglomeration, too. Mathematically it is easily verified that $K(\phi)>P(\phi)$ if, and only if, $1-\phi^{\mu}>0$, which is true for all $\phi<1$ since $\mu<1$. This result is quite tautological: if people in the periphery are better-off under agglomeration than under dispersion, then of course the 'winners' in the core are able to compensate the 'losers' in the periphery—because even the losers gain and hence do not require being compensated. The converse, of course, is not true: there is an intermediate range of trade costs such that Kaldors' criterion selects agglomeration even though this outcome does not Pareto-dominate dispersion.[20]

Four further points deserve being mentioned here. First, ϕ^{Kaldor} is always positive and less than unity, even if $\alpha<1/2$. Second, when vertical linkages are stronger (α increases), agglomeration is more likely to dominate dispersion (ϕ^{Kaldor} decreases) in the sense of Kaldor. Third, since P(ϕ) is larger than K(ϕ), it must be that $\phi^{Kaldor}<\phi^{Pareto}$ holds: when a compensation scheme is available agglomeration is sometimes preferable to dispersion when the former does not dominate the latter. Finally, when μ increases, it has an ambiguous effect on ϕ^{Kaldor}. On the one hand, at given ϕ and prices, residents of the core benefit more from lower consumer prices the higher is μ because manufactured goods represent a more item in their expenses, thus they are willing to forego a larger transfer T_K to compensate people in the periphery; this tends to make $\partial\phi^{Kaldor}/\partial\mu$ negative. On the other hand, the effect of a higher μ on the welfare of people in the periphery depends on whether they gain from agglomeration or not. In the former case, the Kaldor criterion is even less binding (as explained in the previous paragraph) and thus $\partial\phi^{Kaldor}/\partial\mu$ tends to be negative because of this effect; since $\phi^{Kaldor}<\phi^{Pareto}$ always holds, however, this case is immaterial. In the latter case, when people in the periphery are worse off under agglomeration than under dispersion, when μ increases they require a larger nominal compensation, given higher c.i.f. manufacturing prices; if that was the only effect, $\partial\phi^{Kaldor}/\partial\mu$ would be positive. The net effect is thus ambiguous (a higher μ requires larger compensations and, simultaneously, enables winners to afford higher giving away a larger T_K). Unreported numerical simulations suggest that ϕ^{Kaldor} is increasing in μ and that ϕ^{Pareto} is surprisingly insensitive to the value of μ.[21] Together, these two facts are consistent with the fact, outlined in the previous paragraph, that the gap between ϕ^{Pareto} and ϕ^{Kaldor} is increasing in μ.We have thus established:

> **Proposition** 8 (Kaldor improvements). Kaldor's criterion prefers agglomeration to dispersion when the former is Pareto-dominant ($\phi^{Kaldor}<\phi^{Pareto}$). When the two configuration cannot be Pareto-ranked ($\phi<\phi^{Pareto}$), it still favours agglomeration provided vertical linkages are strong and trade costs are low enough ($\phi>\phi^{Kaldor}$, $\phi^{Kaldor}<\phi^{Pareto}$.

In other words, agglomeration with compensation from core to periphery can make both regions better off when trade costs are low enough. The intuition behind this result is straightforward. Agglomeration enhances product variety. This has a positive impact on consumer surplus both in the core and the periphery. If linkages are strong, the impact is large, which makes it easier to compensate the periphery. The more so, the lower the transport costs.

It is interesting to contrast this result with its counterpart derived by Charlot et al. (2006) and Ottaviano and Robert-Nicoud (2006) for CP models and another class of VL models, respectively. Both papers show that compensation from core to periphery is possible provided that transport costs are low enough and that vertical linkages are strong enough, as here. The underlying reason is, nonetheless, quite different. In Ottaviano and Robert-Nicoud (2005), like here, people do not cross borders and firms buy each other's output as intermediates. But there winners can sometimes compensate the losers and still be better off under agglomeration because product variety is larger under agglomeration than under dispersion (the large this difference, the easier it is to compensate the losers from agglomeration). The price of each variety is constant in their model. In stark contrast, in the current model the total mass of variety is constant (N=1) and the effect works by making each variety cheaper under agglomeration than under dispersion. Interestingly, in either case, the effect of enhanced product variety or of lower product price results in a lower price index in agglomeration.

