Trade liberalisation, wage inequality and cross-occupation skill imbalances

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Abstract

This paper proposes a model that, focusing on the imperfect transfer of skill across occupations, is able to explain trade or investment liberalisation-induced increases in inequality independently of technological change or capital flows. It identifies imbalances in skill across occupations as a potential source of rent-seeking behaviour and predicts that when this occurs, firms may choose to employ highly trained individuals to fill roles with both high and low education requirements.

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1. Introduction

Throughout much of the twentieth century, conventional economic wisdom – embodied in the Heckscher-Ohlin model and the forecast of Stolper-Samuelson effects relating the relative prices of goods to those of their principal factors of production – held out a promise to the developing world, not only of increasing income, but also of decreasing inequality if they opened up to trade. With the argument for freer trade won, or at least temporarily accepted, the world has seen trade grow enormously both in nominal and relative terms. As a percentage of global GDP, trade rose from a little over 23% in 1960 to almost 48% in 2003. Low and middle-income countries have experienced an even greater increase, rising from 21% in 1970 to over 64% in 2004 (WDI (2006)).

Despite this explosion in trade, many developing countries have experienced a considerable worsening of inequality, prompting many critics of globalisation to suggest trade or investment liberalisation as the cause. Goldberg and Pavcnik (2006) provide an excellent review of recent literature investigating the distributional effects of globalisation, examining the experiences of some key emerging economies (Argentina, Brazil, Chile, Colombia, India, Hong Kong and Mexico[1]) through their various stages of liberalisation in the 1980s and 1990s. They observe that all of these countries experienced an increase in their skill premium over periods of trade liberalisation, with returns to university education relative to no or primary education increasing by between 10% and 68%. Increases in the returns to education were usually also reflected in general wage inequality, though not in Brazil. Although measures of consumption inequality were rarely available, they note that in Indian urban areas, "consumption inequality moves in the same direction as income and wage inequality." (Goldberg and Pavcnik (2006: p19))

In short, there appears to be a clear and positive correlation between globalisation and inequality and, even allowing for issues of measurement error, the near universality of the experience across countries and all major definitions of inequality suggests the possibility of a common underlying cause. However, Goldberg and Pavcnik

[1] Goldberg and Pavcnik do not include the East Asian countries of China, South Korea, Taiwan and Singapore in their analysis as "neither detailed data on tariffs nor micro surveys [needed for a proper evaluation of inequality] are readily available for these countries." (Goldberg and Pavcnik (2006: p13))

(2006) warn against concluding from this alone that liberalisation causes an increase in inequality, highlighting the difficulty in separating the effects of liberalisation from other contemporaneous changes in the economic environment and the possible endogeneity of trade policy. Nevertheless, various attempts to approach these problems[2] do appear to suggest that the link may be causal; raising the question of how the predicted Stolper-Samuelson effects may have been overturned.

Several possible explanations of this problem have been proposed, including extensions to the basic H-O framework, the role of technology and the impact of production sharing or "outsourcing." The model presented here is inspired principally by Kremer and Maskin (2006) and like their model, does not rely on technological change or capital flows. Unlike Kremer and Maskin (2006), the model presented here is not limited to production sharing as a source of inequality and, unlike much work to-date, allows an explanation of liberalisation-induced increases in inequality in the *short-run*.

The remainder of this paper proceeds as follows: Section 2 provides a critical review of existing attempts to explain the observed increase in inequality. Section 3 presents this paper's model, exploring its implications under both autarky and free trade. Section 4 discusses a number of extensions to the basic model and proposes areas for future work, while section 5 concludes. Proofs of all propositions may be found in the appendix.

2. Review of existing models

Efforts at explaining liberalisation-induced increases in inequality may be grouped into three broad categories: those that extend the original Heckscher-Ohlin model, those that emphasise skill-biased technological change and those that focus on the effects of production sharing or "outsourcing."

2.1. Extensions to the Heckscher-Ohlin model

Firstly, the standard Stolper-Samuelson predictions may not emerge if an additional factor of production (such as land) is introduced into a Heckscher-Ohlin model, the extra factor is assumed to complement skilled labour and poorer countries are assumed abundant in the new factor relative to wealthy countries. In such a case, increasing

[2] For example, Porto (2006) attempts to model the general equilibrium effects of liberalisation, while Goldberg and Pavcnik (2005) examine the impact of liberalisation on Colombian industries with different levels of exposure.

trade openness will induce the poorer countries to increase production of goods that are intensive in the use of the extra factor, leading to a relative increase in demand for skilled labour. However, for most plausible extra factors of production, there is little evidence that both of these assumptions hold.

Alternatively, Wood (1994) and Davis (1996) present similar Heckscher-Ohlin based models that identify three broad classes of countries (rich, middle-income and poor) with comparative advantages according to the skill-intensity (Wood, 1994) or capital-intensity (Davis, 1996) of production. Each shows inequality decreasing in the rich and poor countries, and potentially rising in the middle-income countries.

A final – and arguably more plausible – explanation within a Heckscher-Ohlin framework observes that for a number of developing countries, despite presumably holding a comparative advantage in them, it was the *unskilled*-labour intensive sectors that were most protected prior to liberalisation and consequently were the most exposed by the tariff cuts.[3] In this case, Stolper-Samuelson would (accurately) predict that the overall short-run return to unskilled labour should decrease following liberalisation. However, this pattern of pre-liberalisation protection has not been documented for all developing economies that have liberalised their trade. In those where it has, the increases in inequality appear to be persisting into the longer run when it would be thought that their comparative advantage should have taken hold.

2.2. Skill-biased technological change

Arguably the most popular explanation of the increase in both demand for and the wage premium granted to skilled labour is that of skill-biased technological change, either brought about directly by liberalisation (e.g., through technology transfers as a result of Foreign Direct Investment (FDI)) or indirectly as a response to liberalisation (e.g. in the form of "defensive innovation" (Wood, 1995)). The topic is contested, however, not least because of difficulties in defining, identifying and measuring technological change. Despite this, "the repeated finding of an increase in both the share of skilled workers and their relative wage within fairly narrowly defined industry categories in both developed and developing countries has [typically] been interpreted as evidence for a world-wide skill bias in new technologies." (Goldberg and Pavcnik (2006: p33))

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[3] See, for example, Attanasio, Goldberg and Pavcnik (2004) on Colombia and Hanson and Harrison (1999) on Mexico.

Acemoglu (2003) presents a model in which trade liberalisation leads directly to technological change. He supposes two classes of intermediate machines, complementing skilled and unskilled workers respectively. Foreign-designed machines are less-than-perfectly appropriate to domestic production and are sold at monopoly prices, while domestic machines are sold at cost. Future research will only be undertaken by the world technology leader (the U.S.A.) if, after allowing for these impositions, the quality of machines available from the U.S.A. is still sufficiently high that they are preferred to those available domestically. Trade liberalisation by a developing country then leads to a balanced growth path characterised by an increased equilibrium skill bias, manifested through increased imports of high-technology (and thus skill-complementary) machines. Demand for skilled labour in the developing country will therefore increase most in those industries that import more foreign machinery. However, as Goldberg and Pavcnik (2006) attest, the evidence for this prediction is mixed, with imports of intermediate products being positively associated with productivity improvements following trade liberalisation in Brazil and Mexico, but not in Chile or Colombia.

