Buying frenzies in durable-goods markets

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Abstract

We explain why a durable-goods monopolist would like to create a shortage during the launch phase of a new product. We argue that this incentive arises from the presence of a second-hand market and uncertainty about consumers' willingness to pay for the good. Consumers are heterogeneous and initially uninformed about their valuations but learn about them over time. Given demand uncertainty, first period sales may result in misallocation and lead to active trading on the secondary market after the uncertainty is resolved. Trading on the second-hand market will generate additional surplus. This surplus can be captured by the monopolist ex-ante because consumers are forward-looking, and the price they are willing to pay incorporates the product's resale value. As a consequence, when selling to uninformed consumers, the monopolist faces the trade-off between more sales today and a lower profit margin. Specifically, because the product's resale value is negatively related to the stock of the good in the second-hand market, selling more units today will result in a lower equilibrium price of the product. Therefore, the monopolist may find it optimal to create a shortage and ration consumers to the second period. We characterize conditions under which the monopolist would like to restrict sales and generate buying frenzies.

Keywords: Buying frenzies, second-hand market, durable goods, consumer uncertainty.

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1 No Commitment Case

The following section provides comprehensive proof for proposition 5 in the main paper. The analysis of No Commitment involves several steps where we first solve for the monopolist's second period decision and then we procceed to its decision in period one.

Second period. Given that q_1 units have been sold in period one, the monopolist chooses p_2 and q_2 to maximize its second period profit

$$\pi_2(p_2, q_2) = (p_2 - c)q_2$$

subject to the market clear condition (**??**) and the boundary condition $q_2 \geq 0$. Substitute $q_2 = G(p_2) - q_1$ into the monopolist's profit function and take the derivative with respect to p_2. The interior solutions for p_2 and q_2 are determined by

$$p_2 - c = \frac{1 - G(p_2) - q_1}{g(p_2)} \tag{1}$$

and the market clear condition (**??**).

Lemma 1 *For a given q_1, (1) yields a unique solution for p_2. In addition, p_2 is decreasing in q_1.*

Proof of Lemma 1: We show log concavity of $g(p_2)$ implies the right hand side of (1) is decreasing in p_2. By Theorem 3 in Bagnoli and Bergstrom (2005, Economic Theory), if $g(p_2)$ is log concave, $1 - G(p_2)$ is also log concave. Let $\overline{G}(p_2) \equiv 1 - G(p_2)$. The second derivative $\frac{d^2 \ln \overline{G}(p_2)}{dp_2^2}$ is

$$\frac{-g'(p_2)\overline{G}(p_2) - g(p_2)^2}{\overline{G}(p_2)^2}.$$

So, log concavity of $\overline{G}(p_2)$ implies

$$-g'(p_2)\overline{G}(p_2) \leq g(p_2)^2. \tag{2}$$

Take the derivative of the right hand side of (1) with respect to p_2, we have

$$\frac{-g(p_2)^2 - \left[\overline{G}(p_2) - q_1\right] g'(p_2)}{g(p_2)^2}. \tag{3}$$

So, (3) is nonpositive if and only if

$$-g'(p_2)(\overline{G}(p_2) - q_1) \leq g(p_2)^2. \tag{4}$$

Condition (2) implies condition (4). To see this, suppose $g'(p_2) < 0$. Given

$$0 \leq \overline{G}(p_2) - q_1 \leq \overline{G}(p_2),$$

(4) is satisfied if (2) is satisfied. If $g'(p_2) \geq 0$, (4) is automatically satisfied.

Now, we show there is a unique solution for p_2. The left hand side of (1) is increasing in p_2 and its value falls in the range of $[-c, \overline{\theta} - c]$ as p_2 ranges from 0 to $\overline{\theta}$. The right hand side of (1) is decreasing in p_2 and its value falls in the range of $\left[\frac{1-q_1}{g(0)}, \frac{-q_1}{g(\overline{\theta})}\right]$. Because $\frac{1-q_1}{g(0)} > -c$ and $\frac{-q_1}{g(\overline{\theta})} < \overline{\theta} - c$, (1) has a unique solution for p_2.

Lastly, we show $\frac{dp_2}{dq_1} \leq 0$. Total differentiate (1), it follows that

$$\frac{dp_2}{dq_1} = \frac{-1}{2g(p_2) + (p_2 - c)g'(p_2)}. \tag{5}$$

Substitute $p_2 - c$, the denominator of (5) becomes

$$2g(p_2) + \frac{(\overline{G}(p_2) - q_1)g'(p_2)}{g(p_2)}. \tag{6}$$

By (2), (6) is at least $g(p_2) \geq 0$. Consequently, $\frac{dp_2}{dq_1} \leq 0$. Q.E.D.

