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Optimal monetary policy responses to relative-price changes☆

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Abstract

An optimizing model, with a flexible-price sector and a sticky-price sector, is presented to analyze the effects of relative-price changes on inflation fluctuations. The relative price of the flexible-price good represents a shift parameter of the New Keynesian Phillips curve. The optimal monetary policy is to target sticky-price inflation, rather than a broad inflation measure. Although stabilizing the relative price around its efficient value is one of the appropriate goals of the central bank, stabilizing sticky-price inflation is sufficient for achieving this goal. An optimal monetary policy for a small open economy is also discussed. © 2001 Published by Elsevier Science B.V.

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1. Introduction

Relative prices—for example, of food and energy—, are often discussed in studies of inflation control for two reasons. First of all, relative prices are often

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used as measures of "supply shocks" in "Phillips curve" equations that seek to model the short run output-inflation trade-off.[1] In the empirical literature on the Phillips curve, changes in the relative prices of food and energy are commonly used as a measure of supply shocks, which shift the short-run Phillips curve. Second, many authors have sought to identify a more persistent component of inflation, known as "core inflation".[2] For the conduct of monetary policy, core inflation is considered a more important indicator than broader inflation measures. In this literature, fluctuations in the prices of food and energy are regarded as a transitory component of overall movements in inflation, since they are thought to be caused mainly by temporary, sector-specific shocks. Based on this idea, it is a common practice to subtract the prices of food and energy from an aggregate inflation measure to calculate a measure of core inflation.

However, it is not obvious how changes in relative prices affect aggregate inflation. Strictly speaking, a pure relative disturbance is a change in supply or demand conditions that leaves the appropriately defined aggregate production possibility frontier unchanged. In the absence of price stickiness, this shock should not change aggregate real output and the aggregate price level.[3] It is also not obvious how relative-price changes are related to supply shocks. Large changes in relative prices are not necessarily caused by large supply shocks. Relative prices are affected by several factors other than supply shocks, such as demand shocks and elasticities of substitution among goods. These arguments suggest that the appropriate measures of supply shocks and core inflation should be based on a structural model which identifies the factors that affect relative-price changes and the persistent component of aggregate inflation.

In this paper, we construct a two-sector dynamic general equilibrium model, with a flexible-price sector and a sticky-price sector. The model is a variant of optimizing models with nominal price stickiness, that have recently been used in the literature on inflation dynamics and monetary policy.[4] Using this model we discuss the correct specification of the Phillips curve in the presence of sectoral shocks, and show in what way changes in the relative price of flexible-price good shift the short-run Phillips curve. We also show that the inflation in the sticky-price sector represents a persistent component of inflation, in the

[1] See, for example, Ball and Mankiw (1995) and Roberts (1995).

[2] See, for example, Bryan and Cecchetti (1994), and Cecchetti (1997).

[3] Gordon (1975) is a classic paper which studies the interaction between the relative prices of food and energy and aggregate inflation. A paper by Ball and Mankiw (1995) argues that, under the existence of menu costs for changing prices, the skewness of the distribution of relative price changes is positively related to aggregate inflation.

[4] See, for example, Clarida et al. (1999), Woodford (1996), Rotemberg and Woodford (1997, 1999). Although their focus is different from ours, Ohanian et al. (1995) construct a model with a sticky-price sector and a flexible-price sector.

sense that it responds to smoothed expectations of the future output gaps and relative-price changes. Inflation in the sticky-price sector is therefore a good candidate for a measure of core inflation.

Another important question is how a central bank should conduct monetary policy in the presence of sector-specific shocks that affect the efficient relative prices of the different types of goods. The central bank has a choice among several different possible measures of inflation and output gap, and it must identify which variables are the appropriate goal variables. Using an optimizing model has an important advantage; namely, it allows us to evaluate alternative monetary policies in a welfare-theoretic framework, and to analyze which variables should be stabilized in the optimal equilibrium. The paper shows that the optimal monetary policy is characterized as an inflation targeting regime, that incorporates the correctly chosen inflation measure. It is also found to be desirable to stabilize core inflation, rather than a broader measure of inflation, where core inflation is identified as an index of inflation in the sticky-price sector. The paper demonstrates that stabilizing the relative price of the flexible-price good around its time-varying optimal value is one of the appropriate goals for the central bank. However, the model implies that stabilizing core inflation is sufficient for keeping the relative price at its efficient value. We also address the issue on the output gap-inflation variability trade-off, which has been an important guiding principle in studies of monetary policy. Our model implies that there is a trade-off between stabilizing the aggregate output gap and aggregate inflation, but that there is no trade-off between stabilizing aggregate output gap and stabilizing core inflation. Thus, in this model, whether output gap-inflation variability trade-off exists or not depends on which measures of inflation and output gap the central bank chooses to stabilize.

The organization of the paper is as follows. Section 2 presents a two sector dynamic sticky-price model, and studies the interaction between relative price and inflation determination. Section 3 derives the optimal monetary policy for the economy described in the model, and applies the model to an small open economy setting. Section 4 concludes.

2. Model

2.1. Utility of a representative household

In this section we construct the model which will be used through the paper. Many of the goods that exhibit large price variability are standardized goods, for example, food and energy. Such goods are traded in almost competitive markets, and their prices are adjusted frequently. On the other hand, in markets where goods are differentiated, prices are adjusted slowly. To capture this fact, our model assumes flexible-price goods which are traded in a competitive market, and a continuum of monopolistically produced goods.

The model is a variant of dynamic sticky price model (Clarida et al., 1999, Woodford, 1996; Rotemberg and Woodford, 1997, 1999). There is one type of flexible-price good, denoted by C_F, and a continuum of differentiated goods $c(z)$ indexed in $z \in [0, 1]$. Each household consumes the flexible-price good and all of the differentiated goods, and produces a single good. The objective of household i is to maximize[5]

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u(B_{i,t} C_t^i) - v(A_{i,t} y_t^i) \right], \tag{1}$$

where $u(\cdot)$ represents the utility of consumption and $v(\cdot)$ represents the disutility of production. We make the usual assumptions that $u(\cdot)$ is increasing and concave, and that $v(\cdot)$ is increasing and convex. The constant $\beta \in (0, 1)$ is the discount factor, and the argument C_t^i, which represents an index of household i's purchases of the flexible-price good and all of the continuum of the differentiated goods, is defined as

$$C_t^i = \frac{(C_{S,t}^i)^{\gamma} (C_{F,t}^i)^{1-\gamma}}{\gamma^{\gamma} (1-\gamma)^{1-\gamma}}, \tag{2}$$

where

$$C_{S,t}^i = \left[\int_0^1 c_t^i(z)^{\frac{\theta-1}{\theta}} \, \mathrm{d}z \right]^{\theta/(\theta-1)}. \tag{3}$$

The elasticity of substitution between any two differentiated goods is θ which is assumed to be greater than one.[6] The argument y_t^i is the output of the good that household i produces. The household indexed by i produces one type of good i, where $i = \mathrm{F}$ if it produces the flexible-price good, and $i \in [0, 1]$ if it produces one of the differentiated goods. $A_{i,t}$ and $B_{i,t}$ are stationary random shocks. Since we wish to focus our analysis on the effects of sector-specific supply shocks on the economy, we assume $A_{i,t} = A_{F,t}$ for all households producing the flexible-price good and $A_{i,t} = A_{S,t}$ for all households producing one of the differentiated goods. We also assume that the preference shock $B_{i,t}$ is identical across all households. These assumptions imply that all of the households in the same sector face the same supply shocks, and that there are no idiosyncratic demand shocks in this economy. Eq. (2) also abstracts from any sector-specific demand shocks.