Alternatively, Aghion *et al.* (2005) present a model of indirect technological change. They propose a Schumpeterian growth model of a developing country, with trade liberalisation subjecting firms to a threat of entry to their industry. They conclude that responses will be heterogeneous across firms and that, in particular, "technologically advanced firms and those located in regions with pro-business institutions are more likely to respond to the threat of entry by investing in new technologies and production processes." (Aghion *et al.* (2005: p1)) Although this result is intuitively appealing, once again Goldberg and Pavcnik (2006) note that empirical evidence is inconclusive, with different studies on India coming to different conclusions on whether firm productivity before liberalisation has any significant effect on productivity increases thereafter.

Looking past the particular mechanisms of technology transfer, Zhu and Trefler (2005) present a two-country model examining the distributional impact of Southern technology catch-up. They demonstrate that, even in the absence of any capital flows, technology catch-up can lead to an increase in the average skill intensity of both countries by shifting the production of goods requiring mid-range skill intensity from the North to the South.

In addition to difficulties in defining or measuring technology, one general criticism of skill-biased technological change as a cause of skill upgrading rests on an observation of timescales. Developing new technologies or adapting foreign technologies for local use takes time. Implementing new technologies also takes time because of the need to acquaint the workforce with them. However, as Goldberg and Pavcnik (2006) observe, increases in developing countries' skill premiums are generally contemporaneous with their trade reforms. This raises the question of whether inequality may be rising, at least in the short-term, for reasons other than changes in technology.

2.3. Production sharing

Feenstra and Hanson (1996) stress the role of cross-border production sharing and trade in intermediate goods (commonly referred to as "outsourcing"). In their model, a single final good is produced from a continuum of intermediate goods, indexed by $z \in [0,1]$, that are produced in two countries: the North, with greater capital and skilled worker endowments, and the South, with more unskilled workers. Production of each intermediate good combines capital with some fixed proportion of skilled and unskilled labour. Ordering intermediate goods so that those with a higher z are more skill-intensive in their production, and assuming that trade in intermediate goods is free but that capital flows are restricted, there is an equilibrium cut-off, z^*, such that the South produces goods in the range $z \in [0, z^*]$ and the North produces those in the range $z \in (z^*, 1]$. That is, the North and South specialise in the most and least skill-intensive elements of production respectively. In this framework, investment liberalisation allows capital flows from the North to the South, which increases z^*. Liberalisation in the presence of production sharing therefore causes the average skill intensity of production – and with it, the skill premium – to rise in both countries.

Although outsourcing is certainly an important phenomenon in trade for developing countries, an important question is whether production sharing allows for an increase in inequality in the absence of investment liberalisation. Furthermore, the assumption that each intermediate good requires a fixed and *unchanging* proportion of skilled to unskilled labour seems untenable in the light of work on technology catch-up.

A more recent model of production sharing is presented by Kremer and Maskin (2006). They propose a production function of a single final good with two essential tasks required in equal proportion: a manager and an assistant. The manager's task is more skill-intensive than that of the assistant, so that it is always more efficient for a firm to place a high-skill individual into the manager's position and a low-skill individual into the assistant's position than the other way around.[4] For a single country under autarky with equal numbers of high and low skill workers, total output maximisation then represents a trade-off between the complementarity of the tasks and the asymmetry of the tasks' skill intensities. This is because while the asymmetry favours placing a high-skilled individual in the manager's position irrespective of who plays the role of assistant, the complementarity of the tasks (the marginal product of each is increasing in the skill of the other) favours positive assortative matching. Kremer and Maskin show that for a sufficiently high skill ratio (the ratio of the skill levels of the two groups of individuals), the latter effect dominates and output is maximised by *self-matching*.

Considering trade liberalisation between two countries, Kremer and Maskin assume that "globalisation means that workers from different countries can work together in the same firm," and that the skill level of Southern low-skill workers is sufficiently low that it is never efficient for them to match with anybody from the North. In this scenario, they show that inequality will (weakly) increase in the South following liberalisation, but that the effect on the North is ambiguous. Under the additional assumption that that high-skill Northern workers must be included in cross-border production sharing (thus ruling out the possibility of low-skill Northern workers matching with high-skill Southern workers), inequality (weakly) increases both countries.

One criticism of Kremer and Maskin's model lies in the fact that they assume that skill is perfectly and symmetrically transferable across tasks – that the productive-skill of a labourer would be unchanged if they were to take on the job of an engineer and visa versa – when it is not immediately obvious why this would be the case. One explanation may be to consider skill analogous to innate ability and note that the assumption would hold in the long run as new entrants to the job market choose their occupations and existing workers retrain. However, this explanation requires that the model abandon any claim to explaining short-run increases in inequality resulting from trade liberalisation.

3. The model

The model presented here, like that proposed by Kremer and Maskin (2006), focuses on the differentiation between a person's *occupation* (a choice constrained by minimum education requirements) and their *skill* (a

[4] Kremer and Maskin use a production function of $S_m^2 S_a$, where S_m is the skill of the manager and S_a is the skill of the assistant. Their choice of exponents is necessary for their model to have a simple algebraic solution, but is not required for their argument to hold in general.

function of their ability, education and experience). While there may be a trend for some occupations to attract higher-skill individuals, there is no guarantee that this will always be the case. Phrases such as "high skill jobs" tend to conflate the two concepts. The novel contribution of this paper is to add that, to the extent that a person's education and experience are task-specific, their skill will not be perfectly transferable across occupations in the short term, with the result that sufficient imbalances in skill across occupations may create opportunities for rent extraction that, in turn, cause an increase in inequality.

Consider a model with an arbitrary number of occupations in the economy and with the occupation chosen by each individual taken as fixed. That is, I am concerned with a time-frame sufficiently short as to preclude significant changes to the shape of the labour supply. Nevertheless, a person may still be hired to perform a task other than their chosen occupation. Let $S(i,j)$ represent the skill of a person trained in occupation j but performing task i.[5] Individuals trained in the same occupation in the same country have the same skill level, so that $S(i,i)$ is the same for all individuals trained in occupation i.

Firms are competitive and identical, each characterised by a CRS production process involving n tasks corresponding to each occupation, all of which are essential:

$$Y = \Phi\left(\prod_{i=1}^{n} S(i, j_i)^{\alpha_i}\right)\min\left\{\beta_i L_i^{\gamma_i}\right\}_{i=1}^{n}$$

Other inputs, such as intermediate goods and capital, enter via Φ and are not dealt with in this paper; I therefore set Φ to 1 for simplicity. Also for simplicity, I assume that $\beta_i = \gamma_i = 1 \quad \forall i$ so that labour is required in the same quantities for every task. This means that we can consider a firm to employ a single unit of labour in each task, producing output given by:

$$Y = \prod_{i=1}^{n} S(i, j_i)^{\alpha_i}$$

and treat an expansion of output as the addition of more firms.

[5] There are several examples of individuals choosing to take on a job other than their chosen occupation. Perhaps the clearest example in the developing world is the prevalence of university graduates in India working in call centres that presumably only require a command of English and the most basic of computer proficiency.

Tasks are increasing in their marginal product of skill; i.e. $\alpha_i \leq \alpha_{i+1} \ \forall i$. This implies that, *ceteris paribus*, it is more efficient for a firm to place higher-skilled employees in the more advanced roles. In this context, *perfect cross-matching* occurs when the person performing each task is trained in the corresponding occupation, while *self-matching* occurs when several individuals from the same occupation perform different tasks.