In Charlot et al. (2006) compensation from core to periphery is possible provided that transport costs are low enough for reasons that are deeply different than in the both aforementioned frameworks. First, in CP models product variety is independent from the spatial distribution of economic activities (N=1). Hence, agglomeration cannot be a Pareto improvement with respect to dispersion. Second, migration-driven agglomeration implies that the number of residents in the core is larger than the number of residents in the periphery. Hence, compensation is possible for given product variety.

This discussion is summarized in Table 1. Three models as well as the papers that discuss their properties are listed in the first column. In some of these models, agglomeration might dominate dispersion in the sense of Pareo: the second column informs about why each model has this property or not. Finally, the third column provides the reason that makes agglomeration dominate dispersion in the sense of Kaldor for some (but not all) parameter values.

Hicks's approach

Following Hicks, under dispersion we have to check for the existence of an appropriate redistribution of income from periphery to core that would let the latter region reach the same welfare as under agglomeration. Clearly, for the problem to be nontrivial, $P(\phi)<0$.

It is easy to see that a compensation scheme à la Hicks is not compatible with the location equilibria on which it is built: the payment of any compensation makes the spatial distribution of expenditures uneven ($e_j \neq ½$). Under agglomeration, sales and operating profits do not depend on e_j, ensuring in particular that the material balances hold for any transfer. Under dispersion, by contrast, q_j does depend on e_j and hence, ultimately, on n. This implies that with any transfer supply no longer matches demand at the dispersed market prices, hence dispersion is no longer an equilibrium once transfers are realised. Accordingly, the would-be periphery is unable to compensate the would-be core at the prevailing market prices without destroying the dispersed equilibrium.

Therefore we have:

Proposition 9 (Hicks improvements). Hicks's criterion always prefers agglomeration to dispersion.

This is reminiscent of Charlot et al. (2006) and Ottaviano and Robert-Nicoud (2006), who obtain the same result for CP models and in another class of VL models, respectively.

Insert Table 1 near here

Scitovsky's indetermination

The foregoing analysis has shown that, if trade costs are small and/or linkages are strong, then agglomeration is preferred under both Kaldor's and Hicks's criteria. In this sense, agglomeration is the 'desirable' outcome when $K(\phi)>0$. Otherwise, we are in a typical case of indetermination in the sense of Scitovsky (1941): neither outcome is preferred to the other with respect to both criteria simultaneously. If $\phi<\phi^{Kaldor}$, Kaldor's criterion favours dispersion while Kicks' criterion supports agglomeration.

To summarize, we write:

Proposition 8 (Potential Pareto improvements). Agglomeration is always socially desirable according to Hicks. It is socially desirable also according to Kaldor only when trade costs are low and forward and backward linkages are strong. In this case, the market might deliver inefficient dispersion. Otherwise, the market outcome cannot be improved upon.

This result is reminiscent of Charlot et al. (2006), who find the same sort of indeterminacy in CP models and consider it "as the synthesis of the very contrasted views that prevail in a domain in which the two tenets [those who think agglomeration is efficient against those] have many good reasons to be right (p. 4)."

Insert Figure 1 near here

Turn to, which plots the free-ness of trade ϕ against the strength of vertical linkages α. KK and PP describe normative features of the model; consider them first. The PP-curve maps ϕ^{Pareto} as a function of α. The condition in (17) is violated for all ϕ if $\alpha<½$; $\phi^{Pareto}=1$ if $\alpha=½$ and $\phi^{Pareto}=0$ if $\alpha=1$. More generally, PP is decreasing in α (given ϕ, consumer prices are lower if α is high) and agglomeration Pareto-dominates dispersion for any combination of α and ϕ at the north-east of PP—the doted area labelled 'A'. Turn next to the KK-curve, which plots ϕ^{Kaldor} as a decreasing function of α (as vertical linkages are tighter, consumer prices are relatively lower in the agglomerated outcome and it becomes easier to compensate people in the periphery): even when α is very low, it is feasible to engage in win-win compensation from the core to the periphery provided that ϕ is large enough; to the north-east of KK, all combinations of α and ϕ make agglomeration desirable (I say that agglomeration is "desirable" when it is favoured by both the Kaldor and the Hicks criteria). Between KK and PP, agglomeration is desirable even if it does not Pareto-dominate dispersion. To the south-west of KK, agglomeration is desirable in the sense of Hicks only; it is not so in the sense of Kaldor.

