The Dynamics of Financial Innovation and the Industrial Organization of Risk-sharing Markets

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Abstract

This paper develops a theory of the opening and dynamic development of a futures market with competing exchanges. The optimal contract design involves a trade-off between the hedging potential of a contract and it's degree of substitution with competing contracts. As design costs go down slowly, more exchanges enter, but if costs go down fast or reach zero, markets consolidate (fewer number of exchanges). I develop implications for how the hedging potential and cross-correlation between contracts develop over time.

I extend the model to a case where demand is uncertain before trade has been observed, and perform comparative statics on the social efficiency of market opening. For markets with equivalent expected surplus, the propensity of markets to open are negatively related to the probability of further entry and the ex ante uncertainty, and positively related to the time lag between innovations.

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1. Introduction

Financial markets are far from stagnant and have seen many innovations in the last couple of decades[1]. Futures markets and other derivatives markets are no exceptions. Silber [7] describes 154 contract innovations in futures markets between 1960 and 1980, of which about one third were deemed successful. Technology of trading is constantly improving, and with the rise of the Internet it seems that the costs of setting up markets are lower than ever.

It is important to understand the dynamics of this innovation process and what type of security designs should be expected to be succesful. Also, the driving forces of opening a new market are important to analyze to judge the efficiency of innovation and the type of markets we expect to see open. This paper develops a theoretical model of security design and innovation by exchanges in a competitive dynamic environment. We analyze the opening and dynamic formation of futures markets as technological costs of establishing markets and designing contracts decrease. The model gives implications for how the design and number of contracts evolve over time. We also analyze the driving forces for opening a new market when demand is uncertain before trade has been observed.

The economic role for futures in our model is to allow risk-averse consumers with heterogenous endowments to share their endowment risk. We model exchanges as being risk neutral profit maximizers, who benefit by charging proportional transaction fees.

The demand for a contract will depend on how good a hedging instrument it is and how high the transactions fee is. In the context of our model, the hedging potential of a contract is measured by the correlation of the contract with the *endowment differential* of consumers. The endowment differential is the difference in the stochastic endowments of consumers, and constitutes the hedgeable part of the economy. Therefore, based on the large consumer surplus that can potentially be extracted through contracts with high hedging potential, we might expect exchanges to want to issue these type of contracts. However, two contracts that are good hedging instruments will also be close substitutes, which will drive down profits because of price competition between the exchanges. In the extreme, if contracts are identical, the Bertrand game we propose for fee-setting drives profits to zero. The choice of optimal contract design therefore involves a trade-off between the contract's hedging potential and the degree of substitution with other contracts.

The number of exchanges active in the market will depend on the costs of establishing an exchange or of designing a contract.When the contract innovation cost is diregarded and we take

[1] For a partial list of financial innovations, see Allen and Gale [1] and Duffie and Rahi [6].

the number of exchanges as exogenous, we get the following results on the optimal security design:

When there is only one exchange, he issues a contract which is perfectly correlated with the endowment differential. We call this contract the DJ-contract since it was first identified by Duffie and Jackson [5]. With a monopolist exchange, the substitution effect of contracts is absent, so that maximizing the hedging potential is optimal. When there are two exchanges, there is a unique symmetric equilibrium in which both exchanges issue contracts that are not perfectly correlated with the endowment differential, and where the parts of the contracts that are orthogonal to the endowment differential have correlation minus one with each other. This minimizes the degree of substitution given the hedging potential (in fact, the two contracts are complements in the space orthogonal to the endowment differential). Consumers benefit because a portfolio of the two contracts replicates the DJ-contract but transaction fees are smaller. Finally, when there are three or more exchanges on the market, we get the somewhat surprising result that at least three exchanges issue the DJ-contract, and profits are zero. This is because the increased degree of substitution resulting from an increase in hedging potential has little impact on profits for an exchange when there are already two or more other exchanges offering contracts with high hedging potential, since the competition between the other exchanges has already driven fees down. Therefore, it is relatively more favorable for the first exchange to also increase his hedging potential. Contracts have gone from being *strategic substitutes* (an increase of hedging potential by one exchange results in a decrease by the other exchange, as in the duopoly case) to being *strategic complements*.

We use the insight from these results to study the full dynamic model where there is a cost of introducing or switching contracts that is declining over time (equivalently, the level of demand could be thought of as being increasing over time). The assumption of exogenously decreasing costs (or increased demand) is an attempt at capturing the effects of improved technology and increased sophistication of financial markets over time.

We get the following results: Markets open when costs are low enough for a monopolist to break even, and the monopolist offers the DJ-contract. We might have expected a second exchange to enter when costs are low enough for the symmetric duopoly described above to break even, but it turns out that at this point the gain in profit the incumbent exchange would get from switching to the duopoly contract are not high enough to cover the switching cost. Thus, the incumbent exchange enjoys some extra period of monopoly power. Entry doesn't take place until the encumbent's commitment to the DJ-contract is no longer credible, at which point a period of symmetric duopoly ensues. The further development depends on how fast costs go down. If costs go down slowly, more exchanges can enter, and the hedging potential of contracts will be increasing so that all contracts become closer and closer to the DJ-contract again. Competition between contracts becomes very fierce as the degree of substitution grows, and profits go towards zero. However, there is a discontinuity at zero costs: When costs are zero, profits are also zero when there are three or more exchanges. But in that case it is better for all but two exchanges to stay out of the market, in which case we get back to the symmetric duopoly with positive profits. This also happens if costs go down quickly (without reaching zero) if contracts are not yet close to the DJ-contract.

We go on to study the behavior of an innovating exchange in a version of the model where markets are closed and demand is uncertain before trade has been observed. We show that if there is a reasonable probability that demand will turn out not to be high enough to invite further entry once markets are opened, the DJ contract is an optimal innovation. However, if the probability of entry is sufficiently high, the first mover may do better by offering a contract with slightly lower hedging potential to avoid being forced to change to a worse contract in the case of entry. We also show that the propensity of markets to open (as measured by how high initial developments costs can be) for markets with the same expected demand is negatively related to the probability of a very high demand state (where further entry will take place) and positively related to the time lag between innovation. With longer lags, a first mover enjoys a longer initial period of monopoly which makes it more profitable for him to open the market. Higher probability of very good demand states (holding overall expected demand constant) makes it less profitable to open the market since the first mover has to share the surplus with entrants in these states. Therefore, it is often less likely that a market with very uncertain demand (high variance) will open, even if the expected demand is high.

This is not the first paper to study financial innovation and security design by exchanges, but the first one (to the knowledge of the author) to analyze a dynamic model of competing exchanges. Duffie and Jackson [5] use a static framework very similar to the one in our paper to study optimal futures contract. They derive the optimality of the DJ-contract for a monopolist. They extend the model to a sequential game of competition among exchanges for the case of zero transaction costs, and use volume maximization as the objective function which leads to an equilibrium where contracts are orthogonal to each other. Cuny [4] studies a static model of competition where exchanges get profits from extracting member fees to liquidity-providing traders instead of charging transaction fees. Liquidity, which is absent in our model, plays an important role. In a sequential game, he also derives orthogonal contracts as being optimal, although they are not necessarily perfectly correlated with endowment differentials. However, in a simultaneous move setup, he does get correlated contracts for segments in the market with high hedging demand, as in our paper. Tashjian and Weissman [8] study a monopolistic exchange in a static setting with the same objective function as ours and shows that it may be optimal for the exchange to offer several correlated contracts. This is due to the fact that there are several segments of the market (more than the two consumer types present in this paper). Santos [9] studies a dynamic production economy where a monopolistic exchange can increase the span of the market at a cost. His analysis does not involve security design, however. Che and Rajan [3] analyze a general equilibrium model with market makers who are monopolists in their exogenously specified contract and show which markets will be open in equilibrium. Anderson and Harris [2] study a dynamic model of innovation with demand uncertainty that features an information externality similar to the one in this paper, but innovators only choose the time to adopt an exogenously given innovation and there is no security design. A richer summary of the literature can be found in Duffie and Rahi [6].

The paper proceeds as follows. Section 2 describes the model setup and analyzes consumer demand. Section 3 analyzes equilibrium fees and contract design and solves for the dynamic equilibrium path of innovation when exchanges only plan one period ahead. Section 4 extends this model to one where demand is uncertain and exchanges have a long planning horizon to analyze the trade offs faced by the first innovator on the market in his choice of security design and whether to open the market or not. Section 5 concludes.

2. The Model

Our model builds on the model by Duffie and Jackson [5], and differs mainly in two respects: It is dynamic, and exchanges maximize profits by charging transaction fees instead of maximizing volume. There are τ time periods $t = 1, ..., \tau$. There are two classes of agents in the model: Consumers and exchanges (intermediaries). Consumers are risk-averse and have stochastic endowments that they would like to hedge. Exchanges are risk-neutral intermediaries who can introduce one-period futures contracts at a fixed cost. They strive to maximize profits by charging proportional transaction fees for trade in the contracts.

Consumers have mean-variance utility over consumption each period:

$$U(c_t) = E(c_t) - \sigma^2(c_t)$$

There are two consumer types and a continuum of consumers of each type. Each period, consumer type i gets a realization x_i of her random endowment. For simplicitiy and without loss of generality, each type is of measure 1. Endowments are i.i.d. over time. We assume that x_1 and x_2 are not perfectly correlated, so that there is potential for risk-sharing.

Each period, consumers can hedge their endowment risk by trading futures contracts before the realization of endowments. A futures contract (indexed by k) is a random variable paying out f_k, with transaction cost T_k and futures price p_k. Thus, a consumer who is long one contract gets pay-off $f_k - p_k - T_k$ and a consumer who is short gets pay-off $p_k - f_k - T_k$. All payments are settled at the same time. We denote the column-vector of contracts $1, ..., K$ traded in period t by $\mathbf{F}_t \equiv [f_1, ..., f_K]'$. Likewise, the vectors of prices and transaction fees are denoted by $\mathbf{P}_t$ and $\mathbf{T}_t$, respectively. The variance-covariance matrix of contracts is denoted by Σ.

2.1. The consumer's problem

We now characterize the consumer demand of futures in any given period, and the corresponding volume of futures contracts[2]. Since there is no transfer across periods, the consumer solves a series of one-period optimization problems. He needs to choose his vector of futures holdings $\mathbf{y} \equiv [y_1, ..., y_k]'$ so as to maximize utility of consumption each period. The maximization problem for type 1 is stated formally in 2.1:

$$\max_{\mathbf{y}} E(x_1 + \mathbf{y}'(\mathbf{F} - \mathbf{P}) - |\mathbf{y}|'\mathbf{T}) - \text{var}(x_1 + \mathbf{y}'(\mathbf{F} - \mathbf{P}) - |\mathbf{y}|'\mathbf{T}) \tag{2.1}$$

(where the time subscript has been temporarily dropped to save on overhead). As can be seen in 2.1, consumption stems from the endowment and the futures pay-off net of transaction costs. Here, $|\mathbf{y}|$ denotes component-wise absolute values.

