# Resale and Collusion in a Dynamic Market for Semidurable Goods*

Pasquale Schiraldi (LSE) & Francesco Nava (LSE)

July 4, 2011

**Abstract**

The paper studies the incentives to form collusive agreements when goods can be traded in second-hand markets. It will be shown that such incentives crucially depend on the rate of depreciation of the durable good and on consumer heterogeneity. The main contribution of the paper shows that an active second-hand market may strengthen the incentives to collude, as do policies that affect the functioning of the second-hand market (e.g. leasing policy and buy-back). It will also, be argued that the oligopoly incentives to adopt strategies that strengthen collusion often differ from the monopoly incentives to increase profits.

**Keywords:** Bertrand, collusion, buy-back, leasing, semi-durability, secondary market.

**JEL classification:** D21, D43, L11, L13, L25.

## Contents



*Special thanks to Bart Lipman, Michael Manove for valuable advice and support. We are grateful to Emanuela Ciapanna, Marc Rysman, Bernard Caillaud and participants in several seminars for comments and suggestions. We also thank Kai-Uwe Kuhn and two anonymous referees for their suggestions.Any errors are our own. Email: p.schiraldi@lse.ac.uk, f.nava@lse.ac.uk. Address: Houghton Street, London, WC2A 2AE, UK.

# I Introduction

Durable goods represent a large fraction of the goods produced in industrialized economies. Trade in used markets is dominant feature for many of these goods. We study how second-hand markets affect the ability to sustain collusive agreements. The existence of a second-hand market is endogenously explained by introducing consumers' heterogeneity in the valuation for used durable goods. As units age and quality decreases, goods are traded from high-valuation to low-valuation consumers on a competitive secondary market, allowing owners to update to their preferred quality. The effect of a secondary market on the demand for new products can be decomposed into two components: a positive resale effect due to the option value of selling new units as they become old; and a negative substitution effect due to the imperfect substitutability of new and used units. The first effect introduces a dependence between the current demand for new units and the future price of used units. Producers account for such dependence when choosing the price at which to sell new units. Increasing the level of current production, lowers the price in the secondary market in the following period and erodes consumers' willingness to pay for a new unit by reducing its resale value. Since the value of a new unit depends on its expected price on tomorrow's secondary market, a firm's current profit also depends on its own future production. If firms maintain a collusive price agreement under the threat of a price war, consumers in the wake of a defection will be able to foresee future price wars and may respond by lowering their willingness to pay for new units in the current period. In fact, since a price war will reduce the price of new units after a defection, it will make used goods less desirable, thereby reducing their equilibrium price, due to the substitution effect. This in turn will affect consumers' willingness to pay for new goods due the resale effect, and the punishments will be effective even at the time of the defection. Since the resale effect will be shown to increase in the quality (durability) of a used unit and in the value for quality of a low type consumer, the ability to sustain collusion will be reinforced by quality, but weakened by heterogeneity in consumers' valuations. The collusive force will always prevail when consumers are sufficiently patient, as more patience will lead to higher expected resale values. This in turn will require a deviating firm to price more aggressively if it wants to conquer the entire market. Optimal punishments will be characterized for proposed model. Such punishments are harsher than trigger strategies when marginal costs are positive. Firms will threaten to price below marginal cost in the post-deviation period to drive the expected future price of used units to zero. Such threats will reduce the profit from sales of new units at the defection stage by eliminating the resale effect from the demand.

The second part of the analysis derives necessary and sufficient conditions for an active second-hand market to facilitate collusion. Such conditions will clarify when secondary markets could be used to strengthen the intertemporal linking of consumers decisions (thereby reducing the incentives to deviate). With a secondary market the gains from stealing at the defection stage will be muted by the drop in the resale component, whereas without a second-hand market any action at the defection stage will come from the choice of the target group of buyers. The main result of the section will show that secondary markets facilitate collusion when the quality of a used unit is either sufficiently high or sufficiently low. For intermediate quality values, an active secondary market will instead, restrict the ability to sustain collusive agreements. In fact, when quality is sufficiently high, an active secondary market always strengthens the intertemporal link between consumers' decisions, since the resale effect is significant. Similarly, when quality is sufficiently low without a secondary, market the intertemporal link is small or disappears (as firms sell to all consumers every period). But such a link will remain active and affect decisions whenever consumers are able to resell used units in a secondary market. The analysis will also show that the very same forces that facilitate collusion with open secondary markets, may occasionally reduce the monopoly profits. In fact, the incentives to close an active secondary market or to interfere with it will crucially depend on the market structure. Consequently, the oligopoly incentive to sustain collusion over time and the monopoly incentive to achieve a higher profit will not necessarily coincide.

The analysis will conclude by looking at alternative practices used to cap trade in second-hand markets: namely leasing and buy-back policies. We shall compare the attractiveness of these policies by looking at the monopoly profits and the ability to sustain collusion. A policy aimed at shutting down the secondary market, such as a leasing policy, will eliminate any dynamic consideration in consumers' decisions (since manufacturers only sell the flow of service derived from products) and may thereby decrease the incentives to collude. Specifically, it will be shown that leasing often reduces the range of discount factors which sustain collusion, but always increases monopoly profits. Thus, if multiple firms operate in the market, selling-only policies may be preferred to leasing policies to sustain cooperative outcomes. This result contrasts with most of the existing literature, which has focused only on the profitability of leasing policies, and provides a possible explanation as to why leasing practices are not so commonly used to sell durables. A buy-back policy instead, is aimed at bolstering the intertemporal link in consumers' decisions by increasing the resale price of used units. Such a link will allow manufacturers to punish defections more aggressively and will increase the range of the discount factors which sustain collusion. Buy-back policies however, will be costly to implement (unlike the leasing policy), and might therefore, reduce monopoly profits.

**Literature Review:** The competitive effect of secondary markets in durable goods industries is a long-standing question. However, most of this literature has focused on the effects of the second-hand markets on a monopolist's market power: Anderson & Ginsburgh [1994], Bulow [1982], Hendel & Lizzeri [1999a], Rust [1986], Swan [1980], Waldman [1996, 1997]. This paper differs from the existing literature since it analyzes how the incentives to collude are affected by secondary markets, and by other policies that may cap trade in such markets (leasing & buy-back). The paper is closely related to Ausubel & Deneckere [1987] and Gul [1987]. Such papers develop oligopoly models of durable goods pricing and show that the Coase conjecture fails whenever multiple firms operate in the market, since firms' ability to collude improves. We compare the ability to collude with and without secondary markets and show how a second-hand market can further expand the ability to collude. Moreover, we study the role of consumers' heterogeneity on collusive incentives. Dutta, Matros, & Weibull [2007] consider a related model in which oligopolists sell to overlapping generations of consumers who demand an infinitively durable good. They find that the lower the consumer turnover rate is, the harder it is to sustain prices above marginal cost. Our setup differs in several dimensions since we consider infinitively lived consumers with heterogeneous valuation and durable goods which depreciate over time and last for a finite number of periods. Both papers focus on optimal punishments and show that these may not be triggers. The analysis is also related to the literature on multi-market contracts. As in Bernheim & Whinston [1990], the presence of asymmetries between the primary and secondary markets facilitates collusion. In our analysis asymmetries will be determined by introducing vertically differentiated goods, heterogeneous consumers and different market powers in the primary and secondary markets. In contrast to Bernheim & Whinston, we shall assume that firms cannot directly operate on the secondary market, but are able to affect it either by changing the quantity of new units supplied or by adopting alternative selling practices, such as leasing and buy-back. Finally a recent and closely related work is Dana & Fong [2010] which shows how intertemporal bundling along with staggered long-term contracts may facilitate collusion. Intertemporal bundling can be interpreted as a means to make a product durable. When durable goods are traded, the service provided by a product in each period is exogenously intertemporally bundled. Trade in a secondary market will affect such intertemporal bundling, reinforce the dynamic linking in consumers' decisions and improve the ability to collude. In contrast to their model, consumer heterogeneity will be essential to explain the trade in secondary markets and consequently the ability to collude.

# II A Model with Secondary Markets

Consider an infinite-horizon discrete-time model with infinitely lived consumers and two goods: one durable and the other a numeraire good. The durable good fully depreciates after two periods. The durable good is *new* during the period in which it is produced, and *used* during the following period. Its quality $q \in \{\alpha, 1\}$ determines its value to consumers.