Turn now BB and SS, which summarize the positive properties of the model. Consider first the doted curve denoted BB, which plots the break point in (14) as a decreasing function of α; to the north-east of BB parameters of the model are such that agglomeration is the unique equilibrium. In the configuration shown in the diagram, which holds for low values of σ, the BB-curve starts below the KK-curve for low values of α and, as α increases, crosses first the KK-curve and then the PP-curve (not obvious in the diagram) from below. Before addressing the generality of this configuration, let me first observes that this divides the parameter space in two cut-off values for α, denoted by α_{BK} and α_{BP}, respectively; clearly, $\alpha_{BK}<½<\alpha_{BP}$ is always true. When vertical linkages are weak ($\alpha<\alpha_{BK}$) then agglomeration does not dominate dispersion in the sense of Pareto and is not desirable in the sense of Kaldor (though it is so in the sense of Hicks). As it turns out, dispersion is also a market outcome in that case. For intermediate values of ϕ, the market is biased towards agglomeration: agglomeration is the decentralized outcome, even though it is not desirable in the sense of Kaldor and does not dominate dispersion. For yet larger values of trade freeness ϕ, the market delivers the desirable outcome (in the sense of both Hicks and Kaldor). When firms are strongly bound with each others ($\alpha>\alpha_{BP}$) then agglomeration is simultaneously the market outcome, efficient and desirable if ϕ is sufficiently large.

In all other cases, it is useful to bring in the sustain point in order to avoid a profusion of cases and topology. The SS-curve, not shown on the graph for the sake of readability, would plot ϕ^S from as a decreasing function of α; for parameter combinations to the south-east of SS, dispersion is the unique market outcome by (11). SS has the same shape as BB (except that it crosses PP in no circumstance), it is everywhere below it (and to the left of it), except at the endpoints (where they are confounded). Define α_{SK} as the value of α such that $\phi^S=\phi^K$ (graphically, this point is to the left of α_{BK} but looks similar to it if the doted line described SS rather than BB). When $\alpha>\alpha_{SK}$, then for high transportation costs (low ϕ), the market outcome—dispersion—agrees with the Pareto criterion and is desirable, too. When ϕ takes intermediate values, then the market is biased towards dispersion: the equilibrium is Pareto efficient, however, agglomeration is desirable.

A third general case arises when SS and KK never intercept: for parameter combinations to the south-west of SS (itself at the south-west of KK), dispersion is the market outcome; also, it is dominated in neither the sense of Pareto nor of Kaldor (but it is in the sense of Hicks). In this restricted sense, the market delivers a socially optimal outcome.

To sum up: the market delivers the desirable outcome when trade is quite free (graphically: this is the case whenever (α,ϕ) is in the north-west of KK): this desirable outcome is agglomeration; in addition, provided that α is large enough, it Pareto-dominates dispersion. The market also delivers a socially optimal outcome in the opposite case (when vertical linkages are tiny and when trade is near prohibitive).

Briefly, let me turn to the generality of the situation described here. Turn first to the role of β (which is a function of the elasticity of substitution only, viz. $\beta \equiv 1\text{-}1/\sigma$). Conveniently, in the (α,ϕ)-space β affects curves describing the positive features of the model only, that is, BB and SS. Specifically, ϕ^S and ϕ^B are both decreasing in β (and thus in σ, the elasticity of substitution); σ has no impact on KK or PP. When σ increases towards infinity, $\alpha_{SK} \rightarrow \alpha_{BK} \rightarrow \alpha_{BP} \rightarrow 1$, that is, SS and BB are both positioned to the south-west of KK for all α in (0,1). In English, as goods become closer substitutes and thus when markups shrink, vertical linkages are weakened; this makes it less likely that the market delivers agglomeration when this outcome is neither desirable nor dominates dispersion. Second, and lastly, consider the role of μ. Conveniently, μ has no influence on the positive threshold values of ϕ, that is, BB and SS are unaffected by a change in the value of this parameter. By contrast, I had stressed that and increase in consumers' expenditures on manufactured goods, μ, expands the set of (α,ϕ) for which agglomeration is desirable; more precisely, KK shifts outwards (but at the endpoints). A change in μ has no meaningful impact on PP, as we have seen earlier. Thus, an increase in μ thus makes it less likely that the market delivers agglomeration when this outcome is neither desirable nor dominates dispersion, that is, it makes it more likely that KK crosses SS or BB or both.