Denote the vector of contract volumes as $\mathbf{V} \equiv \{V_1, ..., V_K\}'$, where V_k denotes volume for contract k. We have that $\mathbf{V} = |\mathbf{y}_1| + |\mathbf{y}_2| = 2|\mathbf{y}_1|$, where the last equality follows from the market clearing condition $\mathbf{y}_2 = -\mathbf{y}_1$.

[2]Much of the analysis in this section follows closely that of Duffie and Jackson. The volume equation we derive is a multi-contract version of their single-contract result, for the special case of two consumer types.

We start out assuming that agent 1 will choose a strictly positive position in all of the contracts, so that $\mathbf{V} = 2\mathbf{y}_1$. This assumption turns out to be without loss of generality: In equilibrium, transaction fees will always be set to ensure that some trade take place between the two types. Assuming that type 1 is always long is also innocuous; just multiply a contract by -1 if he is not and replace the old contract with this contract, which is equivalent for risk-sharing purposes and gives the same profit to the exchange.

We also assume that the variance-covariance matrix Σ of contracts is invertible, so that there are no redundant contracts. When there are redundant contracts, either one contract will have zero volume, or demand will be such that an arbitrarily small decrease of the transaction fee will lead to a discrete upward jump in volume (and corresponding profits). This will result in equilibrium profits in the fee-setting game (see below) being zero for at least one contract, so that a contract set with redundant contracts cannot constitute an ex ante equilibrium. Thus, we can concentrate on cases where Σ is invertible.

Following Duffie and Jackson [5] we call the difference in endowments between agent-types 1 and 2 the *endowment differential* and denote it by $e \equiv x_2 - x_1$. Lemma 1 describes the demand-function for type 1 and the corresponding contract volume.

Lemma 1. *Demand for type 1 consumers is given by*

$$\mathbf{y}_1 = \frac{1}{2}\Sigma^{-1}\left(cov(e, \mathbf{F}) - \mathbf{T}\right) \tag{2.2}$$

and contract volumes are given by

$$\mathbf{V} = \Sigma^{-1}\left(cov(e, \mathbf{F}) - \mathbf{T}\right) \tag{2.3}$$

where $cov(e, \mathbf{F})$ is a $K \times 1$ column vector with k:th element $cov(e, f_k)$

Proof. The first order conditions of 2.1 become:

$$E(\mathbf{F}) - \mathbf{P} - \mathbf{T} - 2\left(\Sigma\mathbf{y} + cov(x_1, \mathbf{F})\right) = 0$$

Solving for $\mathbf{y}_1$ gives

$$\mathbf{y}_1 = \Sigma^{-1}\left(\frac{1}{2}\left(E(\mathbf{F}) - \mathbf{P} - \mathbf{T}\right) - cov(x_1, \mathbf{F})\right)$$

Now, through the market clearing condition $\mathbf{y}_1 = -\mathbf{y}_2$ we get the price vector as

$$\mathbf{P} = E(\mathbf{F}) - (cov(x_1, \mathbf{F}) + cov(x_2, \mathbf{F}))$$

Plugging this into the demand function, we get $\mathbf{y}_1$ and $\mathbf{V} \equiv |\mathbf{y}_1| + |\mathbf{y}_2| = 2\mathbf{y}_1$ as in the proposition.
■

The demand-function 2.2 is intuitive. The potential for hedging stems from differences in endowments as captured by the endowment differential e. Type 1 consumers want to off-set their endowment risk by taking positive positions in contracts that have negative correlation with their endowment. Demand is proportional to the extent of this correlation as captured by $cov(e, \mathbf{F})$ and negatively proportional to the cost of hedging as captured by the transaction fees $\mathbf{T}$.

2.2. The exchange's problem

We now turn to a characterization of the problem of introduction of futures contracts faced by issuers. We assume there is an infinite number of potential exchanges that can design and open trade in one futures contract each. There is a fixed cost C_t of introducing or changing a contract. Before the start of period 1, there are no contracts on the market. Each period, the following three-stage game is played out: First, there is a simultaneous-move introduction stage, where exchanges decide whether to enter/stay on the market or not. After the number of entrants has been observed, there is a simultaneous-move design stage where exchanges design their contracts f_k. After the contract set F has been observed, a simultaneous-move fee-setting stage follows. After fees have been set, consumers form their demand, uncertainty is revealed, and consumption takes place. The timing of the game is illustrated in Figure 2.1. We use subgame perfection as our equilibrium concept.

Exchanges are risk-neutral and maximize profit. The pay-off function $\Pi_{k,t}$ as of period t for exchange k is the discounted revenue each period minus switching costs:

$$\Pi_{k,t} = \sum_{i=t}^{\tau} \beta^{i-t} \left(\pi_{k,t} - I_{f_{k,t} \neq f_{k,t-1}} C_t \right)$$

where β is the discount parameter and $I_{f_{k,t} \neq f_{k,t-1}}$ is an indicator function which is one if a new contract is being introduced and zero otherwise, and $\pi_{k,t}$ is the one period revenue function defined

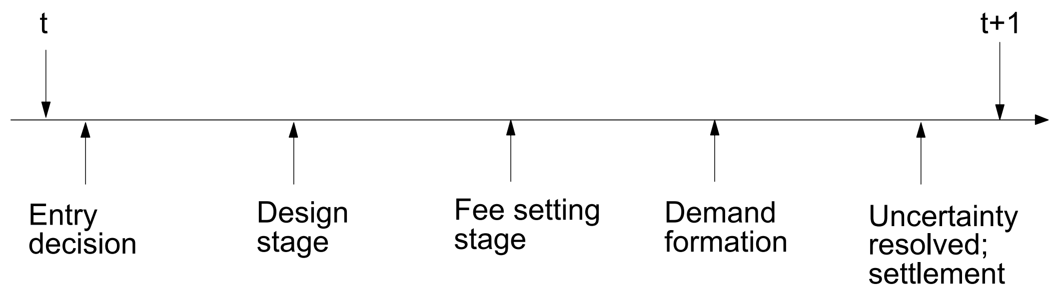

Figure 2.1: Timeline of events in period t.

by

$$\begin{aligned}\pi_{k,t} &\equiv V_{k,t}T_{k,t} \\ &= \Sigma_k^{-1}\left(cov(e,\mathbf{F})-\mathbf{T}\right)T_{k,t}\end{aligned}$$

where Σ_k^{-1} denotes the k:th row of Σ^{-1}.

A strategy for exchange k consists of a sequence of decisions

$$\{E_{k,t}(\mathbf{F}_{t-1},C_t),\ f_{k,t}(\mathbf{F}_{t-1},C_t,\{E_{1,\ldots,}E_\infty\}),\ T_{k,t}\left(\mathbf{F}_t\right)\}$$

at each time period and each stage of the game. Here, $E_{k,t}(\mathbf{F}_{t-1},C_t)\epsilon\,\{0,1\}$ is an indicator function for the decision of staying in the market/entering or not. Note that there is a dynamic link in that E_k will be contingent not only on the cost of entering/switching but also on the previous contract set. This is because the benefits of switching depends on a players previous contract and the benefits of being active on the market depends on whether the entry cost has already been put down or not. Since the set of participants is revealed before the contract-design stage, f_k will also be a function of the participation-set $\{E_{1,\ldots,}E_\infty\}$. Finally, the fee T_k will only be a function of the current contract set.[3] We define a continuation strategy $s_{k,t}^a$ for exchange k at period t and stage a (where $a\in\{1,2,3\}$ indicates the entry stage, the design stage, and the fee-setting stage, respectively) as the sequence of state-contingent decisions from then on and denote the vector of continuation strategies for all exchanges as $\mathbf{s}_t^a=\left\{s_{k,t}^a\right\}_{k=1}^{\infty}$. Similarly, denote the set of continuation

[3]In principle, strategies could be contingent on the whole history of events up until the current period, but since the time horizon is finite backward induction rules out contingencies on past variables that do not enter the profit function. Dynamic punishment strategies that support the large number of equilibria in infinite horizon games do not work in this finite horizon framework.

strategies for all exchanges excluding exchange k as $\mathbf{s}^a_{-k,t}$.

We define a *subgame perfect equilibrium* $\mathbf{s}^1_1$in the following way:

Definition 1. *The set of strategies $\mathbf{s}^1_1$ constitute a subgame perfect equilibrium if, for all t, a, and k, there does not exist a strategy $s'^a_{k,t} \neq s^a_{k,t}$ such that*

$$\Pi_{k,t}\left(\mathbf{s}^a_{-k,t}, s'^a_{k,t}\right) > \Pi_{k,t}\left(\mathbf{s}^a_t\right)$$

Thus, $\mathbf{s}^1_1$ has to constitute a Nash equilibrium at every proper subgame.

In principle, $f_{k,t}$ can take values in an infinite-dimensional state space, which makes the maximization problem for the exchange intractable. Fortunately, we can reduce the dimensionality by noting that volume is only dependent on the variance-covariance matrix Σ and the covariance $cov(e, \mathbf{F})$ of contracts with the endowment differential. Thus, choosing f_k amounts to choosing the covariance with the other variables in the economy. In fact, the following lemma which states the maximization problem of the exchange, shows that $f_{k,t}$ can be fully specified by it's correlation structure with all other variables in the model.

Lemma 2. *Denote by $\boldsymbol{\rho}(e, \mathbf{F}_i)$ the vector of correlation coefficients between the endowment differential and the contracts with individual elements ρ_{ke}. Denote by Σ the correlation matrix of contracts with individual elements ρ_{kj}. Without loss of generality, the maximization problem of an exchange at period t in stage a can be written as*

$$\begin{aligned} \max_{s^a_{k,t}} \Pi_{k,t}\left(s^a_{k,t}, \mathbf{s}^a_{-k,t}\right) &= \sum_{i=t}^{\tau} \beta^{i-t}\left(\pi_{k,i} - I_{f_{k,i}\neq f_{k,i-1}} C_i\right) \\ \text{where} \qquad \pi_{k,i} &= \Sigma^{-1}_{k,i}\left(\boldsymbol{\rho}(e, \mathbf{F}_i) - \mathbf{T}_i\right) T_{k,i} \end{aligned} \tag{2.4}$$

and the contract choice f_k is restricted to have unit standard deviation and is defined by the vector $\left\{\rho_{ke}, \{\rho_{kj}\}_{j\neq k}\right\}$.