The quality of a new good is normalized to 1 and the quality of a used unit is $\alpha \in (0, 1)$. We refer to $\alpha$ as durability. Let there be $G$ firms producing durable goods. The firms face no capacity constraints and produce durable goods at a constant marginal cost $c > 0$. All firms simultaneously set their price at the beginning of each period and are committed to sell at that price to all interested consumers during that period. Let $p^n_{g,t}$ be the price of a new unit set by firm $g$ in period $t$. The lowest price in a period is the market price in that period $p^n_t = \min\{p^n_{1,t}, ..., p^n_{G,t}\}$. All consumers buy only from firms charging that price.[1] If more than one firm sets a price equal to the market price, then sales are split equally between all such firms. All firms are risk-neutral and discount future profits by a common discount factor $\varphi$. The profits of a firm in any given period are its sales multiplied by unit markup. Let $d^n_t$ denote the demand for new goods in period $t$. The industry profit function in that period is $\Pi_t = (p^n_t - c)\, d^n_t$.

On the demand side there are two groups of consumers, denoted by $h$ and $l$. There is a unit mass of consumers of type $h$ and mass $\lambda$ of consumers of type $l$. Every consumer demands at most one unit of the durable good in each period. Consumers differ as to their valuation of the good: consumers in group $h$ have a higher willingness to pay $v_h$ than those in group $l$, $v_l < v_h$. The consumption decision of a buyer is a function of current prices, price history and the stock of used goods available in each period. Consumers form correct expectations about future prices and anticipate that by buying a new product in period $t$ they can collect its resale value at time $t+1$ as extra income if resale is feasible. The current utility of a consumer of type $i$ is quasi-linear in the numeraire good: $u_{it} = qv_i + y_t$, where $y_t$ denotes consumption of numeraire good in period $t$. The lifetime utility of a consumer is given by $U_i = \sum_{t=0}^{\infty} \delta^{t-1} u_{it}$, where $\delta$ is the discount factor common to all consumers. All parameter values are common knowledge to players.

In each period there are two markets: an imperfectly competitive market for new goods and a perfectly competitive market for used goods. All consumers have access to both markets. There are no transaction costs in either market. The timing of the game within each period proceeds as follows: (i) first all firms simultaneously announce their prices for a new unit of output; (ii) then a second-hand market opens where prices equate demand and supply; (iii) finally, consumers decide whether to purchase new units from any one of the firms and whether to buy or sell units in the second hand market. The price of a used unit in the second-hand market is denoted by $p^u_t$.

**Equilibrium:** Firms know all past prices announced in all earlier periods (and hold correct expectations along the induced price path and after unilateral deviations from the path), the stock of used goods available at the beginning of each period, as well as the actions chosen by consumers in the past. This information defines the state of the game played by the

[1] Firms cannot price discriminate among consumers.

firms. Thus, a (pure) behavioral strategy for a firm specifies in each period the price to be set, conditional upon the state in that period. Firms' and consumers' strategies constitute a subgame perfect equilibrium if, in all periods and states, each firm maximizes its expected discounted future stream of profits and each consumer maximizes his own utility, given all other players' strategies. Notice that if the current state depends on the stock of goods available in the secondary market, then the strategic interaction is not repeated.[2] Because consumers are small, we will assume throughout that individual consumption decisions are not observed by producers. In general, any price between marginal cost and the monopoly price can be sustained as a subgame perfect equilibrium in trigger strategies. However, for simplicity and without loss of generality, we shall focus on the implementation of the monopoly outcome. To highlight the most relevant cases, we will make use of the following assumptions:

(a1) $\lambda > 1$

(a2) $c \in ((1-\alpha)\, v_l, (1-\alpha)\, v_h + \alpha v_l)$

(a3) $2\,((1-\alpha)\, v_h + \alpha\,(1+\delta)\, v_l - c) > (1+\lambda)((1+\alpha\delta)\, v_l - c)$

Assumption (a1) allows the equilibrium price of used units to be positive; (a2) implies that it is efficient to sell any new units only to high value consumers when used units are available; and finally (a3) dictates that a monopolist would have no incentives to sell new units to low-value consumers. The presence of vertically differentiated goods (new and used goods), along with consumer heterogeneity, creates the opportunity for trade in a decentralized secondary market. Such trading opportunities constrain the price that can be charged by a monopolist on the primary market, since used units represent a cheaper alternative to the new products. Because utility functions are quasi-linear and because there are no transaction costs, consumers separate their current decision from future decisions and determine their optimal consumption by comparing the utility flows of each possible choice: consuming a new unit (possibly trading in a used unit); consuming a used unit; or consuming no durable at all. Let $d_i(\mathbf{p}^t) = \max\{v_i - p_t^n + \delta p_{t+1}^u, \alpha v_i - p_t^u, 0\}$.

**Lemma 1** *At time $t$ a buyer of type $i$ demands: a new unit iff $v_i - p_t^n + \delta p_{t+1}^u = d_i(\mathbf{p}^t)$; a used unit iff $\alpha v_i - p_t^u = d_i(\mathbf{p}^t)$; and no unit otherwise.*

The previous lemma implies that consumers' choices are identical at every date along the equilibrium path. In particular, our assumptions imply that in every period high value consumers will buy new units while selling their used units to low value consumers.

**Monopoly:** If a single producer operates in such a market, assumptions (a1-3) imply that it is optimal for him to sell every period only to high value consumers.

[2] See Dutta, Matros & Weibull [2007] for further discussion.

**Proposition 1** *If (a1-3) hold, the subgame perfect equilibrium (SPE) price that maximizes the monopoly present discounted profit is* $p_t^m = (1-\alpha)\, v_h + p_t^u + \delta p_{t+1}^u$ *for* $\forall t$*. The SPE price on the secondary market is* $p_t^u = \alpha v_l$ *for* $\forall t$*.*

Thus, the monopoly price is increasing in both the current and future price of used units.

**Oligopoly:** If $G$ firms operate in the market, an equilibrium without reputations exists in which firms always price at marginal cost. In such a Markovian SPE, the net present value of all firms coincides with their security value, namely zero. If firms are sufficiently patient, however, more profitable equilibria can be achieved. In particular, the monopoly outcome can be implemented as SPE in trigger strategies of the dynamic oligopoly game. If such strategies are employed, all firms charge the monopoly price $p^m$ in every period provided that no deviation has taken place and if a firm ever does undercut, all firms price at marginal cost in all the remaining periods. Since demand depends on the stock of used units and on the current and future prices of new and used units, if a defection is observed, forward-looking consumers will anticipate a price war in the following period and will update their beliefs about future prices on the secondary market. Because a price war leads to overproduction of new units, it may depress the demand for used goods thereby reducing their equilibrium price. In turn a lower expected price for used units reduces the demand for new units at the defection stage. Because the punishment can affect demand in the current period, the monopoly price may be easier to enforce. The next proposition summarizes such observations:

**Lemma 2** *In the described dynamic game:*

*(1) the security payoff of any firm is zero.*

*(2) always pricing new units at the marginal cost is a SPE for any* $\varphi \in [0,1]$*.*

*(3) always pricing new units as a monopolist is a SPE in trigger strategies if* $\varphi \geq 1 - \frac{1}{G\gamma^c}$*.*

*(4) in any subgame in which new units are always sold at c, used units are sold at:*

$$p_c^u = \max\left\{\min\left\{\alpha v_l, \frac{c-(1-\alpha)\, v_l}{(1+\delta)}\right\}, 0\right\} \tag{1}$$

where $\gamma^c = \frac{(1-\alpha)v_h+\alpha v_l+\alpha\delta p_c^u - c}{(1-\alpha)v_h+\alpha(1+\delta)v_l - c} \leq 1$ represents the fraction of the monopoly profits captured by a defector. Part (4) implies that in a competitive subgame the resale price $p_c^u$ may fall strictly below $\min\{\alpha v_l, c\}$. Such a price, however, will always be positive if (a2) holds.