An intuitive discussion of what *S(i,i)* actually represents may be in order. *S(engineer,engineer)* represents the skill of a person trained as an engineer in *performing the role* of an engineer. Assuming that there is a minimum level of education required to become an engineer, I regard *S(engineer,engineer)* as a positive function of the engineer's innate ability, experience and amount of *excess* education – the amount of education they possess beyond the minimum required. A concept used frequently in what follows is that of a *skill ratio*, being the ratio between the skills of two occupations in performing their own role. For example, *S(engineer,engineer) / S(labourer,labourer)* represents the skill of an engineer in performing the job of an engineer divided by the skill of a labourer in performing the role of a labourer. Because a person's skill within their chosen occupation relies on their level of excess education and not their education overall, then it is perfectly possible for this ratio to equal or even be less than unity if ability and experience are shared equally across the two occupations. However, to the extent that higher-ability people are attracted to more skill-intensive occupations in the long run, we may expect that the ratio would tend to be greater than one.

When performing a task other than their occupation, a person's skill is affected multiplicatively:

$$S(i,j) = \varphi(i,j)S(j,j)$$

The functional form of $\varphi(i,j)$ is of key importance. While Kremer and Maskin (2006) can be interpreted as imposing the restriction that $\varphi(i,j)=1 \ \forall i,j$, I instead assume a one-dimensional classification of occupations and that when performing a task of higher (lower) marginal productivity than their occupation, a person's skill typically falls (rises).[6] That is, in general:

$$\varphi(i,j) \begin{cases} \geq 1 & j > i \\ = 1 & j = i \\ \leq 1 & j < i \end{cases} \text{ s.t. } \varphi(i,j) \in \left[0, \overline{\varphi}\right]$$

[6] Note that a one-dimensional classification of occupations is still quite a strong assumption. A true categorisation would involve an arbitrary *m* dimensions, with ***i*** and ***j*** being *m*-length vectors and $\varphi(i,j)$ being considerably more complex. In particular, it might realistically have multiple local maxima.

The adjustment in a person's skill is treated as a function solely of the size of the change from occupation to role and not of the occupation itself, so that $\varphi(i+s,i)=\varphi(i'+s,i')\ \forall i,i',s$. This means that we can rewrite $\varphi(i,j)$ as:

$$\varphi(i,j)=\lambda(i-j)$$

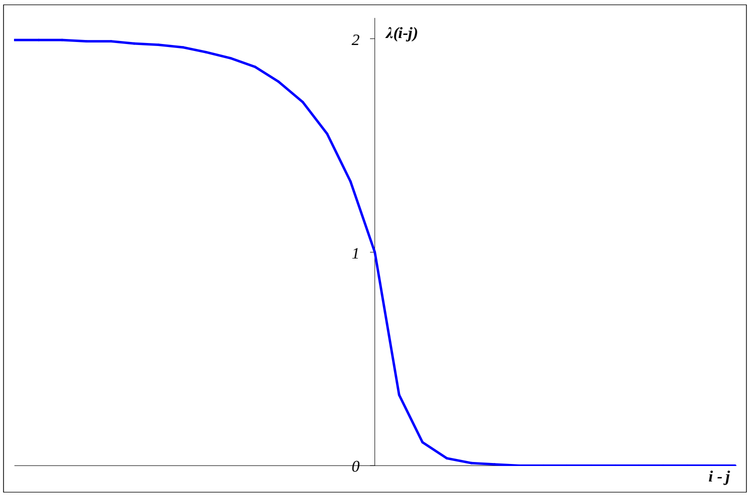

Figure 1 – Stylised graph of $\lambda(i-j)$, being the factor by which a person's skill is multiplied when they are trained in occupation *j* but performing task *i*

For simplicity, I impose that a person's skill in performing tasks higher than their occupation is zero,[7] with the implication that every firm must employ staff qualified at the highest relevant task for their production function.

$$\lambda(i-j)\begin{cases}\geq 1 & j>i\\ =1 & j=i\\ =0 & j<i\end{cases}$$

The distribution of labour across occupations is the same as that demanded by the production function, so that equal numbers of individuals are trained in each occupation.[8] Finally, individuals not taking part in the production process have access to a subsistence wage ($\underline{w}$), normalised to zero.

3.1. Output maximisation

Consider a two-occupation economy with two people trained in each occupation. Total output with cross-matching will be:

$$2S(1,1)^{\alpha_1}S(2,2)^{\alpha_2}$$

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[7] Although this assumption may appear non-ideal, as it means that $\lambda(i-j)$ is not parametrically smooth, my results are broadly robust to its relaxation. See section 4 for more detail.

[8] I explore the implications of relaxing this assumption in section 4.

When type-2 individuals self-match, type-1 individuals will be excluded from the production process altogether (as they are unable to perform the higher task) and output will be:

$$\begin{aligned} S(1,2)^{\alpha_1} S(2,2)^{\alpha_2} &= \left(\lambda(1-2)S(2,2)\right)^{\alpha_1} S(2,2)^{\alpha_2} \\ &= \lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2} \end{aligned}$$

Total output under self-matching will therefore be higher than that under cross-matching when:

$$\frac{S(2,2)}{S(1,1)} \geq \frac{2^{1/\alpha_1}}{\lambda(-1)}$$

That is, output will be maximised by type-2 individuals working on their own when the type-2's skill in their own occupation is sufficiently greater than the type-1's skill in theirs.

3.2. *Wages and rent-extraction*

If type-2 individuals choose to cross-match with type-1s and wages are determined competitively, I assume that they will each receive their productive share of total output:

$$\tilde{w}_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} S(1,1)^{\alpha_1} S(2,2)^{\alpha_2} \qquad \tilde{w}_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} S(1,1)^{\alpha_1} S(2,2)^{\alpha_2}$$

If type-2 individuals choose to self-match, type-1 individuals will be forced to accept the reservation or subsistence wage, while the two type-2 individuals will split the output of their single firm:

$$\hat{w}_1 = \underline{w} = 0 \qquad \hat{w}_2 = \frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$$

Although type-1 individuals will clearly wish to cross-match, type-2 individuals will seek to maximise their wages:

$$w_2 = \max\{\hat{w}_2, \tilde{w}_2\} = \max\left\{\frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}, \frac{\alpha_2}{\alpha_1+\alpha_2} S(1,1)^{\alpha_1} S(2,2)^{\alpha_2}\right\}$$

Therefore, type-2s will wish to self-match when:

$$\frac{S(2,2)}{S(1,1)} \geq \frac{2^{1/\alpha_1}}{\lambda(-1)} \left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}$$

Since $\alpha_2/(\alpha_1+\alpha_2) < 1$, this implies that type-2 individuals will move to self-match at skill ratios lower than maximising output would warrant, leading to an economy-wide productive inefficiency.