Proof. The one-period revenue π_k for an issuer with a contract set (f_k, T_k) on the market can be rewritten as

$$\pi_k = \Sigma^{-1}_k\left(\sigma_e \boldsymbol{\sigma}_{\mathbf{F}_t} \cdot \boldsymbol{\rho}(e, \mathbf{F}_t) - \mathbf{T}\right) T_k$$

Here, σ_e denotes the standard deviation of the endowment differential e, and $\boldsymbol{\sigma}_{\mathbf{F}_t}$ denotes the vector of standard deviations of the contracts. We assume w.l.o.g. that $\sigma_e = 1$. We want to show that $\Pi_{j,t}\left(s^a_{j,t}, \mathbf{s}^a_{-j,t}\right)$ is homogeneous of degree zero in $\{f_{k,t}, T_{k,t}\}$ for all j (including $j = k$) so that a change of length (or standard deviation) of $f_{k,t}$ does not change the profit of any exchange when accompanied by a similar rescaling of the fee. Thus, if $\{f_{k,t}, T_{k,t}\}$ is an optimal choice, so is $\{af_{k,t}, aT_{k,t}\}$ for any multiplier a. Then we can restrict contracts to have unit standard deviation without loss of generality, and all covariances become correlations.

This holds since

$$\begin{aligned}
\pi_k\left(\mathbf{F}_{-k}, \mathbf{T}_{-k}, af_k, aT_k\right) &= \left[\frac{1}{a^2}, \frac{1}{a}, ..., \frac{1}{a}\right] \cdot \Sigma_k^{-1}\left([a, 1, ..., 1]' \cdot \boldsymbol{\sigma}_{\mathbf{F}_t} \cdot \boldsymbol{\rho}(e, \mathbf{F}_t) - [a, 1, ..., 1] \cdot \mathbf{T}\right) aT_k = \pi_k\left(\mathbf{F}_t, \mathbf{T}\right) \\
\pi_j\left(\mathbf{F}_{-k}, \mathbf{T}_{-k}, af_k, aT_k\right) &= \left[\frac{1}{a}, 1, ..., 1\right] \cdot \Sigma_j^{-1}\left([a, 1, ..., 1]' \cdot \boldsymbol{\sigma}_{\mathbf{F}_t} \cdot \boldsymbol{\rho}(e, \mathbf{F}_t) - [a, 1, ..., 1] \cdot \mathbf{T}\right) T_j = \pi_j\left(\mathbf{F}_t, \mathbf{T}\right)
\end{aligned}$$

where $\cdot$ denotes element-by-element multiplication. ■

The intutition behind this is that the consumer can rescale any contract by buying $1/a$ of the upscaled contract. Since the scaling possibility was open to the consumer in the first place, the optimality of the consumer choice will guarantee that the new position in the contract is $1/a$ times the old so profits are unaffected. From now on, Σ therefore denotes the correlation matrix of contracts.

We also have to determine what choices of $\left\{\rho_{ke}, \left\{\rho_{kj}\right\}_{j\neq k}\right\}$ given Σ_{-k} and $\boldsymbol{\rho}_{-k}(e, \mathbf{F})$ are feasible, since it is clear that these parameters cannot be chosen freely and independently of each other. For example, if there are two contracts with $\rho_{1e} = \rho_{2e} = 1$, the cross correlation ρ_{12} will also have to be one. The following lemma gives bounds on the correlation structure between two random variables given their correlation with another set of random variables, which makes it possible for us to define feasible choices of $\left\{\rho_{ke}, \left\{\rho_{kj}\right\}_{j\neq k}\right\}$.

Lemma 3. *Assume x and y are two random variables in infinite dimensional state-spaces, and that $\mathbf{Z}$ is a vector of other random variables. Assume that the vector of correlations $\boldsymbol{\rho}(x, \mathbf{Z})$ between x and the variables in $\mathbf{Z}$ are given, and likewise for $\boldsymbol{\rho}(y, \mathbf{Z})$. Then $\rho(x, y)$ is a feasible correlation coefficient between x and y if and only if*

$$\begin{aligned}
\rho(x, y) &\leq \boldsymbol{\rho}(x, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(y, \mathbf{Z}) + \sqrt{1 - \boldsymbol{\rho}(x, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(x, \mathbf{Z})}\sqrt{1 - \boldsymbol{\rho}(y, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(y, \mathbf{Z})} \\
\rho(x, y) &\geq \boldsymbol{\rho}(x, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(y, \mathbf{Z}) - \sqrt{1 - \boldsymbol{\rho}(x, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(x, \mathbf{Z})}\sqrt{1 - \boldsymbol{\rho}(y, \mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(y, \mathbf{Z})}
\end{aligned}$$

where $\Sigma_{\mathbf{Z}}$ is the correlation matrix of $\mathbf{Z}$.

Proof. In Appendix A. ■

Thus, for example, if ρ_{1e} and ρ_{2e} are given, the lowest possible correlation between contracts 1 and 2 is $\rho_{12} = \rho_{1e}\rho_{2e} - \sqrt{1-\rho_{1e}^2}\sqrt{1-\rho_{2e}^2}$.

3. The $\beta = 0$ case

For the purposes of this section, we analyze the "myopic" case $\beta = 0$ where exchanges only care about pay-off in the current period. This case is substantially easier to characterize than the long-term strategic case $\beta > 0$ but still captures many of the interesting effects. A special case of a model with $\beta > 0$ is analyzed in the following section. When $\beta = 0$, the profit function becomes

$$\Pi_{k,t} = E_{k,t}\left(\pi_{k,t} - I_{f_{k,t} \neq f_{k,t-1}} C_t\right)$$

Participating exchanges maximize the pay-off $\pi_{k,t} - I_{f_{k,i} \neq f_{k,i-1}} C_i$ each period. Each period-game equilibrium has to be subgame perfect in isolation. The equilibrium analysis is performed by backward induction each period. Given a contract set $\mathbf{F}_t$, we find the equilibrium set of fees $\mathbf{T}(\mathbf{F}_t)$, and go backwards to find the equilibrium contract set and participation decisions. We start by describing the equilibrium at the fee setting stage. The equilibrium we develop also holds for the case $\beta > 0$, since past fees do not affect future profits.

3.1. Equilibrium Fees

All participating exchanges maximize $\pi_{k,t} = \Sigma_k^{-1}(\boldsymbol{\rho}(e, \mathbf{F}_t) - \mathbf{T}) T_k$ over T_k. Equilibrium fees are given in the following proposition.

Proposition 1. *Equilibrium fees each period are given by*

$$\mathbf{T}(\mathbf{F}_t) = \left[\Sigma^{-1} + \operatorname{diag}\left(\Sigma^{-1}\right)\right]^{-1} \Sigma^{-1} \boldsymbol{\rho}(e, \mathbf{F}) \tag{3.1}$$

where $\operatorname{diag}\left(\Sigma^{-1}\right)$ *has diagonal elements equal to* Σ^{-1} *and zeros off the diagonal.*

Proof. The first order condition for exchange k is given as

$$\Sigma_k^{-1}(\boldsymbol{\rho}(e, \mathbf{F}_t) - \mathbf{T}) - \Sigma_{k,k}^{-1} T_k = 0$$

This condition has to hold for all k in equilibrium. Putting the first order conditions for all exchanges together gives the system

$$\Sigma^{-1}\left(\boldsymbol{\rho}(e,\mathbf{F}_t)-\mathbf{T}\right)-\operatorname{diag}\left(\Sigma^{-1}\right)\mathbf{T}=0$$

Solving for $\mathbf{T}$ gives the result. ■

Example 1. *(Uncorrelated contracts) If Σ is the identity matrix, so that all contracts are orthogonal to each other, or if there is only one contract on the market so that $\Sigma = 1$, we see from 3.1 that $T_k(\mathbf{F}_t) = \frac{\rho_k}{2}$ and $\pi_k = (\rho_k - T_k)\,T_k = \frac{\rho_k^2}{4}$. This is intuitive: The larger the correlation of a contract with the endowment differential, the higher the fee that can be extracted and the higher the profit. Of course, contracts cannot be orthogonal to each other if two or more contracts have $\rho_k > 0$. For orthogonal contracts, all but one exchange will have zero profits. Thus, we would never expect contracts to be orthogonal (cf Duffie and Jackson [5], where the volume maximizing objective function and zero transaction fees lead to orthogonal contracts).*

Example 2. *(Perfectly correlated contracts) Assume there are two contracts on the market with $\rho_{1e} = \rho_{2e}$ and cross-correlation $\rho_{12} = 1$. (Thus, Σ is not invertible.) Since contracts are perfect substitutes, the contract with the lowest transaction fee will get volume $\rho_{1k} - T_k$ and the other zero. If $T_1 = T_2$ we assume that each exchange gets volume $\frac{\rho_{1k}-T_k}{2}$. The fee-setting game is thus equivalent to Bertrand competition, and the only equilibrium is $T_1 = T_2 = 0$ so that profits are zero.*

3.2. Equilibrium in the Contract Design Stage

At the contract design stage, participants have to decide $f_{k,t}(\mathbf{F}_{t-1}, C_t, \{E_{1,\ldots,}E_\infty\})$, anticipating the equilibrium in the fee-setting stage. From expression 2.4, the maximization problem for participating exchange k given the strategies $\mathbf{F}_{-k,t}$ of the other participants becomes

$$\max_{f_{k,t}=\left\{\rho_{ke},\left\{\rho_{kj}\right\}_{j\neq k}\right\}} \Sigma_k^{-1}\left(\boldsymbol{\rho}(e,\mathbf{F})-\mathbf{T}\left(\mathbf{F}_t\right)\right)T_k\left(\mathbf{F}_t\right)-I_{f_{k,t}\neq f_{k,t-1}}C_t \tag{3.2}$$

Note that for new entrants, C_t is a sunk cost ($I_{f_{k,t}\neq f_{k,t-1}}$ is always one) so that entrants solve the smooth maximization problem $\max_{f_{k,t}} \Sigma_k^{-1}\left(\boldsymbol{\rho}(e,\mathbf{F})-\mathbf{T}\left(\mathbf{F}_t\right)\right)T_k\left(\mathbf{F}_t\right)$. Incumbents switch to their smooth best reply if the extra profits make up for the switching cost.

We start by describing equilibria in a one-period game where all exchanges are entrants so that they choose their smooth best reply. We then use these results to solve for the full dynamic equilibrium.

