Implementing International Monetary Cooperation through Inflation Targeting

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Implementing International Cooperation
Abstract: This paper presents a two-country dynamic general equilibrium model with imperfect competition and nominal price rigidities in which productivity shocks coexist with mark-up shocks. After analysing the features of the optimal cooperative solution, we show that this allocation can be implemented in a strategic context through inflation-targeting regimes. Under these regimes each monetary authority minimizes a quadratic loss function that targets only domestic targets, namely GDP inflation and the output gap.

Keywords: international monetary cooperation, inflation targeting.
In this paper we examine the international monetary policy design problem in a simple two-country dynamic general equilibrium model with imperfect competition, nominal price rigidities and producer currency pricing. As discussed in a related work (Benigno and Benigno, 2005), the structure of the model allows for gains from international monetary policy cooperation. The focus in this work is to study if it is possible to design institutions that can implement the cooperative allocation. In Benigno and Benigno (2005), we have shown how to design targeting rules – i.e. linear combinations of target variables. Here we study how flexible inflation targeting policies (see Svensson, 2002) could be adopted in a framework where policymakers still interact strategically.

There are two classes of exogenous disturbances in the model, productivity and mark-up shocks. Following mark-up shocks, we describe two possible transmission mechanisms whose operation depends on the values of key structural parameters; a “positive-correlated” scenario in which mark-up shocks generate co-movement of inflation rates across countries and a “negative-correlated” scenario in which inflation rates move in opposite directions.

In analyzing the implementation of the cooperative allocation, our main finding is that is sufficient for this purpose that each policymaker commits to minimize a quadratic loss function which appropriately weighs only domestic targets – the output gap and the domestic producer inflation rate. Our model supports the adoption of flexible inflation-targeting regimes (see Svensson, 2002) even in an international context.

Furthermore, the design of the relative weights in the objective functions of the monetary policymakers depends on the scenario that we consider. Under a “positive-correlated scenario” it is optimal to design institutions with a relatively higher weight on the domestic output gap in order to correct for the ‘contractionary bias’ that would arise in a strategic framework. On the other hand, in the “negative correlated scenario” the corresponding ‘expansionary bias’ should be corrected by assigning a higher relative weight to domestic producer inflation in both countries.

The paper is structured as follows. Section 1 presents the model and Section 2 the log-linear approximation of the structural equilibrium conditions. Section 3 studies the cooperative solution. Section 4 shows how to implement the cooperative solution in a decentralized context. Section 5 concludes.
1 The Model

In this section we present our two-country dynamic general equilibrium model with imperfect competition and nominal price rigidities along the lines of Benigno and Benigno (2002, 2005), Clarida Gali and Gertler (2002), and Svensson (2000).

Households’ preferences

We consider a two-country economy, Home (H) and Foreign (F). The population on the segment $[0, n)$ belongs to the Home country while the one on the segment $[n, 1]$ belongs to the Foreign country. Each individual maximizes the following utility function:

$$U^j_t = E_t \left\{ \sum_{s=t}^{\infty} \beta^{s-t} \left[ U(C^j_s) - V(y_s(j), \xi_{Y,s}) \right] \right\},$$

where the index $j$ denotes a variable that is specific to household $j$; $E_t$ denotes the expectation conditional on the information set at date $t$, while $\beta$ is the intertemporal discount factor, with $0 < \beta < 1$. Individuals obtain utility from consumption while they receive disutility from producing a single differentiated good which is subject to a country-specific shock, $\xi_Y$, that we interpret as a productivity shock. Country’s $F$ variables are denoted with a star index. $U$ is an increasing concave function of the index $C^j$ defined as

$$C^j = n \frac{\bar{\beta}^\theta}{\bar{\sigma} + 1} (C^j_H)^{\frac{\bar{\beta} - 1}{\bar{\sigma} + 1}} + (1 - n) \frac{\bar{\beta}^\theta}{\bar{\sigma} + 1} (C^j_F)^{\frac{\bar{\beta} - 1}{\bar{\sigma} + 1}},$$

where $C^j_H$ and $C^j_F$ are consumption sub-indexes of the continuum of differentiated goods produced respectively in country $H$ and $F$

$$C^j_H = \left( \frac{1}{n} \right)^{\frac{1}{\bar{\sigma}}} \int_0^n c^j(h)^{\frac{\bar{\beta} - 1}{\bar{\sigma}} - 1} dh, \quad C^j_F = \left( \frac{1}{1 - n} \right)^{\frac{1}{\bar{\sigma}}} \int_0^1 c^j(f)^{\frac{\bar{\beta} - 1}{\bar{\sigma}} - 1} df,$$

where $\bar{\sigma} > 1$ is the elasticity of substitution across goods produced within a country and $\bar{\theta}$ is the elasticity of substitution between the bundles $C^j_H$ and $C^j_F$; $n$ is a parameter such that $0 \leq n \leq 1$.

