October 2003

Companion Appendix to "Real exchange rate persistence and monetary policy rules"

Appendix

In this Appendix we define the symmetric steady state around which we will approximate the economy. We will analyze the behavior of the economy in the neighborhood of a symmetric deterministic steady state in which inflation and depreciation rates are zero. Monetary policy shocks are assumed to be zero and there are no productivity shocks. Nominal interest rates are equal to the preferences' discount rate. From (7) in the main text and the foreign counterpart we obtain

$$\beta = \frac{1}{1+\bar{i}} = \frac{1}{1+\bar{i}^*}$$

The firm's discount factors are

$$\Xi_{t,t+s} = \Xi^*_{t,t+s} = \beta^s$$

Then from the firm's first order conditions we have that in steady state each firm charge the same price in the domestic and foreign markets.

$$\overline{p}_H = \overline{S}\overline{p}^*_H = \overline{P}_H = \frac{\sigma}{\sigma-1}\frac{\overline{W}}{\overline{X}} \tag{A.1}$$

$$\overline{p}^*_F = \frac{\overline{p}_F}{\overline{S}} = \overline{P}^*_F = \frac{\sigma}{\sigma-1}\frac{\overline{W}^*}{\overline{X}^*} \tag{A.2}$$

Note that in steady state the law of one price holds. Moreover we can write the price indices as

$$\overline{P}^{1-\varsigma} = n\overline{P}_H^{1-\varsigma} + (1-n)\left(\overline{SP}^*_F\right)^{1-\varsigma}$$

$$\overline{P}^{*^{1-\varsigma}} = n\left(\frac{\overline{P}_H}{\overline{S}}\right)^{1-\varsigma} + (1-n)\left(\overline{P}^*_F\right)^{1-\varsigma}$$

which implies that in the steady state the real exchange rate is pegged to one (the steady state value for the real exchange rate is well defined):

$$\overline{P} = \overline{SP}^* \Rightarrow \overline{RS} = 1$$

from which using the optimal risk sharing condition it follows that

$$\overline{C} = \overline{C}^*$$

In steady state, the demand for domestic and foreign produced goods are given by

$$\overline{Y^H} = \left[\frac{\overline{P}_H}{\overline{P}}\right]^{-\varsigma}\left[n\overline{C}+(1-n)\overline{C}^*\right] = \left[\frac{\overline{P}_H}{\overline{P}}\right]^{-\varsigma} C \tag{A.3}$$

$$\overline{Y^F} = \left[\frac{\overline{P}_F^*}{\overline{P}^*}\right]^{-\varsigma}\left[n\overline{C}+(1-n)\overline{C}^*\right] = \left[\frac{\overline{P}_F}{\overline{P}}\right]^{-\varsigma} C \tag{A.4}$$

In equilibrium (8) in the main text and the corresponding foreign condition imply that

$$V_L\left(\frac{\overline{Y^H}}{\overline{X}}\right) = U_C\left(\overline{C}\right)\frac{\overline{W}}{\overline{P}}$$

$$V_L\left(\frac{\overline{Y^F}}{\overline{X}^*}\right) = U_C\left(\overline{C}\right)\frac{\overline{W}^*}{\overline{P}^*}$$

Substituting (A.1), (A.2), (A.3) and (A.4) we obtain:

$$V_L\left(\left[\frac{\overline{P}_H}{\overline{P}}\right]^{-\varsigma}\frac{\overline{C}}{\overline{X}}\right) = U_C\left(\overline{C}\right)\left(\frac{\sigma-1}{\sigma}\right)\overline{X}\frac{\overline{P}_H}{\overline{P}}$$

$$V_L\left(\left[\frac{\overline{P}_F}{\overline{P}}\right]^{-\varsigma}\frac{\overline{C}}{\overline{X}^*}\right) = U_C\left(\overline{C}\right)\left(\frac{\sigma-1}{\sigma}\right)\overline{X}^*\frac{\overline{P}_F}{\overline{P}}$$

Note that $\frac{\overline{P}_H}{\overline{P}}$ and $\frac{\overline{P}_F}{\overline{P}}$ are both function of $T \equiv \frac{P_F}{P_H}$ so that the previous two equations uniquely determine T and C.

Aggregate Supply

In this appendix, we present the log-linear approximation of the supply equations (17) and (18) in the main text. The derivation of (19) and (20) follows similarly. Since marginal costs are constant we can separate the maximization problem in the domestic and in the foreign market. When a domestic firm maximizes in the domestic market, the optimal price $\widetilde{p}_t\left(h\right)$ is given by

$$\widetilde{p}_t\left(h\right) = \frac{\sigma}{\sigma-1}\frac{E_t\sum_{s=0}^{\infty}\left(\alpha^H\right)^s\Xi_{t,t+s}\frac{W_{t+s}^h}{X_{t+s}}\widetilde{y}_{t,t+s}\left(h\right)}{E_t\sum_{s=0}^{\infty}\left(\alpha^H\right)^s\Xi_{t,t+s}\widetilde{y}_{t,t+s}\left(h\right)} \tag{A.5}$$

where

$$\widetilde{y}_{t,t+s}\left(h\right) = \left(\frac{\widetilde{p}_t\left(h\right)}{P_{H,t+s}}\right)^{-\sigma}\left(\frac{P_{H,t+s}}{P_{t+s}}\right)^{-\varsigma}\left(nC_{t+s}\right)$$

and

$$P_{H,t}^{1-\sigma} = \left[\left(\frac{1}{n} \right) \int_0^n p\left(z\right)^{1-\sigma} dz \right] = \alpha^H P_{H,t-1}^{1-\sigma} + \left(1-\alpha^H\right) \widetilde{p}_t\left(h\right)^{1-\sigma} \tag{A.6}$$

Rearranging (A.5), we obtain that

$$E_t \sum_{s=0}^{\infty} (\alpha^H \beta)^s \frac{U_C(C_{t+s}) P_{H,t+s}}{P_{t+s}} \left\{ \left[\begin{array}{c} \frac{\widetilde{p}_t(h)}{P_{H,t+s}} - \\ -\left(\frac{\sigma}{\sigma-1}\right) \frac{W_{t+s}^h}{P_{H,t+s} X_{t+s}} \end{array} \right] \widetilde{y}_{t,t+s}^d(h) \right\} = 0$$