Proposition 2. *When one exchange enters, the unique static equilibrium is* $\rho_{1e} = 1$ *and the exchange gets revenue* $\pi_1 = \frac{1}{4}$

Proof. Since $\Sigma = 1$ we get $T_1 = \frac{\rho_{1e}}{2}$ from 1 so that $\pi_1 = \left(\rho_{1e} - \frac{\rho_{1e}}{2}\right)\frac{\rho_{1e}}{2} = \frac{\rho_{1e}^2}{4}$. This is maximized at $\rho_{1e} = 1$. ■

This result was first derived by Duffie and Jackson for a volume maximizing monopolist, and we therefore call a contract with $\rho_{ke} = 1$ the DJ-contract. A monopolist chooses to issue a contract which is maximally correlated with the endowment differential, since this is the most valuable hedging contract for consumers and allows him to extract the most consumer surplus.

Proposition 3. *When two exchanges enter, the unique static equilibrium is* $\rho_{1e} = \rho_{2e} = \sqrt{\frac{5}{6}}$ *and* $\rho_{12} = \frac{2}{3}$ *(the minimal feasible correlation). Revenues are* $\pi_1 = \pi_2 = \frac{3}{32}$.

The details of the proof of Proposition 3 are given in Appendix B. We first show in Lemma 10 that, when there are two contracts with correlations ρ_{1e} and ρ_{2e} with the endowment differential, revenues are maximized when ρ_{12} is set at the minimal feasible value. This is intuitive: Given a certain correlation with the endowment differential, which determines the hedging potential of a contract, an exchange would like to be as far as possible from other contracts, since the higher the degree of substitution between contracts, the fiercer the competition at the fee setting stage. Substituting for ρ_{12} and $\mathbf{T}(\mathbf{F}_t)$ in the profit-function, we can solve for best replies $\rho_{ke} = b\left(\rho_{je}\right)$ by setting the first order condition with respect to ρ_{ke} to zero. The first order condition is a high degree polynomial and cannot be solved analytically in general, but we show in the appendix that there must be a fixed point $\rho_{ke} = b\left(b\left(\rho_{je}\right)\right)$ and, in fact, it is unique at the symmetric solution $\rho_{1e} = \rho_{2e} = \sqrt{\frac{5}{6}}$. This can be seen in Figure 3.1 which shows the best reply functions.

Note that the portfolio $\sqrt{\frac{3}{10}}\left(f_1 + f_2\right)$ is equal to the DJ-contract[4], and it's cost can be calculated to be $\frac{1}{4}$ which is less than the price $\frac{1}{2}$ in the monopolist case. Thus, consumers are better off.

Proposition 4. *When there are more than two exchanges entering, the unique symmetric static equilibrium is for everyone to issue the DJ contract* $\rho_{ke} = 1$*, and revenues are zero. For* $K = 3$*, this is the unique equilibrium.*

---

[4] $\sigma^2\left(\sqrt{\frac{3}{10}}\left(f_1 + f_2\right)\right) = \frac{3}{10}\left(1 + 1 + 2 * \frac{2}{3}\right) = 1$, $\rho\left(\sqrt{\frac{3}{10}}\left(f_1 + f_2\right), e\right) = \operatorname{cov}\left(\sqrt{\frac{3}{10}}\left(f_1 + f_2\right), e\right) = \sqrt{\frac{3}{10}} * 2 * \sqrt{\frac{5}{6}} = 1$

Figure 3.1: Best reply functions $b(\rho_{1e})$ and $b(\rho_{2e})$ when two contracts are on the market.

Proof. In Appendix C. ■

The intuition behind Proposition 2 is the following: There are two countervailing forces in the choice of ρ_{ke}. On the one hand, an exchange wants ρ_{ke} to be as high as possible because this increases the hedging potential of the contract. On the other hand, he wants it to be as far away from other contracts as possible to decrease the degree of substitution, since close substitutes will yield low fees in the fee-setting stage. Proposition 3 shows how the substitution effect leads to contracts other than the DJ-contract when there are two exchanges ($b\left(\rho\right)$ is decreasing in ρ). With three or more exchanges, the incremental effect on competition for an exchange that increases his ρ_{ke} becomes smaller, since he cannot affect the degree of substitution between other contracts. The hedging potential therefore becomes relatively more important, and in particular, if all other exchanges use the same ρ, we can show that $b\left(\rho\right)$ becomes increasing in ρ and $b\left(\rho\right) > \rho$. Contracts go from being strategic substitutes to being strategic complements. The only fixed point is at the DJ-contract.

It is strongly conjectured that all equilibria with $K > 3$ also have the feature that at least three contracts have $\rho_{ke} = 1$, but checking for asymmetric equilibria has to be done computationally and is infeasible for arbitrarily large Σ matrices so this has only been checked for the case $K = 3$.

Turning to the case where everyone is not an entrant, so that incumbent exchanges have to decide whether to change their contract or not, we can get many more equilibria if we look at a

Table 3.1: **Evolution of Market Structure**

The table describes the dynamic time path of the equilibrium market structure when costs go down slowly. The number of exchanges increases until costs reach zero, at which point all but two exchanges drop out of the market. The first contract is a perfect hedge for consumers. Then follows a period of differentiation and subsequent homogenization of contracts, until costs reach zero where the symmetric duopoly gets reestablished.

	ρ_{1e}	ρ_{2e}	ρ_{3e}	ρ_{12}	ρ_{13}	ρ_{23}	π_1	π_2	π_3
$C_t \geq \frac{1}{4}$	–	–	–	–	–	–	–	–	–
$\frac{1}{4} > C_t \geq .027$	**1**	–	–	–	–	–	$\frac{1}{4}$	–	–
$.027 > C_t \geq .016$	$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{5}{6}}$	–	$\frac{2}{3}$	–	–	$\frac{3}{32}$	$\frac{3}{32}$	–
$.016 > C_t \geq .009$	$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{5}{6}}$	**.894**	$\frac{2}{3}$	.816	.816	.039	.039	.016
$.009 > C_t \geq .0013$	**.977**	$\sqrt{\frac{5}{6}}$	.894	.823	.816	.816	.048	.021	.016
$.0013 > C_t \geq 9 * 10^{-5}$	.977	**.986**	**.985**	.948	.949	.952	.006	.009	.009
$C_t \rightarrow 0$	$\rightarrow 1$	$\rightarrow 1$	$\rightarrow 1$	$\rightarrow 1$	$\rightarrow 1$	$\rightarrow 1$	$\rightarrow 0$	$\rightarrow 0$	$\rightarrow 0$
$C_t = 0$	$\sqrt{\frac{5}{6}}$	$\sqrt{\frac{5}{6}}$	–	$\frac{2}{3}$	–	–	$\frac{3}{32}$	$\frac{3}{32}$	–

one period game in isolation. Any arbitrary past contract set $\mathbf{F}_{t-1}$ can be an equilibrium at t if switching costs are high enough. However, each period game is not played in isolation, and the contract set $\mathbf{F}_{t-1}$ is not arbitrary, and in the following section we describe the dynamic path of the equilibrium contract set.

3.3. The full dynamic equilibrium

We assume that the initial contract design cost C_1 is bigger than the highest profit that anyone can get so that markets do not open immediately. When markets open, the starting equilibrium contract set has to be one of the equilibria identified in the previous section where everyone is at their best smooth reply, since everyone will be an entrant. Once there are incumbents on the market, they may stick with their old contracts if switching is too expensive to make up for the maximized revenue at the smooth best reply.

Table 3.1 describes the dynamic evolution of the market structure when switching costs are going down smoothly and there are three potential exchanges. For the cases where three exchanges are active on the market, we solve numerically for equilibria. We describe the equilibrium path by breaking it into a number of subperiods:

1. $C_t \geq \frac{1}{4}$: Markets remain unopened since the cost of innovating is larger than maximal monopolist profits.

2. $\frac{1}{4} > C_t \geq .027$: One exchange enters with the DJ-contract when $C_t = \frac{1}{4}$ since this is equal

to the monopoly profit he enjoys. Two exchanges cannot enter since profits in the symmetric duopoly (Proposition 3) are not enough to cover the costs.

3. $.027 > C_t \geq .016$: A second exchange enters when costs reach .027. The two possible equilibria when a second exchange enters are the symmetric duopoly from Proposition 3 (if switching costs for the incumbent are low enough to make it profitable for him to change) or that the incumbent stays with his old contract and the entrant chooses his smooth best reply. At $f_2 = b(DJ)$, profits for exchange 2 are only $\frac{1}{48}$, so the second equilibrium cannot happen before costs go down below this number. The symmetric equilibrium becomes possible at $C_t = .027$, since that is the threshold cost at which the incumbent finds it profitable to switch when the entrant plays the symmetric equilibrium strategy. Profits at $\frac{3}{32}$ are more than enough to make up for the entrants design cost.

4. $.016 > C_t \geq .009$: A third exchange enters when C_t reaches .016, when profits just cover his cost of innovating His optimal contract has correlation $\rho_{k3} = \rho_{3e}\rho_{ke}$ with contracts one and two. This is the minimal possible correlation given the hedging potential. The other exchanges do not find it worthwhile to switch contracts.

5. $C_t \to 0$: As C_t continues to decrease, exchanges find it worthwhile to switch to their smooth best reply against other contracts in lock-step. The tendency is for all exchanges to increase the hedging potential of their contracts, for reasons explained in Proposition 4. Even though exchanges strive to minimize the correlation of their contract with other contracts, the increased hedging potential of all contracts makes the degree of substitution between contracts go up and profits go down.

6. $C_t = 0$: When switching costs are zero, Proposition 4 tells us that profits must be zero if more than two exchanges participate. Anticipating this at the entry stage, all but two exchanges will keep out of the market. Therefore, the symmetric duopoly ensues, and profits become positive.

If the innovation cost C_t features sudden downward jumps, it may happen that a third exchange drops out or never enters even if C_t does not reach zero. For example, imagine a situation where C_t suddenly jumps from .02 to .001 . The initial market structure is the symmetric duopoly, which remains to be an equilibrium if no other exchange enters. If a third exchange enters, it will be profitable for both incumbents to change and the only equilibrium possible is the zero profit equilibrium. This is a sufficient entry-deterrant to keep the profitable duopoly intact.

4. Innovation with demand uncertainty and $\beta > 0$

We now study a case where demand (or equivalently, the relative innovation cost) is uncertain before markets have opened. Once markets are open, demand is revealed and stays constant throughout time. We also set $\beta > 0$, so that exchanges have to worry about how their decision today affects profits tomorrow. This gives us a chance to analyze under what circumstances we expect markets to open, and what type of intertemporal trade offs face a first mover.