[5] Following Rotemberg and Woodford (1997), we can safely abstract from the liquidity services provided by money when monetary policy takes a form of interest rate rule, as we consider later.

[6] Here, the elasticity of substitution between the flexible-price good and the composite differentiated good is unity.

2.2. The consumption decision

The model assumes complete financial markets with no obstacles to borrowing against future income, so that each household faces a single intertemporal budget constraint. The model further assumes that households can insure one another against idiosyncratic income risk. These assumption imply that, if all households have identical initial wealth, they will choose identical consumption plans.[7] The optimal allocation for a given level of nominal spending across the flexible-price good and all of the differentiated goods at time t leads to the Dixit–Stiglitz demand relations as functions of relative prices. For the following the index i is suppressed, since the consumption decision is identical across all households. The total expenditure required to obtain a given level of consumption index C_t is given by P_tC_t, where P_t is defined as

$$P_t = (P_{S,t})^{\gamma}(P_{F,t})^{1-\gamma}. \tag{4}$$

Here $P_{S,t}$ is the price index of the composite differentiated good defined below, and $P_{F,t}$ is the price of the flexible-price good. Demand for the flexible-price good is then given by

$$C_{F,t} = (1-\gamma)\left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t. \tag{5}$$

On the other hand, demand for the composite differentiated good is given by

$$C_{S,t} = \gamma\left(\frac{P_{S,t}}{P_t}\right)^{-1} C_t, \tag{6}$$

where $P_{S,t}$ is the Dixit–Stiglitz price index defined as

$$P_{S,t} = \left[\int_0^1 X_t(z)^{1-\theta}\,\mathrm{d}z\right]^{1/(1-\theta)}, \tag{7}$$

where $X_t(z)$ is the price of differentiated good indexed as z at time t. Demand for each differentiated good z is given by

$$\begin{aligned} c_t(z) &= \left(\frac{X_t(z)}{P_{S,t}}\right)^{-\theta} C_{S,t} \\ &= \gamma\left(\frac{P_{S,t}}{P_t}\right)^{-1}\left(\frac{X_t(z)}{P_{S,t}}\right)^{-\theta} C_t. \end{aligned} \tag{8}$$

[7] Insurance contracts are assumed to be made before households know in which sector they are. By making insurance contracts, the households can insure one another against the difference of revenues that they could receive in future states. The insurance contracts make the marginal utility of nominal income identical across the households at any time t.

The optimal consumption plan of the household must satisfy

$$\frac{B_t u'(B_t C_t)}{P_t} = \Lambda_t, \tag{9}$$

where Λ_t is marginal utility of nominal income. The marginal utility of nominal income must satisfy

$$\Lambda_t \delta_{t,t+1} = \beta \Lambda_{t+1} \tag{10}$$

at each possible state at time $t+1$, and $\delta_{t,t+1}$ is a stochastic discount factor which satisfies

$$R_t = (E_t[\delta_{t,t+1}])^{-1}, \tag{11}$$

where R_t is the risk-free nominal interest rate at time t.

2.3. The production decision

2.3.1. The production decision of firms in the flexible-price sector

Sellers of the flexible-price good are assumed to be price takers.[8] Given a market price $P_{F,t}$, the sellers set

$$\Lambda_t P_{F,t} = v'(A_{F,t} Y_{F,t}) A_{F,t},$$

where $Y_{F,t}$ is the production of the flexible-price good. Using Eq. (9) and the market clearing conditions, the above condition implies that

$$P_{F,t} = A_{F,t} v'(A_{F,t} Y_{F,t}) \frac{P_t}{B_t u'(B_t Y_t/2)} \tag{12}$$

must hold. The right hand side can be interpreted as the marginal cost of production in the flexible-price sector. In deriving (12) we assume that there are mass of one of households producing the flexible-price good and mass of one of households producing the differentiated goods. Therefore market clearing conditions are $Y_{S,t} = 2C_{S,t}$, $Y_{F,t} = 2C_{F,t}$, and hence $Y_t = 2C_t$ at all time t.

2.3.2. The production decision of firms in the sticky-price sector

We assume that prices of the differentiated goods are sticky. Following Calvo (1983) and Woodford (1996), prices in the sticky-price sector are changed at exogenous random intervals. A fraction $1-\alpha \in (0,1)$ of the sellers in

[8] The important assumption here is that the price of the flexible-price good can be adjusted every period. Whether that sector is competitive or not does not affect the subsequent analysis.

the sticky-price sector can change their prices at each period t. The others must continue charging their old prices.

When sellers get a chance to change their prices at time t, they choose their price X_t in order to maximize the following objective[9]

$$\sum_{k=0}^{\infty} \alpha^k [\Lambda_t E_t[R_{t,t+k} X_t y_{t+k}(X_t)] - \beta^k E_t[v(A_{S,t+k} y_{t+k}(X_t))]],$$

where $R_{t,t+k} \equiv \prod_{i=0}^{k} R_{t+i-1}$ with the initial condition $R_{t,t} \equiv 1$. Here, production must equal the demand facing the sellers

$$y_{t+k}(X_t) = 2\gamma \left(\frac{P_{S,t+k}}{P_{t+k}}\right)^{-1} \left(\frac{X_t}{P_{S,t+k}}\right)^{-\theta} C_{t+k}.$$

The sticky-price index is expressed as

$$P_{S,t} = [\alpha P_{S,t-1}^{1-\theta} + (1-\alpha) X_t^{1-\theta}]^{1/(1-\theta)}. \tag{13}$$

The optimal price X_t solves the following first order condition:

$$\sum_{k=0}^{\infty} \alpha^k \Lambda_t E_t \left[\gamma(1-\theta) R_{t,t+k} \left(\frac{P_{S,t+k}}{P_{t+k}}\right)^{-1} \left(\frac{X_t}{P_{S,t+k}}\right)^{-\theta} C_{t+k}\right]$$
$$= \sum_{k=0}^{\infty} \alpha^k \beta^k E_t \left[-\gamma\theta A_{S,t+k} v'(A_{S,t+k} y_{t+k}) \left(\frac{X_t}{P_{S,t+k}}\right)^{-\theta} X_t^{-1} C_{t+k}\right]. \tag{14}$$

Eqs. (9), (10), and (11) imply that

$$R_{t,t+k} = \beta^k \frac{B_{t+k} U'(B_{t+k} C_{t+k})}{B_t U'(B_t C_t)} \frac{P_t}{P_{t+k}}. \tag{15}$$

Substituting (15) into (14) and arranging terms, (14) reduces to

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[R_{t,t+k} \left(\frac{P_{S,t+k}}{P_{t+k}}\right)^{-1} \left(\frac{X_t}{P_{S,t+k}}\right)^{-\theta} C_{t+k} \left\{X_t - \frac{\theta}{\theta-1} S_{S,t+k}\right\}\right] = 0, \tag{16}$$

where

$$S_{S,t+k} = A_{S,t+k} v'(A_{S,t+k} y_{t+k}) \frac{P_{t+k}}{B_{t+k} u'(B_{t+k} C_{t+k})}$$

is interpreted as the marginal cost of production, and $\theta/(\theta-1)$ is the constant markup of price over marginal cost. Eq. (16) implies that sellers in the sticky-price sector set their price equal to a weighted average of their marginal cost

[9] We suppress index z for the simplicity of notation, since all producers in the sticky-price sector are assumed to be symmetric.

multiplied by the constant markup. Substituting the market clearing condition, $Y_{t+k}/2 = C_{t+k}$, yields

$$\sum_{k=0}^{\infty} \alpha^k E_t \left[R_{t,t+k} \left(\frac{P_{S,t+k}}{P_{t+k}} \right)^{-1} \left(\frac{X_t}{P_{S,t+k}} \right)^{-\theta} Y_{t+k} \left\{ X_t - \frac{\theta}{\theta - 1} S_{S,t+k} \right\} \right] = 0, \tag{17}$$

where

$$S_{S,t+k} = A_{S,t+k} v'(A_{S,t+k} y_{t+k}) \frac{P_{t+k}}{B_{t+k} u'(B_{t+k} Y_{t+k}/2)}. \tag{18}$$

Eqs. (9), (10), (12), (17), (18), the definitions of the price indices in Eqs. (4) and (13), and the monetary policy function which chooses the risk-free nominal interest rate, jointly determine the equilibrium path of aggregate consumption, aggregate output, and the price levels in the two sectors. The demand for each sector is determined by (5) and (6).