We now take a log linear approximation around the steady state previously defined. We define $\widehat{p}(h)_{t,t+s} = \ln(\widetilde{p}_t(h)/P_{H,t+s})$ and we obtain

$$E_t \sum_{s=0}^{\infty} (\alpha^H \beta)^s \left[\widehat{p}(h)_{t,t+s} - \left(\frac{\widehat{W_{t+s}^h}}{P_{H,t+s} X_{t+s}} \right) \right] = 0$$

Note now that

$$\frac{\widehat{W_{t+s}^h}}{P_{H,t+s} X_{t+s}} = \frac{\widehat{W_{t+s}^h}}{P_{t+s} X_{t+s}} \frac{\widehat{P_{t+s}}}{P_{H,t+s}} \tag{A.7}$$

Log linearizing (8) and (12) in the main text and substituting into (A.7)

$$\frac{\widehat{W_{t+s}^h}}{P_{H,t+s} X_{t+s}} = \eta \left(\widehat{y}_{t+s}(h) - \widehat{X}_{t+s} \right) + \rho \widehat{C}_{t+s} - \widehat{X}_{t+s} + (1-n)\, \widehat{T}_{t+s}$$

with $\eta \equiv \frac{\overline{L} V_{LL}\left(\overline{L}\right)}{V_L\left(\overline{L}\right)}$ and $\rho \equiv \frac{-\overline{C} U_{CC}\left(\overline{C}\right)}{U_C\left(\overline{C}\right)}$ and where

$$\widehat{y}_{t+s}(h) = -\sigma n \widehat{p}(h)_{t,t+s} + \varsigma n\,(1-n)\, \widehat{T}_{t+s} + \widehat{C}_{t+s}^W - \sigma(1-n) \widehat{p}(h)_{t,t+s}^* - \varsigma\,(1-n)^2\, \widehat{T}_{t+s}^*$$

and we have defined $\widehat{C}_{t+s}^W \equiv n\widehat{C}_t + (1-n)\,\widehat{C}_t^*$ and $\widehat{p}(h)_{t,t+s}^* = \ln(\widetilde{p}_t^*(h)/P_{H,t+s}^*)$. Note now that

$$\widehat{p}(h)_{t,t+s} = \widehat{p}(h)_{t,t} - \sum_{j=1}^{s} \pi_{H,t+j}$$

$$\widehat{p}(h)_{t,t+s}^* = \widehat{p}(h)_{t,t}^* - \sum_{j=1}^{s} \pi_{H,t+j}^*$$

and from log linearizing (A.6)

$$\widehat{p}(h)_{t,t} = \frac{\alpha^H}{1-\alpha^H} \pi_{H,t}$$

and similarly for $\widehat{p}(h)_{t,t}^*$:

$$\widehat{p}(h)_{t,t} = \frac{\alpha^{H^*}}{1-\alpha^{H^*}} \pi_{H,t}^*$$

Substituting all these expression in the log-linearized first order condition, we get

$$\begin{aligned}\pi_{H,t}-\beta E_t\pi_{H,t+1} &= \pi^*_{H,t}\frac{\alpha^H-\alpha^{H^*}}{\alpha^H}\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{\sigma(1-n)\eta}{1+\sigma\eta n}+ \\ &\widehat{G}_t\frac{\left(1-\alpha^H\beta\right)\left(1-\alpha^H\right)}{\alpha^H\left(1+\sigma\eta n\right)}+ \\ &+\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{\sigma(1-n)\eta}{1+\sigma\eta n}\left[\beta E_t\pi^*_{H,t+1}-\pi^*_{H,t}\right]\end{aligned} \tag{A.8}$$

where

$$\begin{aligned}G_t &\equiv (\eta+\rho)\,\widehat{C}_t-\frac{\eta(1-n)}{\rho}\widehat{RS}_t-(1+\eta)\,\widehat{X}_t+ \\ &(1-n)\,(1+\varsigma\eta n)\,\widehat{T}_t-\varsigma\,(1-n)^2\,\eta\widehat{T}^*_t\end{aligned}$$

Note the dependence on the pricing decision in the foreign country. Similar steps are needed in order to compute the supply curve for goods sold in the foreign country. The only difference is the presence of a term that depends on the deviations from the law of one price; rearranging the first order condition we get

$$E_t\sum_{s=0}^{\infty}(\alpha^{H^*}\beta)^s\frac{U_C(C_{t+s})P_{H,t+s}}{P_{t+s}}\left\{\begin{bmatrix}\frac{\widetilde{p}^*_t(h)}{P^*_{H,t+s}}\frac{P^*_{H,t+s}S_{t+s}}{P_{H,t+s}}- \\ -\left(\frac{\sigma}{\sigma-1}\right)\frac{W^h_{t+s}}{P_{H,t+s}X_{t+s}}\end{bmatrix}\widetilde{y}^{f^*}_{t,t+s}(h)\right\}=0$$

Let $RSh\equiv\frac{SP^*_H}{P_H}$ then, log-linearizing around the steady state we obtain

$$E_t\sum_{s=0}^{\infty}(\alpha^{H^*}\beta)^s\left[\widehat{p}(h)^*_{t,t+s}+\widehat{RSh}_{t+s}-\left(\frac{\widehat{W^h_{t+s}}}{P_{H,t+s}X_{t+s}}\right)\right]=0$$

Following the same steps as before we have

$$\begin{aligned}\pi^*_{H,t}-\beta E_t\pi^*_{H,t+1} &= \frac{\alpha^{H^*}-\alpha^H}{\alpha^{H^*}}\frac{1-\alpha^{H^*}}{1-\alpha^H}\frac{\sigma n\eta}{1+\sigma\eta(1-n)}\pi_{H,t}+ \\ &+\left(\widehat{G}_t-\widehat{RSh}_t\right)\frac{\left(1-\alpha^{H^*}\beta\right)\left(1-\alpha^{H^*}\right)}{\alpha^{H^*}\left(1+\sigma\eta(1-n)\right)} \\ &\frac{1-\alpha^{H^*}}{1-\alpha^H}\frac{\sigma n\eta}{1+\sigma\eta(1-n)}\left[\beta E_t\pi_{H,t+1}-\pi_{H,t}\right]\end{aligned} \tag{A.9}$$