In a standard Bertrand game, setting future prices at marginal cost yields the worst punishment that can be inflicted on a deviator. A future price below marginal cost would not have any further impact, since a deviator would simply withhold production. In a durable goods setup however, the willingness to pay for new units at the defection stage can be further reduced if a more pronounced drop in the price of new units (below marginal cost) is expected after a unilateral price cut. Because of this dynamic effect (absent in repeated games) even harsher punishments than trigger strategies are possible. Let us show explicitly how this may reduce the incentives to deviate. First notice that the expected utility flow of any buyer of new units is $v_h - p^m + \delta p^u_{t+1}$. Thus, if a producer wants to attract all high type consumers by undercutting the monopoly price by $\varepsilon$, it must be that $v_h - p^m + \varepsilon + \delta \overline{p}^u_{t+1} > 0$. By proposition 1 this requires that:

$$\alpha v_h - p^u_t + \varepsilon + \delta \Delta p^u_{t+1} > 0 \quad \Rightarrow \quad \varepsilon > \alpha(v_l - v_h) - \delta \Delta p^u_{t+1}$$

where $\Delta p^u_{t+1}$ is the expected change in the price of used units due to the deviation in the market for new units. The largest price change that can take place in the secondary market following a deviation reduces the trading price from $\alpha v_l$ to zero. If $(1+\delta)\, v_l - v_h \leq 0$, any undercutting of the monopoly price attracts all consumers, and secondary markets do not affect the incentives to defect.[3] Otherwise the price cut necessary to attract any consumers has to be positive, $\varepsilon = \alpha[(1+\delta)\, v_l - v_h] > 0$, since for any smaller deviation no buyer would want to purchase new units at current prices due to the anticipated reduction in the future prices. In this scenario, a deviator would have to reduce the price significantly below the monopoly price to win over the same number of consumers and incentives to deviate would be reduced. In fact, defection would be less profitable because resale values would drop discretely after any arbitrarily small deviation. Thus, to find the optimal punishment strategy for the proposed dynamic game we need to depart from the standard trigger strategy by providing a strategy with stronger short term incentives. Define a *sharp* trigger strategy as follows: initially all the firms set the monopoly price and do so as long as no firm sets a different price; in the first period after deviation all firms post a price $p^b_{t+1} \in [0, (1-\alpha)v_l]$ to induce $p^u_{t+1} = 0$;[4] finally in the second period after a defection, based on the realization of a publicly observable signal, all firms revert to collusive pricing in every period with probability $\mu$, and price at the marginal cost in every period with probability $1-\mu$. If a firm does not obey the punishment pricing, the punishment sequence is restarted. Formally, a sharp trigger strategy consists of two maps: a behavioral strategy, $\pi$, mapping states into prices and a stochastic state transition rule, $\sigma$, mapping from the current state and posted prices into the future state:

(2)

$$\pi(z_t) = \begin{cases} p^m_t & \text{if} \quad z_t = 0 \\ p^b_t & \text{if} \quad z_t = 1 \\ c & \text{if} \quad z_t = 2 \end{cases} \qquad \sigma(p^n_t | z_t) = \begin{cases} 1 & \text{if} \quad \mathbf{p}^n_t \neq \pi(z_t) \\ z_t & \text{if} \quad \mathbf{p}^n_t = \pi(z_t) \ \cap \ z_t \in \{0, 2\} \\ 0 \cdot \mu \oplus 2 \cdot (1-\mu) & \text{if} \quad \mathbf{p}^n_t = \pi(z_t) \ \cap \ z_t = 1 \end{cases}$$

[3] In this scenario relevant discount factor remains $(G-1)/G$.

[4] Any price $p^b_{t+1} \in [0, (1-\alpha)v_l]$ guarantees that both types of buyer purchase new units. If $c > (1-\alpha)\, v_l$, such a price guarantees that firms do not run positive profits. If $c \leq (1-\alpha)\, v_l$, trigger strategies suffice to drive $p^u_{t+1} = 0$, since by lemma 2 the post-deviation equilibrium price on the secondary market satisfies (1).

where $\mathbf{p}_t^n$ is the firms' price vector at time $t$ and the equality holds componentwise. Notice that driving the expected price to zero completely removes the resale value component of demand in the defection period. Reversion to collusive prices after a deviation must occur with some probability, since profits are negative in the period following a defection by assumption (a2) and since the security value of every producer is zero. If players adhere to the sharp trigger strategy, the period $t$ expectation regarding the evolution of future prices along the equilibrium path satisfies: $p_s^n = p^m$ and $p_s^u = \alpha v_l$ for any $s > t$. If, instead, a deviation is observed the expectation regarding future prices must satisfy:

(3)

| $z$ | 1 | 0 | 0 | 2 | 2 |
|---|---|---|---|---|---|
| $p \backslash s$ | $t+1$ | $t+2$ | $> t+2$ | $t+2$ | $> t+2$ |
| $p_s^n$ | $p_{t+1}^b$ | $p_{t+2}^m$ | $p_s^m$ | $c$ | $c$ |
| $p_s^u$ | 0 | 0 | $\alpha v_l$ | 0 | $p_c^u$ |

The sharp trigger strategy supports $p^m$ as a subgame perfect equilibrium if and only if the following two conditions hold:

$$\Pi_t^d + \varphi \frac{\Pi_{t+1}^b}{G} + \mu\varphi^2 \left[ \frac{\Pi^m}{G(1-\varphi)} - \frac{\alpha v_l}{G} \right] \leq \frac{\Pi^m}{G(1-\varphi)} \tag{4}$$

$$\frac{\Pi_{t+1}^b}{G} + \mu\varphi \left[ \frac{\Pi^m}{G(1-\varphi)} - \frac{\alpha v_l}{G} \right] = 0 \tag{5}$$

The first term in equation (4), $\Pi_t^d$, is the profit that the defector gains at the deviation stage and satisfies $\Pi_t^d \in [(1-\alpha) v_h - c, (1-\alpha) v_h + \alpha v_l - c]$, since a defector prefers not to sell the new units to low value consumers. The second term, $\Pi_{t+1}^b / G$, is negative and defines the loss that each firm incurs in the first punishment period. In particular, because all consumers purchase new units at $p_{t+1}^b$, profits must satisfy $\Pi_{t+1}^b \leq (1+\lambda)((1-\alpha)v_l - c)$. The last term on the left-hand side is the expected share of the monopoly profit earned after the first punishment period, if the system reverts to collusion. The profit in each period depends on the stock of used goods available. In period $t+2$, the monopoly profit differs from the monopoly profit in equilibrium. In fact, there is an excess supply of used units that reduces the equilibrium prices both in the primary and in the secondary market. However, from $t+3$ onwards the monopoly profits coincide with equilibrium. The last term on the left-hand side of equations (4) accounts for such an effect since profits drop by $\alpha v_l / G$. On the right-hand side is the share of the equilibrium profits that a deviating firm would have earned if it had not defected, $\Pi^m = ((1-\alpha) v_h + \alpha (1+\delta) v_l - c)$. Equation (5) guarantees that such a harsh punishment strategy remains individually rational, since no firm could guarantee itself a better payoff by pricing at marginal cost forever. The latter equation also requires the left-hand side to be non-positive in order to punish the defector most effectively.[5] For the dynamic game presented, sharp trigger strategies minimize the incentive to defect:

**Proposition 2** *Sharp grim strategies maximize the range of discount factor values for which collusion can be sustained.*

The next proposition characterizes such range. Let $\gamma^*$ denote the fraction of the monopoly profits captured by a defector, $\gamma^* = \Pi_t^d/\Pi^m$:

**Proposition 3** *Collusion on the monopoly price $p^m$ is a SPE of the dynamic oligopoly game if $\varphi \geq \varphi^* = 1 - \frac{1}{G\gamma^*}$. Moreover, $\varphi^*$ decreases with consumers' discount factor $\delta$ and with the quality of a used unit $\alpha$, but increases with the distance between consumers' valuations $v_h - v_l$.*

In this setup, goods of higher quality lead to harsher punishments in case of a defection due to the increase in resale values. This finding suggests that increasing the number of competitors in a market might lead to improvements in the quality of goods to facilitate collusion. The literature on investment in R&D[6] finds that competition among firms often leads to overinvestment in R&D with respect to the level that maximizes aggregate profitability. The results of the model appear to confirm this finding, since attempts to collude in large markets might lead to overinvestment in durability with respect to the level that maximizes monopoly profit. More generally, two forces influence the choice of the durability and are not necessarily aligned: (i) the firms' incentives to collude which favor overinvestment; and (ii) the effect on the monopoly profits whose sign depends on the relative importance of the resale and substitution effects. Proposition 3, also, shows that increasing the heterogeneity in consumers' valuations diminishes the importance of the resale effect for high value consumers (since the equilibrium price on the secondary market is a function of $v_l$) and makes punishments less effective. Finally, an increase in consumer patience will facilitate collusion, since resale values would increase and the expectation of a price war would lead to considerable adjustments in the willingness to pay for new units, further deterring defections from a collusive agreement.