Given Assumption **??**, by Theorem 3 in Bagnoli and Bergstrom (2005), the function $1 - G(p_2)$ is also log concave, which implies the right hand side of (1) is decreasing in p_2. Hence, for a given q_1, (1) yields a unique solution for p_2. In addition, equations (1) implies that p_2 is decreasing in q_1 because the monopolist has to face more intense competition against resellers when it sells more in the first period. We summarize the monopolist's optimal second-period decisions and the corresponding profit by the following lemma. Denote $\widetilde{q}_1 \equiv 1 - G(c)$.

Lemma 2 *When $q_1 \geq \widetilde{q}_1$, the monopolist does not produce and sell in period two and will make zero second period profit. The equilibrium secondary market price is determined by $p_2 = G^{-1}(1 - q_1)$. When $q_1 < \widetilde{q}_1$, the monopolist charges p_2 and produce q_2 units, where p_2 and q_2 are determined by (1) and (**??**). Its corresponding profit is $\pi_2 = (p_2 - c)^2 g(p_2)$.*

Proof of lemma 2: Step 1. We show $q_2 = 0$, for $q_1 \geq \widetilde{q}_1$ by contradiction. Suppose $q_2 = q_2^* > 0$ when

$q_1 \geq \widetilde{q}_1$. The second-period price is pinned down by the market clear condition

$$1 - G(p_2) = q_1 + q_2^*.$$

Because $q_1 \geq \widetilde{q}_1$, it follows that

$$\begin{aligned} \beta[1 - F_o(p_2)] + (1-\beta)[1 - F_p(p_2)] &\geq \beta(1 - F_o(c)) + (1-\beta)(1 - F_p(c)) + q_2^* \\ \beta[F_o(c) - F_o(p_2)] + (1-\beta)[F_p(c) - F_p(p_2)] &\geq q_2^*. \end{aligned} \tag{7}$$

Inequality (7) will hold only if $p_2 < c$. To see this, suppose $p_2 \geq c$. Because $F_o(\theta)$ and $F_p(\theta)$ are strictly increasing, we have $F_o(c) \leq F_o(p_2)$ and $F_p(c) \leq F_p(p_2)$. As a consequence

$$\beta[F_o(c) - F_o(p_2)] + (1-\beta)[F_p(c) - F_p(p_2)] \leq 0.$$

This implies $q_2^* \leq 0$. A contradiction. We conclude that if $q_2 > 0$, p_2 must be lower than c. However, given $p_2 < c$, the monopolist must choose $q_2 = 0$. A contradiction. When $q_2 = 0$, the monopolist makes zero second-period profit and p_2 is pinned down by $1 - G(p_2) = q_1$.

Step 2. Consider $q_1 < \widetilde{q}_1$. First, ignore the constraint $q_2 \geq 0$. We will show the constraint $q_2 \geq 0$ is not binding at the optimal p_2. The second-period price and quantity are determined by (1) and (**??**). Condition (1) implies that p_2 is decreasing in q_1. By step 1, we have $p_2 = c$ at $q_1 = \widetilde{q}_1$. Hence, $p_2 > c$ when $q_1 < \widetilde{q}_1$. Given that the profit margin is positive, the monopolist must choose $q_2 > 0$. Substitute $q_2 = (p_2 - c)g(p_2)$, the equilibrium profit becomes $\pi_2 = (p_2 - c)^2 g(p_2)$, where p_2 is determined by (1). Q.E.D.

The monopolist will stop producing the good in period two if q_1 is at least $\widetilde{q}_1$. The reallocation of q_1 units through the secondary market will drive the marginal consumer's maximum willingness to pay to be, at most, the marginal cost. As a result, the monopolist will make a loss from production. When q_1 is less than $\widetilde{q}_1$, the second-period price is higher than the marginal cost and therefore the monopolist will continue to produce in the second period.

First Period We are now ready to solve for the monopolist's optimal strategy in the first period. A type-i, $i = o, p$, consumer's purchase decision is the same as that in the case when the monopolist has the commitment power. So, the consumer will buy the good in period one if and only if

$$p_1 \leq E_i(\theta) + p_2. \tag{8}$$

It is worthwhile to emphasize that when the consumer decides whether to purchase the good in period one, he needs to correctly form the expectation of the second period price based on the monopolist's announcement of q_1. Specifically, the consumer expect p_2 to be determined by $1 - G(p_2) = q_1$ when the announced q_1 is at least $\widetilde{q}_1$. And, they expect p_2 to be determined by (1) when the announced q_1 is smaller than $\widetilde{q}_1$.