Combining (A.8) and (A.9) we obtain the supply curve reported in the text

$$\begin{aligned}\pi_{H,t} &= k^H_\pi\pi_{H,t}+k^H_{\pi^*}\pi^*_{H,t}+k^H_C\left(\widehat{C}_t-\widetilde{C}_t\right)+k^H_T\left(\widehat{T}_t-\widetilde{T}_t\right) \\ &+k^H_{T^*}\left(\widehat{T}^*_t-\widetilde{T}^*_t\right)+k^H_{RS}\widehat{RS}_t+\beta E_t\pi_{H,t+1}\end{aligned}$$

$$
\begin{aligned}
\pi^*_{H,t} &= k_\pi^{H^*}\pi_{H,t} + k_{\pi^*}^{H^*}\pi^*_{H,t} + k_C^{H^*}\left(\widehat{C}_t - \widetilde{C}_t\right) + k_T^{H^*}\left(\widehat{T}_t - \widetilde{T}_t\right) \\
&\quad + k_{T^*}^{H^*}\left(\widehat{T}^*_t - \widetilde{T}^*_t\right) + k_{RS}^{H^*}\widehat{RS}_t + \beta E_t \pi^*_{H,t+1}
\end{aligned}
$$

where

$$k_\pi^H \equiv \frac{\alpha^H - \alpha^{H^*}}{\alpha^{H^*}}\frac{\sigma^2(1-n)n\eta^2}{1+\sigma\eta}$$

$$k_\pi^{H^*} \equiv \frac{\alpha^{H^*} - \alpha^H}{\alpha^{H^*}}\frac{1-\alpha^{H^*}}{1-\alpha^H}\frac{(1+\sigma n\eta)\,\sigma n\eta}{1+\sigma\eta}$$

$$k_{\pi^*}^H \equiv \frac{\alpha^H - \alpha^{H^*}}{\alpha^H}\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{(1+\sigma(1-n)\eta)\,\sigma(1-n)\eta}{1+\sigma\eta}$$

$$k_{\pi^*}^{H^*} \equiv \frac{\alpha^{H^*} - \alpha^H}{\alpha^H}\frac{\sigma^2(1-n)n\eta^2}{1+\sigma\eta}$$

$$k_C^H \equiv \left(1 + (1-n)\,\sigma\eta\left(1 - \frac{\zeta^{H^*}}{\zeta^H}\frac{1-\alpha^H}{1-\alpha^{H^*}}\right)\right)\zeta^H\left(\frac{\eta+\rho}{1+\sigma\eta}\right);$$

$$k_C^{H^*} \equiv \left(1 + n\sigma\eta\left(1 - \frac{\zeta^H}{\zeta^{H^*}}\frac{1-\alpha^{H^*}}{1-\alpha^H}\right)\right)\zeta^{H^*}\left(\frac{\eta+\rho}{1+\sigma\eta}\right);$$

$$k_T^H \equiv \frac{k_C^H\,(1-n)\,(1+\varsigma n\eta)}{\eta+\rho} + \zeta^{H^*}\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{(1-n)\,\sigma(1-n)\eta}{1+\sigma\eta}$$

$$k_T^{H^*} \equiv \frac{k_C^{H^*}\,(1-n)\,(1+\varsigma n\eta)}{\eta+\rho} - \zeta^{H^*}\frac{(1-n)\,(1+\eta\sigma n)}{1+\sigma\eta}$$

$$k_{T^*}^H \equiv \zeta^{H^*}\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{(1-n)\,\sigma(1-n)\eta}{1+\sigma\eta} - \frac{k_C^H\varsigma\,(1-n)^2\,\eta}{\eta+\rho}$$

$$k_{T^*}^{H^*} \equiv -\zeta^{H^*}\frac{(1-n)\,(1+\eta\sigma n)}{1+\sigma\eta} - \frac{k_C^{H^*}\varsigma\,(1-n)^2\,\eta}{\eta+\rho}$$

$$k_{RS}^H \equiv \zeta^{H^*}\frac{1-\alpha^H}{1-\alpha^{H^*}}\frac{\sigma(1-n)\eta}{1+\sigma\eta} - \frac{k_C^H\,(1-n)\,\eta}{(\eta+\rho)\,\rho};$$

$$k_{RS}^{H^*} \equiv -\zeta^{H^*}\frac{1+\sigma n\eta}{1+\sigma\eta} - \frac{k_C^{H^*}\,(1-n)\,\eta}{(\eta+\rho)\,\rho};$$

Similar steps are needed in order to get (19) and (20). Here we report the coefficients of the two supply relations.

$$k_\pi^F \equiv \frac{\alpha^F - \alpha^{F^*}}{\alpha^{F^*}}\frac{\sigma^2(1-n)n\eta^2}{1+\sigma\eta}$$

$$k_\pi^{F^*} \equiv \frac{\alpha^{F^*} - \alpha^F}{\alpha^{F^*}}\frac{1-\alpha^{F^*}}{1-\alpha^F}\frac{(1+\sigma n\eta)\,\sigma n\eta}{1+\sigma\eta}$$

$$k_{\pi^*}^F \equiv \frac{\alpha^F - \alpha^{F^*}}{\alpha^F}\frac{1-\alpha^F}{1-\alpha^{F^*}}\frac{(1+\sigma(1-n)\eta)\,\sigma(1-n)\eta}{1+\sigma\eta}$$

$$k_{\pi^*}^{F^*} \equiv \frac{\alpha^{F^*} - \alpha^F}{\alpha^F} \frac{\sigma^2(1-n)n\eta^2}{1+\sigma\eta}$$

$$k_C^F \equiv \left(1 + (1-n)\sigma\eta \left(1 - \frac{\zeta^{F^*}}{\zeta^F} \frac{1-\alpha^F}{1-\alpha^{F^*}}\right)\right) \zeta^F \left(\frac{\eta+\rho}{1+\sigma\eta}\right);$$

$$k_C^{F^*} \equiv \left(1 + n\sigma\eta \left(1 - \frac{\zeta^F}{\zeta^{F^*}} \frac{1-\alpha^{F^*}}{1-\alpha^F}\right)\right) \zeta^{F^*} \left(\frac{\eta+\rho}{1+\sigma\eta}\right);$$

$$k_T^F \equiv -\zeta^F \frac{(1+\sigma(1-n)\eta)\, n}{1+\sigma\eta} - \frac{k_C^F \varsigma n^2 \eta}{\eta+\rho}$$

$$k_T^{F^*} \equiv -\frac{k_C^{F^*} \varsigma n^2 \eta}{\eta+\rho} + \zeta^F \frac{1-\alpha^{F^*}}{1-\alpha^F} \frac{n\sigma n\eta}{1+\sigma\eta}$$

$$k_{T^*}^F \equiv -\zeta^F \frac{(1+\sigma(1-n)\eta)n}{1+\sigma\eta} + \frac{k_C^F n(1+\eta\varsigma(1-n))}{\eta+\rho}$$

$$k_{T^*}^{F^*} \equiv \zeta^F \frac{1-\alpha^{F^*}}{1-\alpha^F} \frac{n\sigma n\eta}{1+\sigma\eta} + \frac{k_C^{F^*} n(1+\eta\varsigma(1-n))}{\eta+\rho}$$

$$k_{RS}^F \equiv \zeta^F \frac{1+\sigma(1-n)\eta}{1+\sigma\eta} - \left(1 + \frac{(1-n)\,\eta}{\rho}\right) \frac{k_C^F}{(\eta+\rho)};$$

$$k_{RS}^{F^*} \equiv -\zeta^F \frac{1-\alpha^{F^*}}{1-\alpha^F} \frac{\sigma n\eta}{1+\sigma\eta} - \left(1 + \frac{(1-n)\,\eta}{\rho}\right) \frac{k_C^{F^*}}{(\eta+\rho)};$$