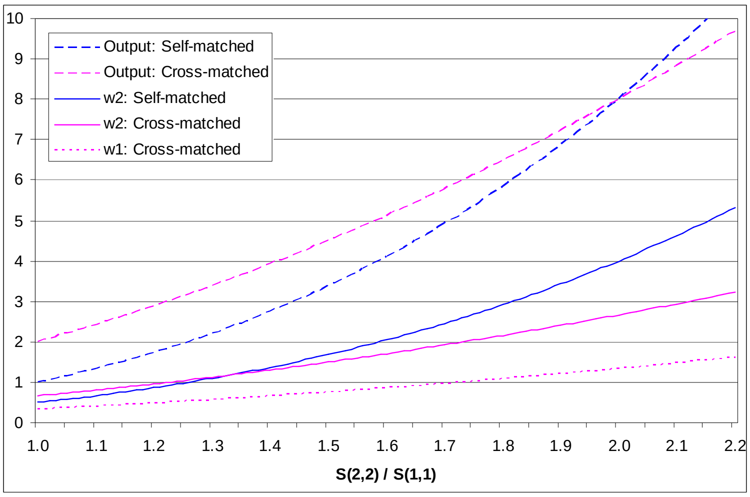

Figure 2 – Characterisation of output and wages under self- and cross-matching. Parameters used are: $S(1,1) = 1$, $\lambda(-1) = 1$, $\alpha_1 = 1$ and $\alpha_2 = 2$

For skill ratios between these two thresholds, type-2 individuals are thus in a position to extract rents from the type-1 individuals by being willing to cross-match, but demanding to receive a wage no less than they would have attained under self-matching. Faced with an outside option of zero, type-1 individuals will clearly accept.[9] Output is therefore maximised (productive efficiency is obtained), but the upper-limit for type-1 individuals' wage will fall as the skill ratio increases. If output is maximised by self-matching, the type-2 self-matching wage will exceed the output of any one firm under cross-matching. Type-1 individuals will therefore be unable to compensate the type-2s sufficiently and so will accept self-matching over a negative wage.

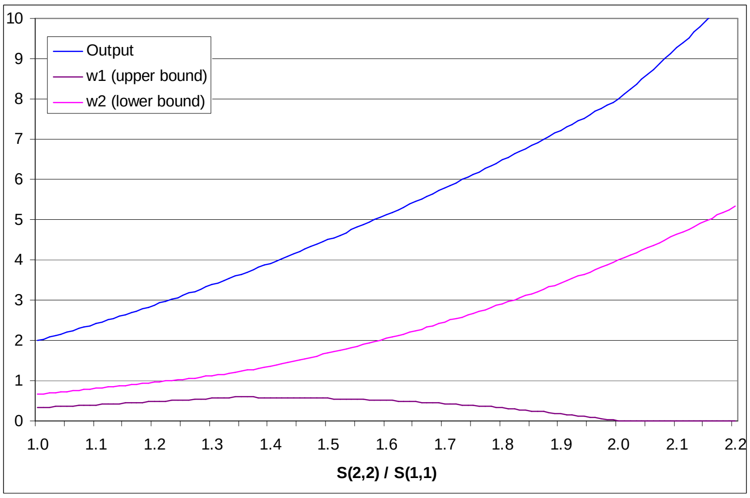

Figure 3 – Characterisation of output and wages with rent extraction. Parameters used are: $S(1,1) = 1$, $\lambda(-1) = 1$, $\alpha_1 = 1$ and $\alpha_2 = 2$

[9] Note that it is optimal for type-1 individuals to accept any wage greater than the subsistence wage, implying that for skill ratios in between the two thresholds, the self-matching wage represents a lower bound for type-2 individuals.

We then have that if *S(2,2)* increases relative to *S(1,1)*, inequality will rise if the new skill ratio exceeds the lower, "rent seeking" threshold. Note that for certain parameters, the rent-seeking threshold may be less than unity.[10]

$\frac{S(2,2)}{S(1,1)} \in$	$\left[0, \frac{2^{1/\alpha_1}}{\lambda(-1)}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}\right)$	$\left[\frac{2^{1/\alpha_1}}{\lambda(-1)}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}, \frac{2^{1/\alpha_1}}{\lambda(-1)}\right)$	$\left[\frac{2^{1/\alpha_1}}{\lambda(-1)}, \infty\right)$
Type of production	Cross-matching		Self-matching
Result	All receive their productive share of output	Rent extraction	Type-1 individuals excluded
$\frac{\Delta\text{Inequality}}{\Delta\text{Skill Ratio}}$	$= 0$ (constant)	> 0 (increasing)	
w1	$= \frac{\alpha_1}{\alpha_1+\alpha_2} S(1,1)^{\alpha_1} S(2,2)^{\alpha_2}$	$\leq S(1,1)^{\alpha_1} S(2,2)^{\alpha_2} - \frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$	$= \underline{w} = 0$
w2	$= \frac{\alpha_2}{\alpha_1+\alpha_2} S(1,1)^{\alpha_1} S(2,2)^{\alpha_2}$	$\geq \frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$	$= \frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$

Table 1 – Summary of results under autarky

3.3. Globalisation and inequality

The model presented here can provide an explanation of increasing inequality following liberalisation in two different ways. Firstly and most simply, even without changes to trade barriers, liberalisation that attracts an inflow of foreign expertise will increase inequality if that expertise increases the skill-level of higher occupations by more than it does for lower occupations and the new skill ratio is above the rent-seeking threshold. An obvious candidate for such an inflow is technology transfer resulting from Foreign Direct Investment (FDI).

In considering FDI-induced technology transfers, it is important here to differentiate between changes in the *production process* or choice of *capital* or *intermediate goods*, which would alter the parameters of production (here Φ, α_i, β_i and γ_i), and increases in the underlying *skills* or quality of the workforce. Multinational Corporations (MNCs) that provide host-country workers with additional or improved training, a wider variety of work experience, or a management style that encourages greater self-directed learning will lead to increases in the underlying skills of domestic workers. To the extent that individuals in higher occupations are more receptive to this training, or are granted greater exposure to it, the country's skill ratio will rise. Note that although this effect is distinct from other forms of technology transfer, it may be exacerbated by them if, as is usually observed, they

[10] In particular, this may occur when tasks are quite close in their marginal products (α_1 is quite close to α_2), or when $\lambda(-1)$ is large. For example, if α_1 = 1.0 and α_2 = 1.1, $\lambda(-1)$ need only equal 1.05 for this to occur.

lead to an increase in demand for high-occupation workers and thus an imbalance in exposure to foreign training.

There exists a large body of literature examining interactions between human capital levels and FDI (in both directions), but relatively little on differentiating types of technology transfer or on training in particular. Eicher and Kalaitzidakis (1997) offer one such model, in which an MNC decides where to locate a subsidiary on the basis on human capital levels precisely *because* they will need to train local staff. They suggest that an MNC brings with it a superior production process or blueprint, but needs to invest in local staff, increasing their skills in order to make use of the MNC's particular technology. They assume that training efficiency – the receptiveness of workers to training or, inversely, the cost required to train them – is based on worker quality (analogous to skill in my model), which is at least partially unobservable. This information asymmetry then leads firms to use human capital levels as a signal for the expected quality of the workers in a particular location.

A second form of globalisation that is relevant to the model developed here is that of production sharing, as examined in Kremer and Maskin (2006). Suppose that there are two countries, north and south, with skill levels in the north being higher for all occupations: $S(i,i,n) > S(i,i,s)\ \forall i$. The countries, initially in a state of autarky, become open, with the effect being to allow matching of occupations across the two countries. I assume that that the production process does not require that tasks be performed in the same location. Assume that the skill ratio in both countries is low enough that before liberalisation, type-2 individuals prefer to cross-match in both countries. Then initially we have:

$$w_i^q = \tilde{w}_i^q = \frac{\alpha_i}{\alpha_1 + \alpha_2} S(1,1,q)^{\alpha_1} S(2,2,q)^{\alpha_2}$$

Where: i indexes the occupation; and
q indexes the country

Wages after liberalisation will be designated with a prime: $w_1^{s\prime}, w_2^{s\prime}, w_1^{n\prime}, w_2^{n\prime}$

Because they are essential to the production process, no change to the production arrangements will occur after liberalisation unless type-2 individuals from both countries are weakly better off.