We assume that the end period τ is very large (going to infinity).The cost C can take on three values: $C \in \{C_H, C_M, C_L\}$ with probabilities p_H, p_M, and p_L. Costs are only measured relative to demand, so rather than interpreting this as uncertainty about cost we interpret it as uncertainty about demand. We assume that $C_H > C_M > C_L$. The values are set such that at H there will be no further entry regardless of which contract the first mover chooses to launch, at M there may be entry depending on the choice of contract, and at L entry from a second exchange is certain.

We also assume that there is an added fixed development cost D for any exchange to enter the market when the market is closed. This is supposed to capture the stylized fact that the first innovation generally requires much more research and development than subsequent contract-introductions (see Tufano [10] and Silber [7] for empirical support of this assumption). We use D as a slack variable such that the expected returns from innovating are zero. This assures that there will be only one exchange innovating in the first period (we would expect the first exchange to come in at a point where he breaks even in a world of free entry).

There are two disadvantages of being a first mover: The added development cost, and the information externality provided to other exchanges who can tailor their strategy based on the revealed demand structure in the market. The first mover advantages that we identify are threefold: Firstly, the extra profits in the brief period of monopoly that an innovating exchange enjoys before competitors enter when the demand level is high. Secondly, demand may turn out to be high enough to sustain profits for a monopolist, but not for a duopoly, in which case the first mover becomes the sole market participant. Thirdly, the first mover may be able to position a better contract on the market than followers. In the next subsection we characterize the optimal contract choice of the innovator given that he chooses to open the market. We show that he will issue the DJ-contract if the probability of the low cost state is not too big, and that he will choose a contract with slightly lower hedging potential otherwise. The following subsection uses these results and analyzes the conditions that affect whether a market opens or not.

4.1. Optimal innovation contracts

It is clear that the DJ-contract is optimal for the innovating exchange if cost does not turn out to be C_L, since the DJ-contract gives the highest monopoly profit.

What happens when the high demand state L occurs? We assume that C_L is such that an incumbent with the DJ contract just finds it profitable to change to the symmetric duopoly contract (which we denote the SD contract) if the entrant plays the SD contract forever:

$$\begin{aligned} C_L &= \sum_{t=0}^{\infty} \beta^t \left(\pi_{1t} \left(f_{1t} = SD, f_{2t} = SD \right) - \pi_{1t} \left(f_{1t} = DJ, f_{2t} = SD \right) \right) \\ &= \frac{\pi_1 \left(SD, SD \right) - \pi_1 \left(DJ, SD \right)}{1-\beta} \\ &= \frac{\frac{3}{32} - \frac{24}{361}}{1-\beta} \left(\approx \frac{0.027}{1-\beta} \right) \end{aligned}$$

(from substitution into the profit function). If f_{11} (the contract introduced in period 1 by the innovating exchange) is the DJ-contract, the symmetric duopoly is in fact the dynamic equilibrium in state L, as shown in the following lemma:

Lemma 4. *When $C_L = \frac{\pi_1(SD,SD) - \pi_1(DJ,SD)}{1-\beta}$ and f_{11} is the DJ-contract, the unique equilibrium from period 2 onwards is the symmetric duopoly.*

Proof. In Appendix D. ■

The expected profits gross of initial expected innovation cost $D + p_H C_H + p_M C_M + p_L C_L$ for the innovating exchange from introducing f_{11} is therefore

$$\begin{aligned} \Pi_1 \left(f_{11} = DJ \right) &= \left(p_H + p_M \right) * \left(\frac{1}{1-\beta} \frac{1}{4} \right) \\ &\quad + p_L * \left(\frac{1}{4} + \frac{\beta}{1-\beta} \frac{3}{32} - \beta C_L \right) \end{aligned} \tag{4.1}$$

Here, $\frac{1}{1-\beta}\frac{1}{4} = \sum_{t=0}^{\infty} \beta^t \frac{\rho_{1e}^2}{4}$ is the monopolist profit with the DJ contract. The term $\frac{\beta}{1-\beta}\frac{3}{32} = \sum_{t=1}^{\infty} \beta^t \frac{3}{32}$ in the last line is the discounted profit from period 2 onwards of a symmetric duopolist, and βC_L measures the extra design cost of switching contract in period 2.

However, the DJ contract may not be optimal. We can show that if exchange 1 introduces a contract with a slightly lower hedging potential (but not as low as $\sqrt{\frac{5}{6}}$), the resulting equilibrium in the low cost state is for the incumbent to stay with his contract and for the entrant to choose his short-term best reply. This increases profits in the low cost state for the incumbent, because we can show that for a contract set $\{f_1, f_2 = b(f_1)\}$ where $b(f_1)$ is the short-term best reply against f_1 as in Figure 3.1, the one-period profit $\pi_{1,t}(\{f_1, f_2 = b(f_1)\})$ of the incumbent is increasing in the hedging potential ρ_{1e} of his contract. Thus, in a Stackelberg game where the first exchange introduces his contract without any possibility of changing it and the second exchange chooses his best reply, the DJ-contract is in fact optimal for the first mover. The problem in the dynamic game that we are studying is that the first mover cannot credible commit to staying with the DJ-contract. The second exchange can "prey" it by introducing the symmetric duopoly contract, which is not a short term best reply against the DJ-contract but forces the first exchange to change to an equilibrium which is more beneficial for the entrant. The incumbent can avoid this by introducing a "non-preyable" contract in period 1, as outlined in the next lemma.

Lemma 5. *When* $C_L = \frac{\pi_1(SD,DJ)-\pi_1(SD,SD)}{1-\beta}$ *and* f_{11} *has* $\rho_{1e} \approx 0.9324$*, the unique equilibrium from period 2 onwards is* $\{f_1 = f_{11}, f_2 = b(f_{11})\}$*.(where* $\rho_{2e} \approx 0.8984$ *and* ρ_{12} *is minimal). Also,* f_{11} *has the highest hedging potential of all non-preyable contracts.*

Proof. In Appendix D. ■

We call this contract the *NP contract*, for non-preyable. By using the NP contract the incumbent can insure himself of having a better hedging instrument that generates more profit than the contract of a follower. The one-period profit from the NP-contract for a monopolist is $\frac{\rho^2_{NP,e}}{4} \approx 0.217$ (as compared to .25 for the DJ-contract) and the profit in a duopoly when $f_2 = b(NP)$ is $\pi_1(NP, b(NP)) \approx 0.1008$ (as compared to $\frac{3}{32} = 0.09375$ for the symmetric duopoly).

If the NP-contract is used and state M occurs, there is a chance that it may be profitable for a second exchange to enter whereas it would not be if the DJ contract was used. This cut-off point is $C^*_M = \frac{\pi_2(NP,b(NP))}{1-\beta} \approx \frac{0.08324}{1-\beta}$, which is the profit a second exchange would enjoy by entering with $f_2 = b$. For higher values of C_M, he will not enter. The expected profits gross of initial expected innovation cost from innovating with NP for this case are:

$$\Pi_1(f_{11} = NP) = (p_H + p_M) * \left(\frac{\rho^2_{NP,e}}{4}\frac{1}{1-\beta}\right) + p_L * \left(\frac{\rho^2_{NP,e}}{4} + \pi_1(NP, b(NP))\frac{\beta}{1-\beta}\right) \tag{4.2}$$

and for the case $C_M < C^*_M$:

$$\Pi_1(f_{11} = NP) = p_H * \left(\frac{\rho^2_{NP,e}}{4}\frac{1}{1-\beta}\right) + (p_M + p_L) * \left(\frac{\rho^2_{NP,e}}{4} + \pi_1(NP, b(NP))\frac{\beta}{1-\beta}\right) \tag{4.3}$$

The following proposition shows when the DJ contract and the NP contract are optimal innovations, respectively.

Proposition 5. *When $C_M \geq C^*_M$, the DJ contract is preferred to the NP contract if and only if $p_L < p^*_L$ where $p^*_L = \frac{\frac{1-\rho^2_{NP,e}}{4}}{\beta\left(\frac{1-\rho^2_{NP,e}}{4}+\pi_1(NP,b(NP))-\frac{24}{361}\right)} \approx \frac{1}{2\beta}$.*

When $C_M < C^*_M$, the DJ contract is preferred to the NP contract if and only if $p_L < p^{**}_L$ where $p^{**}_L = p^*_L + 1.741p_M$

Proof. In Appendix E. ■

For both cases of C_M, the NP contract is more attractive the higher p_L is, since it gives higher profits than the DJ contract only in the L state. If C_M is large, the NP contract is equally inferior in both the H and the M state, but if C_M is small enough it is excessively unattractive in the M state since there will be entry in this state if the incumbent has the NP contract. Therefore, if p_M is not too small relative to p_L, the DJ contract is preferred. The discount factor β has an unambiguous effect: The smaller it is, the more favorable the DJ contract is. This is because of the first-period monopoly profits that have more relative weight the smaller the discount factor. They are higher for the DJ contract, since this is the optimal monopolist contract. The NP contract is only attractive to an exchange that worries relatively more about future profits in the state where there is entry and wants to avoid the symmetric duopoly.

Proposition 5 is also interesting because it shows that we may very well observe a suboptimal monopolist contract as the only contract on the market. This contract may look suboptimal ex post (when we end up in a no-follower state), but is of course optimal ex ante.

4.2. Opening of the market

In this section we perform comparative statics on the conditions that affect the opening of the market. To this extent, we fix expected innovation cost normalized by the discount parameter:

$$p_H c_H + p_M c_M + p_L c_L = K \tag{4.4}$$

where the relationship between the normalized cost c_S and the actual cost C_S is given by $C_S = \frac{c_S}{1-\beta}$. This is to hold the social value of opening the market constant, since a social planner would only care about the expected innovation cost (or inversely, demand) when deciding whether to open the market or not. By keeping this constant, we can compare the efficiency of innovation under different market conditions. The reason for the normalization is that we view the discount parameter as a measure of the length of a time-period. Similarly, we normalize $D = \frac{d}{1-\beta}$ and keep d constant.

Using the results from the previous section, the profit of an innovator is

$$\begin{aligned}
\Pi_1 = & \left(I_{C_M \geq C_M^*} I_{p_L < p_L^*} + \left(1 - I_{C_M \geq C_M^*}\right) I_{p_L < p_L^{**}}\right) * \Pi_1 \left(f_{11} = DJ\right) \\
& + \left(1 - \left(I_{C_M \geq C_M^*} I_{p_L < p_L^*} + \left(1 - I_{C_M \geq C_M^*}\right) I_{p_L < p_L^{**}}\right)\right) * \Pi_1 \left(f_{11} = NP\right) \\
& - \frac{K}{1-\beta} - \frac{d}{1-\beta}
\end{aligned}$$

In the following we perform comparative statics on Π_1 subject to restriction 4.4. Markets open when Π_1 is positive, or equivalently when $(1-\beta)\Pi_1$ is positive. An increase in $(1-\beta)\Pi_1$ subject to 4.4 indicates a higher propensity for markets to open (can open for higher d).