6. Asymmetric regions in the absence of linkages: The positive and normative properties of the FHMR model

For lack of space, I do not provide a comprehensive description of the FHMR model here. Fortunately, the properties of this model—both positive and normative—can be found in several sources.[22] This model is a special case of the model developed in sections 2 and 3: it holds for $\alpha=0$. The advantage of this model is that it is fully tractable, even if $s\neq½$; thus, we can relax the assumption $s=½$ in this section. Its main positive features are the following. First, the spatial equilibrium is unique because market sizes are exogenous: from (5) it is easy to see that $e_1=s$ and $e_2=1-s$, irrespective of n. This property allows us to introduce all sorts of exogenous asymmetries, like different endowment sizes ($s\neq½$), asymmetric trade costs ($\phi_2\neq\phi$) and even asymmetric relative endowments (whereby $s_L\neq s_K$). Then, solving the model for the spatial equilibrium n, we find:

(23) $$n = s + (s-½)\frac{2\phi}{1-\phi_2} + \frac{\phi_2-\phi}{(1-\phi)(1-\phi_2)} - (1-2\phi)\frac{(s-½)(\phi_2-\phi)}{(1-\phi)(1-\phi_2)}$$

or n=0 or n=1 in an obvious manner if the solution to the expression above does not belong to [0,1]. This expression shows that region 1's manufacturing share is more likely to be larger than its income share s if (a) its GNP is larger than region 2's (this is a manifestation of Krugman's (1980) celebrated Home market effect), and (b) the more open is its foreign partner: in DSK monopolistic competition, unilateral protection always lowers the price index because the elasticity of delocation is high (see Baldwin et al. 2003). Multilateral trade liberalisation (simultaneous reduction of $\phi_2=\phi$) increases the gap between n and s, and hence n=1 for all ϕ larger than ϕ_1^{sust}=(1-s)/s<1; see (9). In words, the larger country attracts the whole of the manufacturing sector when trade barriers are low enough. This benefits the large country's residents more than the small country's. In this model, the nominal rewards are invariant in the spatial equilibrium of the model, hence all welfare effects operate through the price index, or Δ. As it turns out, the reduced-form of Δ shows that both countries benefit from a reciprocal increases in ϕ, even though 1-n decreases.

Baldwin et al. also study the normative properties of the model using the Pareto and utilitarian criteria. In particular, they show (this lemma is related to Lemma 4):

> Lemma 10. In the FC model conflicts of interests arise on the spatial dimension only. Indeed, any spatial reallocation of capital (industry) benefits one region at the expense the other. Moreover, there is no conflict between capital owners and workers who live in the same region.

This implies that no Pareto improvement is feasible but it is legitimate to ask whether a planner with a utilitarian social welfare function can improve on the decentralized equilibrium. Baldwin et al. show that there is a 'social home market effect' whereby the planner would provide the larger region with a more than proportional share of manufactures; however, the planner would chose a more even spatial equilibrium than the market. In this sense:

> Lemma 11 (Utilitarian SWF). When regions are asymmetric, the market outcome has too many firms in the region that has the highest per capita income.

In other words, the market is biased in favour of agglomeration; the utilitarian planner implements the market outcome only when per capita incomes are equal. Note that this result depends crucially on the absence of vertical linkages. More generally, using a CES-type welfare function that encompasses the utilitarian criterion as a special case, we find that as the degree of aversion towards inequality increases (which is equal to one over the elasticity of substitution between individuals' welfare; the case of the utilitarian criterion corresponds to a unit elasticity), the gap between the market outcome and the social optimum widens. More generally :[23]

> **Proposition** 12 (Social welfare function). Assume that all agents have the same income per capita. Then the market is biased in favour of agglomeration if, and only if, the SWF planner is more averse to inequalities than the utilitarian planner. Conversely, the market is biased against agglomeration if, and only if, the SWF planner is less averse to inequalities than the utilitarian planner.

7. Summary and concluding remarks

This paper put forth a 'New Trade, New Economi Geography' model in which agglomeration is driven by the interaction of increasing returns at the firm level, trade/transportation costs, free capital mobility and vertical linkages among firms –that is, firms use each other's output as an input. I have sketched the ways the present model behaves in a very similar fashion to already well-established economic geography models. In particular, it shares the features of the original model developed by Venables (1996). However, it is more tractable and hence allows for easier extensions and less reliance on simulations. I have also shown that this model nests the 'footloose capital' trade model of Flam and Helpman (1987) and Martin and Rogers (1995).