First, consider the possibility of individuals cross-matching across countries. If type-2 individuals from the south were to match with type-1 individuals from the north, w_2^s would rise because $S(1,1,n) > S(1,1,s)$, but w_1^n would fall because *S(2,2,s) < S(2,2,n)*. Similarly, if type-2 individuals from the north were to match with type-1 individuals from the south, type-2 individuals from the north would see their wages fall and type-1 individuals from would see their wages rise. Although this situation would be better for all individuals from the south, both types from the north remain better off by working together and so this will not happen.

Next, consider the possibility of matching between type-2 individuals from the north and the south. We need to be careful to allow for the fact that although they come from the same occupation, which might suggest an equal share of output, those from the north have higher skill and so may demand a higher share of output. Defining the output of their firm to be $\ddot{Y}_2 = (\lambda(-1)S(2,2,s))^{\alpha_1} S(2,2,n)^{\alpha_2}$, their new wages would be:

$$\ddot{w}_2^{n\,\prime} = \eta \ddot{Y}_2 \qquad \ddot{w}_2^{s\,\prime} = (1-\eta)\ddot{Y}_2 \qquad \text{for some } \eta \in [0,1]$$

Proposition 1: There exists a range of values for η that would allow type-2 individuals from both countries to be better off by working together iff. $\ddot{Y}_2 \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\tilde{Y}_n + \tilde{Y}_s\right)$, where $\tilde{Y}_n$ and $\tilde{Y}_s$ represent the per-firm cross-matching output under autarky in the north and south respectively.[11]

When this condition holds, and presuming that they can agree on a value of η within that range, type-1 individuals from both countries will be excluded from the production process and inequality will increase in both countries.

The following numeric example illustrates this possibility.

α_1	α_2	$\lambda(-1)$
1	2	1.2

Skill	S(1,1)	S(2,2)
North	1.2	1.3
South	1.0	1.1

The rent-seeking threshold is:

$$\frac{2^{1/\alpha_1}}{\lambda(-1)}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1} = \frac{2^{1/1}}{1.2}\left(\frac{2}{1+2}\right)^{1/1} = 1.1111\ldots$$

[11] Proofs of this and all subsequent propositions may be found in the appendix to this paper.

The skill ratio is below this threshold in both countries, so before opening to trade type-2 individuals in both countries prefer to cross-match. After liberalisation, type-2 individuals from the north will prefer to work with type-2 individuals from the south if they can negotiate a share of output such that:

$$\eta \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,n)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1} = \frac{2}{3}\left(\frac{1.2}{1.2\times 1.1}\right) = 0.6060...$$

Type-2 individuals from the south would likewise prefer to work with their northern counterparts if they will receive at least:

$$1-\eta \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,s)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}\left(\frac{S(2,2,s)}{S(2,2,n)}\right)^{\alpha_2} = \frac{2}{3}\left(\frac{1}{1.2\times 1.1}\right)\left(\frac{1.1}{1.3}\right)^2 = 0.3616$$

Therefore, type-2 individuals from both countries will be better off by working together (and excluding type-1 individuals), if they can agree on a value for η in the range: $\eta \in [60.61\%, 63.84\%]$.

4. Extensions and future work

The model presented above made a number of assumptions for the purposes of clarity. I here briefly explore the consequences those assumptions, finding that my results appear broadly robust to their relaxation, and identify areas for future research.

4.1. Discussion of φ(i,j)

One of the least intuitive assumptions made in section three was that a person's skill falls to zero when performing a task above their occupation. Consider the case that type-1 individuals are able to perform the higher task at merely a reduced level of skill:

$$\lambda(i-j)\begin{cases} \geq 1 & j > i \\ = 1 & j = i \\ \leq 1 & j < i \end{cases}$$

but impose the lesser assumption that:

$$\lambda(i-j)\lambda(j-i) \leq 1$$

That is, a person's skill falls faster by performing a higher task than it rises by doing a lower task. As I illustrate below, this assumption is needed to ensure that cross-matching will be more efficient than self-matching for at least *some* skill ratios, but it also seems quite reasonable. A manual labourer may be able to perform less than half the duties of an engineer, but an engineer working *as* a labourer is unlikely to be twice as productive as a typical labourer is.

4.1.1. Output maximisation

For a two-occupation model with two individuals trained in each, output under self-matching is:

$$S(1,2)^{\alpha_1} S(2,2)^{\alpha_2} + S(1,1)^{\alpha_1} S(2,1)^{\alpha_2} = \lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2} + \lambda(+1)^{\alpha_2} S(1,1)^{\alpha_1+\alpha_2}$$

Denoting the skill ratio by $R = S(2,2)/S(1,1)$ and the output difference between self-matching and cross-matching as ***f(R)***, we have:

$$f(R) = \lambda(-1)^{\alpha_1} R^{\alpha_1+\alpha_2} - 2R^{\alpha_2} + \lambda(+1)^{\alpha_2}$$

Output will be maximised under self-matching rather than cross-matching when $f(R) \geq 0$. Solving this will generally require numerical methods. The inclusion of the last term, $\lambda(+1)^{\alpha_2}$, has the effect that in addition to high skill ratios, self-matching will also be more productive for very low skill ratios. This makes intuitive sense. A highly experienced nurse may be more productive than an inexperienced junior doctor in both of their roles, but as the doctor gains experience and her skill rises, it becomes more efficient for the two to work together.

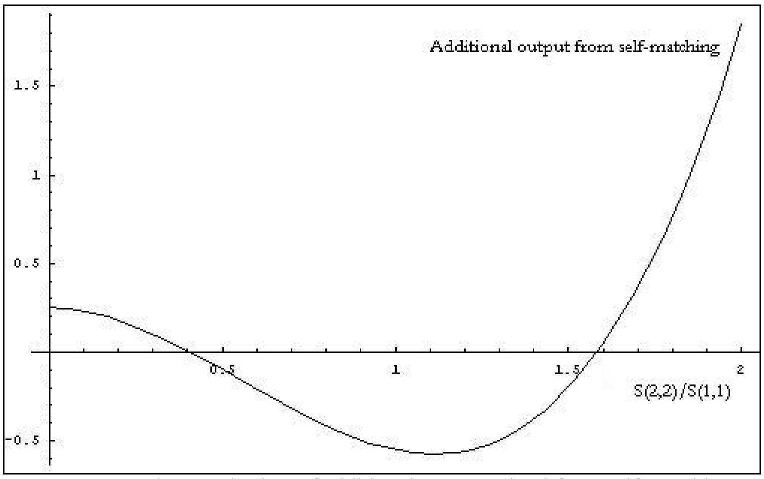

Figure 4 – Characterisation of additional output gained from self-matching
Parameters used are: $\lambda(-1) = 1.2$, $\lambda(+1) = 0.5$, $\alpha_1 = 1$ and $\alpha_2 = 2$

4.1.2. Wages and rent-extraction

Wages for each type under self-matching will be:

$$\hat{w}_1 = \frac{1}{2}\lambda(+1)^{\alpha_2} S(1,1)^{\alpha_1+\alpha_2} \qquad \hat{w}_2 = \frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$$

Corresponding to the lower self-matching range, type-1 individuals will prefer self-matching if the skill ratio is sufficiently *low*:

$$R \le R_1^{sm} \equiv \frac{\lambda(+1)}{2^{1/\alpha_2}}\left(\frac{\alpha_1+\alpha_2}{\alpha_1}\right)^{1/\alpha_2}$$

As in section three, type-2 individuals will prefer self-matching if the skill ratio is sufficiently high:

$$R \ge R_2^{sm} \equiv \frac{2^{1/\alpha_1}}{\lambda(-1)}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}$$

Proposition 2: With $\lambda(+1) > 0$ and assuming that $\lambda(-1)\lambda(+1) \le 1$, it remains the case that R_2^{sm} represents a *rent-seeking* threshold for type-2 workers, such that for some range $R \in \left[R_2^{sm}, ?\right)$, type-2 individuals are willing to cross-match, but demand a wage no less than the type-2 self-matching wage.