3. Inflation and relative prices

3.1. Log-linearized system

In this section we log-linearize the system around the steady state with constant prices. The stationary variables are defined as follows. Aggregate inflation is defined by $\Pi_t \equiv P_t/P_{t-1}$, while $\Pi_{i,t} = P_{i,t}/P_{i,t-1}$ for $i = \mathrm{S, F}$ denotes the inflation rate in each of the sticky-price and flexible-price sectors. The relative price charged at time t by firms with new prices in the sticky-price sector is denoted by $x_t = X_t/P_{S,t}$, while $x_{i,t} = P_{i,t}/P_t$ for $i = \mathrm{S, F}$ denotes the relative price in each of the sticky-price and flexible-price sectors. The price indices thus satisfy the following relationships.[10] From Eq. (4) we have

$$\hat{\Pi}_t = \gamma \hat{\Pi}_{S,t} + (1 - \gamma) \hat{\Pi}_{F,t}. \tag{19}$$

Eq. (4) also implies

$$\hat{x}_{S,t} = -\frac{1 - \gamma}{\gamma} \hat{x}_{F,t}. \tag{20}$$

If we notice that $x_{S,t} = \frac{P_{S,t}}{P_{S,t-1}} \frac{P_{S,t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}$, we obtain

$$\hat{\Pi}_t = \hat{\Pi}_{S,t} + \hat{x}_{S,t-1} - \hat{x}_{S,t}. \tag{21}$$

[10] A variable $\hat{x}_t$, for example, represents the percentage deviation from its stationary value $\bar{x}$.

Using these identities, aggregate inflation is given by

$$\hat{\Pi}_t = \hat{\Pi}_{S,t} + \frac{1-\gamma}{\gamma}\Delta\hat{x}_{F,t}, \tag{22}$$

$$\Delta\hat{x}_{F,t} \equiv \hat{x}_{F,t} - \hat{x}_{F,t-1}.$$

The aggregate demand equation is the log-linearized Euler conditions (9) and (10), where aggregate demand is equated to aggregate supply,

$$\hat{Y}_t = E_t\hat{Y}_{t+1} - \sigma[R_t - E_t\hat{\Pi}_{t+1} - \hat{B}_t + E_t\hat{B}_{t+1}]. \tag{23}$$

The term $-\hat{B}_t + E_t\hat{B}_{t+1}$ can be interpreted as an aggregate demand shock. From (6) and (5) we obtain the output for both sectors as

$$\hat{Y}_{S,t} = \hat{Y}_t + \frac{1-\gamma}{\gamma}\hat{x}_{F,t}, \tag{24}$$

$$\hat{Y}_{F,t} = \hat{Y}_t - \hat{x}_{F,t}. \tag{25}$$

The supply equation of the flexible-price sector, which is obtained by log-linearizing (12), is given by[11]

$$\hat{x}_{F,t} = \kappa(\hat{Y}_t - \hat{Y}^n_{F,t}), \tag{26}$$

where

$$\kappa \equiv \frac{\omega^{-1}+\sigma^{-1}}{1+\omega^{-1}}, \tag{27}$$

and

$$\hat{Y}^n_{F,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{A}_{F,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{B}_t. \tag{28}$$

$\hat{Y}^n_{F,t}$ is interpreted as a supply shock in the flexible-price sector. It also represents a change in the natural rate of output in the flexible-price sector, in the sense that it is the level of supply required to keep certain marginal cost in the flexible-price sector, in the absence of price variability. Eq. (26) implies that

[11] Here $\sigma = -u''BC/u'$ and $\omega = v_i''A_iY_i/v_i'$ for $i = \mathrm{F,S}$, evaluated at the steady state values. For simplicity we assume that ω is the same across the all sellers. This is true when v takes a form $v = (Ay)^{1+\omega}/(1+\omega)$. Allowing ω to vary across the two sectors would slightly complicate the definition of the "aggregate output gap" that is discussed in Section 3.2, but the analysis below would remain unchanged.

the relative price of the flexible-price sector depends only on the output gap of that sector, defined by $\hat{Y}_t - \hat{Y}^n_{F,t}$.

It can be shown that the supply equation of the sticky-price sector is given by[12]

$$\begin{aligned}\hat{\Pi}_{S,t} &= \kappa_1(\hat{Y}_t - \hat{Y}^n_{S,t}) + \beta E_t \hat{\Pi}_{S,t+1} - \kappa_2 \hat{x}_{S,t} \\ &= \kappa_1(\hat{Y}_t - \hat{Y}^n_{S,t}) + \beta E_t \hat{\Pi}_{S,t+1} + \frac{1-\gamma}{\gamma}\kappa_2 \hat{x}_{F,t},\end{aligned} \tag{29}$$

where

$$\kappa_1 \equiv \frac{1-\alpha}{\alpha}(1-\alpha\beta)\frac{\omega^{-1}+\sigma^{-1}}{1+\theta/\omega} > 0,$$

$$\kappa_2 \equiv \frac{1-\alpha}{\alpha}(1-\alpha\beta)\frac{\omega^{-1}+1}{1+\theta/\omega} > 0,$$

$$\hat{Y}^n_{S,t} \equiv -\frac{1+\omega^{-1}}{\sigma^{-1}+\omega^{-1}}\hat{A}_{S,t} - \frac{\sigma^{-1}-1}{\sigma^{-1}+\omega^{-1}}\hat{B}_t. \tag{30}$$

$\hat{Y}^n_{S,t}$ is regarded as a supply shock in the sticky-price sector. It also represents a change in the natural rate of output in the sticky-price sector, in the sense that it is the level of supply that would arise in the absence of price rigidity. Eqs. (23), (26), (29), the definitions of price indices, and the monetary policy function which chooses $\hat{R}_t$ jointly determine the equilibrium path of $\hat{Y}_t$, $\hat{R}_t$, $\hat{x}_{F,t}$, $\hat{\Pi}_t$ and $\hat{\Pi}_{S,t}$. We now focus on the analysis of inflation determination.