$P$ denotes the consumption-based price index associated with $C$, defined as

$$P \equiv \left[ n(P_H)^{1-\theta} + (1 - n)(P_F)^{1-\theta} \right]^{\frac{1}{1-\sigma}},$$

with

$$P_H = \left[ \left( \frac{1}{n} \right) \int_0^n p(h)^{1-\sigma} dh \right]^{\frac{1}{1-\sigma}}, \quad P_F = \left[ \left( \frac{1}{1 - n} \right) \int_0^1 p(f)^{1-\sigma} df \right]^{\frac{1}{1-\sigma}},$$
where \( p(h) \) and \( p(f) \) are prices in units of domestic currency of the Home-produced and Foreign-produced goods, respectively.

All consumption goods are traded. Prices are set in the currency of the producer and the law of one price holds: \( p(h) = S \cdot p^*(h) \) and \( p(f) = S \cdot p^*(f) \), where \( S \) is the nominal exchange rate (the price of foreign currency in terms of domestic currency). Given these assumptions and the structure of preferences, purchasing power parity holds, i.e. \( P = S \cdot P^* \). The terms of trade are defined as the relative price of foreign goods in terms of home goods expressed in the domestic currency, i.e. \( T \equiv \frac{P_F}{P_H} \).

Finally \( V \) is an increasing convex function of household \( j \)'s supply of the differentiated good \( y(j) \).

Total demands of the home and foreign differentiated goods are then given by

\[
y^d(h) = \left( \frac{p(h)}{P_H} \right)^{-\sigma} \left( \frac{P_H}{P} \right)^{-\theta} C^W, \quad y^d(f) = \left( \frac{p(f)}{P_F} \right)^{-\sigma} \left( \frac{P_F}{P} \right)^{-\theta} C^W, \tag{1}
\]

where \( C^W \) is the aggregate consumption in the whole economy. Applying the appropriate aggregate operators we obtain the domestic and foreign aggregate demand as

\[
Y^H = \left( \frac{P_H}{P} \right)^{-\theta} C^W, \quad Y^F = \left( \frac{P_F}{P} \right)^{-\theta} C^W. \tag{2}
\]

From (2), it follows that changes in the terms of trade create dispersion of output across countries.

As in Chari, Kehoe and McGrattan (1998), we assume that markets are complete both at a domestic and international level so that households have access to a complete set of state-contingent one-period nominal securities denominated in the currency of the Home country. Along with the assumption that initial wealth is identical among all the household, it follows that marginal utilities of income are equalized across and within countries at all times and across all states of nature:

\[
U_C(C_t) = U_C(C^*_t) \tag{3}
\]

Equation (3) is derived from the set of optimality conditions that characterize the optimal allocation of wealth among the state-contingent securities.

Our model is in the class of the cashless-limiting economies discussed in Woodford (2003).

**Price-Setting Behavior**

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\(^1\)With producer currency pricing, and the law of one price holding, this is equivalent if expressed in foreign currency.
Each producer of a single differentiated good acts in a monopolistic-competitive market. The demand of the differentiated good, \( p(h) \), depends on the pricing decision \( p \), for the generic home-produced good, and \( p^*(f) \), for the foreign-produced good, respectively.\(^2\) On the other hand, producers take as given \( P, P_H, P_F \) and \( C \). The price-setting behavior is modelled following the Calvo-Yun partial adjustment rule, under which each producer has the opportunity to change its price with a fixed probability \( 1 - \alpha \) at each point in time.\(^3\) We allow this probability to be different across countries.