Now define the index as $T^R \equiv \frac{P_H^*}{P_H} \frac{P_F}{P_F^*}$ so that

$$\widehat{T}_t^R = \widehat{T}_t + \widehat{T}_t^* = \widehat{T}_{t-1}^R + \left(\pi_{F,t} - \pi_{F,t}^*\right) + \left(\pi_{H,t}^* - \pi_{H,t}\right)$$

The two terms in parenthesis measure the extent of pricing-to-market for the foreign and the Home firm respectively (the extent of pricing to market measure the difference of the inflation rate for goods sold in the foreign market and the inflation rate of goods sold on the domestic market). As we noted before a genuine pricing to market component emerges in this context if a firm, in setting its prices, faces different degrees of nominal price rigidities depending on the destination of the good. When $\alpha^H = \alpha^{H^*}$ then, for supply considerations, the domestic and foreign market are seen as identical for the Home firm and the extent of pricing to market measures the local currency pricing dimension. As long as prices are preset in the currency of the buyer, then an unanticipated movement in the exchange rate will drive a temporary wedge between the home and the foreign price. But in this case there is no pricing to market behavior because firms do not choose to set different prices. Similarly for the Foreign firm when $\alpha^F = \alpha^{F^*}$.

$$\pi_{H,t} - \pi_{H,t}^* = \zeta^H \widehat{RSh}_t + \beta \left(E_t \pi_{H,t+1}^* - E_t \pi_{H,t+1}\right)$$

$$\pi_{F,t} - \pi^*_{F,t} = -\zeta^F \widehat{RSf}_t + \beta \left(E_t \pi_{F,t+1} - E_t \pi^*_{F,t+1} \right)$$

where $RSh \equiv \frac{SP^*_H}{P_H}$, $RSf \equiv \frac{P_F}{SP^*_F}$ and $\zeta^j \equiv \frac{(1-\alpha^j\beta)(1-\alpha^j)}{\alpha^j}$ for $j = H, H^*, F, F^*$. When $\alpha^H \neq \alpha^{H^*}$, then the firms takes into account the asymmetric effects of shocks in their pricing decision. They optimally choose to charge a different price for the same good in different markets since prices have different degrees of nominal price stickiness according to the destination of the good. Indeed

$$\begin{aligned} \pi_{H,t} - \pi^*_{H,t} &= \zeta^{H^*} \widehat{RSh}_t + f^H \left(\widehat{C}, \widehat{X}, \widehat{T}, \widehat{T}^*, \widehat{RS}, \widehat{RSh}_t, \alpha^H - \alpha^{H^*} \right) \\ &\quad + \beta \left(E_t \pi_{H,t+1} - E_t \pi^*_{H,t+1} \right) \end{aligned}$$

$$\begin{aligned} \pi_{F,t} - \pi^*_{F,t} &= -\zeta^F \widehat{RSf}_t + f^F \left(\widehat{C}, \widehat{X}^*, \widehat{T}, \widehat{T}^*, \widehat{RS}, \widehat{RSf}_t, \alpha^F - \alpha^{F^*} \right) + \\ &\quad \beta \left(E_t \pi_{F,t+1} - E_t \pi^*_{F,t+1} \right) \end{aligned}$$

where f^H and f^F are two functions equal to zero when $\alpha^H = \alpha^{H^*}$ and $\alpha^F = \alpha^{F^*}$ and they measure the pricing to market dimension of the pricing decision. From this it follows that identical CES preferences do not necessarily rule out pricing to market. In particular, we have that

$$\begin{aligned} \pi_{H,t} - \pi^*_{H,t} &= \zeta^{H^*} \widehat{RSh}_t + \zeta^{H^*} \widehat{RSh}_t \left(\frac{1-\alpha^H}{1-\alpha^{H^*}} \frac{\sigma(1-n)\eta}{1+\sigma\eta} + \frac{(1+\eta\sigma n)}{1+\sigma\eta} - 1 \right) \\ &\quad \left(k^H_\pi - k^{H^*}_\pi \right) \pi_{H,t} + \left(k^H_{\pi^*} - k^{H^*}_{\pi^*} \right) \pi^*_{H,t} + \left(k^H_C - k^{H^*}_C \right) \left(\widehat{C}_t - \widetilde{C}_t \right) \\ &\quad \left(k^H_C - k^{H^*}_C \right) \frac{(1-n)(1+\varsigma\eta n)\left(\widehat{T}_t - \widetilde{T}_t \right)}{(\eta+\rho)} - \\ &\quad \left(k^H_C - k^{H^*}_C \right) \left[\frac{\varsigma (1-n)^2 \eta \left(\widehat{T}^*_t - \widetilde{T}^*_t \right) + \frac{n(1-n)}{\rho} \widehat{RS}_t}{(\eta+\rho)} \right] \\ &\quad + \beta \left(E_t \pi_{H,t+1} - E_t \pi^*_{H,t+1} \right) \end{aligned}$$

where $f^H \begin{pmatrix} \widehat{C}, \widehat{X}, \widehat{T}, \widehat{T}^*, \\ \widehat{RS}, \widehat{RSh}_t, \alpha^H - \alpha^{H^*} \end{pmatrix} \equiv \begin{bmatrix} \pi_{H,t} - \pi^*_{H,t} - \zeta^{H^*} \widehat{RSh}_t \\ -\beta \left(E_t \pi_{H,t+1} - E_t \pi^*_{H,t+1} \right) \end{bmatrix}$. Note that $f^H = 0$ when $\alpha^H = \alpha^{H^*}$.