3.2. *The Phillips curve and relative price as a proxy for supply shocks*

Sticky-price inflation (29) takes a form of the expectation-augmented Phillips curve. It depends on the output gap in the sticky-price sector ($\hat{Y}_t - \hat{Y}^n_{S,t}$), its inflation expectation, and the relative price of the flexible-price good. Here the relative price of the flexible-price good represents a shift parameter of the Phillips curve. This is a variant of the aggregate supply equations obtained from dynamic sticky-price models, called by Roberts (1995) the New Keynesian Phillips curve. Since firms cannot change their prices every period, they change prices on the basis of the expectations of future demand and cost conditions. This results in the expected inflation term.[13] The sticky-price

[12] *Details of derivation can be found in the following web page: http: //www.bankofengland.co.uk/ workingpapers/external/index.html.*

[13] On the other hand, inflation in the flexible-price sector does not have a Phillips-curve representation. This is because the price of the flexible-price good can be adjusted each period. This does not necessarily mean that the relative price of the flexible-price good does not depend on any expectations about the future. For example, if the good were storable, its current price would depend on its expected future prices. However, all goods are assumed to be perishable in this model.

inflation depends also on the relative price of the flexible-price good because the demand for the composite sticky-price good depends on the relative price as well as aggregate demand conditions (See Eq. (24).)

It is of interest to compare these results with those of Gordon's (1975) classic paper on relative prices and aggregate inflation. In that paper Gordon considers a model in which the prices of farm goods are flexible while those of nonfarm goods are fixed, and shows that an increase in the relative price of farm goods is inflationary. An increase in the relative price of farm goods causes the nominal price of the farm goods to rise while other prices remain constant, so raising the aggregate price level. However, Gordon (1975) assumes that the price in the nonfarm sector is fixed forever, and does not analyze how the relative price changes over time affect the pricing behavior of the firms in the nonfarm sector. In contrast, the model of this paper shows that, holding other conditions constant, an increase in the relative price of the flexible-price good leads an increase in the prices of sticky-price goods. It is a well known fact that the increase in the prices of food and energy during the 1970s was associated with increases in the prices of other goods. This implication of our model agrees with this historical episode.

It is of great interest to examine the implications of this model for the econometric specification of aggregate inflation, since much of the empirical literature on the Phillips curve uses aggregate data. Using Eqs. (22) and (29), aggregate inflation is given by

$$\hat{\Pi}_t = \kappa_1(\hat{Y}_t - \hat{Y}^n_{S,t}) + \beta E_t \hat{\Pi}_{t+1} + \frac{1-\gamma}{\gamma}(\kappa_2 \hat{x}_{F,t} + \triangle \hat{x}_{F,t} - \beta E_t \triangle \hat{x}_{F,t+1}),$$

or equivalently,

$$\hat{\Pi}_t = \kappa_1 \hat{Y}_t + \beta E_t \hat{\Pi}_{t+1} + \frac{1-\gamma}{\gamma}(\kappa_2 \hat{x}_{F,t} + \triangle \hat{x}_{F,t} - \beta E_t \triangle \hat{x}_{F,t+1}) - \kappa_1 \hat{Y}^n_{S,t}. \tag{31}$$

Eq. (31) is the form of the Phillips curve that is estimated by Roberts (1995). Here aggregate inflation depends on detrended aggregate output, aggregate inflation expectations, the relative price of the flexible-price good, and the $\hat{Y}^n_{S,t}$ disturbance. Roberts (1995) uses changes in real oil prices as a proxy for supply shocks when he presents his estimates of the Phillips curve. He finds that the effect of the relative price of crude oil on aggregate (CPI) inflation is statistically significant. Our expression for aggregate inflation Eq. (31) agrees with Roberts' (1995) results once the way the relative price enters into the Phillips curve equation is correctly specified. However, one sector models, from which his estimation equations are derived, do not explicitly explain how the price of oil affects the pricing behavior of the firms in the other industries. The mechanism considered in our model is the substitution between the flexible-price good and the sticky-price goods. When the price of the flexible-price good increases the households increase the demand for the sticky-price goods. Facing an increase in demand, the sellers in the sticky-price sector raise their prices.

Note that (26) shows that the relative price of the flexible-price sector is affected not only by the supply shock in the flexible-price sector but also by aggregate demand. Thus, if the current level of consumption and the relative price of the flexible-price good are observed, then the supply shock in the flexible-price sector can be identified. In this sense, relative prices can be used as proxies for supply shocks. However, they do not provide complete information about supply shocks. In particular, it is not possible to extract information about the supply shock in the sticky-price sector by observing the relative price.[14] Therefore, $\hat{Y}^n_{S,t}$ from the estimation equation of the Phillips curve even if one adds a relative-price term as a proxy for supply shocks.

3.3. Inflation in the sticky-price sector as a measure of core inflation

We can solve (29) forward to obtain

$$\hat{\Pi}_{S,t} = E_t \sum_{i=0}^{\infty} \beta^i \left[\kappa_1 (\hat{Y}_{t+i} - \hat{Y}^n_{S,t+i}) + \frac{1-\gamma}{\gamma} \kappa_2 \hat{x}_{F,t+i} \right]. \tag{32}$$

Therefore, inflation in the sticky-price sector represents a relatively persistent component of aggregate inflation because it responds to smoothed expectations of future output gaps and relative-price changes. Hence (32) can be interpreted as a measure of core inflation. Since the distributed lead coefficients in (32) die out only very slowly as i increases, sticky-price inflation depends mainly upon the relatively persistent components of variations in the output gap and relative-price changes. Sticky-price inflation will itself be a persistent variable, since it is a good predictor of its own future values.

Alternatively, by substituting (26) into (29), sticky-price inflation can be expressed as

$$\hat{\Pi}_{S,t} = \frac{1}{\gamma} \kappa_1 [\gamma(\hat{Y}_t - \hat{Y}^n_{S,t}) + (1-\gamma)(\hat{Y}_t - \hat{Y}^n_{F,t})] + \beta E_t \hat{\Pi}_{S,t+1}. \tag{33}$$

Here,

$$\hat{G}_t \equiv \gamma(\hat{Y}_t - \hat{Y}^n_{S,t}) + (1-\gamma)(\hat{Y}_t - \hat{Y}^n_{F,t}) \tag{34}$$

[14] The supply shock in the sticky-price sector affects the relative price of the flexible-price sector trough changes in aggregate consumption. Given changes in aggregate consumption, changes in the relative price does not have further information about the supply shock in the sticky-price sector.

represents the *aggregate* output gap, where the relative weights on each output gap are equated to the proportion of spending on the goods in each sector.[15] Therefore, sticky-price inflation is determined by smoothed expectations of current and future aggregate output gaps

$$\hat{\Pi}_{S,t} = \frac{1}{\gamma}\kappa_1 E_t \sum_{i=0}^{\infty} \beta^i \hat{G}_{t+i}. \tag{35}$$

Much literature on core inflation is mainly concerned with how to remove temporary "noise" in order to get better forecasts of future inflation. For example, Bryan and Cecchetti (1994) suggest trimmed mean inflation as a measure of core inflation.[16] Their approach is based on the hypothesis that large price changes are caused mainly by large sector-specific shocks whose effects on aggregate inflation are temporary. By trimming these outliers, they argue, the remaining inflation measure represents the persistent part of aggregate inflation that monetary policy can control. However, the effect of a sector-specific shock (a supply shock to food and energy markets, for example) on aggregate inflation would not always be transitory. Eq. (32) implies that if a shock is persistent, then it could change inflation expectations and thus affect aggregate inflation for a long period of time. Moreover, the sticky-price firms may change their prices largely when they get a chance to do so. This is because the firms need to take into account the fact that they would have to keep charging their current price in the future. Suppose, for example, a firm expects that inflation may be higher in the near future. Since the firm changes its price on the basis of future demand and cost conditions, it would increase its price largely today in order to keep its future relative prices near its desired levels. For example, prices of magazines are kept for a long period of time, but when their prices are adjusted, the percentage changes in the price are large. Large price changes of this kind are due to price stickiness, and therefore it is not desirable to exclude goods with large price changes from the components of core inflation simply because their price changes are large. The nature of pricing behavior is as important as the nature of shocks in constructing a good measure of core inflation.