Similar to the main model with Commitment, the monopolist either charges $E_p(\theta) + p_2$ to attract both types of consumers or $E_o(\theta) + p_2$ to sell to optimistic consumers exclusively. Based on the insight of the main model, buying frenzies occur only when it is optimal for the monopolist to attract both types of consumers. So, we focus on this case and study how the condition for buying frenzies to occur change when the monopolist lacks commitment power.

At price $p_1 = E_p(\theta) + p_2$, the monopolist chooses q_1 to maximize its expected profit. Its profit function is continuous in q_1 but has a kink at $\widetilde{q}_1$. So, it is not differentiable everywhere. We divide the monopolist's first period problem into two regimes, Regime A and B. Regime A is the range $q_1 \in [0, \widetilde{q}_1]$ and regime B is the range $q_1 \in (\widetilde{q}_1, 1]$. Let q_1^i, $i = A, B$, denote the optimum in regime i. The monopolist will choose the first period output $q_1^* \in \{q_1^A, q_1^B\}$ that results in the highest profit.

The monopolist's optimization problem in regime A is to choose q_1 to maximize

$$\pi_A(q_1) = (E_p(\theta) + p_2 - c)q_1 + (p_2 - c)^2 g(p_2), \tag{9}$$

where p_2 is a function of q_1 and is implicitly determined by (1) subject to $0 \leq q_1 \leq \widetilde{q}_1$. In Regime B, the monopolist will not produce the good in period two. Anticipating this, it chooses q_1 to maximize

$$\pi_B(q_1) = (E_p(\theta) + p_2 - c)q_1, \tag{10}$$

subject to

$$1 - G(p_2) = q_1. \tag{11}$$

and

$$\widetilde{q}_1 \leq q_1 \leq 1.$$

Next, we characterize conditions under which buying frenzies occur.

Assumption 1 $-(p_2 - c)[\beta f_o^{''}(p_2) + (1 - \beta) f_p^{''}(p_2)] \leq 3[\beta f_o'(p_2) + (1 - \beta) f_p'(p_2)]$.

Assumption 1 implies the second period price p_2 in Regime A is concave in q_1. This is a sufficient condition for the concavity of $\pi_A(q_1)$. Now we are ready to proof the proposition **??**. First, we characterizes the sufficient and necessary condition for buying frenzies to arise and then we prove that the monopolist will ration at least the same number of consumers, q_{1c}^r, to the second period as it would like to when it has commitment power. Notice, that assumption 1 is violated, the condition characterized in Proposition **??** is sufficient but may not be necessary.

Proof of Proposition ?? Let q_1^* denote the seller's optimal output in period one. We first show $E_p(\theta) - \dfrac{1}{g(0)} < c$ is a sufficient condition for $q_1^* < 1$. By lemma 2, when $\widetilde{q}_1 \leq q_1$, the seller earns zero profit in period two. The derivative of $\pi_B(q_1)$ with respect to q_1 is

$$\pi'_B(q_1) = E_p(\theta) + p_2 - c + \frac{dp_2}{dq_1} q_1.$$

Total differentiate (11), we have

$$\frac{dp_2}{dq_1} = \frac{-1}{g(p_2)} \leq 0. \tag{12}$$

Substitute (12), it follows that

$$\pi'_B(q_1) = E_p(\theta) + p_2 - c - \frac{q_1}{g(p_2)}. \tag{13}$$

Evaluate $\pi'_B(q_1)$ at $q_1 = 1$. By (11), p_2 is decreasing in q_1 and becomes zero when $q_1 = 1$. Hence, $\pi'_B(q_1)|_{q_1=1} = E_p(\theta) - c - \dfrac{1}{g(0)}$. Given $E_p(\theta) - \dfrac{1}{g(0)} < c$, $\pi'_B(q_1)|_{q_1=1} < 0$. Consequently, the seller can make more profit by undercutting q_1 below 1.

Next, we show $E_p(\theta) - \dfrac{1}{g(0)} < c$ is a necessary condition for $q_1^* < 1$. First, we show the seller's total profit function is concave in q_1, for $q_1 \in (\widetilde{q}_1, 1]$ (Regime B). Take the second derivative $\pi_B(q_1)$ and substitute q_1 defined by (11), $\pi''_B(q_1)$ becomes

$$\frac{dp_2}{dq_1}\left[1 + \frac{g'(p_2)[1 - G(p_2)]}{g(p_2)^2}\right] - \frac{1}{g(p_2)}.$$

Given $\dfrac{dp_2}{dq_1} \leq 0$ and (2), we have $\pi''_B(q_1) \leq 0$.