Aggregate Demand

Log linearizing (11) in the main text we have an equilibrium relation between domestic and foreign nominal interest rates that links the interest rate differential to the differences of expected consumption growth and consumer based-inflation indexes among countries:

$$\widehat{i}_t - \widehat{i}_t^* = \rho \left[\left(E_t \widehat{C}_{t+1} - \widehat{C}_t \right) - \left(E_t \widehat{C}_{t+1}^* - \widehat{C}_t^* \right) \right] + E_t \left(\pi_{t+1} - \pi_{t+1}^* \right) \tag{A.10}$$

We can obtain a similar log-linearized equilibrium condition in a incomplete market setting in which each country issues its nominal bond denominated and traded in the country of origin. If we interpret (A.10) in this way, we can see that, in a stochastic setup with pricing to market, the linearized uncovered interest rate parity does not hold. Note the following considerations:

- When there are complete markets then (10) holds. Using (10) and (23), we can rewrite (A.10) in the familiar way

$$\widehat{i}_t - \widehat{i}_t^* = E_t \left(RS_{t+1} - RS_t \right) + E_t \left(\pi_{t+1} - \pi_{t+1}^* \right) = \tag{A.11}$$

$$= E_t \left(RS_{t+1} - RS_t \right) + E_t \left(RS_t - RS_{t+1} + \Delta S_{t+1} \right) = E_t \Delta S_{t+1}$$

  Note that the linearized stochastic uncovered interest parity holds independently from the fact that the law of one price holds;

- In an incomplete market setting, when the law of one price holds, with unit intratemporal elasticity of substitution between home and foreign goods then the stochastic uncovered interest parity holds. Indeed, unit intratemporal elasticity of substitution insures that expected consumption growth is the same across countries (see Corsetti and Pesenti, 2001).

Taylor Rules

Here we explicitly analyze the dynamics of the system under the Taylor rules. In the case in which the degrees of rigidity are equal across countries, we can restrict the analysis to the following equilibrium conditions[1]

$$\pi_t - \pi_t^* = \zeta \widehat{RS}_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*),$$

$$\widehat{RS}_t = \widehat{RS}_{t-1} + \pi_t^* - \pi_t + \Delta S_t,$$

$$E_t \Delta S_{t+1} = \phi(\pi_t - \pi_t^*) + \psi \left(\widehat{T}_t^W - \widetilde{T}_t \right) + \widehat{\varepsilon}_t^R,$$

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W - \left(\zeta^H + \frac{\zeta^H \eta (\varsigma - \sigma)}{1 + \sigma\eta} \right) \left(\widehat{T}_t^W - \widetilde{T}_t \right) + \beta \left(\widehat{T}_{t+1}^W - \widehat{T}_t^W \right)$$

from which it is possible to determine the equilibrium paths of the real exchange rate, the nominal exchange rate, the inflation rate differential and the relative price index $\widehat{T}_t^W$. Given the Markovian nature of the process $\widetilde{T}_t$ and $\widehat{\varepsilon}_t^R$, a rational expectations equilibrium assumes the following form

$$\widehat{RS}_t = a_1 \widehat{RS}_{t-1} + b_1 \widehat{T}_{t-1}^W + c_1 \widetilde{T}_t + d_1 \widehat{\varepsilon}_t^R,$$

$$\pi_t - \pi_t^* = a_2 \widehat{RS}_{t-1} + b_2 \widehat{T}_{t-1}^W + c_2 \widetilde{T}_t + d_2 \widehat{\varepsilon}_t^R,$$

$$\Delta S_t = a_3 \widehat{RS}_{t-1} + b_3 \widehat{T}_{t-1}^W + c_3 \widetilde{T}_t + d_3 \widehat{\varepsilon}_t^R$$

$$\widehat{T}_t^W = a_4 \widehat{RS}_{t-1} + b_4 \widehat{T}_{t-1}^W + c_4 \widetilde{T}_t + d_4 \widehat{\varepsilon}_t^R.$$

From (A.14), we have that the relative price index $\widehat{T}_t^W$ has always a unique and stable rational expectation solution of the form

$$\widehat{T}_t^W = \lambda_1 \widehat{T}_{t-1}^W + \lambda_1 \left(\zeta + \frac{\zeta\eta(\varsigma - \sigma)}{1 + \sigma\eta} \right) E_t \sum_{j=0}^{+\infty} (\beta\lambda_1)^j \widetilde{T}_{t+j},$$

where λ_1 is the stable eigenvalue of (A.14), with $0 < \lambda_1 < 1$. Furthermore if the process followed by $\widetilde{T}_t$ is Markovian of the form

$$\widetilde{T}_t = \rho_2 \widetilde{T}_{t-1} + \nu_{2,t}$$

with $0 < \rho_1 < 1$, then

$$\widehat{T}_t^W = \lambda_1 \widehat{T}_{t-1}^W + \upsilon \widetilde{T}_t,$$

[1] We have defined $\widehat{\varepsilon}_t^R \equiv \widehat{\varepsilon}_t^H - \widehat{\varepsilon}_t^F$.

where we have defined

$$\upsilon \equiv \frac{\lambda_1\left(\zeta + \frac{\zeta\eta(\varsigma-\sigma)}{1+\sigma\eta}\right)}{1-\beta\lambda_1\rho_2}, \qquad\qquad 0 < \nu < 1.$$

Moreover, in a unique and stable rational expectations equilibrium, it is always the case that $a_1 = a_2 = a_4 = 0$, $a_3 = -1$ and

$$\begin{aligned}
b_1 &= \frac{b_4\psi(\beta b_4 - 1)}{1-\beta b_4 - \zeta b_4 + \beta b_4^2 - b_4 + \zeta\phi}, \\
b_2 &= \frac{b_4\psi\zeta}{1-\beta b_4 - \zeta b_4 + \beta b_4^2 - b_4 + \zeta\phi}, \\
b_3 &= \frac{b_4\psi(\beta b_4 - 1 - \zeta)}{1-\beta b_4 - \zeta b_4 + \beta b_4^2 - b_4 + \zeta\phi}, \\
b_4 &= \lambda_1
\end{aligned}$$