[15] When ω takes different values between the sticky-price sector and the flexible-price sector, then (33) is given by

$$\hat{\Pi}_{S,t} = \frac{1}{\gamma}\kappa_1 \left[\gamma(\hat{Y}_t - \hat{Y}^n_{S,t}) + (1-\gamma)\frac{\omega_F^{-1} + \sigma^{-1}}{\omega_S^{-1} + \sigma^{-1}}\frac{\omega_S^{-1} + 1}{\omega_F^{-1} + 1}(\hat{Y}_t - \hat{Y}^n_{F,t})\right] + \beta E_t \hat{\Pi}_{S,t+1},$$

where the subscripts F is for the flexible-price sector and S for the sticky-price sector. Therefore, the definition of the aggregate output gap would be slightly different. However, this does not change the analysis below.

[16] Bryan and Cecchetti (1994) apply Ball and Mankiw's (1995) model with a menu cost of changing prices.

4. Optimal monetary policy

We now characterize the optimal monetary policy for the economy described by the model. Much of the literature on optimal monetary policy assumes that the objective of a central bank is to minimize the squared deviations of certain measures of inflation and output from their target values.[17] In our model the central bank has a choice between several different possible measures of inflation and output gap, and it is of great interest to study which variables are the appropriate goal variables of the central bank.

The objective of the central bank here is assumed to be to maximize the ex-ante utility of the households. Setting the central bank's objective as the maximization of the households' utility implicitly assumes that the central bank is not responsible for the welfare loss due to factors which are not assumed in the model.[18] The idea is that the government could use other instruments to remove welfare loss caused by these factors, and the central bank is responsible only for the welfare loss due to price stickiness which is the propagation mechanism of business cycles in our model. Furthermore, following Rotemberg and Woodford (1997, 1999), we take a second order Taylor series approximation of this welfare measure around the steady state values with constant prices. The important advantage of this approach is that we can derive a loss function for the central bank that is theoretically justified in terms of individual welfare. Using this loss function we can evaluate alternative monetary policies and analyze which variables should be stabilized in the optimal equilibrium. It will be shown that the optimal monetary policy for the economy in this setting is the complete stabilization of sticky-price inflation.

4.1. Welfare of the economy

As stated above, we look for an optimal monetary policy which maximizes the welfare of the households. Following Rotemberg and Woodford (1997, 1999), the welfare measure we consider is the ex-ante expected utility given by [19]

$$W \equiv E[W_t] \equiv E\left[2U(B_t Y_t/2) - \int_0^1 v(A_{S,t} y_{S,t}(z))\,dz - v(A_{F,t} Y_{F,t})\right] \tag{36}$$

[17] See, for example, Clarida et al. (1999).

[18] The costs of inflation that are often discussed are, for example, the distortions caused by un-indexed tax system and the redistribution of income and wealth over heterogeneous agents. These costs are typically due to the failure of nominal contract to adjust inflation.

[19] Here $U(B_t Y_t/2)$ is multiplied by two because we assume that there are mass of one of agents in each of the flexible-price and sticky-price sectors.

in the stationary equilibrium, where aggregate consumption is equated to aggregate supply. It should be noted that our specification of the utility of the representative household (1) implies that we do not consider the so-called shoe-leather cost of inflation, which is the welfare cost of inflation due to the decrease in money holding.[20] We take a second order Taylor series approximation of (36) around the steady state where all prices are constant. We assume that this steady state involves a tax rate which is set such that the steady state level of output in the sticky-price sector is efficient. Thus, monetary policy is not responsible for the welfare loss that arises from the distortion caused by monopoly power.[21]

It can be shown that a second order Taylor expansion of W_t is given by[22]

$$\begin{aligned} W_t = & -\frac{1}{2}U'\bar{Y}(\sigma^{-1}+\omega^{-1})\hat{G}_t^2 \\ & -\frac{1}{2}U'\bar{Y}(1-\gamma)\gamma(\omega^{-1}+1)\{(\hat{Y}_{S,t}-\hat{Y}_{F,t})-\kappa(\hat{Y}^n_{S,t}-\hat{Y}^n_{F,t})\}^2 \\ & -\frac{1}{2}U'\bar{Y}\gamma(\theta^{-1}+\omega^{-1})Var_z[\hat{y}_t(z)]+t.i.p.+\mathcal{O}(3). \end{aligned} \quad (37)$$

In Eq. (37) U' and $\bar{Y}$ are evaluated at their steady state values. Here the aggregate output gap $\hat{G}_t$ is defined by (34), and the term $\kappa(\hat{Y}^n_{S,t}-\hat{Y}^n_{F,t})$ represents the efficient fluctuation in relative output where κ is defined by (27).[23] The term $Var_z[\hat{y}_t(z)]$ represents the output dispersion in the sticky-price sector. The term $t.i.p.$ represents terms that are independent of policy, which consist of variations in exogenous variables, and the term $\mathcal{O}(3)$ indicates that we neglect terms that are of third or higher order in the deviations of variables from their steady state values. Eq. (37) shows that welfare depends only on real factors: namely the aggregate output gap, the deviation of relative output from its efficient value, and the dispersion of the level of output in the sticky-price sector.

Alternatively, one can write welfare as a function of relative prices. From Eqs. (24) and (25) we have a relationship between relative price

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[20] See, also, Section 4.4

[21] It is often argued that expansionary monetary policy is welfare improving since it can reduce the market power of monopolistic producers. However, there are better policy variables such as tax system with which to address the distortion caused by monopoly power. Here, we focus our analysis on the welfare loss due to the failure of the firms to adjust their prices to the exogenous shocks.

[22] *Details of calculations in this section can be found in the following web page: http://www.bankofengland.co.uk/workingpapers/external/index.html.*

[23] Using (28) and (30), this simplifies to

$$\kappa(\hat{Y}^n_{S,t}-\hat{Y}^n_{F,t})=(-\hat{A}_{S,t})-(-\hat{A}_{F,t}).$$

Therefore the efficient relative output depends solely on the relative random variations in the disutility of production in each sector.

and relative output

$$\hat{x}_{F,t} = \gamma(\hat{Y}_{S,t} - \hat{Y}_{F,t}). \tag{38}$$

Similarly,

$$\hat{x}^*_{F,t} = \gamma\kappa(\hat{Y}^n_{S,t} - \hat{Y}^n_{F,t}) \tag{39}$$

is the efficient relative-price changes. It is also the change in the level of the relative price that would arise in the absence of price stickiness. Substituting these into (37) we obtain

$$\begin{aligned} W_t = & -\frac{1}{2}U'\bar{Y}(\sigma^{-1} + \omega^{-1})\hat{G}_t^2 \\ & -\frac{1}{2}U'\bar{Y}\frac{1-\gamma}{\gamma}(\omega^{-1} + 1)(\hat{x}_{F,t} - \hat{x}^*_{F,t})^2 \\ & -\frac{1}{2}U'\bar{Y}\gamma(\theta^{-1} + \omega^{-1})Var_z[\hat{y}_t(z)] + t.i.p. + \mathcal{O}(3). \end{aligned} \tag{40}$$