The analysis has focused on environments in which there are no transaction costs, $\tau$. But it is straightforward to extend the analysis to allow for such costs on the secondary market. If transaction costs are sufficiently small, the second-hand market still opens. In this case, the equilibrium price of used units is a decreasing function of the transaction cost, $p_t^u = \alpha v_l - \tau$. Thus, a marginal increase in the cost to transact will reduce the resale effect and the ability to punish a defector, $\partial\gamma^*/\partial\tau > 0$. Hence, the presence of transaction costs

[5] Lemma 4 in the appendix provides the necessary and sufficient conditions for the existence of a sharp trigger strategy. Such conditions are easier to satisfy when $p_{t+1}^b \in [0, (1-\alpha)v_l]$ is high.

[6] See Waldman (2003) for a survey.

will not only reduce a firm's profitability, but also its ability to collude.[7] This result is in line with the main findings of the present work, since firms may prefer to have an active and frictionless secondary market. Another important assumption invoked in all our results was $\lambda > 1$. Notice that, if such assumption were violated, used units would always be sold for free in a secondary market as supply would exceed demand. If so, the intertemporal link in the demand function would vanish and new units would be sold in every time period, to all types of consumers as if the goods were non durable. Thus, the ability to collude would not be affected by the option value of trading in such markets.

# III Closing Secondary Markets

Since used units are imperfect substitutes for new units, closing the secondary market may in principle benefit a monopoly producer. However, many industries can be found in which manufacturers appear to intervene to foster trade in secondary markets. This section considers the incentives of a firm to interfere with transactions in the secondary market. In particular, it compares the environment developed in section 2, to one in which used units cannot be traded, as commonly assumed in the extensive literature on durable goods and collusion (Gul [1987] and Ausubel & Deneckere [1987]).[8] Hence, the analysis provides necessary and sufficient conditions on the quality of a used unit for an active secondary market to facilitate collusion.

Consider the same setup discussed in section 2, but assume that consumers cannot trade in secondary markets. Closing such markets amounts to reducing consumers' choices to either scrapping or keeping their used units. Thus, consumers will only decide whether to buy a new unit, every two periods, or each and every period while scrapping their used unit. Let $q^n$ denote the price of new units without secondary markets.

**Lemma 3** *A consumer of type $i$ prefers to buy new units every two periods iff $q^n > (1-\alpha)v_i$.*

To begin with assume that a single firm operates in the market and that it runs higher profits by not selling to low type consumers. Both assumptions shall be dismissed momentarily. Two scenarios need to be considered. In the first, the monopolist charges a lower price and sells a new unit to each high type consumer in every period at price $q^m = (1-\alpha)v_h$. Consumers buy the new product every period and scrap the used one. In the second scenario, the monopolist extracts all the rents from type $h$ consumers by charging a price $q^m = (1+\alpha\delta)v_h$ and, by lemma 3, type $h$ consumers respond by purchasing a new unit every two periods.

[7] See for example Porter & Sattler (1999), Stolyarov (2002), Anderson & Ginsburgh (1994) for a formal derivation of the equilibrium with transaction costs, and Schiraldi (2008) for an empirical analysis.

[8] Any discussion about the alternative strategies to cap transactions in the secondary market will be deferred to section 4.

In this scenario, for notational convenience, we shall split the population of high types into two groups with mass 1/2, one purchasing every odd period, the other purchasing every even period.[9] The period-profits in either scenario are respectively: $((1-\alpha)v_h - c)$ and $((1+\alpha\delta)v_h - c)/2$. The first strategy is preferred to the second if and only if $\alpha > \frac{v_h - c}{v_h(2+\delta)}$.

**Proposition 4** *If (a1-3) hold, the monopoly profits are higher without a secondary market iff* $\alpha[(2+\delta)v_h - 2(1+\delta)v_l] > v_h - c$.

A monopolist always prefers to keep the secondary market open while selling only to high types, rather than selling to both types with a closed secondary market. This follows by (a2-3), just as in proposition 1. Moreover, a monopolist always prefers the secondary market to be open whenever it plans to sell to all high types in every period, since a high type's willingness to pay increases when units can be resold. Finally, when without secondary markets a monopolist chooses to sell every two periods to the high types only, it would prefer the secondary market to be open if and only if costs are sufficiently low as increased volumes may hinder profits otherwise. Intuitively, a monopolist may achieve higher profits with an active secondary market, because aggregate surplus may grow and because the secondary market may be exploited to price discriminate consumers.

**Oligopoly:** Consider now a setting with $G$ firms that want to sustain the monopoly outcome as a subgame perfect equilibrium in sharp trigger strategies. As in Gul [1987] and Ausubel & Deneckere [1987] the option to substitute consumption over time constrains the maximum price that a defector can charge in the deviation period, and thus reduces the incentives to defect. In fact, intertemporal linking of consumers' decisions leads to such a conclusion, since consumers may prefer not to buy new products if they expect the prices to drop in the future. As was the case with secondary markets, such linking in the demand, expand the scope of punishments at defection stage. However, intertemporal linking will lead to different implications on the incentives to collude when secondary markets are closed, since the resale value effect is muted in this scenario.

**Proposition 5** *An active secondary market increases the range of discount factors for which collusion can be sustained if and only if* $\alpha \notin [k(c,v,\delta), g(c,v_h)]$*, for* $g(c,v_h) = \frac{v_h - c}{v_h}$ *and for some* $k(c,v,\delta) \in \left[\frac{g(c,v_h)}{2+\delta}, \frac{g(c,v_h)}{2}\right]$.

The collective ability of firms to sustain collusion depends on the durability of the good. If the quality of the good exceeds the short term gains from trade with a high type, an open secondary market reduces the incentives to defect, since a drop in prices has a larger effect on aggregate profits at the defection stage, $\frac{(1-\alpha)v_h + \alpha v_l - c}{(1-\alpha)v_h + \alpha(1+\delta)v_l - c} < \frac{v_h - c}{(1+\alpha\delta)v_h - c}$. Hence, provided that

[9] All results are easily modified to account for any timing.

the quality is high enough, collusion will be sustained under a broader range of parameter values with an open secondary market. Similarly, an open secondary market will facilitate collusion when the quality is not too high, because it enables harsher short term punishments by strengthening the intertemporal link in firms' decision problems. In fact, if the secondary market were closed and if the quality were below $g(c, v_h)/(2+\delta)$, firms would achieve the highest collusive profit by selling the new unit to all high type consumers in every period. All consumers would buy new units, possibly scrapping used ones, and any incentive for intertemporal substitution would disappear. The lack of the intertemporal link in demand would, therefore, completely destroy the possibility of extra punishment in the defection period. Similarly, if quality were below $g(c, v_h)/2$, a defector would still benefit by selling the good to all type $h$ consumers at the defection stage. Thus, for sufficiently low quality, the intertemporal link would remain sufficiently weak and an active secondary market would still foster collusion. For intermediate quality levels, the short term incentives point in the opposite direction and collusion is facilitated by closing the secondary market. Thus, if firms were to choose quality, tension may arise between the incentives to maximize aggregate profits and the incentives to sustain collusion, as quality certainly improves stability, but does not necessarily improve monopoly profits. Figure 1 plots the critical discount factors for different parameter specifications satisfying (a1-3) and highlights how such values vary with quality.