Now, we show the monopolist's profit function is concave in q_1, for $q_1 \in [0, \widetilde{q}_1)$ (Regime A). Take the derivative of $\pi_A(q_1)$ with respect to q_1, it follows that

$$\pi'_A(q_1) = E_p(\theta) + p_2 - c + [q_1 + 2(p_2 - c)g(p_2) + (p_2 - c)^2 g'(p_2)] \frac{\partial p_2}{\partial q_1}.$$

By (5), we can rearrange $\pi'_A(q_1)$ as

$$E_p(\theta) + q_1 \frac{\partial p_2}{\partial q_1}.$$

The second derivative is therefore

$$\pi''_A(q_1) = \frac{\partial p_2}{\partial q_1} + q_1 \frac{\partial^2 p_2}{\partial q_1^2}.$$

By Lemma 1, $\frac{dp_2}{dq_1} \leq 0$. Hence, a sufficient condition for $\pi''_A(q_1) \leq 0$ is $\frac{\partial^2 p_2}{\partial q_1^2} \leq 0$. From (5), we derive

$$\frac{d^2 p_2}{dq_1^2} = \frac{3g'(p_2) + (p_2 - c)g''(p_2)}{\{2g(p_2) + (p_2 - c)g'(p_2)\}^2} \frac{dp_2}{dq_1}.$$

Given Assumption 1, $\frac{d^2 p_2}{dq_1^2} \leq 0$.

Finally, we show the monopolist's total profit function is continuous and globally concave in q_1. Notice that in Regime A, when $q_1 = \widetilde{q}_1$, it must be true that $p_2 = c$. To see this, suppose $p_2 > c$ at $q_1 = \widetilde{q}_1$. Since $F_o(.)$ and $F_p(.)$ are increasing functions, $1 - G(p_2) < \widetilde{q}_1$. Hence, the right hand side of (1) is negative which contradicts the assumption that $p_2 > c$. Because the seller makes zero profit in the second period when $q_1 = \widetilde{q}_1$, his profit in Regime A at $\widetilde{q}_1$ is the same as his profit in regime B at the same value of q_1. Hence, the seller's profit function is continuos at $\widetilde{q}_1$.

Given that the seller's profit function is concave in regimes A and B, it is globally concave if $\pi'_A(q_1)|_{q_1=\widetilde{q}_1} \geq \pi'_B(q_1)|_{q_1=\widetilde{q}_1}$. Evaluate 13 at $q_1 = \widetilde{q}_1$ and substitute $p_2 = c$, it follows that

$$\pi'_B(q_1)|_{q_1=\widetilde{q}_1} = E_p(\theta) - \frac{\widetilde{q}_1}{g(c)}.$$

Similarly,

$$\pi'_A(q_1)|_{q_1=\widetilde{q}_1} = E_p(\theta) - \frac{\widetilde{q}_1}{2g(c)}.$$

Clearly, $\pi'_A(q_1)|_{q_1=\widetilde{q}_1} \geq \pi'_B(q_1)|_{q_1=\widetilde{q}_1}$. Because the profit function is globally concave in q_1, $E_p(\theta) - \frac{1}{g(0)} < c$ is a necessary condition for $q_1^* < 1$.

So far, we have shown that when it is optimal for the monopolist to attract both types of consumers, the condition for buying frenzies occur remains unchanged even when the monopolist lacks commitment power. To prove the second part of the proposition, we first, suppose that the optimal sales in buying frenzies is $q_1^B \in [\widetilde{q}_1, 1)$ (Regime B). By (13) and the market clear condition (11), the first order condition is

$$E_p(\theta) + G^{-1}(1 - q_1^B) - c = \frac{q_1^B}{g(G^{-1}(1 - q_1^B))}.$$

Consider that monopolist can commit to future price and quantity, buying frenzies occurs when the monopolist sells to both types of consumers in period one and ration consumers. Let q_{1c} denote the optimal sales in buying frenzies when the monopolist has the commitment power. By Lemma **??**, when the monopolist rations consumers in period one, the optimal p_2 is determined by $E_p(\theta)+p_2-c=\frac{1-G(p_2)}{g(p_2)}$. Using the market clear condition $G(p_2)=1-q_1$, the optimal sales in period one is pinned down by $E_p(\theta)+G^{-1}(1-q_{1c})-c=\frac{q_{1c}}{g(G^{-1}(1-q_{1c}))}$ and is identical to q_1^B. Therefore, in this case, the same number of consumers are rationed to the second period as in the case when the monopolist has commitment power.

Now, suppose that the optimal sales in buying frenzies is $q_1^A \in (0, \widetilde{q}_1)$ (Regime A). Since $q_1^A < \widetilde{q}_1 < q_{1c} = q_1^B$, it follows that $1-q_1^A > 1-q_{1c} = q_{1c}^r$. Q.E.D.