The main contribution of the paper is to study the normative implications of the model. First, **Proposition** 5 established that agglomeration might benefit everybody, including residents of the periphery, provided that vertical linkages are strong enough (so that producer prices are low) and trade barriers are low enough (so that consumer prices are close to producer prices). Together, **Proposition** 8 and **Proposition** 9 convey a related message: they shows that, under qualitatively similar but less stringent conditions, agglomeration is the most desirable market outcome when compensations are allowed for. Second, when vertical linkages are absent and when international per capita incomes are equalised, **Proposition** 12 claims that the market delivers the social optimum only if the planner uses the utilitarian criterion. The market is biased in favour of (resp. against) agglomeration if the planner has an aversion to inequalities that is larger than unity, which corresponds to the utilitarian case.

The paper also provide a thorough description of the effects of the respective normative and positive role of each parameter of the model. The distinction between the share of income that consumers spend on manufactured and the share of variable costs that firms spend on other firms' output is usually blurred in the literature because people make the working assumption that they are the same. I show here that the former has no positive implication on the model but has a crucial impact on its normative ones, whereas the value of the latter is relatively more important for the positive implications of the model. Likewise, I show that the main role of the elasticity of substitution is to change the range of parameter values for which agglomeration is desirable but not a market equilibrium (and the other way round).

A final remark is in order here. One central prediction of the model is that agglomeration is more likely when trade costs fall. In this model, agglomeration also means that one country specialises in manufacturing, thereby eliminating all intra-industry and firm-to-firm international trade, which is clearly counterfactual. However, it is well known that when NEG models are enriched to allow for different intensities in immobile primary factor across sectors (Epiphani 2005), decreasing returns to labour in the numéraire sector, or when the no-specialisation condition is violated, all predict that the industrial structure of the two regions or countries converge when trade is free enough (chapter 14 in Fujita et al. 1999). This model is no exception: an increase in trade freeness, for large values of ϕ to start with, increase the volume of intra-industry and firm to firm trade. The welfare properties of the model in this case are outside the scope of the present paper (see Gaigné 2006 for a contribution along this line).

8. Acknowledgements

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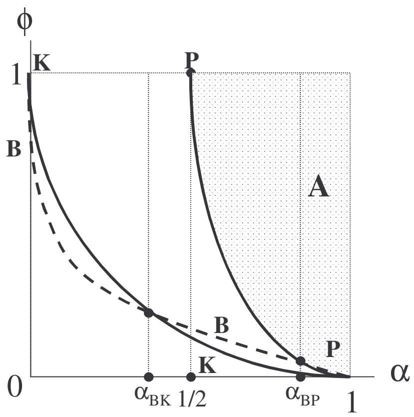
TABLES AND FIGURES

Table 1 : Pareto criterion and Kaldor compensation in various models

Model	Agglomeration might Pareto-dominate dispersion	The mechanics of Kaldor compensation
CP model: Migration-driven (Charlot et al. 2006)	**No**: N is fixed, price of each variety is fixed	Winners are more numerous than losers
VL models (Ottaviano Robert-Nicoud 2006)	**(Yes:)** N is larger under agglomeration	
FCVL model (Current paper)	**(Yes:)** price of each variety is lower under agglomeration	

Figure 1 : The positive and normative properties of the model



Appendix & Guide to calculations

Lemma 7

As mentioned in the text, 5 cases can occur. From the point of view of region 2's residents:

1. If $\phi<\phi^{sust}$ the market outcome (n=½) is the 'second best';
2. If $\phi>\phi_P,\phi^{break}$ the market outcome (n=1) is the 'second best';
3. If $\phi_P>\phi>\phi^{break}$ the market outcome (n=1) provides <u>too much</u> agglomeration;
4. If $\phi_P<\phi<\phi^{break}$ the market outcome is the second best if n=1but provides <u>too little</u> agglomeration if n=½;
5. If $\phi^{sust}<\phi<\phi_P$ the market outcome is the second best if n=1but provides <u>too little</u> agglomeration if n=½.