4.1.3. Globalisation and inequality

As in section three, an inflow of foreign expertise that raises the skill ratio will raise inequality if the new ratio is above R_2^{sm}. In considering cross-border self-matching, it likewise remains the case that type-2 individuals from both countries will be better off if:

$$\ddot{Y}_2 \ge \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\tilde{Y}_n + \tilde{Y}_s\right)$$

However, with $\lambda(+1) > 0$ we also have the possibility of type-1 individuals from both countries working together. If, like Kremer and Maskin (2006), we mandate that cross-border production sharing must involve high-occupation individuals from the North,[12] then the results are unchanged from section three, with inequality weakly increasing in both countries.

If we allow this possibility though, and using the same logic applied in proposition 1 for type-2 individuals, it is possible for type-1 individuals from both countries to be better off by working together if and only if:

$$\ddot{Y}_1 \ge \frac{\alpha_1}{\alpha_1+\alpha_2}\left(\tilde{Y}_n + \tilde{Y}_s\right)$$

If both of these conditions hold, then it is possible that liberalisation will benefit everybody. Alternatively, cross-border self-matching may increase inequality if:

[12] As Kremer and Maskin (2006) note, “this is consistent with stylized facts on employment and globalization. Nike’s designers and marketers can live in the U.S. and work with Vietnamese workers, but Nike cannot hire U.S. factory supervisors to oversee workers in Vietnam.”

$$\ddot{Y}_2 \geq \frac{\alpha_2}{\alpha_1 + \alpha_2}\left(\tilde{Y}_n + \tilde{Y}_s\right) \text{ and } \ddot{Y}_1 \leq \frac{\alpha_1}{\alpha_1 + \alpha_2}\left(\tilde{Y}_n + \tilde{Y}_s\right)$$

$$\ddot{Y}_2 \geq \frac{\alpha_2}{\alpha_1}\ddot{Y}_1$$

4.2. Imbalanced labour supply

One of the key assumptions made in section three was that the distribution of the labour supply over occupations was the same as that required by the production process. I here demonstrate that with non-equal numbers of type-1 and type-2 individuals, conditions for output maximisation do not change, but there are increased opportunities for rent-seeking behaviour and the effects of globalisation may potentially become ambiguous.

4.2.1. Output maximisation

Proposition 3: For a two-occupation model with $\beta_i = \gamma_i = 1 \forall i$, it remains the case that output is maximised by having greater than the minimum amount of self-matching when $\frac{S(2,2)}{S(1,1)} > \frac{2^{1/\alpha_1}}{\lambda(-1)}$ irrespective of the number of individuals trained in each occupation.

4.2.2. Wages and rent-extraction

Proposition 4: Under a two-occupation model with $\beta_i = \gamma_i = 1 \forall i$, for skill ratios above the rent-seeking threshold, wages remain unchanged from the situation presented in section three. When the skill ratio is lower than the rent-seeking threshold, the individuals in shorter supply are in a position to extract rents, with those in greater supply potentially reduced to their outside option.

4.2.3. Globalisation and inequality

Given propositions 3 and 4, opportunities for increased inequality from an inflow of foreign expertise remain unchanged from section three. However, the results are mixed when considering cross-border production sharing. Consider a country where the skill ratio is below the rent-seeking threshold. If type-2 individuals outnumber type-1s (so that type-1s hold rent-seeking power), type-2 individuals will be more inclined to work with their counterparts from the other country, as their pre-liberalisation wage was lower. The impact on inequality will be ambiguous, however, depending on whether the pre-liberalisation wage of type-2 individuals was greater than that of type-1s. Alternatively, if type-1 individuals outnumber type-2s, production sharing with another country can only increase inequality, as it will further reduce the number of type-2 individuals available to work with type-1s.

4.2.4. Imbalanced labour supply across many occupations

These results need not necessarily hold when the number of occupations increases above two. For example, consider a three-occupation model with $\beta_i = \gamma_i = 1 \forall i$ where the highest occupation has the lowest number of individuals ($X_3 < min\{X_1, X_2\}$). In that case, for output to be maximised by using up to ($X_2 - X_3$) type-2 individuals in the lowest task, we only require that $\lambda(-1)S(2,2) \geq S(1,1)$ because there is no reduction in the number of firms as a result. A full analysis of these possibilities is beyond the scope of this paper and clearly calls for future work.

4.3. Many occupations

One intuitive criticism of the model presented here may be that it is implausible to suppose that high-occupation individuals would be willing to take on lower tasks. We do not observe engineers working as janitors or taxi drivers, except perhaps as recent immigrants to developed countries, when language problems or a lower-than-market level of skill in the engineer are arguably better explanatory factors. This concern may be addressed by analysing the model with an arbitrary number of (n) tasks under the more plausible restriction that individuals may only take on a task that is, at most, one step down from their chosen occupation. In this situation, an individual qualified as a medical registrar (mid-range doctor) may act as a resident (junior doctor), for example, but not as a nurse. Although a complete analysis of this extension is beyond the scope of this paper, I present here the following proposition:

Proposition 5: In an *n*-occupation model with $\beta_i = \gamma_i = 1 \forall i$, when firms each make a single, one-step deviation from perfect cross-matching, it remains the case that total output will be maximised if $\frac{S(k+1,k+1)}{S(k,k)} \geq \frac{2^{1/\alpha_k}}{\lambda(-1)}$.

Furthermore, *additional* one-step deviations elsewhere in the production chain require only that $\frac{S(j+1,j+1)}{S(j,j)} \geq \frac{1}{\lambda(-1)}$ to be more efficient.

5. Conclusion

The model presented here provides a simple and intuitive explanation for why trade or investment liberalisation may cause inequality to rise without resorting to technology changes or relying on FDI-induced changes in product specialisation. Unlike most previous attempts at explaining this phenomenon, this model can also explain the fact that we observe inequality rising contemporaneously with liberalisation. To the extent that this model's results are correct, we would predict that the increased use of highly-trained individuals by developing-nation firms is not to perform the roles for which they are trained, but to take the place of their lesser-educated compatriots in "lower" roles. That is, we would expect to see the use of a more educated workforce with little change in the production process itself.

There is a clear implication that, independently of trade, if policymakers wish to minimise wage inequality then they must pay attention to equality of skills across occupations. If a person's skill within their occupation is partially determined by their "excess" education then it may not be enough to guarantee that everyone in society receives at least a basic level of education. For example, if the absolute minimum educational requirements for manual labour and engineering are 5 and 12 years respectively, but engineers typically obtain 16, then manual labourers may require more than 5 years of education in order to equalise skill across the two occupations.