Lemma 6. *Increasing β (decreasing length of time periods) makes markets less prone to open.*

Proof. Substituting for the profit-functions 4.1, 4.2 and 4.3 in the previous subsection shows and taking the derivative of $(1-\beta)\Pi_1$ shows this. ■

Thus, longer time periods makes it easier for an innovator to open the market. This is because of the longer initial period of monopoly before another exchange can enter. We may expect markets with longer innovation lags to have more innovation.

Lemma 7. *Increasing C_M and decreasing C_H makes markets more prone to open*

Proof. Increasing C_M and decreasing C_H has no effect except if C_M is pushed above C_M^* and the NP contract is optimal. In that case, $\Pi_1(f_{11} = NP)$ increases. ■

By increasing the entry cost in state M without changing overall expected innovation cost, it becomes more profitable ex ante for the first mover to innovate, since the risk of entry by a competitor goes down.

Lemma 8. *Increasing p_L and decreasing p_M and p_H makes markets less prone to open*

Proof. Both $\Pi_1(f_{11} = DJ)$ and $\Pi_1(f_{11} = NP)$ are decreasing in p_L. The added increase of C_M or C_H either has no effect or increases profits if C_M is pushed above C_M^* (see Lemma 7). ■

If we increase the probability of the most favorable state for a follower (state L) without changing overall expected innovation cost, profits for the first mover goes down.

Lemma 9. *Increasing p_M and decreasing p_L and p_H (decreasing variance) makes markets more prone to open unless C_M is small and p_L is big so that the NP contract is preferred.*

Proof. Increasing p_M and decreasing p_H has no effect unless $C_M < C_M^*$ and the NP contract is optimal, in which case it has negative effect. Increasing p_M and decreasing p_L increases value unless $C_M < C_M^*$ and the NP contract is optimal, in which case it has no effect. ■

Thus, increasing the probability of the middle state is good because it reduces the probability of entry, unless there is entry in the middle state which is when C_M is low and the NP contract is used. In that case, the decreased probability of the high state (where there is no entry) is bad for the innovator. This shows that we may expect markets where there is a lot of uncertainty about demand (high variance) to be less prone to open, unless it is more or less certain that another exchange will follow.

This is because the innovator fails to capture a lot of the surplus in state L, where demand is high. Comparing two situations with equal expected innovation cost (demand) shows this. First, imagine a situation where it is certain that state M will occur and no entry will take place, and that profits are positive in this state. Next, imagine a situation where costs will either be very low or very high (the H state). Losses occur in the high cost state, and they are not made up for by the profits in the low cost state since these profits have to be shared with a competitor.

5. Conclusion

I have developed a theory of the opening and dynamic development of a risk-sharing market with competing exchanges. In a pure exchange setting, commission revenue-maximizing exchanges can introduce futures contracts to facilitate risk-sharing. There is a fixed contract-design cost. I show that the optimal contract design involves a trade-off between the hedging potential of a contract and it's degree of substitution with competing contracts. I show that as design costs go down slowly, markets become more fragmented (higher number of exchanges), but if costs go down fast or reach zero, markets consolidate (fewer number of exchanges). The hedging potential and cross-correlation between contracts follow a non-linear path: The first contract on the market is a perfect hedging instrument, but as competition increases contracts go through a period of differentiation with lower hedging potential, followed by a period of homogenization with higher hedging potential.

I extend the model to a case where demand is uncertain before trade has been observed, and show that a first innovator may choose to issue a contract with less than perfect hedging potential to be able to capture a favorable niche of the market if demand turns out to be high enough for competing exchanges to enter. I also perform comparative statics on the social efficiency of market opening and show that for markets with equivalent expected surplus, the propensity of markets to open are negatively related to the probability of further entry and the ex ante uncertainty, and positively related to the time lag between innovations.

The paper is unfortunately lacking in empirical observations. Hopefully, the empirical implications from the model in the form of hedging potential and degree of substitution between contracts and the initial development for newly opened markets can be tested. This would be a fruitful direction of future research.

Appendices

A. Proof of Lemma 3

Assume w.l.og. that $E(x) = E(y) = E(\mathbf{Z}) = 0$ and that all variables are of unit standard deviation (since means and standard deviations do not affect correlations). Rewrite x and y in the projection form:

$$
\begin{aligned}
x &= \boldsymbol{\beta}_1 \mathbf{Z} + \varepsilon_1 \\
y &= \boldsymbol{\beta}_2 \mathbf{Z} + \varepsilon_2
\end{aligned}
$$

where we have that

$$
\begin{aligned}
\boldsymbol{\beta}_1 &= E(x\mathbf{Z})' E(\mathbf{Z}\mathbf{Z}')^{-1} \\
&= \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \\
\boldsymbol{\beta}_2 &= E(y\mathbf{Z})' E(\mathbf{Z}\mathbf{Z}')^{-1} \\
&= \boldsymbol{\rho}(y,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \\
E(\varepsilon_1 \mathbf{Z}) &= E(\varepsilon_2 \mathbf{Z}) = E(\varepsilon_1) = E(\varepsilon_1) = 0
\end{aligned}
$$

We then have the correlation between x and y as $\rho(x,y) = E(xy) = E((\boldsymbol{\beta}_1\mathbf{Z} + \varepsilon_1)(\boldsymbol{\beta}_2\mathbf{Z} + \varepsilon_2)) = E(\boldsymbol{\beta}_1\mathbf{Z}\boldsymbol{\beta}_2\mathbf{Z}) + E(\varepsilon_1\varepsilon_2)$

$= E(\boldsymbol{\beta}_1\mathbf{Z}\mathbf{Z}'\boldsymbol{\beta}_2') + E(\varepsilon_1\varepsilon_2) = \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(y,\mathbf{Z}) + E(\varepsilon_1\varepsilon_2)$. The third equality follows since $E(\varepsilon_1\mathbf{Z}) = E(\varepsilon_2\mathbf{Z}) = 0$, the fifth from $E(\mathbf{Z}\mathbf{Z}') = \Sigma_{\mathbf{Z}}$ and from the expressions for $\boldsymbol{\beta}_1$ and $\boldsymbol{\beta}_2$ above.

We have $E(\varepsilon_1\varepsilon_2) = \rho(\varepsilon_1,\varepsilon_2)\,\sigma(\varepsilon_1)\,\sigma(\varepsilon_1)$. The correlation $\rho(\varepsilon_1,\varepsilon_2)$ can be anything in the interval $[-1,1]$ since the projections put no restrictions on the errors except that they be orthogonal to $\mathbf{Z}$ and have the appropriate standard deviation. The variance of ε_1 is calculated as follows:

$\sigma^2(x) = 1 \Longleftrightarrow \sigma^2(\boldsymbol{\beta}_1\mathbf{Z} + \varepsilon_1) = 1 \Leftrightarrow \sigma^2(\varepsilon_1) = 1 - \sigma^2(\boldsymbol{\beta}_1\mathbf{Z})$

$= 1 - \sigma^2\left(\boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\mathbf{Z}\right) = 1 - \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(x,\mathbf{Z})$ so that $\sigma(\varepsilon_1) = \sqrt{1 - \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(x,\mathbf{Z})}$. The standard deviation of ε_2 is calculated in the same way to be $\sqrt{1 - \boldsymbol{\rho}(y,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(y,\mathbf{Z})}$. Summarizing, we get

$$
\begin{aligned}
\rho(x,y) &= \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(y,\mathbf{Z}) + E(\varepsilon_1\varepsilon_2) \\
&= \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(y,\mathbf{Z}) + \rho(\varepsilon_1,\varepsilon_2)\,\sigma(\varepsilon_1)\,\sigma(\varepsilon_1) \\
&= \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(y,\mathbf{Z}) + \rho(\varepsilon_1,\varepsilon_2)\sqrt{1 - \boldsymbol{\rho}(x,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(x,\mathbf{Z})}\sqrt{1 - \boldsymbol{\rho}(y,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1}\boldsymbol{\rho}(y,\mathbf{Z})}
\end{aligned}
$$

where $\rho(\varepsilon_1,\varepsilon_2) \in [-1,1]$. ■

B. Proof of Proposition 3

Lemma 10. *When there are two contracts with correlations ρ_{1e} and ρ_{2e} with the endowment differential, revenues are maximized when ρ_{12} is set minimal at $\rho_{12} = \rho_{1e}\rho_{2e} - \sqrt{1-\rho_{1e}^2}\sqrt{1-\rho_{2e}^2}$.*

Proof. Substituting for equilibrium fees in the profit function of exchange 1 when there are two exchanges gives

$$\begin{aligned}\pi_1 &= \frac{1}{1-\rho_{12}^2}\left(\rho_{1e} - \frac{2\rho_{1e}-\rho_{12}^2\rho_{1e}-\rho_{12}\rho_{2e}}{4-\rho_{12}^2} - \rho_{12}\left(\rho_{2e} - \frac{2\rho_{2e}-\rho_{12}^2\rho_{2e}-\rho_{12}\rho_{1e}}{4-\rho_{12}^2}\right)\right)\frac{2\rho_{1e}-\rho_{12}^2\rho_{1e}-\rho_{12}\rho_{2e}}{4-\rho_{12}^2} \\ &= \frac{\left(-2\rho_{1e}+\rho_{12}^2\rho_{1e}+\rho_{12}\rho_{2e}\right)^2}{\left(1-\rho_{12}^2\right)\left(\rho_{12}^2-4\right)^2}\end{aligned}$$