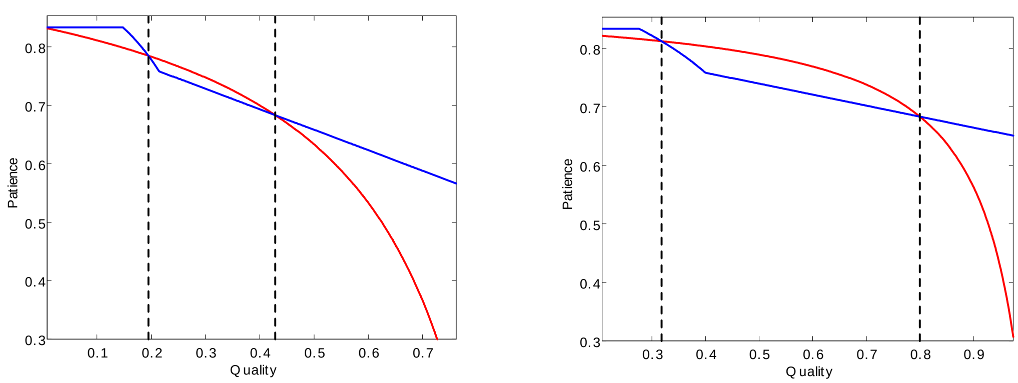


Figure 1: In red the critical discount factor with secondary markets, in blue without; on the horizontal axis quality, on the vertical patience; for $c = 2$, $\delta = 0.9$ and on the left $v_h = 3.5$, $v_l = 2$, $\lambda = 1.5$, on the right $v_h = 10$, $v_l = 2.5$, $\lambda = 1.1$.

# IV Limiting Trade in Secondary Markets

The internet revolution, lower transportation costs and trade agreements have expanded the scope of secondary markets, by reducing the transaction costs. When firms cannot prevent the possibility of reselling used goods on a decentralized market, it becomes relevant to study alternative practices that can be used to cap trade in such markets. This section briefly analyzes two mechanisms that are commonly used to limit resale: leasing policies and buy-back policies.

**The Leasing Policy:** A leasing policy in our environment consists of a pair of rental prices $\{l^n, l^u\}$ that have to be paid to the manufacturer to lease respectively a new or used unit for one period. Leasing contracts effectively cap transactions in secondary markets, since firms retain ownership of all goods. Consumers' decision to rent new products does not depend on their resale value, but only on the substitution effect (the option to rent a used unit instead). Thus, the demand for new products does not display any intertemporal linking and can be determined for a type $i$ consumer by solving:

$$\max\{v_i - l_t^n, \alpha v_i - l_t^u, 0\} \tag{6}$$

**The Buy-Back Policy:** A buy-back policy consists of an amount of money $b$ offered by a manufacturer to buy back used units from consumers who are willing to purchase new ones.[10] The aim of such a policy is to interfere with the secondary market by raising the equilibrium price of used products. If a such policy were adopted, the willingness to pay for new products would increase due to an increase in resale values and a reduction in the substitution effect. This preserves the intertemporal link in the demand for new products. The demand of a type $i$ consumer can be determined by solving:

$$\max\{v_i - p_t^n + \delta b_{t+1}, \alpha v_i - b_t, 0\} \tag{7}$$

**Monopoly:** The next proposition characterizes the effects of either policy on the profits of a monopolist. Even though similar results have already been proven in the literature, they shall be developed in this context to benchmark the oligopoly case. For convenience, we will assume that firms can commit to buying and scrapping used units for a given price.[11]

**Proposition 6** *The monopoly profit:*

*(1) with leasing is higher than selling.*

*(2) with leasing is higher than with buy-back.*

*(3) with buy-back is higher than selling if $\delta v_h > (1+\delta)v_l$.*

A monopolist always prefers to lease units, since he can acquire complete control of the secondary market at no cost.[12] If he is prevented from doing so, he will choose either to sell

[10] This is a common practice in the car industry.

[11] Any SPE with commitment can be obtained in a game without commitment in which a monopolist establishes a reputation for eliminating the secondary market. The threat of moving to an equilibrium with an active secondary market provides incentives to repurchase any used unit, see Waldman [1997].

[12] Waldman (1997) and Hendel & Lizzeri (1999a) show that a monopolist always benefits by leasing new units rather than selling them in a similar context.

directly or with a buy-back policy depending on the fundamentals of the economy. With a buy-back, a manufacturer indirectly controls the number of used units available at the cost of buying them back, but leaves the secondary market active. Buy-backs are more profitable when the difference in consumers' valuations is significant, since the cost of implementing the policy is smaller than the benefit derived by increasing resale values. The optimal buy-back policy for a monopolist consist of a buy-back price $b^m = \alpha v_h$ and of a price for new units $p^m = (1 + \alpha\delta)\, v_h$. In such an equilibrium, all high value consumers buy new units in each period and trade them in the next period; and the buy-back price always exceeds the market price of used units.

**Oligopoly:** Leasing breaks the intertemporal link in the demand function, since consumers only respond to current prices. However, firms' decisions remain intertemporally linked, as the leasing price for new units offered by a firm determines its stock of used units in the following period. The optimal strategy to sustain a constant collusive leasing price pair $\{l^n, l^u\}$ is a sharp trigger strategy. When such strategy is employed, all firms set the collusive price pair $\{l^n, l^u\}$ in the initial period and continue to do so as long as no firm defects; in the first period after deviation all firms post a punishment price for new units $l^n_{t+1} \leq (1-\alpha)v_l$ to induce $l^u_{t+1} = 0$; in the second period after a defection with probability $\mu$ all firms revert to the collusive equilibrium and offer $\{l^n, l^u\}$ from that period onwards and with probability $1 - \mu$ they go to a competitive Markovian equilibrium $\{l^n_c, l^u_c\}$ in which all firms make zero profits. If any firm deviates from the punishment pricing at any given period, the others restart the punishment sequence. If more used units are demanded that are those that are available, consumers are supplied on a first-come first-served basis and all remaining consumers may lease new units. Even when buy-back policies are employed, the optimal punishment strategy to sustain collusion remains in sharp trigger strategies. When such strategy is adopted, all firms set the price of new units to $p^m$ and offer to buy back used units at $b^m$, as long as no firm defects; in the first period after deviation all firms post a punishment price for new units $p^b_{t+1} \leq c$ to induce $p^u_{t+1} = 0$ and stop offering the buy-back policy; in the second period after a defection with probability $\mu$ all firms revert to the collusive equilibrium and offer $\{p^m, b^m\}$ from that period onwards and with probability $1 - \mu$ they go to a competitive Markovian equilibrium in which all new units are sold at marginal cost without the buy-back policy. If any firm deviates from the punishment pricing at any given period, the others restart the punishment sequence. In a defection period the undercutting firm continues to offer the same buy-back price $b^m$, since any lower buy-back price would further reduce the price at which new units are sold. The competitive Markov perfect equilibria for leasing and buy-back are characterized in the proof of the next result. In such equilibria, all firms run zero profits and with leasing both rental prices $\{l^n_c, l^u_c\}$ are positive. The next result shows when leasing and buy-back policies can foster or prevent collusion.

**Proposition 7** *The minimum discount factor necessary to sustain collusion:*

*(1) with leasing is higher than selling if $v_h > 2v_l$ or if $1 - 1/G < \delta$.*

*(2) with buy-back is lower than selling if $\delta v_h > v_l$.*

*(3) with buy-back is lower than leasing if $\delta v_h > v_l$.*

The first part of the result implies that leasing might jeopardize a collusive agreement, even if it increases aggregate profits. If $v_h > 2v_l$, the minimum discount factor necessary to sustain collusion with leasing remains $\varphi^L = 1 - \frac{1}{G}$ (as for non-durable goods). Since used units are scrapped in equilibrium, no extra punishment can be inflicted to a defector. Similarly if $1 - 1/G < \delta$, intertemporal linking in firms' decisions due to the stock of used units remains weak and more patience is required by firms to enforce collusive agreements. Hence, firms might prefer sales-only policies to leasing policies in order to facilitate collusion. Any buy-back policy that makes non-negative profits (i.e. $\delta v_h > v_l$) instead, always expands the scope for collusion, since it magnifies intertemporal linking. Thus, such a policy may be used in oligopolistic environments to facilitate collusion even when it reduces aggregate profits. Collective and individual incentives to adopt such practices may not be aligned with leasing, but are aligned with buy-back if $\delta v_h > (1 + \delta)v_l$. Leasing policies are profitable, but may reduce a firm's ability to collude. Buy-back policies instead, are costly to implement and may not necessarily increase the monopoly profits. But, if profitable, such policies always exacerbate the intertemporal link in the demand for new units and further facilitate collusion.