The coefficients on the shocks are such that $d_4 = 0$, $c_4 = \upsilon$ and

$$\begin{aligned}
c_1 &= -\frac{(\phi-\rho_2)\beta b_2 c_4 + b_3 c_4\left(\beta\rho_2 - 1\right) + \psi(1-\beta\rho_2)\left(c_4 - 1\right)}{(\phi-\rho_2)\zeta + (1-\rho_2)(1-\beta\rho_2)}, \\
c_2 &= \frac{(1-\rho_2)\beta b_2 c_4 + b_3 c_4\zeta + \psi\zeta\left(1 - c_4\right)}{(\phi-\rho_2)\zeta + (1-\rho_2)(1-\beta\rho_2)}, \\
c_3 &= \frac{(1-\phi)\beta b_2 c_4 + \left(1-\beta\rho_2+\zeta\right)\left(b_3 c_4 + \psi\left(1-c_4\right)\right)}{(\phi-\rho_2)\zeta + (1-\rho_2)(1-\beta\rho_2)}, \\
d_1 &= \frac{\rho_R\beta - 1}{(\phi-\rho_R)\zeta + (1-\rho_R)(1-\beta\rho_R)}, \\
d_2 &= \frac{-\zeta}{(\phi-\rho_R)\zeta + (1-\rho_R)(1-\beta\rho_R)}, \\
d_3 &= -\frac{\zeta + 1 - \rho_R\beta}{(\phi-\rho_R)\zeta + (1-\rho_R)(1-\beta\rho_R)}
\end{aligned}$$

Note that both countries follow an inflation-targeting instrument rules, i.e. $\psi = 0$, we will have that the real exchange rate is completely isolated from productivity shocks. In the case in which there are only monetary shocks (i.e.var($\widetilde{T}$) $= 0$) or there is no weight on output stabilization (i.e. $\psi = 0$) then we have the following expression for the real exchange rate variance

$$\text{var}(\widehat{RS}_t) = d_1^2\,\text{var}(\widehat{\varepsilon}_t^R)$$

From the expression for d_1, we can see that an increase in the inflation stabilization parameter reduces this variance. Indeed for $\phi \to \infty$, var($\widehat{RS}_t$)$= 0$.

Managed Exchange Rate Regimes

In the case of managed exchange rate, our equilibrium conditions change into

$$\pi_t - \pi_t^* = \zeta \widehat{RS}_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*),$$

$$\widehat{RS}_t = \widehat{RS}_{t-1} + \pi_t^* - \pi_t + \Delta S_t,$$

$$\pi_t^{index} = \left(\zeta^H + \frac{\zeta^H \eta\left(\varsigma - \sigma\right)}{1+\sigma\eta}\right)\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \beta E_t \pi_{t+1}^{index}$$

$$E_t \Delta S_{t+1} = \phi(\pi_t - \pi_t) + \psi\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \mu \Delta S_t + \widehat{\varepsilon}_t^R,$$

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W + \pi_t^{index}$$

which can be compacted in a system of the form ($\pi_t^{index} \equiv n\pi_{F,t} + (1-n)\,\pi_{F,t}^* - (1-n)\,\pi_{H,t}^* - n\pi_{H,t}$)

$$E_t \begin{bmatrix} y_{t+1}^m \\ z_t^m \end{bmatrix} = \begin{bmatrix} M_1^m & M_2^m \\ M_3^m & M_4^m \end{bmatrix} \begin{bmatrix} y_t^m \\ z_{t-1}^m \end{bmatrix} + \begin{bmatrix} m_{1T}^m \\ \mathbf{0} \end{bmatrix} \widetilde{T}_t + \begin{bmatrix} m_{2\varepsilon}^m \\ \mathbf{0} \end{bmatrix} \widehat{\varepsilon}_t^R, \tag{A.12}$$

where $y_t^{m\prime} = [\pi_t^R \quad \Delta S_t, \pi_t^{index}]$, $z_{t-1}^{m\prime} = [\widehat{RS}_{t-1},\ \widehat{T}_{t-1}^W]$, M_1^m is a 3×3 matrix, M_2^m, $M_3^{m'}$ are 2×2 matrices, M_4^m is a 2×2 matrix, m_j^m are 3×1 vectors and 0 is 2×1 vector. Under the condition for determinacy, there are three eigenvalues outside the unit circle. Let denote these eigenvalues as ω_1, ω_2 ω_3 and collected in the diagonal matrix Υ. Let V a 3×5 matrix of the left eigenvectors associated with the unstable eigenvalues, with the property that $VM = \Upsilon V$. Furthermore we decompose V in two matrices with $V = [V_1\ V_2]$ where V_1 is 3×3 and V_2 is 3×2. Now, if $\widetilde{T}_t$ and $\widehat{\varepsilon}_t^R$ follow a Markovian process we have that the solution for y_t is of the form

$$y_t^m = \Psi_1^m z_{t-1}^m + \Psi_{2T}^m \widetilde{T}_t + \Psi_{2\varepsilon}^m \widehat{\varepsilon}_t^R,$$

where $\Psi_1^m \equiv -V_1^{-1}V_2$ and $\Psi_{2T}^m \equiv V_1^{-1}(I\rho_1 - \Upsilon)^{-1}Vm_T^m$ and $\Psi_{2\varepsilon}^m \equiv V_1^{-1}(I\rho_2 - \Upsilon)^{-1}Vm_\varepsilon^m$. Furthermore from the system (A.12), we have

$$\begin{aligned} z_t^m &= M_3^m y_t^m + M_4^m z_{t-1}^m, \\ &= (M_3^m \Psi_1^m + M_4^m) z_{t-1}^m + (M_3^m \Psi_{2T}^m \widetilde{T}_t + M_3^m \Psi_{2\varepsilon}^m \widehat{\varepsilon}_t^R), \\ &= Z_1^m z_{t-1}^m + Z_{2T}^m \widetilde{T}_t + Z_{2\varepsilon}^m \widehat{\varepsilon}_t^R, \end{aligned}$$

where Z_1^m and Z_2^m have been appropriately defined. Reminding that $z' = [\widehat{RS}_t \;\; \widehat{T}_t^W]$, it can be possible to rewrite the solution

$$\begin{aligned} A^m(L)\widehat{RS}_t &= R_T^m(L)\widetilde{T}_t + R_\varepsilon^m(L)\widehat{\varepsilon}_t^R, \\ A^m(L)\widehat{T}_t^W &= U_T^m(L)\widetilde{T}_t + U_\varepsilon^m(L)\widehat{\varepsilon}_t^R, \end{aligned}$$

where $A(L)$ is a second-order polynomial with $A^m(L) = \det[I - LZ_1^m]$ and $R_T^m(L)$, $R_\varepsilon^m(L)$, $U_T^m(L)$, $U_\varepsilon^m(L)$ are first-order polynomials. The real exchange rate is then a stationary variable and behaves according to an ARMA process in which the AR component is of the second order (for μ,$\psi \neq 0$).