Taking unconditional expectation of (40) we obtain[24]

$$\begin{aligned} W = & -\frac{1}{2}U'\bar{Y}(\sigma^{-1} + \omega^{-1})[Var[\hat{G}_t] + (E[\hat{G}_t])^2] \\ & -\frac{1}{2}U'\bar{Y}\frac{1-\gamma}{\gamma}(\omega^{-1} + 1)[Var[\hat{x}_{F,t} - \hat{x}^*_{F,t}] + (E[\hat{x}_{F,t} - \hat{x}^*_{F,t}])^2] \\ & -\frac{1}{2}\gamma U'\bar{Y}(\theta^{-1} + \omega^{-1})E[Var_z[\hat{y}_t(z)]]. \end{aligned} \tag{41}$$

Furthermore, $E[Var_z[\hat{y}_t(z)]]$ can be written as a function of the sticky-price inflation process. This is because the dispersion of output levels in the sticky-price sector directly corresponds to the degree of price dispersion in this sector. Price dispersion, in turn, depends on the process of the sticky-price inflation in this model. With this substitution, we obtain

$$\begin{aligned} W = & -\frac{1}{2}U'\bar{Y}(\sigma^{-1} + \omega^{-1})[Var[\hat{G}_t] + (E[\hat{G}_t])^2] \\ & -\frac{1}{2}U'\bar{Y}\frac{1-\gamma}{\gamma}(\omega^{-1} + 1)[Var[\hat{x}_{F,t} - \hat{x}^*_{F,t}] + (E[\hat{x}_{F,t} - \hat{x}^*_{F,t}])^2] \\ & -\frac{1}{2}\gamma U'\bar{Y}(\theta^{-1} + \omega^{-1})\frac{\alpha}{(1-\alpha)^2}\{Var[\hat{\Pi}_{S,t}] + (E[\hat{\Pi}_{S,t}])^2\}. \end{aligned} \tag{42}$$

Eq. (42) clarifies which kinds of stabilization are important for the social welfare. It shows that the relevant variables for welfare are the variations

[24] We suppress *t.i.p.* and $\mathcal{O}(3)$ here.

of the *aggregate* output gap, *core* inflation (i.e., sticky-price inflation), and the deviation of the relative price from its efficient value.[25] Eq. (42) also identifies the proper weights on the variabilities of each variable, as functions of the preference parameters (σ, ω, γ, θ) and the degree of price stickiness (α).

We can furthermore express our welfare function as a function of only core inflation. Eq. (33) implies that the aggregate output gap is given by

$$\hat{G}_t = \frac{\gamma}{\kappa_1}(\hat{\Pi}_{S,t} - \beta E_t \hat{\Pi}_{S,t+1}).$$

Using (26) and (39), we can express the deviation of the relative price as a function of the aggregate output gap

$$\begin{aligned}\hat{x}_{F,t} - \hat{x}^*_{F,t} &= \kappa(\hat{Y}_t - \hat{Y}^n_{F,t}) - \gamma\kappa(\hat{Y}^n_{S,t} - \hat{Y}^n_{F,t}) \\ &= \kappa\{\hat{Y}_t - \gamma\hat{Y}^n_{S,t} - (1-\gamma)\hat{Y}^n_{F,t}\} = \kappa\hat{G}_t. \qquad (43)\end{aligned}$$

Therefore, the deviation of the relative price reduces to

$$\hat{x}_{F,t} - \hat{x}^*_{F,t} = \frac{\gamma}{\kappa_1}\kappa(\hat{\Pi}_{S,t} - \beta E_t \hat{\Pi}_{S,t+1}).$$

Substituting these into (42) and arranging terms, we obtain

$$W = -\lambda_1 Var[\hat{\Pi}_{S,t} - \beta E_t \hat{\Pi}_{S,t+1}] - \lambda_2 Var[\hat{\Pi}_{S,t}] - \lambda_3 (E[\hat{\Pi}_{S,t}])^2, \qquad (44)$$

where λ_1, λ_2, λ_3 are defined as

$$\lambda_1 \equiv \frac{1}{2}\gamma U' \bar{Y}\left[\frac{\kappa}{\kappa_1^2}\{\gamma(1+\omega^{-1}) + (1-\gamma)(\omega^{-1}+\sigma^{-1})\}\right],$$

$$\lambda_2 \equiv \frac{1}{2}\gamma U' \bar{Y}\left[(\theta^{-1}+\omega^{-1})\frac{\alpha}{(1-\alpha)^2}\right],$$

$$\lambda_3 \equiv \frac{1}{2}\gamma U' \bar{Y}\left[(1-\beta)^2\frac{\kappa}{\kappa_1^2}\{\gamma(1+\omega^{-1}) + (1-\gamma)(\omega^{-1}+\sigma^{-1})\} + (\theta^{-1} + \omega^{-1})\frac{\alpha}{(1-\alpha)^2}\right].$$

[25] It can be also shown that average aggregate output gap $E[\hat{G}_t]$ and relative-price deviation $E[\hat{x}_{F,t} - \hat{x}^*_{F,t}]$ can be expressed as functions of only $E[\hat{\Pi}_{S,t}]$. We use this fact to derive (44) below. Therefore, our loss function is

$$W = -L - \lambda_3(E[\hat{\Pi}_S])^2,$$

where L similar to a conventional quadratic loss function with a relative-price term

$$L = a_y Var[\hat{Y} - \gamma\hat{Y}^n_S - (1-\gamma)\hat{Y}^n_F] + a_\pi Var[\Pi_S] + a_x Var[\hat{x}_{F,t} - \hat{x}^*_{F,t}],$$

where a_y, a_π, and a_x are relative weights on the variabilities in output gap, inflation and relative prices, respectively.

Eq. (44) attains its theoretical maximum when $\hat{\Pi}_{S,t} = 0$ at all t, implying that complete stabilization of core inflation maximizes the social welfare. We discuss the properties of the optimal equilibrium in the next section.

4.2. Complete stabilization of sticky-price inflation

Eq. (35) implies that, if the central bank chose the interest rate $\hat{R}_t$ such that the aggregate output gap $\hat{G}_t$ is zero at any time t, then in a rational expectations equilibrium core inflation would be completely stabilized, i.e., $\hat{\Pi}_{S,t} = 0$ at all times. Note that these responses are also equivalent to those that would arise in an economy where the prices in both sectors are completely flexible. This is an extension of the results obtained by several authors who use one-sector sticky-price models (Ireland, 1996; King and Wolman, 1999; Rotemberg and Woodford, 1997, 1999). In a one-sector sticky-price model where only the market friction is price stickiness, complete stabilization of aggregate inflation is desirable,[26] and so, monetary policy should target aggregate inflation. By stabilizing aggregate inflation the central bank can remove market distortion due to price stickiness. In the model considered here, it is sticky-price inflation that should be targeted, because price stickiness in the sticky-price sector is the only market friction. The central bank should pay attention to core inflation not because it is useful for forecasting future aggregate inflation (which many authors suggest it should target), but because it is core inflation that should be targeted in order to achieve the socially optimal allocation of resources.

Eq. (43) implies that the change in the relative price would also be efficient in the optimal equilibrium,

$$\hat{x}_{F,t} = \hat{x}^*_{F,t}. \tag{45}$$

Therefore if core inflation is completely stabilized, the changes in the relative price solely depends on changes in the technology in both sectors. Demand factors do not affect these changes in the relative prices. This result implies that, even though the stabilization of the relative price around its time-varying optimal level is an appropriate goal for the central bank, (see Eq. (42)), stabilizing core inflation at all times is sufficient for keeping the relative price at its efficient level. From (22) and (45) aggregate inflation in this equilibrium is given by

$$\begin{aligned} \hat{\Pi}^*_t &= \frac{1-\gamma}{\gamma}(\hat{x}^*_{F,t} - \hat{x}^*_{F,t-1}) \\ &= (1-\gamma)\kappa_3\{(\hat{Y}^n_{S,t} - \hat{Y}^n_{F,t}) - (\hat{Y}^n_{S,t-1} - \hat{Y}^n_{F,t-1})\}. \end{aligned} \tag{46}$$

This represents the optimal aggregate inflation rate.