A home producer, that sets a new price in period \( t \), maximizes the expected discounted value of her net profits\(^4\)

\[
E_t \sum_{k=0}^{\infty} (\alpha \beta)^k \left[ \frac{U_C(C_{t+k})}{P_{t+k}} (1 - \tau_{t+k}) \bar{p}_t(h) \bar{y}_{t,t+k}(h) - V(\bar{y}_{t,t+k}(h), \xi_{Y,t+k}) \right],
\]

where \( \tau_t \) is a time-varying tax on firms’ revenues. With \( \bar{p}_t(h) \) we have denoted the price of the good \( h \) chosen at date \( t \) and with \( \bar{y}_{t,t+k}(h) \) the total demand of good \( h \) at time \( t + k \) conditional on the fact that the price \( \bar{p}_t(h) \) has not changed,

\[
\bar{y}_{t,t+k}(h) = \left( \frac{\bar{p}_t(h)}{P_{H,t+k}} \right)^{-\sigma} \left[ \left( \frac{P_{H,t+k}}{P_{t+k}} \right)^{-\theta} C_{t+k} \right].
\]

The optimal choice of \( \bar{p}_t(h) \) is

\[
\bar{p}_t(h) = \frac{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k V_y(\bar{y}_{t,t+k}(h), \xi_{Y,t+k}) \bar{y}_{t,t+k}(h)}{E_t \sum_{k=0}^{\infty} (\alpha \beta)^k (1 - \mu_{t+k}) \frac{U_C(C_{t+k})}{P_{t+k}} \bar{y}_{t,t+k}(h)},
\]

where

\[
(1 - \mu_t) \equiv \frac{(1 - \tau_t)(\sigma - 1)}{\sigma}.
\]

The Calvo-style price-setting mechanism implies the following state equation for \( P_{H,t} \) :

\[
(P_{H,t})^{1-\sigma} = \alpha (P_{H,t-1})^{1-\sigma} + (1 - \alpha) \bar{p}_t(h)^{1-\sigma}.
\]

\(^2\)There is indeed producer currency pricing.

\(^3\)This probability is the same for each producer and is independent of the amount of time elapsed since her last change of price.

\(^4\)It is important to note that all the producers that belong to the same country and that can modify their price at a certain time will face the same discounted value of the streams of current and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.
2 Log-Linear Approximation to the Equilibrium Conditions

We log-linearize the model around the deterministic steady state in which monopolistic distortions are eliminated by a taxation subsidy, i.e. $\bar{\mu} = 1$. By log-linearizing (5), (6) and their foreign correspondents, we obtain the two aggregate supply (AS) equations for the Home and Foreign country:

$$\pi_{H,t} = \kappa[(\hat{Y}_{H,t} - \bar{Y}_{H,t}) + (1-n)\psi(\hat{T}_t - \bar{T}_t) + u_t] + \beta E_t \pi_{H,t+1},$$  \hspace{1cm} (7)

$$\pi_{F,t}^* = \kappa^*[\hat{Y}_{F,t}^* - \bar{Y}_{F,t}^*] - n\psi(\hat{T}_t - \bar{T}_t) + u_t^*] + \beta E_t \pi_{F,t+1}^*,$$ \hspace{1cm} (8)

where $\kappa^i \equiv (1 - \alpha^i)(1 - \alpha^i\beta)(\rho + \eta)/[\alpha^i(1 + \alpha^i\eta)]$ and $\psi \equiv (1 - \rho\theta)/(\rho + \eta)$. Variables with the hat denotes log-deviations of the respective variable from the steady state value ($\bar{T}_t$, $\bar{Y}_{H,t}$ and $\bar{Y}_{F,t}$ are the deviation of the terms of trade, domestic and foreign output from their steady state.\(^5\)) while we have defined $\pi_{H,t} \equiv \ln \Pi_{H,t}$, $\pi_{F,t}^* \equiv \ln \Pi_{F,t}^*$ and $T_t = P_{F,t}/P_{H,t}$. The variables $\bar{C}_t$, $\bar{T}_t$, $\bar{Y}_{H,t}$, $\bar{Y}_{F,t}$ are the following functions of the shocks\(^6\)

$$\bar{C}_t \equiv \frac{\eta}{(\eta + \rho)} \hat{a}_{W,t}, \hspace{1cm} \bar{T}_t = \frac{\eta}{(1 + \theta\eta)} \hat{a}_{R,t},$$ \hspace{1cm} (9)

$$\bar{Y}_{H,t} \equiv \bar{C}_t + (1-n)\theta \bar{T}_t, \hspace{1cm} \bar{Y}_{F,t} \equiv \bar{C}_t - n\theta \bar{T}_t,$$ \hspace{1cm} (10)