We want to show first that $\frac{d\pi_1}{d\rho_{12}}$starts out negative and stays positive whenever it becomes positive (for high enough values). Thus the lower contour sets are convex in ρ_{12} and the maximum must be at an extreme point. The derivative of the profit function w.r.t. ρ_{12} can be calculated to be
$\frac{d\pi_1}{d\rho_{12}} = \frac{2\left(\rho_{1e}-T_i-\rho_{12}\left(\rho_{2e}-\frac{2\rho_{2e}-\rho_{12}^2\rho_{2e}-\rho_{12}\rho_{1e}}{4-\rho_{12}^2}\right)\right)T_1\rho_{12}}{\left(1-\rho_{12}^2\right)^2} +$
$\frac{1\left(-\rho_{2e}+\frac{2\rho_{2e}-\rho_{12}^2\rho_{2e}-\rho_{12}\rho_{1e}}{4-\rho_{12}^2}-\rho_{12}\left(-\frac{-2\rho_{12}\rho_{2e}-\rho_{1e}}{4-\rho_{12}^2}-2\frac{2\rho_{2e}-\rho_{12}^2\rho_{2e}-\rho_{12}\rho_{1e}}{\left(4-\rho_{12}^2\right)^2}\rho_{12}\right)\right)T_1}{1-\rho_{12}^2}$ which divided by $\frac{T_1}{1-\rho_{12}^2}$ can be simplified to
$-2\frac{4\rho_{12}\rho_{1e}-2\rho_{12}^3\rho_{1e}+\rho_{1e}\rho_{12}^5-\rho_{12}^2\rho_{2e}+2\rho_{2e}\rho_{12}^4-4\rho_{2e}}{\left(-4+\rho_{12}^2\right)^2\left(-1+\rho_{12}^2\right)}$ which has the same sign as $\rho_{12}\rho_{1e}\left(4-2\rho_{12}^2+\rho_{12}^4\right) - \rho_{2e}\left(4+\rho_{12}^2-2\rho_{12}^4\right)$. We first study the case where $T_1 < 0$ (which is equivalent to consumer type 1 being short) in which case we have $\rho_{1e} < \frac{\rho_{12}\rho_{2e}}{2-\rho_{12}^2} < \rho_{2e}$. Thus, $\rho_{12}\rho_{1e} < \rho_{2e}$. We also have that $0 < 4 -2\rho_{12}^2 + \rho_{12}^4 < 4 + \rho_{12}^2 - 2\rho_{12}^4$, so that the derivative is positive (since the term is negative but we divided by the negative T_1). This happens when $\rho_{1e} < \frac{\rho_{12}\rho_{2e}}{2-\rho_{12}^2}$ or $\rho_{12} > \frac{1}{2\rho_{1e}}\left(-\rho_{2e}+\sqrt{\left(\rho_{2e}^2+8\rho_{1e}^2\right)}\right)$. If $T_1 > 0$, the expression is positive whenever $\rho_{12} > \frac{\rho_{2e}\left(4+\rho_{12}^2-2\rho_{12}^4\right)}{\rho_{1e}\left(4-2\rho_{12}^2+\rho_{12}^4\right)} > 0$ and negative otherwise. This condition cannot be satisfied for the same values of ρ_{1e}, ρ_{2e} for which T_1 can be negative. We have thus established that $\frac{d\pi_1}{d\rho_{12}}$ starts out negative and stays positive whenever it becomes positive. Thus the lower contour sets are convex in ρ_{12} and the maximum must be at an extreme point. We have verified that this extreme point is the lowest value by checking the profit function numerically at the two extreme values of ρ_{12}. ■

When analyzing the choice of ρ_{1e} and ρ_{2e} we can thus replace ρ_{12} with $\rho_{12L} = \rho_{1e}\rho_{2e} - \sqrt{1-\rho_{1e}^2}\sqrt{1-\rho_{2e}^2}$. This yields the profit function as:

$$\pi_1 = \frac{1}{1-\rho_{12L}^2}\left(\rho_{1e} - T_1\left(\rho_{1e}, \rho_{2e}, \rho_{12L}\right) - \rho_L\left(\rho_{2e} - \frac{2\rho_{2e} - \rho_{12}^2\rho_{2e} - \rho_{12}\rho_{1e}}{4-\rho_{12L}^2}\right)\right) T_1\left(\rho_{1e}, \rho_{2e}, \rho_{12L}\right)$$

To calculate best replies, we take the derivative $\frac{d\pi_1}{d\rho_{1e}}$:

$$\begin{aligned}
\frac{d\pi_1}{d\rho_{1e}} &= \frac{\partial\pi_1}{\partial\rho_{1e}} + \frac{\partial\pi_1}{\partial\rho_{12L}}\frac{\partial\rho_{12L}}{\partial\rho_{1e}} + \frac{\partial\pi_1}{\partial T_1}\frac{\partial T_1}{\partial\rho_{1e}} = \frac{\partial\pi_1}{\partial\rho_{1e}} + \frac{\partial\pi_1}{\partial\rho_{12L}}\frac{\partial\rho_{12L}}{\partial\rho_{1e}} \\
\frac{\partial\pi_1}{\partial\rho_{1e}} &= \frac{T_1}{1-\rho_{12L}^2}\left(1 - \frac{\rho_{12L}^2}{4-\rho_{12L}^2}\right) > 0 \\
\frac{\partial\pi_1}{\partial\rho_{12L}} &= 2T_1\frac{\rho_{12L}\rho_{1e}\left(4-2\rho_{12L}^2+\rho_{12L}^4\right) - \rho_{2e}\left(4+\rho_{12L}^2-2\rho_{12L}^4\right)}{\left(4-\rho_{12L}^2\right)^2\left(1-\rho_{12L}^2\right)^2} < 0 \\
\frac{\partial\rho_{12L}}{\partial\rho_{1e}} &= \rho_{2e} + \rho_{1e}\frac{\sqrt{1-\rho_{2e}^2}}{\sqrt{1-\rho_{1e}^2}} > 0
\end{aligned}$$

The first relation follows from the envelope theorem ($\frac{\partial\pi_1}{\partial T_1} = 0$), and the third from the proof of lemma 10. As ρ_{1e} goes to 1, $\frac{d\pi_1}{d\rho_{1e}}$ goes to $-\infty$. As ρ_{1e} goes to 0, $\frac{d\pi_1}{d\rho_{1e}}$ is strictly positive. Thus we have an interior best response for all values of ρ_{2e} . Furthermore, these are continuous, proving that there is a fixed point to the best reply function such that $\rho_{1e}\left(\rho_{2e}\right) = \rho_{2e}$. This is in fact the unique equilibrium in pure strategies, as is seen in Figure 3.1. The solution to the fixed point is given by setting the first order condition to zero at $\rho_{1e} = \rho_{2e}$ and $\rho_{12L} = 2\rho_{1e}^2 - 1$:

$$\begin{aligned}
4 - 2\rho_{12L}^2 &= -4\rho_{1e}\frac{\rho_{12L}\rho_{1e}\left(4-2\rho_{12L}^2+\rho_{12L}^4\right) - \rho_{1e}\left(4+\rho_{12L}^2-2\rho_{12L}^4\right)}{\left(4-\rho_{12L}^2\right)\left(1-\rho_{12L}^2\right)} \Leftrightarrow \\
\left(1-4\rho_{1e}^4+4\rho_{1e}^2\right)\left(3-2\rho_{1e}^2\right) &= 8\rho_{1e}^6 - 8\rho_{1e}^4 + 3 \Rightarrow \\
\rho_{1e} &= \sqrt{\frac{5}{6}}
\end{aligned}$$

This concludes the proof of Proposition 3. ■

C. Proof of Proposition 4

First, we calculate the minimal cross-correlation between a set of symmetric contracts given that they all have the same correlation with the endowment differential. This will be the cross-correlations in a candidate symmetric equilibirum. We then look at a best reply of an exchange to this strategy of the other players, where he still keeps a minimal cross-correlation with the other contracts. We show that the best reply features a higher correlation with the endowment differential than that of the other contracts, except when they play the DJ-contract. This proves that the only symmetric equilibrium is for everyone to offer the DJ-contract.

Lemma 11. *The minimal cross-correlations x between a symmetric set of $k+2$ contracts each with correlation ρ with the endowment differential is* $x(k,\rho) = \frac{(k+2)\rho^2-1}{k+1}$.

Proof. Using Lemma 3, we get that the minimal correlation between two contracts f_1 and f_2 is

$$\begin{aligned} x &= \boldsymbol{\rho}(f_1,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(f_2,\mathbf{Z}) - \sqrt{1-\boldsymbol{\rho}(f_1,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(f_1,\mathbf{Z})}\sqrt{1-\boldsymbol{\rho}(f_2,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(f_2,\mathbf{Z})} \\ &= 2\boldsymbol{\rho}(f_1,\mathbf{Z})' \Sigma_{\mathbf{Z}}^{-1} \boldsymbol{\rho}(f_1,\mathbf{Z}) - 1 \end{aligned}$$

where $\mathbf{Z} = \{e, f_3, ..., f_{k+2}\}'$ and $\boldsymbol{\rho}(f_i,\mathbf{Z}) = \{\rho, x, ..., x\}'$ (both $(k+1)\times 1$ vectors) and the second equality follows since $\boldsymbol{\rho}(f_1,\mathbf{Z}) = \boldsymbol{\rho}(f_2,\mathbf{Z})$ in the symmetric case. We have that $\Sigma_{\mathbf{Z}}$ is of the form

$$\begin{bmatrix} 1 & \rho & \rho & \rho & \cdots \\ \rho & 1 & x & x & \\ \rho & x & 1 & x & \\ \rho & x & x & 1 & \\ \vdots & & & & \ddots \end{bmatrix}$$

(with $k+1$ number of rows and columns). The inverse $\Sigma_{\mathbf{Z}}^{-1}$ will be of the form

$$\begin{bmatrix} a & b & b & b & \cdots \\ b & c & d & d & \\ b & d & c & d & \\ b & d & d & c & \\ \vdots & & & & \ddots \end{bmatrix}$$

where a, b, c, d solve

$$\begin{aligned}
a+bk\rho &= 1\\
a\rho+b+b(k-1)x &= 0\\
b+c\rho+d(k-1)\rho &= 0\\
b\rho+c+d(k-1)x &= 1\\
b\rho+cx+d+d(k-2)x &= 0
\end{aligned}$$

This yields the unique solution:

$$\begin{aligned}
a &= -\frac{1+x\,(k-1)}{k\rho^2-1-x\,(k-1)}\\
b &= \frac{\rho}{k\rho^2-1-x\,(k-1)}\\
c &= \frac{x\,(k-2)-(k-1)\,\rho^2+1}{\left(k\rho^2-1-x\,(k-1)\right)(x-1)}\\
d &= \frac{\rho^2-x}{\left(k\rho^2-1-x\,(k-1)\right)(x-1)}
\end{aligned}$$

Plugging this in we get

$$x=2\left[(a\rho+bkx)^2+2\,(a\rho+bkx)\,(b\rho+cx+d(k-1)x)\,k\rho+(b\rho+cx+d(k-1)x)^2\left(k+x\left(k^2-k\right)\right)\right]-1$$

Substituting for the coefficients, the two solutions to this equation are $x=\frac{1}{2(k+1)}\left(2k\rho^2+4\rho^2-2\right)$ and $x=1$, and we choose the first since it is minimal:

$$x=\frac{(k+2)\,\rho^2-1}{(k+1)}$$

■

Assume that everyone use contracts with $\rho_{ie}=\rho$ and the minimal cross-correlation $x=\rho_{ij}=\frac{k\rho^2-1}{k-1}$ when there are k contracts on the market, as calculated in Lemma 11 above. Given a deviation ρ_{1e} from this strategy by exchange 1, the optimal cross-correlation $x_1\equiv\rho_{1i}$ with other contracts is given uniquely (he will choose it minimal). We can then calculate profits and a best reply $\rho_{1e}(\rho)$ for exchange 1. If this function has a fixed point, we are done!