# V Conclusion

The aim of the present work was to analyze how secondary markets would influence the ability of firms to collude. A simple model of competition with semi-durable goods was proposed to tackle the question. It was shown that both the durability of the good and the presence of a secondary market would strengthen intertemporal linking in consumers' decisions. Optimal punishment strategies were derived for the proposed setup. Such strategies would enhance short term competition and foster collusion, by reducing resale values and consequently the gains of a deviating firm. The results developed with open secondary markets were then compared to the model without secondary trades. The analysis implied that an active secondary market would favor collusion when durability was either sufficiently high or sufficiently low, since in either scenario aggregate profits would drop more at the defection stage with an open secondary market. The analysis showed that a trade-off might arise between the incentives to sustain collusive agreements and the incentives to maximize aggregate profits. Alternative schemes to interfere with an active second-hand market were also considered. In particular, it was shown that leasing policies, by eliminating any dynamic link in the demand, would reduce the incentives to collude, but increase monopoly profits. Buy-back policies instead, were shown to favor collusive behavior by increasing the market value of used units and to have an ambiguous effect on monopoly profits. All the results developed in this analysis could be interpreted in terms of Shapiro's Topsy-Turvy principle. In this interpretation, the analysis would clarify when second-hand markets might be used to enhance short-term competition. The analysis provided several motives for durable good manufacturers to have stakes in the interplay between primary and secondary markets. A further enquiry would be desirable to understand such interplay in more general setups. All the results presented relied on a few strong assumptions on consumer behavior. Namely, we did assume consumers to be small and unable to establish reputations and, more importantly, we did assume that consumers were rational and forward looking. The second assumption, however, was stronger than necessary. In fact, all that was required to prove our results was the rationality of the high type consumers, while the rationality of low types was superfluous.

# VI Appendix

**Proof of Lemma 1.** The optimal consumption problem faced by a consumer of type $i$ at period $t$ given the prices of new and used units can be formulated via Bellman's equation. There are two relevant cases:

- If consumer $i$ has no good at the beginning of the period, he can: either buy a new or a used unit, or to remain without any durable:

$$V_{it}^{u} = \max\{v_i - p_t^n + \delta V_{it+1}^{n}, \alpha v_i - p_t^u + \delta V_{it+1}^{u}, \delta V_{it+1}^{u}\} \tag{8}$$

- If consumer $i$ owns a used unit, he can: either keep the used unit; or sell the used unit on the secondary market and buy a new unit; or remain without any durable:

$$V_{it}^{n} = p_t^u + \max\{v_i - p_t^n + \delta V_{it+1}^{n}, \alpha v_i - p_t^u + \delta V_{it+1}^{u}, \delta V_{it+1}^{u}\} \tag{9}$$

  (8) and (9) imply $V_{it}^{n} - p_t^u = V_{it}^{u}$. Thus, the consumer maximization problem is independent of his endowment:

$$V_{it}^{u} = \delta V_{it+1}^{u} + \max\{v_i - p_t^n + \delta p_{t+1}^{u}, \alpha v_i - p_t^u, 0\}$$

■

**Proof of Proposition 1.** If the monopolist sells only to high types, there is a unit mass of used goods available on the secondary market in every period. Given that $\lambda > 1$, high-valuation types capture the entire rent from low-valuation types. In any period the competitive price on the secondary market is $p^u = \alpha v_l$. The maximum price that the monopolist can charge must leave the $h$ types indifferent as whether to buy a new good or keep the used good:

$$v_h - p_t^n + p_t^u + \delta p_{t+1}^{u} = \alpha v_h \quad \Rightarrow \quad p_t^n = (1-\alpha)\, v_h + (1+\delta)\, \alpha v_l$$

Moreover, the monopolist prefers to sell only to $h$ types, because (a2) implies that he would run a loss by selling to both types every period and because (a3) implies that he cannot benefit by selling to both types every two periods.

**Proof of Lemma 2.** (1) The security payoff of each firm in this dynamic game is zero, because any firm can guarantee itself such a payoff by always pricing at marginal cost and because no firm can improve on such a payoff if all others price at marginal cost.
(2) If all firms always price at $c$, no firm can benefit by undercutting if the others adhere to equilibrium, since it would run a loss. Moreover, no firm would profit by increasing prices

since, no consumer would buy from it.
(3) Let $\Pi^d = p^d - c$ denote the optimal profit at the defection stage and let $\Pi^m = (1-\alpha)v_h + \alpha(1+\delta)v_l - c$ denote the monopoly profit. Trigger strategies sustain collusion iff:

$$\Pi^d \leq \frac{\Pi^m}{G\left(1-\varphi\right)} \quad \Rightarrow \quad \varphi \geq 1 - \frac{\Pi^m}{G\Pi^d} = 1 - \frac{1}{G\gamma^c}$$

Notice that the optimal deviation price $p^d$ must satisfy $p^d = (1-\alpha)v_h + \alpha v_l + \alpha\delta p^u$ to attract all high types, where $p^u$ is the price of used units in a competitive subgame defined in (4). ∎
(4) There are two relevant cases to consider: if $c > (1+\alpha\delta)\,v_l$, buying new goods cannot be optimal for type $l$ and the equilibrium price on the secondary market remains $\alpha v_l$; if instead, $c \leq (1+\alpha\delta)\,v_l$, the maximum price that type $h$ can charge on the secondary market must leave low value consumers indifferent between buying a new unit and keeping it or buying a used unit every period:

$$\frac{(1+\alpha\delta)v_l - c}{1-\delta^2} = \frac{\alpha v_l - p^u}{1-\delta}$$

The previous two observations together imply that the price of a used unit satisfies:

$$p^u = \max\left\{\min\left\{\alpha v_l, \frac{c-(1-\alpha)\,v_l}{1+\delta}\right\}, 0\right\}$$

which is strictly positive iff $c > (1-\alpha)\,v_l$. ∎

**Lemma 4** *If $G\left(1-\varphi\right)\Pi^d_t \leq \Pi^m$ and $\left(1-\varphi\right)\Pi^b_{t+1} + \varphi\Pi^m > (1-\varphi)\varphi\alpha v_l$, a sharp trigger strategy exists.*

**Proof of Lemma 4.** The first inequality is clearly necessary and is derived by substituting equation (5) in (4). The sufficient condition holds because of the following reasoning. Notice that the left-hand side of (5) is increasing in $\mu \in [0,1]$, with $f(0) = \frac{\Pi^b_{t+1}}{G}$ and $f(1) = \frac{\Pi^b_{t+1}}{G} + \varphi\left[\frac{\Pi^m}{G(1-\varphi)} - \frac{\alpha v_l}{G}\right]$. By the intermediate value theorem, $f(0) < 0 < f(1)$ is sufficient for the existence of a probability $\mu \in (0,1)$ such that condition (5) holds: $f(0) < 0$ is satisfied by assumption (a2), since $\Pi^b_{t+1} \leq \left(\left(1-\alpha\right)v_l - c\right)\left(1+\lambda\right)$; and $f(1) > 0$ is satisfied whenever the second condition holds. Moreover, when assumption (a3) holds the second condition can be simplified to $\left(1-\varphi\right)\Pi^b_{t+1} + \varphi^2\Pi^m > 0$ if $\Pi^m > \alpha v_l$. ∎

**Proof of Proposition 2.** Condition (5) guarantees that the present discount payoff after a defection is equal to zero. This is the maximum credible punishment that can be inflicted through the continuation payoff after a defection that still satisfies individual rationality. By substituting equation (5) in (4), the minimum discount factor, $\varphi^*$, that sustains the equilibrium collusion can be determined:

$$\Pi_t^d \leq \frac{\Pi^m}{G(1-\varphi)} \quad \Rightarrow \quad \varphi \geq \varphi^* = 1 - \frac{1}{\gamma^* G} \tag{10}$$

$\varphi^*$ is increasing in $\gamma^*$, where $\gamma^* = \frac{\Pi_t^d}{\Pi^m} = \frac{(1-\alpha)v_h + \alpha v_l - c}{(1-\alpha)v_h + \alpha(1+\delta)v_l - c}$. Given that $\Pi^m$ is the monopoly profit, the minimum $\varphi^*$ is achieved by minimizing $\Pi_t^d$. For $\varepsilon$ very small, a deviator can price at $p_t^m - \varepsilon$ and guarantee a payoff of almost $(1-\alpha)\, v_h + \alpha v_l + \alpha\delta p_{t+1}^u - c$. Since the competitors can only affect such a payoff through the resale value component of the demand, the harshest punishment occurs when $p_{t+1}^u = 0$ and $\Pi_t^d = (1-\alpha)\, v_h + \alpha v_l - c$. This can be achieved by setting $p_{t+1}^n = p_{t+1}^b < (1-\alpha) v_l$ since all consumers will prefer buying new products at these prices. The excess supply of used goods would then drive $p_{t+1}^u$ to zero. ■