In the case of managed real exchange rate regime, our equilibrium conditions change into

$$\pi_t - \pi_t^* = \zeta \widehat{RS}_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*),$$

$$\widehat{RS}_t = \widehat{RS}_{t-1} + \pi_t^* - \pi_t + \Delta S_t,$$

$$\pi_t^{index} = \left(\zeta^H + \frac{\zeta^H \eta\,(\varsigma - \sigma)}{1 + \sigma\eta}\right)\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \beta E_t \pi_{t+1}^{index}$$

$$E_t \Delta S_{t+1} = \phi(\pi_t - \pi_t) + \psi\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \vartheta \widehat{RS}_t + \widehat{\varepsilon}_t^R,$$

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W + \pi_t^{index}$$

which can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^r \\ z_t^r \end{bmatrix} = \begin{bmatrix} M_1^r & M_2^r \\ M_3^r & M_4^r \end{bmatrix} \begin{bmatrix} y_t^r \\ z_{t-1}^r \end{bmatrix} + \begin{bmatrix} m_{1T}^r \\ \mathbf{0} \end{bmatrix} \widetilde{T}_t + \begin{bmatrix} m_{2\varepsilon}^r \\ \mathbf{0} \end{bmatrix} \widehat{\varepsilon}_t^R,$$

where $y_t^{r\prime} = [\pi_t^R \;\; \Delta S_t, \pi_t^{index}]$, $z_{t-1}^{r\prime} = [\widehat{RS}_{t-1}, \; \widehat{T}_{t-1}^W]$, M_1^r is a 3×3 matrix, M_2^r, $M_3^{r^{\prime}}$ are 2×2 matrices, M_4^r is a 2×2 matrix, m_j^r are 3×1 vectors and 0 is 2×1 vector. Doing all the steps as before we obtain that a solution for $\widehat{RS}_t$ is given

$$\begin{aligned} A^r(L)\widehat{RS}_t &= R_T^r(L)\widetilde{T}_t + R_\varepsilon^r(L)\widehat{\varepsilon}_t^R, \\ A^r(L)\widehat{T}_t^W &= U_T^r(L)\widetilde{T}_t + U_\varepsilon^r(L)\widehat{\varepsilon}_t^R, \end{aligned}$$

where $A^r(L)$, $U_T^r(L)$ are first-order polynomial (the second column Z_1^r is a zero vector) with $A^r(L) = \det[I - LZ_1^r]$ and $R_T^r(L)$, $R_\varepsilon^r(L), U_T^r(L), U_\varepsilon^r(L)$ are first-order polynomials. As before, the real exchange rate is then a stationary variable and behaves according to an ARMA process in which the AR component is of first order. Note that this AR component would be zero in the case in which there is no weight on output gap stabilization.

Interest Rate Smoothing

Under the class of interest-rate smoothing rules, the relevant equilibrium conditions for the determination of the terms of trade and the exchange rate are

$$\pi_t - \pi_t^* = \zeta \widehat{RS}_t + \beta E_t(\pi_{t+1} - \pi_{t+1}^*),$$

$$\widehat{RS}_t = \widehat{RS}_{t-1} + \pi_t^* - \pi_t + \Delta S_t,$$

$$\pi_t^{index} = \left(\zeta^H + \frac{\zeta^H \eta (\varsigma - \sigma)}{1 + \sigma\eta}\right)\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \beta E_t \pi_{t+1}^{index}$$

$$E_t \Delta S_{t+1} = \gamma \left(\widehat{i}_{t-1} - \widehat{i}_{t-1}^*\right) + \phi(\pi_t - \pi_t) + \psi \left(\widehat{T}_t^W - \widetilde{T}_t\right) + \widehat{\varepsilon}_t^R,$$

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W + \pi_t^{index}$$

$$\widehat{i}_t - \widehat{i}_t^* = \gamma \left(\widehat{i}_{t-1} - \widehat{i}_{t-1}^*\right) + \phi(\pi_t - \pi_t) + \psi \left(\widehat{T}_t^W - \widetilde{T}_t\right) + \widehat{\varepsilon}_t^R$$

which can be compacted in a system of the form

$$E_t \begin{bmatrix} y_{t+1}^s \\ z_t^s \end{bmatrix} = \begin{bmatrix} M_1^s & M_2^s \\ M_3^s & M_4^s \end{bmatrix} \begin{bmatrix} y_t^s \\ z_{t-1}^s \end{bmatrix} + \begin{bmatrix} m_{1T}^s \\ \mathbf{0} \end{bmatrix} \widetilde{T}_t + \begin{bmatrix} m_{2\varepsilon}^s \\ m_{3\varepsilon}^s \end{bmatrix} \widehat{\varepsilon}_t^R,$$

where $y_t^{r\prime} = [\pi_t^R \;\; \Delta S_t, \pi_t^{index}]$, $z_{t-1}^{r\prime} = [\widehat{RS}_{t-1}, \widehat{T}_{t-1}^W, \widehat{i}_{t-1} - \widehat{i}_{t-1}^*]$, M_j^s is a 3×3 matrix, while m_j^s is a 3×1 vector. Following the same steps as in the case of the managed-exchange rate regimes, we obtain

$$\begin{aligned} y_t^s &= \Psi_1^s z_{t-1} + \Psi_{2T}^s \widetilde{T}_t + \Psi_{2\varepsilon}^s \widehat{\varepsilon}_t^R, \\ z_t^s &= Z_1^s z_{t-1}^s + Z_{2T}^s \widetilde{T}_t + Z_{2\varepsilon}^s \widehat{\varepsilon}_t^R, \end{aligned}$$

where the matrices have the same interpretations as in the previous case, clearly starting from different matrices M^s.

The second column of Z_1^s is of zeros. It follows that the only source of inertia in the system is coming only from the interest rate smoothing component. We can rewrite the solution as

$$\begin{aligned} A^s(L)\widehat{RS}_t &= R_T^s(L)\widetilde{T}_t + R_\varepsilon^s(L)\widehat{\varepsilon}_t^R, \\ A^s(L)\widehat{T}_t^W &= U_T^s(L)\widetilde{T}_t + U_\varepsilon^s(L)\widehat{\varepsilon}_t^R, \\ A^s(L)\left(\widehat{i}_t - \widehat{i}_t^*\right) &= F_T^s(L)\widetilde{T}_t + F_\varepsilon^s(L)\widehat{\varepsilon}_t^R, \end{aligned}$$

where $A^s(L)$ is a first order polynomial, $R_T^s(L)$, $R_\varepsilon^s(L), U_T^s(L), U_\varepsilon^s(L)$, $F_T^s(L)$ and $F_\varepsilon^s(L)$ are second-order polynomials. The inertia component is only of the first order.