Likewise, policymakers that seek to raise their country's human capital in order to take advantage of international trade would be advised to ensure that this accrual occurs across all occupations and not only in the higher professions. If some aspect of globalisation has already caused inequality to increase through the mechanisms described in this paper, the best response would be to increase the skill levels of those people excluded from production.

Finally, note that this model need not be representative of all industries and particularly not in those that – at least in the short term – do not require people to perform certain roles in fixed proportion. Nevertheless, the presence of at least some industries in developing countries that employ people for jobs that do not require the education levels they possess suggests the relevance of this mechanism in overcoming the much hoped-for equalising effects of trade.

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Appendix: Proofs

Proof of Proposition 1: Consider first type-2 individuals from the north. They will be better off by working with type-2 individuals from the south if:

$$\eta(\lambda(-1)S(2,2,s))^{\alpha_1} S(2,2,n)^{\alpha_2} \geq \frac{\alpha_2}{\alpha_1+\alpha_2} S(1,1,n)^{\alpha_1} S(2,2,n)^{\alpha_2}$$

$$\eta \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,n)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}$$

Likewise, southern type-2 individuals will be better off by working with type-2 individuals from the north if:

$$(1-\eta)(\lambda(-1)S(2,2,s))^{\alpha_1} S(2,2,n)^{\alpha_2} \geq \frac{\alpha_2}{\alpha_1+\alpha_2} S(1,1,s)^{\alpha_1} S(2,2,s)^{\alpha_2}$$

$$\eta \leq 1-\frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,s)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}\left(\frac{S(2,2,s)}{S(2,2,n)}\right)^{\alpha_2}$$

Type-2 individuals from both countries will therefore be better off by working together if it is possible for them to agree on a value of η that satisfies both of the above requirements. This will only be the case if:

$$1-\frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,s)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}\left(\frac{S(2,2,s)}{S(2,2,n)}\right)^{\alpha_2} \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\frac{S(1,1,n)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}$$

$$1 \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left[\left(\frac{S(1,1,n)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}+\left(\frac{S(1,1,s)}{\lambda(-1)S(2,2,s)}\right)^{\alpha_1}\left(\frac{S(2,2,s)}{S(2,2,n)}\right)^{\alpha_2}\right]$$

$$(\lambda(-1)S(2,2,s))^{\alpha_1} \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left[S(1,1,n)^{\alpha_1}+S(1,1,s)^{\alpha_1}\left(\frac{S(2,2,s)}{S(2,2,n)}\right)^{\alpha_2}\right]$$

$$(\lambda(-1)S(2,2,s))^{\alpha_1} S(2,2,n)^{\alpha_2} \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left[S(1,1,n)^{\alpha_1} S(2,2,n)^{\alpha_2}+S(1,1,s)^{\alpha_1} S(2,2,s)^{\alpha_2}\right]$$

$$\ddot{Y}_2 \geq \frac{\alpha_2}{\alpha_1+\alpha_2}\left(\tilde{Y}_n+\tilde{Y}_s\right)$$

Q.E.D.

Proof of Proposition 2: First, note that the proposed rent-seeking threshold corresponds to the critical point that minimises *f(R)* in the region $R \in [0,\infty)$:

$$\frac{df(R)}{dR}=0 \Rightarrow \quad \lambda(-1)^{\alpha_1}(\alpha_1+\alpha_2)R^{*\alpha_1+\alpha_2-1}-2\alpha_2 R^{*\alpha_2-1}=0$$

$$R^{*\alpha_2-1}\left(\lambda(-1)^{\alpha_1}(\alpha_1+\alpha_2)R^{*\alpha_1}-2\alpha_2\right)=0$$

$$R^*=\frac{2}{\lambda(-1)}^{1/\alpha_1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1} \quad \text{OR } 0 \text{ (if } \alpha_2>1)$$

Therefore, provided that $f(R^*)<0$ (i.e. provided that cross-matching is superior to self-matching at R_2^{sm}), the proof is complete and $R \in \left[R_2^{sm}, \vec{R}\right)$ (where $\vec{R}$ is the highest root of $f(R)$) will represent the rent-seeking range for type-2 workers. Substituting R^* back into $f(R)$:

$$f(R^*)=\lambda(-1)^{\alpha_1}\left(\frac{2}{\lambda(-1)}^{1/\alpha_1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}\right)^{\alpha_1+\alpha_2}-2\left(\frac{2}{\lambda(-1)}^{1/\alpha_1}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{1/\alpha_1}\right)^{\alpha_2}+\lambda(+1)^{\alpha_2}$$

$$=\frac{2^{\frac{\alpha_1+\alpha_2}{\alpha_1}}}{\lambda(-1)^{\alpha_2}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{\frac{\alpha_2}{\alpha_1}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}-1\right)+\lambda(+1)^{\alpha_2}$$

$$=\lambda(+1)^{\alpha_2}-\frac{2^{\frac{\alpha_1+\alpha_2}{\alpha_1}}}{\lambda(-1)^{\alpha_2}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{\frac{\alpha_2}{\alpha_1}}\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)$$

Therefore,

$$f(R^*)\le 0 \quad \Leftrightarrow \quad \lambda(-1)\lambda(+1)\le 2^{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}}\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_2}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_1}}$$

Define $g(\alpha_1,\alpha_2)=2^{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}}\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_2}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_1}}$. We have that $\frac{\partial g(\alpha_1,\alpha_2)}{\partial\alpha_1}=0\Rightarrow \alpha_1{}^*=\alpha_2$ and $\frac{\partial g(\alpha_1,\alpha_2)}{\partial\alpha_2}=0\Rightarrow \alpha_2{}^*=\alpha_1$.[13] Then,

$$\begin{aligned} g(\alpha,\alpha) &= 2^{\frac{1}{\alpha}+\frac{1}{\alpha}}\left(\frac{\alpha}{\alpha+\alpha}\right)^{\frac{1}{\alpha}}\left(\frac{\alpha}{\alpha+\alpha}\right)^{\frac{1}{\alpha}} \\ &= 2^{\frac{2}{\alpha}}\left(\frac{\alpha}{2\alpha}\right)^{\frac{2}{\alpha}} \\ &= 1 \end{aligned}$$

[13] I did not derive this algebraically, but instead used Mathematica version 5.2. Instructions used were:

```
Solve[D[2^((x + y)/(x*y))*(x/(x + y))^(1/y)*(y/(x + y))^(1/x), x] == 0, x]
Solve[D[2^((x + y)/(x*y))*(x/(x + y))^(1/y)*(y/(x + y))^(1/x), y] == 0, y]
```

That is, $\inf\left\{2^{\frac{1}{\alpha_1}+\frac{1}{\alpha_2}}\left(\frac{\alpha_1}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_2}}\left(\frac{\alpha_2}{\alpha_1+\alpha_2}\right)^{\frac{1}{\alpha_1}}\right\}=1$ and since we have that $\lambda(-1)\lambda(+1)\leq 1$, the proof is complete.

Proof of Proposition 3: Suppose first that the number of type-2 individuals (X_2) is smaller than or equal to the number of type-1s (X_1). Because type-1 individuals are unable to perform the higher task, this means that X_2 provides the upper limit for the number of firms and whatever happens, at least $(X_1 - X_2)$ type-1 individuals will be excluded from the production process. For the purposes of output maximisation, we can therefore ignore those people and the results will be the same as in section 3.