**Proof of Proposition 3.** Under sharp grim trigger strategies, consumers' expectations about future prices in period $t$ are described by table 3. Thus, profits in the defection period are strictly less than the monopoly profit in the industry and collusion is sustained if and only if equation 10 holds. Thus $\partial\varphi^*/\partial\gamma^* = 1/G(\gamma^*)^2$ where $\gamma^* = \frac{\Pi_t^d}{\Pi^m} = \frac{(1-\alpha)v_h + \alpha v_l - c}{(1-\alpha)v_h + \alpha(1+\delta)v_l - c} < 1$. Let $W = (\Pi^m)^2 \geq 0$ and observe that:

$$\frac{\partial\gamma^*}{\partial\alpha} = -\frac{\alpha v_l\,(v_h - c)}{W} < 0 \quad \& \quad \frac{\partial\gamma^*}{\partial\delta} = -\frac{\alpha v_l\,((1-\alpha)\, v_h + \alpha v_l - c)}{W} < 0$$

$$\frac{\partial\gamma^*}{\partial v_h} = \frac{(1-\alpha)\,\alpha\delta v_l}{W} > 0 \quad \& \quad \frac{\partial\gamma^*}{\partial v_l} = -\frac{((1-\alpha)\, v_l - c)\alpha\delta}{W} < 0$$

■

**Proof of Lemma 3.** If the price of new units on the equilibrium path is equal to $p^n$, consumers can choose either to buy the new good and keep it for two periods, or to buy the new good every period and scrap the used one. The continuation values for the two scenarios for a consumer of type $i$ are respectively $V_i^2 = \frac{(1+\alpha\delta)v_i - p^n}{1-\delta^2}$ and $V_i^1 = \frac{v_i - p^n}{1-\delta}$. Thus, the first strategy is preferred by type $i$ whenever $p^n > (1-\alpha)v_i$. ■

**Proof of Proposition 4.** As argued in proposition 1, if (a2-3) hold, a monopolist always prefers to keep the secondary market open while selling only to high types, rather than selling to both types with a closed secondary market. Moreover a monopolist, who chooses to sell every period to all high types when the used market is closed, always prefers to keep the secondary market open, since he would be able to sell the same number of units while increasing his price by $(1+\delta)\alpha v_l > 0$. If, instead, the monopolist charges a price of $(1+\alpha\delta)v_h$ in order to sell every two periods to the high types when the secondary market is closed, he prefers to keep the secondary market closed if $((1+\alpha\delta)\, v_h - c)\,/2 > ((1-\alpha)\, v_h + \alpha\,(1+\delta)\, v_l - c)$ which requires $c > (1-\alpha(2+\delta))v_h + 2\alpha(1+\delta)v_l$. ■

**Proof of Proposition 5.** By lemma 3, if the price of a new unit in equilibrium is $q^n = c > (1-\alpha)v_h$, consumers prefer buying a new unit every two periods. Consider four cases:

1. If $\alpha > g(c, v_h)$, the highest profit is achieved by setting $q^m = (1+\alpha\delta)\, v_h$. If firms follow a standard trigger strategy, and charge a price equal to $c$ from $t+1$ onward, the maximum price $q_t^d$ that a defector can charge must leave consumers without the good indifferent between buying the good today and keeping it for two periods, or buying the good tomorrow at a lower price:

$$(1+\alpha\delta)v_h - q_t^d + \delta^2 V_h^2 = \delta((1+\alpha\delta)v_h - c + \delta^2 V_h^2) \quad \Rightarrow \quad q_t^d = \frac{(1+\alpha\delta)v_h + \delta c}{1+\delta}$$

recall that $V_h^2 = ((1+\alpha\delta)v_h - c)/(1-\delta^2)$. The lower the price charged after a defection, the higher is the punishment inflicted to the defector. The lowest price that a defector can charge for any harsher punishment strategy is $p_t^d = v_h$. At this price, a defector sells the good only to consumers without the good, who then scrap the unit to purchase new units free of charge. The defector, however, may have an incentive to further lower the price of new units to sell to all high type consumers. If so, the defection price should make the consumers with a used good indifferent between buying a new unit today (and buy it again tomorrow) or keeping the used unit, i.e. $q_t^d = (1-\alpha)v_h$. The profit in the first scenario is higher if $c > (1-2\alpha)v_h$, which is satisfied whenever $c > (1-\alpha)v_h$. In this scenario, the smallest $\gamma$ that sustains collusion without second-hand market is:

$$\gamma^\circ = \frac{v_h - c}{(1+\alpha\delta)v_h - c} > \frac{(1-\alpha)\, v_h + \alpha v_l - c}{(1-\alpha)\, v_h + \alpha\,(1+\delta)\, v_l - c} = \gamma^*$$

Thus, the first part of the proposition is proved. Notice that $\gamma^* = \gamma^\circ$ if $c = (1-\alpha)v_h$.

2. If $\alpha \in [g(c, v_h)/2, g(c, v_h)]$, provided that $q_{t+1}^n \leq c$, consumers with a used unit always prefer to buy a new unit and scrap the used one. Hence, the maximum price that a defector can charge is $q_t^d = v_h$. The defector does not have any incentive to further lower the price to sell to all consumers, because of part 1. Thus, $\gamma^* > \gamma^\circ$.

3. If $\alpha \in [g(c, v_h)/(2+\delta), g(c, v_h)/2]$, a defector has an incentive to further lower the price of new unit to sell to all consumers with high valuation (including those who own a used unit). If so, $\gamma^\circ = \frac{2((1-\alpha)v_h - c)}{(1+\alpha\delta)v_h - c} > \gamma^*$ defines a quadratic inequality in $\alpha$. Such inequality must have a root, $k(c, v, \delta)$, in the relevant interval since it is satisfied at one extreme point of the interval and fails on the other. Thus, $\gamma^\circ > \gamma^*$ for any $\alpha < k(c, v, \delta)$.

4. If $\alpha < g(c, v_h)/(2+\delta)$, firms maximize profits by selling units to all high value consumers in every period at price $q^m = (1-\alpha)v_h$. If so, any intertemporal link in the demand

disappears. Thus, no extra punishment can be inflicted to a defector after a deviation and $\gamma^{\circ} = 1 > \gamma^{*}$.

■

**Proof of Proposition 6.** To prove (1), two relevant scenarios have to be considered. In the first, the monopolist leases both new and used units. The highest leasing fee that a monopolist can charge for a used unit leaves low value types indifferent between renting a used unit or remaining without. Thus, $l^u = \alpha v_l$ and the leasing fee for a new product can be at most $l^n = (1-\alpha)v_h + \alpha v_l$. If so, monopoly profits with leasing exceed those from sales:

$$(1-\alpha)v_h + 2\alpha v_l - c > (1-\alpha)\, v_h + (1+\delta)\, \alpha v_l - c$$

In the second scenario, the monopolist chooses not to rent used units and charges $l^n = l^u = v_h$. The latter strategy is preferred if $v_h > (1-\alpha)v_h + 2\alpha v_l$ iff $v_h \geq 2v_l$.[13] In this case no used units are sold to low types. Again the monopoly profits under leasing are strictly higher than with selling.