[26] This conclusion assumes that the zero-bound of nominal interest rates does not prevent such a policy. Our result also abstracts this issue. See Rotemberg and Woodford (1997) and Woodford (1999a) for a discussion on the optimal policy under the presence of the zero bound.

How should the central bank choose the nominal interest rate in order to achieve the optimal equilibrium? The interest rate which is consistent with the optimal equilibrium can be calculated by using Eqs. (22), (23), (34), and (45). It is given by

$$\begin{aligned}\hat{R}_t^* =& \frac{1}{\sigma}[\gamma(E_t\hat{Y}^n_{S,t+1} - \hat{Y}^n_{S,t}) + (1-\gamma)(E_t\hat{Y}^n_{F,t+1} - \hat{Y}^n_{F,t})] \\ &+ (1-\gamma)\kappa_3\{E_t[\hat{Y}^n_{S,t+1} - \hat{Y}^n_{F,t+1}] - (\hat{Y}^n_{S,t} - \hat{Y}^n_{F,t})\} \\ &+ (E_t\hat{B}_{t+1} - \hat{B}_t). \end{aligned} \tag{47}$$

This is a variant of the "Wicksellian" natural rate of interest.[27] It is the optimal real interest rate, in terms of the sticky-price goods, which is determined solely by real factors. It is also the equilibrium real interest rate in terms of the sticky-price goods that would arise in the absence of price stickiness. Using (46), we can also derive the equivalent natural interest rate in terms of the complete basket of goods

$$\begin{aligned}\hat{r}_t^* &\equiv \hat{R}_t^* - E_t\hat{\Pi}^*_{t+1} \\ &= \frac{1}{\sigma}[\gamma(E_t\hat{Y}^n_{S,t+1} - \hat{Y}^n_{S,t}) + (1-\gamma)(E_t\hat{Y}^n_{F,t+1} - \hat{Y}^n_{F,t})] + (E_t\hat{B}_{t+1} - \hat{B}_t).\end{aligned} \tag{48}$$

The notion of the natural interest rate is useful to judge whether monetary policy is too inflationary or deflationary. We can rewrite the aggregate demand Eq. (23) as

$$\hat{G}_t = E_t\hat{G}_{t+1} - \sigma[(\hat{R}_t - E_t\hat{\Pi}_{t+1}) - \hat{r}_t^*]. \tag{49}$$

Eq. (49) implies that the current aggregate output gap is determined as a distributed leads of the expected deviations of the current and future real interest rates from the natural interest rates,

$$\hat{G}_t = -E_t\sum_{i=0}^{\infty}\sigma[(\hat{R}_{t+i} - \hat{\Pi}_{t+i+1}) - \hat{r}^*_{t+i}]. \tag{50}$$

If the real interest rates were chosen lower than the natural rates, then the current aggregate output gap would be positive, requiring a higher core inflation as a result of (35). On the contrary, if the real interest rates were above the natural rates, then the current aggregate demand gap would be negative and the policy would be deflationary. By choosing a nominal interest rate equal

[27] For the recent discussion of this concept, see Blinder (1998) and Woodford (1999a).

to the natural rates at all times, the central bank can completely stabilize core inflation, maximizing the social welfare.[28]

4.3. Core inflation targeting and the output gap-inflation variability trade-off

The previous section has shown that the central bank should stabilize the aggregate output gap in order to stabilize core inflation. The optimal aggregate inflation rate is given by (46), which is not in general equal to zero. This implies that stabilizing aggregate inflation and stabilizing the aggregate output gap are not consistent each other in this model. The existence of output gap-inflation variability trade-off is originally emphasized by Taylor (1979) and has been an important guiding principle in studies of monetary policy. However, standard (one sector) New Keynesian sticky-price models predict that there is no output gap-inflation variability trade-off when the output gap is defined as a deviation from the efficient level of output, rather than a deviation from trend.[29] In other words, they predict that stabilizing output gap at all times would lead to the complete stabilization of inflation. However, bringing down inflation is costly in the real world,[30] and this is regarded as a challenge to the New Keynesian models. Clarida et al. (1999) use a one-sector sticky-price model and argue that a certain kind of cost-push shocks could induce the output gap-inflation variability trade-off. Their specification of the Phillips curve is

$$\hat{\Pi}_t = \kappa(\hat{Y}_t - \hat{Y}^n_t) + \beta E_t \hat{\Pi}_{t+1} + u_t,$$

where $\hat{\Pi}_t$ is a measure of aggregate inflation, $\hat{Y}_t - \hat{Y}^n_t$ is a measure of the aggregate output gap, and u_t is a cost-push shock. Note that their specification is very similar to our inflation equation in the sticky-price sector (29). The relative-price term corresponds to their cost-push disturbance. Their notion of cost-push disturbance can be interpreted as a factor that influences the relevant inflation measure but is not included in their definition of the output gap. Furthermore, our two-sector model predicts that there would be no trade-off between stabilizing *core* inflation and stabilizing the aggregate output gap, but

[28] An important question is how the central bank can implement the optimal policy given by (47). Implementation of the optimal monetary policy requires the knowledge of the demand and supply shocks, but it is impossible for the central bank to directly observe them in the real world. Furthermore, the interest rate policy given by (47) is not a good candidate for the policy recommendation. One reason is that the policy (47) would result in the indeterminacy of equilibria, since (47) does not depend on any nominal variables. For these reasons, the construction of a feedback rule which depends only on observable variables and which results in a unique equilibrium remains to be an important problem. See Rotemberg and Woodford (1999) for the construction of the feedback rules that implement the desirable equilibrium.

[29] See, for example, Clarida et al. (1999), Goodfriend and King (1997), and Rotemberg and Woodford (1999).

[30] For example, Ball (1994) documented costly disinflation in 19 countries.

there would be a trade-off between stabilizing *aggregate* inflation and stabilizing the aggregate output gap. Thus, whether output gap-inflation variability trade-off exists or not would depend on which measures of inflation and the output gap are relevant for the central bank.

An important question for the central bank is which variables should be the appropriate goal variables. For example, suppose there is an increase in the price of food and energy, as observed in the 1970s, putting an upward pressure on aggregate inflation. Is it desirable for the central bank to stabilize aggregate inflation? The central bank could respond with a sharp contractionary policy and reduce aggregate demand by a large amount so as to decrease prices in the sticky-price sector. By doing so, the central bank could offset the effect of the increase in the price of food and energy on aggregate inflation. However, our model shows that such a policy is not optimal. The optimal policy is to stabilize core inflation.[31] Furthermore, we have shown that stabilizing the relative prices around their efficient level is one of the appropriate goals of the central bank, and that stabilizing core inflation is sufficient for this goal. Of course it does not imply that the central bank need not pay attention to any observed relative-price changes. As discussed in Section 3, the relative-price changes convey information about the supply shocks to the flexible-price sector, and the central bank needs this information in order to stabilize core inflation.

4.4. Discussions

Here we discuss briefly how our utility-based welfare analysis is modified if we incorporate two complications from which we abstracted in the analysis. First of all, we consider implications of transactions frictions for optimal policy. Next we consider lags in the effects of monetary policy actions on the sticky-price inflation.