By contrast, anyone's welfare is maximised when firms cluster in one's region. Hence, from the point of view of region 1's residents, three cases can occur:

1. If $\phi>\phi^{break}$ then the decentralised outcome (n=1) delivers the first best;
2. If $\phi<\phi^{sust}$ then the decentralised outcome (n=½) delivers <u>too little</u> agglomeration;

If $\phi^{sust}<\phi<\phi^{break}$ then the decentralised outcome delivers the first best if n=1but provides <u>too little</u> agglomeration if n=½.

10. Endnotes

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[1] This NEG model is being sketched in Baldwin et al. (2003), who attributed it the acronym ‘FCVL’ model.

[2] As it turns out, one possible equilibrium of the FCVL model, agglomeration, is counterfactual in this respect. I will mention in the concluding remarks how to enrich the model to reconcile it with these stylised facts.

[3] See also Englmann and Walz (1995) (on geography and growth), Faini (1984) (on geography and vertical linkages), and Ottaviano et al. (2002) and Pflüger (2004) (on geography and embodied factor mobility).

[4] In Ethier (1982), the upstream sector is monopolistically competitive à-la Dixit and Stiglitz (1977) and the downstream sector is perfectly competitive and operate under constant returns to scale. In Venables (1996), both sectors are monopolistically competitive and operate under increasing returns to scale. In Krugman and Venables (1995) and Ottaviano and Robert-Nicoud (2005), as well as in the current paper, the downstream and upstream sectors are merged into a single one.

[5] We had to wait 15 zears after Krugman’s (1991) seminal contribution to have an exhaustive charaterisation of its properties. Mossay (2006) formally shows that the short-run (or instantaneous) equilibrium exists in the Krugman model; in Robert-Nicoud (2005), I provide an analytical characterisation of the long-run properties of the model.

[6] See Charlot et al. (2006) for a detailed discussion of this issue in CP models.

[7] Baldwin and Robert-Nicoud (2000) relax the assumption of identical relative endowments.

[8] What 'long run' means in the context of this model will be made clear in Section 4.

[9] More on this in footnote 10 below.

[10] Economic consistency imposes $\pi>0$, which requires $(1-\alpha)\sigma+\alpha-\mu>0$. This always holds because $\sigma>1$ and $1>\mu$. If K had not been normalised to unity, the expression for π in the text should be divided by K.

[11] Total operating profits are equal to total expenditure over σ. Since there is a unit mass of firms, total operating profits are equal to π, hence the expression in the text. Also, the use of the 'q' notation in this static model is deliberate: in a straightforward dynamic extension of the model, π would play the role of the replacement cost of capital.

[12] This reduced form can be obtained as the result of a well-specified dynamic programm, as I did in my thesis (calculations available upon request). The original source is Baldwin (2001), in the context of the Krugman (1991) model.

[13] Baldwin (2001) formally assesses the validity of these methods.

[14] See Baldwin et al. (2003) for the analysis of a NEG model with $s\neq½$.

[15] Or, which is equivalent by the symmetry of the model when $s=½$, to signing $d(q_1-q_2)/dn$ at $n=½$.

[16] These terminology and acronyms are due to Baldwin et al. (2003).

[17] That is, I follow Charlot et al. (2006) in that I compare market equilibria between them: $n\in\{0,½,1\}$. There are several justification for this, one of them being that in a decentralised economy, the planner cannot force the firms to locate in region 2, say, if it would be more profitable for them to locate in region 1. They write: “We believe that this approach, which does not involve any interpersonal comparison and rests on market prices and incomes determined in a general equilibrium context, is superior to many others (p. 4).”

[18] This is also the route I took in the thesis version of this paper. Details available from the author upon request.

[19] See Charlot et al. (2006) for a detailed discussion of this issue in CP models.

[20] Note that the gap between the two criteria is increasing in μ (and $P(\phi)=K(\phi)\to 0$ when $\mu\to 0$): when people in the periphery benefit from agglomeration because manufactured c.i.f. prices are low, they benefit more, the larger is μ (the share of income they spend on manufactured goods)—and the less Kaldor’s criterion is binding.

[21] For any given value of α in $\{.11,.51,.91\}$, varying μ from .11 to .91, changes ϕ^{Pareto} at the ninth decimal or beyond only.

[22] See e.g. Baldwin et al. (2003) chapters 3 and 11 for the positive and normative properties, respectively.

[23] I am not including this material to save on space. Details available upon request from the author.