With a buy-back policy the monopolist commits to a buy-back price $b$. Such a price cannot be smaller than $\alpha v_l$, since no consumer would sell a used good to a firm below this price, and the policy would not be binding. The maximum price for new units that a high type is willing to pay is defined by:

$$v_h - p^n + \delta b = \max\{\alpha v_h - b, 0\}$$

There are two relevant cases. If the monopolist commits to $b \in [\alpha v_l, \alpha v_h]$, then in each period he sets a price $p^n = (1-\alpha)\, v_h + (1+\delta)b$ and his profits are $p^n - c - b$ where $b$ is the cost of buying back all the used goods available. The profit function is strictly increasing in $b$, since $\partial[p^n - c - b]/\partial b = \delta > 0$. Hence, within this range the optimal buy-back price is $b = \alpha v_h$. If $b > \alpha v_h$, the monopoly price becomes $p^n = v_h + \delta b$, the cost of the policy remains $b$ and profits are strictly decreasing in $b$. Thus, the optimal strategy for a monopolist is to set $b^m = \alpha v_h$ and $p^m = (1+\alpha\delta)\, v_h$. In each period, high value consumers trade in used units for new ones. The monopoly profits are $(1-\alpha+\alpha\delta)\, v_h - c$. Monopoly profits are higher than selling if $(1-\alpha+\alpha\delta)\, v_h \geq (1-\alpha)\, v_h + (1+\delta)\, \alpha v_l$ iff $\delta v_h \geq (1+\delta)v_l$, which establishes (3). To prove part (2) consider two scenarios: if $\delta v_h < (1+\delta)v_l$ then the statement follows from part (1); and if $\delta v_h \geq (1+\delta)v_l$ then we have $v_h > (1-\alpha+\alpha\delta)\, v_h$. ■

**Proof of Proposition 7.** With leasing there is a competitive MPE in which all firms run zero profits. In such a MPE, high types lease new units, low types lease old units, there is

[13] Again we shall impose restrictions on primitives to guarantee that a monopolist never finds it optimal to sell to both types.

always a unit stock of used units and prices satisfy:

$$l_c^u = \min\left\{\alpha v_l, \frac{c-(1-\alpha)\,v_l}{1+\varphi}\right\} \quad \& \quad l_c^n = c - \varphi l_c^u$$

No firm could benefit by undercutting on the rental price of used units price, as it leases all of its stock. Similarly no firm could benefit by increasing the price of used units, since its demand would plummet to zero. As for the price of new units: no firm could benefit by undercutting since it would run a loss on all units leased for both periods; and no firm could benefit by increasing the price, since it would sell no units. If after a deviation the stock of used units goes to $1+\lambda$, we shall impose $l_c^u = 0$, since an excess supply of used units would flood the rental market. With buy-back, the competitive MPE prescribes to sell at marginal cost without buy-back policy. Clearly, such a policy is optimal if employed by others, since no deviation by a competitor could ever be profitable.

For notational convenience, let $x = (1-\alpha)v_h - c$ and let $y = \alpha v_l$. Recall that assumption (a2) implies $x + y > 0$. As before we shall measure the incentives to collude by the fraction of the collusive profits that can be captured by a defector at the deviation stage, $\gamma = \Pi^d/\Pi^m$. This is again the relevant measure of the incentives to collude, since a deviating player is guaranteed a present discounted payoff of 0 after a defection by the sharp trigger strategy. First, consider an environment with leasing. If $v_h \geq 2v_l$, in the collusive outcome firms scrap old units and only lease new ones to high types. Thus, an arbitrarily small deviation from $l^n = v_h$ captures the entire surplus. If so, intertemporal linking breaks down ($\gamma^L = 1$) and leasing reduces the incentives to collude compared to selling. If $v_h < 2v_l$ however, some intertemporal linking remains since no deviation can appropriate the entire surplus in the used goods markets as the stock of used good owned by a firm is sunk. Thus the relevant profit ratio becomes:

$$\gamma^L = \frac{x + (1 + 1/G)y}{x + 2y}$$

Thus, leasing reduces the incentives to collude compared to selling if $1 - 1/G < \delta$, since:

$$\gamma^L = \frac{x + (1 + 1/G)y}{x + 2y} > \frac{x + y}{x + (1+\delta)y} = \gamma^*$$

In an environment with buy-back instead, a deviator cannot appropriate the entire collusive surplus since the profits of a deviating firm are reduced by the expected drop in buy-back prices in the first period after a defection. Given assumption (a2), the profits of the monopoly buy-back policy are positive if $\delta v_h > v_l$, since $x + (v_h/v_l)\delta y > 0$. If so, the relevant profit ratio becomes:

$$\gamma^B = \frac{x}{x + (v_h/v_l)\delta y}$$

Hence, if $\delta v_h > v_l$, a buy-back policy increases the incentives to collude both compared to selling and compared to leasing since:

$$\gamma^B = \frac{x}{x + (v_h/v_l)\delta y} < \frac{x+y}{x+(1+\delta)y} = \gamma^*$$
$$\gamma^B = \frac{x}{x + (v_h/v_l)\delta y} < \frac{x+(1+1/G)y}{x+2y} = \gamma^L$$

The proof did not explicitly establish that both sharp trigger strategies exist (i.e. that such harsh short term punishments exist). However, it would be immediate to provide sufficient conditions for the existence of such strategies, as done in lemma 4. ■

# References


[1] Anderson, S. and V. Ginsburgh, 1994, "Price discrimination via Second-Hand Markets", European Economics Review, 38:1, pp 23-44.

[2] Ausubel, L. and R. Deneckere, 1987, "One is almost enough for monopoly", *RAND Journal of Economics*, 18:2, pp 255-274.

[3] Bernheim, D. and M. D. Whinston, 1990, "Multimarket Contact and Collusive Behavior", *RAND Journal of Economics*, 21:1, pp 1-26.

[4] Bresnahan, T. F., 1987, "Competition and Collusion in the American Automobile Industry: 1955 Price War". *RAND Journal of Economics*, 25:4, pp 457-482.

[5] Bullow, J., 1982, "Durable Goods Monopolists", *Journal of Political Economy*, 90:2, pp 314-332.

[6] Coase, R., 1972, "Durability and Monopoly", *Journal of Law and Economics*, 15:1, pp 143-149.

[7] Dana, J. D. and Y. F. Fong, 2006, "Long-Lived Consumers, Intertemporal Bundling, and Tacit Collusion", *Mimeo.*

[8] Dutta, P., A. Matros and J. Weibull, 2007, "Long-run price competition", *RAND Journal of Economics*, 38:2, pp 291-313.

[9] Fudenberg, D. and J. Tirole, 1998, "Upgrades, Trade-ins, and Buybacks", *RAND Journal of Economics*, 29:2, pp 235-258.

[10] Gul, F., 1987, "Noncooperative Collusion in Durable Goods Oligopoly", *RAND Journal of Economics*, 18:2, pp 248-254.

[11] Hendel, I. and A. Lizzeri, 1999a, "Interfering with Secondary Markets", *RAND Journal of Economics*, 30:1, pp 1-21.

[12] Hendel, I. and A. Lizzeri, 1999b, "Adverse Selection in Durable Goods Market", *American Economic Review*, 89, pp 1097-1115.

[13] Johnson, J. and M. Waldman, 2002, "Leasing, Lemons, and Buy-Backs", *RAND Journal of Economics*, 34:2, pp 247-265.

[14] Nava, F. and Schiraldi P., 2011, "Sales and Collusion in a Dynamic Market with Heterogeneous Storage Technologies", *Mimeo LSE.*

[15] Porter, R. and P. Sattler, 1999, "Pattern of Trade in the Market for Used Durables: Theory and Evidence", *NBER Working Paper*, no 7149.

[16] Rust, J., 1986, "When Is It Optimal to Kill Off the Market for Used Durable Goods", *Econometrica*, 54:1.

[17] Schiraldi, P., 2006, "Buyback Policy and the Second-Hand Market for Durable Goods", *Ph.D. Thesis* Boston University.

[18] Schiraldi, P., 2011, "Automobile Replacement: A Dynamic Structural Approach", RAND Journal of Economics, 42:2, pp 266-291.

[19] Stolyarov, D., 2002, "Turnover of Used Durables in a Stationary Equilibrium: Are Older Goods Traded More?", *Journal of Political Economy*, 110:6, pp 1390-1413.

[20] Swan, P., 1980, "Alcoa: The Influence of Recycling on Monopoly Power", *Journal of Political Economy*, 88:1, pp 76-99.

[21] Waldman, M., 1996, "Durable Goods Pricing When Quality Matters", *Journal of Business*, 69:4, pp 489-510.

[22] Waldman, M., 1997, "Eliminating the Market for Secondhand Goods: An Alternative explanation for Leasing", *Journal of Law and Economics*, 40:1, pp 61-92.

[23] Waldman, M., 2003, "Durable Goods Theory for Real World Markets", *Journal of Economic Prospectives*, 17:1, pp 131-154.