with $\hat{a}_{R,t} \equiv \hat{a}_t - \hat{a}_t^*$ and $\hat{a}_{W,t} \equiv n\hat{a}_t + (1-n)\hat{a}_t^*$. In what follows we will refer to $\bar{T}_t$ as a terms of trade shock (a combination of the home and foreign productivity shocks). More importantly, $u_t$ and $u_t^*$ represent inefficient supply shocks that capture the deviations of the flexible-price allocation from the efficient allocation (for the case in which $\bar{\mu} = 1$, the efficient allocation for the respective variables is characterized by equations (9) to (10)). In our context, these deviations are produced by variations in the distortionary taxes that apply to the firms’ revenues. In particular, in equations (7) and (8) $u_t$ and $u_t^*$ are just proportional to the mark-up shocks

$$u_t = \frac{\hat{\mu}_t}{(\eta + \rho)}, \hspace{1cm} u_t^* = \frac{\hat{\mu}_t^*}{(\eta + \rho)}.$$ \hspace{1cm} (11)

\(^5\)Those deviations arise in the stochastic equilibrium in which prices are subject to the partial adjustment rule a la Calvo.

\(^6\)A variable with a sub-index $W$ denotes a weighted average of the Home and Foreign variables with weights $n$ and $1-n$, respectively. A variable with an sub-index $R$ denotes the difference between the Home and Foreign variables. We have defined $\rho \equiv -U_{CC}/U_C$ and $\eta \equiv V_{yy}/V_y$, while $a_t$ is defined as $V_y\bar{C}a_t = -V_y\xi_t, \xi_{Y_t}$. $a_t^*$ is defined appropriately. The steady-state level of consumption solve the equation $U_C(\bar{C}) = V_y(\bar{C},0)$. 

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In the AS equations (7) and (8) GDP inflation rates depend on the present discounted value of the aggregate real marginal costs. In general, in open economies, real marginal costs are not only proportional to the output gap but they also depend on the terms of trade. (see Svensson, 2000) This dependence captures the expenditure-switching effect; only in the special case in which \( \rho \theta = 1 \) the terms of trade channel disappears.

Finally, after log-linearizing equations (2), we obtain that the terms of trade reflect also the differential in the countries’ output gaps

\[
(\hat{T}_t - \hat{T}_{t-1}) = \theta^{-1}[(\hat{Y}_{H,t} - \hat{Y}_{H,t-1}) - (\hat{Y}_{F,t} - \hat{Y}_{F,t-1})].
\]  

(12)

Using the definition of the terms of trade, we can decompose the changes in the terms of trade between the nominal exchange rate depreciation and the producers’ inflation rate differential

\[
\hat{T}_t = \hat{T}_{t-1} + \Delta S_t + \pi_{F,t}^* - \pi_{H,t}.
\]  

(13)

where \( \Delta S_t = \ln S_t/S_{t-1} \).

3 Optimal Cooperative Solution

In this section we characterize the efficient response of macroeconomic variables to the various shocks that affect the economy, whether in the form of terms of trade or inefficient supply shocks. This problem has been extensively studied in the Keynesian literature on international monetary cooperation, (see Canzoneri and Henderson, 1991, and Persson and Tabellini, 1996). Importantly, we propose both a microfounded model and a welfare criterion, based directly on consumers’ utility.\(^7\)

Our welfare criterion for characterizing the cooperative allocation is represented by the utility of the agents belonging to the world economy. As in Woodford (2003), we rely on a quadratic approximation of the utility-based welfare criterion, so that our framework becomes comparable to the linear-quadratic models that have been extensively used in the Keynesian literature.

The quadratic approximation of the welfare criterion is taken around the steady state in which the taxation subsidy completely offsets the monopolistic distortions in

\(^7\)Our approach follow the recent tradition in the open-economy literature as in Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2003) and Corsetti and Pesenti (2005). However, the latter works have focused on static models, in which prices are pre-set one-period in advance and the only source of uncertainty comes from terms of trade’s shocks.
both countries (as in Benigno, P. 2004). We obtain the following loss function

\[ L^C = (\rho + \eta) \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ n(\tilde{Y}_{H,t} - \tilde{Y}_{H,t})^2 + (1 - n)(\tilde{Y}_{F,t}^* - \tilde{Y}_{F,t})^2 + n(1 - n)\theta \psi[\tilde{T}_t - \tilde{T}_t]^2 \\
+ \frac{n\sigma}{\kappa}(\pi_{H,t})^2 + \frac{(1 - n)\sigma}{\kappa^*}(\pi_{F,t}^*)^2 \right\} \]  

(14)

The variables \( \tilde{Y}_{H,t}, \tilde{Y}_{F,t}, \tilde{T}_t \) can be interpreted as the desired targets that policymakers wish to achieve in a cooperative agreement for domestic output, foreign output and the terms of trade respectively. When \( \mu = 1 \), these targets coincide with the flexible-price allocation assuming no mark-up shocks. (see Benigno and Benigno, 2005 for the generalization to the case in which \( \mu \neq 1 \)).