Next, consider the possibility that there are more type-2 individuals than there are type-1s ($X_2 > X_1$). Provided that $X_2 - X_1 \geq 2$, there will always be some self-matching in this situation. Let q be the minimum number of firms with type-2s self-matching. That is,

$$q \equiv \begin{cases} \dfrac{X_2 - X_1}{2} & \text{if } \dfrac{X_2 - X_1}{2} \text{ is even} \\ \dfrac{X_2 - X_1}{2} - 1 & \text{if } \dfrac{X_2 - X_1}{2} \text{ is odd} \end{cases}$$

Let p be the actual number of firms with type-2s self-matching. If $p > q$ then $X_2 - 2p < X_1$ and so the number of firms with cross-matching will be $X_2 - 2p$. Output will be maximised in this situation when:

$$\begin{aligned}
p\lambda(-1)^{\alpha_1}S(2,2)^{\alpha_1+\alpha_2} + (X_2 - 2p)S(1,1)^{\alpha_1}S(2,2)^{\alpha_2} &\geq q\lambda(-1)^{\alpha_1}S(2,2)^{\alpha_1+\alpha_2} + X_1 S(1,1)^{\alpha_1}S(2,2)^{\alpha_2} \\
(p-q)\lambda(-1)^{\alpha_1}S(2,2)^{\alpha_1+\alpha_2} &\geq (X_1 - (X_2 - 2p))S(1,1)^{\alpha_1}S(2,2)^{\alpha_2} \\
\frac{S(2,2)}{S(1,1)} &\geq \left(\frac{2p - (X_2 - X_1)}{p-q}\right)^{1/\alpha_1}\frac{1}{\lambda(-1)} \\
\frac{S(2,2)}{S(1,1)} &\geq \left(\frac{2p-2q}{p-q}\right)^{1/\alpha_1}\frac{1}{\lambda(-1)} = \frac{2^{1/\alpha_1}}{\lambda(-1)}
\end{aligned}$$

Q.E.D.

Proof of Proposition 4: The first part of the proposition follows from observation. Note that if the skill ratio exceeds the rent-seeking threshold, we have that $\frac{1}{2}\lambda(-1)^{\alpha_1}S(2,2)^{\alpha_1+\alpha_2} \geq \frac{\alpha_2}{\alpha_1+\alpha_2}S(1,1)^{\alpha_1}S(2,2)^{\alpha_2}$. That is, type-2 individuals obtain more by choosing to match with other type-2 individuals, irrespective of the number of each.

As such, their optimal rule will be to demand to receive at least the amount on the left hand side of this inequality, just as in section 3.

For a skill ratio below the rent-seeking threshold, consider first the situation when there are more type-1 individuals than there are type-2s ($X_1 > X_2$). Although output is maximised by having X_2 firms with cross-matching, there will still be ($X_1 - X_2$) type-1 individuals shut out of the production process and forced to accept the subsistence wage. Since skill is constant within each occupation, the optimal choice for type-1 individuals engaged in production is to accept any wage greater than the subsistence wage. The rent-seeking power of type-2 individuals is thus extended below the lower threshold.

Likewise, when there are more type-2 individuals than type-1s ($X_1 > X_2$), there will be ($X_2 - X_1$) individuals forced to self-match and receive a wage of $\frac{1}{2}\lambda(-1)^{\alpha_1} S(2,2)^{\alpha_1+\alpha_2}$. For type-2 individuals engaged in cross-matching, it will be optimal to accept any wage greater than this outside option. Type-1 individuals thus possess rent-seeking power in this case.

Proof of Proposition 5: With equal numbers of individuals trained in each occupation, X_n represents the upper limit on the number of firms. Assuming that p firms choose to make the single, one-step deviation, total output is:

$$\begin{aligned}\hat{Y} &= p\left(\prod_{i=1}^{k-1} S(i,i)^{\alpha_i}\right)\left(\lambda(-1)S(k+1,k+1)\right)^{\alpha_k} S(k+1,k+1)^{\alpha_{k+1}}\left(\prod_{i=k+2}^{n} S(i,i)^{\alpha_i}\right) + (X_n - 2p)\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right) \\ &= p\left(\prod_{i=1}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1,k+1)^{\alpha_k+\alpha_{k+1}}\left(\prod_{i=k+2}^{n} S(i,i)^{\alpha_i}\right) + (X_n - 2p)\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right)\end{aligned}$$

Total output in this case will be higher than under perfect cross-matching if:

$$\begin{aligned}p\left(\prod_{i=1}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1,k+1)^{\alpha_k+\alpha_{k+1}}\left(\prod_{i=k+2}^{n} S(i,i)^{\alpha_i}\right) + (X_n - 2p)\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right) &\geq X_n\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right) \\ p\left(\prod_{i=1}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1,k+1)^{\alpha_k+\alpha_{k+1}}\left(\prod_{i=k+2}^{n} S(i,i)^{\alpha_i}\right) &\geq 2p\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right) \\ \lambda(-1)^{\alpha_k} S(k+1,k+1)^{\alpha_k+\alpha_{k+1}} &\geq 2S(k,k)^{\alpha_k} S(k+1,k+1)^{\alpha_{k+1}} \\ \frac{S(k+1,k+1)}{S(k,k)} &\geq \frac{2^{1/\alpha_k}}{\lambda(-1)}\end{aligned}$$

This is the same result as under two occupations. Note that the *2* here comes from the assumption that the labour supply exists with the same distribution over occupations as the production function does over tasks. This means that for every firm that chooses partial self-matching, there are two that can no longer exist with perfect cross-matching. This assumption leads to a further result: that once it is optimal to make one one-step deviation from perfect cross-matching, the "barrier" to repeating the process anywhere else in the production chain is lowered. Suppose that in addition to employing people trained in occupation $k+1$ to perform tasks k and $k+1$, firms are also considering hiring people trained in occupation $j+1$ to perform tasks j and $j+1$. Output in that scenario will be:

$$\bar{Y} = p\left(\prod_{i=1}^{j-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_j} S(j+1, j+1)^{\alpha_j+\alpha_{j+1}}\left(\prod_{i=j+2}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1, k+1)^{\alpha_k+\alpha_{k+1}}\left(\prod_{i=k+2}^{n} S(i,i)^{\alpha_i}\right)+(X_n-2p)\left(\prod_{i=1}^{n} S(i,i)^{\alpha_i}\right)$$

This will represent a further increase in output when:

$$\begin{aligned}
\bar{Y} &\geq \hat{Y} \\
\lambda(-1)^{\alpha_j} S(j+1, j+1)^{\alpha_j+\alpha_{j+1}}\left(\prod_{i=j+2}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1, k+1)^{\alpha_k+\alpha_{k+1}} &\geq \left(\prod_{i=j}^{k-1} S(i,i)^{\alpha_i}\right)\lambda(-1)^{\alpha_k} S(k+1, k+1)^{\alpha_k+\alpha_{k+1}} \\
\lambda(-1)^{\alpha_j} S(j+1, j+1)^{\alpha_j+\alpha_{j+1}} &\geq S(j, j)^{\alpha_j} S(j+1, j+1)^{\alpha_{k+1}} \\
\frac{S(j+1, j+1)}{S(j, j)} &\geq \frac{1}{\lambda(-1)}
\end{aligned}$$

Q.E.D.