We now report the proofs of the propositions in section 7.

PROPOSITION 1:

Proof. To capture the dynamic of the relative prices we define the following two variables:

$$\begin{aligned} \widehat{T}_t^R &\equiv \widehat{T}_t + \widehat{T}_t^* \\ \widehat{T}_t^W &\equiv n\widehat{T}_t - (1-n)\,\widehat{T}_t^* \end{aligned}$$

From (21) and (22) and substituting (17), (18), (19) and (20) in the main text, we obtain that

$$\widehat{T}_t^R = \widehat{T}_{t-1}^R + \pi_{F,t} - \pi_{F,t}^* + \pi_{H,t}^* - \pi_{H,t}$$

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W + n\pi_{F,t} + (1-n)\,\pi_{F,t}^* - (1-n)\,\pi_{H,t}^* - n\pi_{H,t}$$

Substituting the supply curve expressions

$$\widehat{T}_t^R = \widehat{T}_{t-1}^R - \zeta^H \widehat{T}_t^R + \beta\left(\widehat{T}_{t+1}^R - \widehat{T}_t^R\right) \tag{A.13}$$

and

$$\widehat{T}_t^W = \widehat{T}_{t-1}^W - \left(\zeta^H + \frac{\zeta^H(\varsigma - \sigma)}{1+\sigma\eta}\right)\left(\widehat{T}_t^W - \widetilde{T}_t\right) + \beta\left(\widehat{T}_{t+1}^W - \widehat{T}_t^W\right) \tag{A.14}$$

It is easy to see that from inspection of equation (A.13) and (A.14) that the dynamics of relative prices do not depend on monetary policy and monetary shocks. Note that only relative productivity shocks affect their dynamics. Note that the orthogonality property of the relative prices is robust to the specification of the monetary policy rule. Two are its main determinants: the assumption of full pricing to market and the structure of nominal price rigidities across countries and markets. ■

PROPOSITION 2

Proof. Note that as long as $\alpha^H = \alpha^F = \alpha^{H^*} = \alpha^{F^*}$ we can consider rules with $\psi = 0$ since

$$y_t^H - y_t^F = \varsigma\left(\widehat{T}_t^W - \widetilde{T}_t\right)$$

and because of proposition 1, relative prices are orthogonal to monetary policy and monetary shocks. The system of difference equation that determines the dynamics of the real exchange rate and of the nominal exchange rate is given by

$$E_t\widehat{RS}_{t+1} - \frac{1+\beta+\zeta}{\beta}\widehat{RS}_t + \frac{1}{\beta}\widehat{RS}_{t-1} = E_t\Delta S_{t+1} - \frac{1}{\beta}\Delta S_t$$

We can rewrite the system in a matrix form after having defined $E_t\widehat{Q}_{t+1} = \widehat{RS}_t$.

$$E_t z_{t+1} = B z_t + S\varepsilon_t$$

where $z_t \equiv [RS_t, Q_t, \Delta S_t]$, $\varepsilon_t = \left[\widehat{\varepsilon}_t^H, \widehat{\varepsilon}_t^F\right]$ and

$$B = \begin{bmatrix} \frac{1+\beta+\zeta}{\beta} - \phi & \phi - \frac{1}{\beta} & \phi - \frac{1}{\beta} \\ 1 & 0 & 0 \\ -\phi & \phi & \phi \end{bmatrix}$$

$$S = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

In order for the system to be determined we need two eigenvalues greater than 1 and one less than 1 in absolute value (associated with the state variable Q_{t+1}). The characteristic polynomial associated with the matrix B is given by

$$-\lambda^3 + \lambda^2\frac{1+\beta+\zeta}{\beta} - \lambda\left(\frac{1+\phi\zeta}{\beta}\right) = 0$$

from which it is easy to see that the stable eigenvalue is $\lambda_1 = 0$. The other two eigenvalues are determined by solving the second order polynomial. Note that

$$\lambda^2 - \lambda\frac{1+\beta+\zeta}{\beta} + \left(\frac{1+\phi\zeta}{\beta}\right) = 0$$

When we evaluate this polynomial at 1 and -1 we obtain that

$$P(1) > 0 \;\; P(-1) > 0 \text{ when } \phi > 1 \Longrightarrow \lambda_2, \lambda_3 > 1$$

$$P(1) > 0 \;\; P(-1) < 0 \text{ when } \phi < 1 \Longrightarrow 0 < \lambda_2 < 1, \lambda_3 > 1$$

so that the equilibrium is determined when $\phi > 1$. Let the eigenvector matrix be

$$E = \begin{bmatrix} 0 & e_{1,2} & e_{1,3} \\ -1 & e_{2,2} & e_{2,3} \\ 1 & 1 & 1 \end{bmatrix}$$

For the given eigenvector matrix E, let's define $V \equiv E^{-1}S$ where $v_{i,j}$ are the elements of V. Then we obtain

$$\widehat{RS}_{t+1} = -\sum_{s=t+1}^{\infty}\begin{bmatrix} \left(\frac{1}{\lambda_2}\right)^{t-s} e_{2,1}E_t\left(v_{2,1}\widehat{\varepsilon}_s^H + v_{2,2}\widehat{\varepsilon}_{t+s}^F\right) + \\ \left(\frac{1}{\lambda_3}\right)^{t-s} e_{3,1}E_t\left(v_{3,1}\widehat{\varepsilon}_s^H + v_{3,2}\widehat{\varepsilon}_{t+s}^F\right) \end{bmatrix}$$

$$\widehat{\Delta S}_{t+1} = v_{1,1}\widehat{\varepsilon}_{t+s}^H - v_{1,2}\widehat{\varepsilon}_{t+s}^F - \sum_{s=t+1}^{\infty}\begin{bmatrix} \left(\frac{1}{\lambda_2}\right)^{t-s} e_{2,1}E_t\left(v_{2,1}\widehat{\varepsilon}_s^H + v_{2,2}\widehat{\varepsilon}_{t+s}^F\right) + \\ \left(\frac{1}{\lambda_3}\right)^{t-s} e_{3,1}E_t\left(v_{3,1}\widehat{\varepsilon}_s^H + v_{3,2}\widehat{\varepsilon}_{t+s}^F\right) \end{bmatrix}$$

from which it easy to see that there is no persistence in the real exchange rate unless the monetary shocks are serially correlated. ■