4.4.1. Transactions frictions

In our model, we abstracted from the cost of inflation caused by the transactions frictions that account for the demand for money (i.e., triangles under the money demand curve). If we take the transactions frictions into account, complete stabilization of the sticky-price inflation is no longer optimal. This is because, as Friedman (1969) famously argues, the policy which minimizes welfare loss caused by the transactions frictions would require deflation (this deflation rate is known as the Friedman rule, i.e., minus the rate of time preference).[32] Therefore, there is a conflict between minimizing

[31] See, also, the discussion of Goodfriend and King (1997).

[32] See, also, Wolman (1997) and King and Wolman (1999). Wolman (1997) indicates that the most of the welfare gains from reducing average inflation from 5 percent to the Friedman rule are gained by making inflation zero.

the welfare loss caused by the relative price dispersion among the sticky-price goods and minimizing the welfare loss caused by the transactions frictions.

More specifically, in a framework of a one-sector sticky-price model, Woodford (1999b) shows that the utility-based welfare measure under the presence of the transactions frictions is given by the sum of the variabilities of (i) inflation, (ii) output gap, *and* (iii) the deviations of the nominal rate from its target, whose value is negative. The minimization of the last term requires deflation, as argued by Friedman (1969), so there is a conflict between the rate of inflation needed to minimize the first term and that needed to minimize the third term. If we applied his analysis to our two-sector model, the proper loss function of the central bank should put a positive weight on the interest-rate variability, in addition to the variabilities of the sticky-price inflation, the aggregate output gap, and the deviation of the relative price from its efficient value. An important point to be noted here is that, although complete stabilization of the sticky-price inflation is no longer optimal, the proper measure of inflation to be stabilized is still the sticky-price inflation but not aggregate inflation. This is because the inefficient relative price dispersion among the sticky-price goods is caused by inflation in the sticky-price sector. Therefore, core inflation targeting continues to be a good candidate for a good monetary policy even if the transactions frictions are taken into consideration.

4.4.2. Lags in the effects of monetary policy on inflation

Our model, along with many in this literature, models the sticky prices as forward-looking variables rather than predetermined variables, and assumes that monetary policy actions has an immediate impact on the current output and the sticky-price inflation. However, the empirical literature on inflation finds that there exist lags in the effects of monetary policy on inflation. For example, Christiano et al. (1999) report that an monetary policy shock decreases the GDP deflator with a lag of roughly six quarters. Rotemberg and Woodford (1997) report that, after a contractionary monetary policy shock, the greatest decline in inflation occurs two quarters later. Cochrane (1994) also obtains very similar impulse responses.

The delay in responses of inflation to monetary policy actions can be explained by the existence of decision lags or information lags in price setting. For example, Rotemberg and Woodford (1999) develop a variant of the Calvo model with pricing decision lags. Specifically, they assume pricing decision lags of one and two quarters for the two groups of firms. They argue that these assumed delays can explain why no prices respond in the quarter of a monetary policy shock and why the largest response of inflation to a monetary policy shock takes place two quarters after the shock, as they find in the empirical part of their paper. Under the assumption of the pricing decision lags of this kind, Rotemberg and Woodford (1999) furthermore show that a proper loss function of a central bank should put a positive weight on the variability of an unforecastable component of inflation (namely, $Var[\hat{\Pi}_t - E_{t-2}\hat{\Pi}_t]$ under their model specification), in addition to the variabilities of inflation and the output gap.

Their argument could be directly applied to our two-sector model with a sticky-price sector and a flexible-price sector. The central bank in this case should put a positive weight on the variability of an unforecastable component of the sticky-price inflation, in addition to the variabilities of the sticky-price inflation, the aggregate output gap, and the deviation of the relative price from its efficient value. Furthermore, it should be noted that complete stabilization of the sticky-price inflation at zero can eliminate the variability of its unforecastable component. Therefore, stabilizing the sticky-price inflation continues to be the optimal monetary policy in our model, even if we modify the model in order to capture the observed lags in the effects of monetary policy on the sticky-price inflation.

4.5. *An interpretation: domestic inflation targeting in a small open economy*

In this section, we apply the model to an analysis of a small open economy, and discuss the optimal monetary policy response to changes in the exchange rate. We assume a small open economy which produces differentiated industry goods whose prices are sticky, and imports raw material such as food and energy. Exchange rate often exhibits large variability, which will affect the relative price between domestic and foreign goods, which in turn will affect both domestic and foreign demand for domestic goods. Thus, changes in the exchange rate affect the pricing decisions of domestic firms. At the same time, the exchange rate affects the domestic currency prices of foreign goods, which enter the consumer price index. For example, a currency depreciation puts an upward pressure on aggregate inflation. For this small open economy, an interesting question is whether the central bank should stabilize broad inflation measure, which includes prices of imported goods, or domestic inflation.

Svensson (2000) addresses this question. He assumes that the central bank's loss function is the weighted sum of unconditional variances of measures of inflation, output, and interest rate. Using this loss function, he examines the consequences of strict and flexible inflation targeting, domestic and CPI-inflation targeting, as well as the consequence of the Taylor Rule. He calibrates standard deviations of domestic inflation, CPI-inflation, output, exchange rate, and interest rate for each policy regime. Based on the calibration results, he argues that flexible CPI-inflation targeting stands out as successful in limiting not only the variability of CPI inflation but also the variability of output and the real exchange rate.

The model developed in the previous section could be applied to the analysis of a small open economy with sticky prices.[33] Our model implies that the optimal monetary policy for this economy would be the complete stabilization of domestic inflation. This argument is intuitively verified by noting that, in this open economy, price stickiness of the domestic producers is still the only distortion. Then it would be optimal for the central bank to stabilize inflation in that sector. When domestic inflation is completely stabilized, then the responses of the economy to the various shocks is equivalent to those that would arise in an economy with a domestic sector that has flexible prices. Note also that, in this model, the stabilization of the real exchange rate around its time-varying optimal value would be an appropriate goal of the central bank. Presence of the relative-price term in our welfare function (42) implies this observation. However, our model also implies that there is no trade-off between the goal of domestic inflation stabilization and the goal of exchange rate stabilization, when the latter is properly understood. Stabilizing domestic inflation at all times is sufficient for keeping the real exchange rate at its optimal level at all times.

5. Conclusion

In this paper, we addressed two important questions for a central bank under the existence of sector-specific supply shocks: the relationship between relative-price changes and inflation fluctuations, and the identification of appropriate goal variables for the central bank. We showed that the relative price of the flexible-price good represents a shift parameter of inflation in the sticky-price sector. This feature of the model is in agreement with the findings of the empirical literature on the Phillips curve that the relative price of food and energy is significant source of supply shocks. We also characterized the optimal monetary policy that maximizes the welfare of the representative household. The optimal monetary policy for the economy described in the model is a complete stabilization of the inflation in the sticky-price sector. This result implies that the central bank should target core inflation, defined as inflation in the sticky-price sector, rather than a broad inflation measure. Furthermore, the model predicts that stabilizing *core* inflation and stabilizing the aggregate output gap are consistent each other. The model also predicts that, although stabilizing the relative price around its efficient level is one of the appropriate goals of the central bank, stabilizing core inflation is sufficient for achieving this goal.

[33] In fact, Svensson (2000)'s aggregate supply equation is closely related to our Phillips curve in the sticky-price sector.

The characterization of an optimal monetary policy would critically depend on the central bank's goal variables and propagation mechanism of business cycles. The appropriate goal variables, in turn, depend on the structure of the economy. It is therefore important to use an optimizing model to identify the appropriate goal variables of the central bank and its optimal policy regime.[34]

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[34] An interesting research in this area is Benigno (1999), which characterizes an optimal monetary policy in a monetary union.

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