The minimal x_1 is given as

$$x_1 = \rho_{1e}\rho - \sqrt{\frac{1-\rho_{1e}^2}{k-1}}\sqrt{\left(x\left(k-2\right)-\left(k-1\right)\rho^2+1\right)}$$

We do not supply a proof since it follows by almost identical calculations to the once in the proof of Lemma 11.

Plugging in for x we get

$$x_1\left(k,\rho,\rho_{1e}\right) = \rho_{1e}\rho - \frac{1}{k-1}\sqrt{1-\rho_{1e}^2}\sqrt{1-\rho^2}$$

Having x and x_1 in these simple forms, we can now calculate equilibrium fees given ρ_{1e} and ρ.

We have $\Sigma = \begin{bmatrix} 1 & x_1 & x_1 & \cdots \\ x_1 & 1 & x & \\ x_1 & x & 1 & \\ \vdots & & & \ddots \end{bmatrix}$ (a $k \times k$ matrix) and $\Sigma^{-1} = \begin{bmatrix} a & b & b & \cdots \\ b & c & d & \\ b & d & c & \\ \vdots & & & \ddots \end{bmatrix}$ with

$$\begin{aligned}
a &= -\frac{1+x\left(k-2\right)}{\left(k-1\right)x_1^2-1-x\left(k-2\right)} \\
b &= \frac{x_1}{\left(k-1\right)x_1^2-1-x\left(k-2\right)} \\
c &= \frac{x\left(k-3\right)-\left(k-2\right)x_1^2+1}{\left(\left(k-1\right)x_1^2-1-x\left(k-2\right)\right)\left(x-2\right)} \\
d &= \frac{x_1^2-x}{\left(\left(k-1\right)x_1^2-1-x\left(k-2\right)k\right)\left(x-2\right)}
\end{aligned}$$

From Equation 2.3, volumes then become:

$$\frac{1}{1-\left(k-1\right)x_1^2+x\left(k-2\right)}\begin{bmatrix} 1+x\left(k-2\right) & -x_1 & -x_1 & \cdots \\ -x_1 & \frac{x(k-3)-(k-2)x_1^2+1}{1-x} & \frac{x_1^2-x}{1-x} & \\ -x_1 & \frac{x_1^2-x_2}{1-x} & \frac{x(k-3)-(k-2)x_1^2+1}{1-x} & \\ \vdots & & & \ddots \end{bmatrix}\begin{bmatrix} \rho_{1e}-T_1 \\ \rho-T \\ \rho-T \\ \vdots \end{bmatrix}$$

which gives profits as

$$\pi_1 = \frac{((\rho_{1e} - T_1)(1 + x(k-2)) - (k-1)x_1(\rho - T))T_1}{\left(1 - (k-1)x_1^2 + x(k-2)\right)}$$

$$\pi_{i\neq 1} = \frac{\left((\rho - T)\frac{x(k-3)-(k-2)x_1^2+1}{1-x} + (k-2)\frac{x_1^2-x}{1-x}(\rho - T) - x_1(\rho_{1e} - T_1)\right)T}{\left(1 - (k-1)x_1^2 + x(k-2)\right)}$$

The solution of the first order condition for fees is :

$$T_1^* = \frac{1}{2}\left(\rho_{1e} - (k-1)x_1\frac{\rho - T^*}{1 + x(k-2)}\right)$$

$$T^* = \frac{(1-x)\left(\rho - \frac{\rho_{1e}x_1}{2} - \frac{\rho(k-1)x_1^2}{2(1+x(k-2))}\right)}{\left(x_2(k-4) - x_1^2(k-2) + 2 - \frac{(1-x)(k-1)x_1^2}{2(1+x(k-2))}\right)}$$

We can plug the expressions for x, x_1, T_1^*, and T^* into the profit equation for exchange 1:

$$\pi_1(k, \rho, \rho_{1e}) = \frac{((\rho_{1e} - T_1^*)(1 + x(k-2)) - (k-1)x(\rho - T^*))T_1^*}{\left(1 - (k-1)x_1^2 + x(k-2)\right)}$$

Plotting this for $k \geq 3$ shows that the best response $\rho_1^* = b(\rho)$ has $\rho_1^* > \rho$ except for $\rho = 1$. ■

D. Proof of Lemmas 4 and 5

When $\beta > 0$, an exchange has to evaluate the effect of contract-choice not only on profits today but also on profits in the future. If it's the case that the other exchange will not change his contract in the foreseeable future independent of what the competitors contract choice is, the competitor's best strategy is to choose a short-run best reply as in Figure 3.1. The condition for the second exchange to enter at his best reply rather than staying out of the market is

$$\frac{\pi(b(f_1), f_1)}{(1-\beta)} > C$$

i.e., the discounted profits of entering are bigger than the entry cost. In Figure ? one-period profits of exchange 1 are shown as a function of ρ_{1e} when exchange 2 enters at his best reply. Profits are increasing in ρ_{1e}. Thus, if the game was a Stackelberg game where exchange 1 is commited to stick with his contract, we see that the DJ contract is optimal for him. However, staying with the same contract is not always credible. It may be profitable for the second exchange to forego short-term profits by introducing a contract which forces the first exchange to change his contract next period if the future profits from this strategy are high enough. We refer to this type of contract as a *preying* contract. The conditions for f_2 to be a viable preying contract given f_1 are

1. $\frac{\pi(f_1,f_2)-\pi(b(f_2),f_2)}{(1-\beta)} > C$ and $\frac{\pi(b(b(f_2)),b(f_2))-\pi(f_2,b(f_2))}{(1-\beta)} \leq C$
2. $\frac{\pi(b(f_1),f_1)-\pi(f_2,f_1)}{(1-\beta)} \leq C$
3. $\pi(f_2, f_1) + \frac{\beta}{(1-\beta)}\pi(f_2, b(f_2)) \geq C$
4. f_2 itself is not subject to prey the following period

The first condition is necessary for $\{f_2, b(f_2)\}$ to be a viable equilibrium next period: Exchange 1 will find it profitable to change from f_1 to $b(f_2)$, and exchange 2 does not shift to his best reply against $b(f_2)$. The second condition says that $\{b(f_1), f_1\}$ is not a viable equilibrium next period: Exchange 2 does not find it profitable to change to his best reply against f_1. This condition turns out to be a bit stricter than would be necessary for $\{f_2, b(f_2)\}$ to be a subgame perfect equilibrium; it is however necessary for this to be the *only* such equilibrium, which seems natural to impose as a criterion for preying. The third condition states that it is profitable for exchange 2 to enter the market with f_2 instead of staying out. Finally, the fourth condition is that f_2 cannot be profitably preyed itself in the following period. If an exchange is able to prey, it can "turn the tables" and acquire a role similar to a Stackelberg leader.

It is easy to show that the conditions for preying are satisfied when the incumbent exchange uses the DJ contract and the entrant uses the symmetric duopoly contract. Also, it is easy to show that this is the only possible equilibrium candidate in pure strategies. This proves Lemma 4. Lemma 5 is shown by checking numerically that there is no contract that can prey NP at $C = C_L$, and that this is actually the non-preyable contract with the highest hedging potential and hence the highest profits for the incumbent in case of competitor entry. There may be contracts with higher hedging potential that can be supported in mixed equilibria, but the qualitative implications are the same: There will be some contract with lower hedging potential than the DJ contract but higher than the symmetric duopoly contract that can be sustained on the market once a competitor enters. ■

E. Proof of Proposition 5

Proof. When $C_M \geq C_M^*$, $\Pi_1 (f_{11} = DJ) - \Pi_1 (f_{11} = NP)$ is equal to

$$
\begin{aligned}
& (1-p_L)\left(\frac{1}{1-\beta}\left(\frac{1}{4}-\frac{\rho_{NP,e}^2}{4}\right)\right)+p_L\left(\frac{1}{4}-\frac{\rho_{NP,e}^2}{4}+\frac{\beta}{1-\beta}\left(\frac{3}{32}-\pi_1\left(NP,b\left(NP\right)\right)\right)-\beta\frac{\frac{3}{32}-\frac{24}{361}}{1-\beta}\right) \\
= \quad & \frac{1}{1-\beta}\left(\frac{1-\rho_{NP,e}^2}{4}-p_L\beta\left(\frac{1-\rho_{NP,e}^2}{4}+\pi_1\left(NP,b\left(NP\right)\right)-\frac{24}{361}\right)\right)
\end{aligned}
$$

from simple calculations. The DJ contract is preferred when $\Pi_1 (f_{11} = DJ) - \Pi_1 (f_{11} = NP) > 0$ which is when $\frac{1-\rho_{NP,e}^2}{4} - p_L\beta\left(\frac{1-\rho_{NP,e}^2}{4} + \pi_1 (NP, b(NP)) - \frac{24}{361}\right) > 0$ or $p_L\beta < \frac{\frac{1-\rho_{NP,e}^2}{4}}{\frac{1-\rho_{NP,e}^2}{4}+\pi_1(NP,b(NP))-\frac{24}{361}} \approx 0.48868$.

When $C_M < C_M^*$, $\Pi_1 (f_{11} = DJ) - \Pi_1 (f_{11} = NP)$ is equal to

$$
\begin{aligned}
& (1-p_L)\left(\frac{1}{1-\beta}\frac{1}{4}\right)+p_L*\left(\frac{1}{4}+\frac{\beta}{1-\beta}\frac{3}{32}-\beta\frac{0.027165}{1-\beta}\right)-p_H\left(0.2173\frac{1}{1-\beta}\right)-(1-p_H)\left(0.2173+0.1008\right. \\
= \quad & \frac{1}{1-\beta}\left(0.0327-0.183415p_L\beta+(1-p_H)\,0.1165\beta\right)
\end{aligned}
$$

This is greater than zero if $0.0327 - 0.183415p_L\beta + (1-p_H)\,0.1165\beta > 0$ or $p_L\beta < \frac{0.0327+(1-p_H)0.1165\beta}{0.183415}$ which is equivalent to $p_L\beta < 0.48868 + 1.741p_M\beta$. ■

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