In the case in which \( \mu = 1 \), the quadratic loss function (14) can be evaluated by only a log-linear approximation to the structural constraints (7), (8) and (12). By specifying a path for \( \pi_{H,t} \) and \( \pi_{F,t}^* \), the variables \( \tilde{Y}_{H,t}, \tilde{Y}_{F,t}^* \) and \( \tilde{T}_t \) can be determined by (7)-(8) and this is all that is needed to evaluate (14).

**The Optimal Stabilization Plan Under Commitment**  We study the optimal cooperative policy under commitment from a ‘timeless perspective’ as discussed in Woodford (2003). This optimal plan is obtained by minimizing the loss function of society (14) under the structural equilibrium conditions (7), (8) and (12) together with the constraints implied by the ‘timeless perspective’ commitment on the initial conditions \( \pi_{H,t_0} \) and \( \pi_{F,t_0}^* \) given by \( \pi_{H,t_0} = \tilde{\pi}_{H,t_0} \) and \( \pi_{F,t_0}^* = \tilde{\pi}_{F,t_0}^* \). Equation (13) determines residually the exchange rate. As discussed in Benigno and Benigno (2005), this minimization problem is well-behaved, i.e. the loss function is convex.

Following Currie and Levine (1993) and Woodford (2003), the optimal plan can be described using the Lagrangian method, with multiplier \( \lambda_1 \) and \( \lambda_2 \) associated with the aggregate supply equations (7) and (8) and \( \lambda_3 \) associated with equation (13).

To study the optimal cooperative allocation in this more general case, we write the following Lagrangian:

\[ \mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^{t-t_0} (\rho + \eta) \left\{ \frac{1}{2} n y_{H,t}^2 + \frac{1}{2} (1 - n) y_{F,t}^2 + \frac{1}{2} n(1 - n)\theta \psi q_t^2 + \frac{1}{2} \frac{\sigma n}{\kappa} \pi_{H,t}^2 + \frac{1}{2} \frac{\sigma (1 - n)}{\kappa^*} \pi_{F,t}^2 \right\} + n\lambda_1, [\kappa^{-1}\pi_{H,t} - y_{H,t} - (1 - n)\psi q_t - \beta \kappa^{-1}\pi_{H,t+1}] + \right. \]
\[ + (1 - n)\lambda_2, [\kappa^{-1}\pi_{F,t} - y_{F,t} + n\psi q_t - \beta \kappa^{-1}\pi_{F,t+1}] + n(1 - n)\lambda_3, [q_t + \theta^{-1} y_{H,t} + \theta^{-1} y_{F,t} - n\lambda_{1, t_0 - 1}\kappa^{-1}\pi_{H,t_0} - (1 - n)\lambda_{2, t_0 - 1}\kappa^{-1}\pi_{F,t_0}^*], \]
where we have defined \( y_{H,t} \equiv (\hat{Y}_{H,t} - \bar{Y}_{H,t}) \), \( y_{F,t}^* \equiv (\hat{Y}_{F,t}^* - \bar{Y}_{F,t}) \) and \( q_t \equiv (\hat{T}_t - \bar{T}_t) \) and we have appropriately normalized the Lagrangian multiplier in a way to obtain time-invariant first-order conditions. The first-order condition with respect to \( y_{H,t} \), \( y_{F,t}^* \) and \( q_t \) are

\[
(\rho + \eta) y_{H,t} = \lambda_{1,t} + (1 - n) \theta^{-1} \lambda_{3,t},
\]

(15)

\[
(\rho + \eta) y_{F,t}^* = \lambda_{2,t} - n \theta^{-1} \lambda_{3,t},
\]

(16)

\[
(\rho + \eta) \theta \psi q_t = \psi \lambda_{1,t} - \psi \lambda_{2,t} - \lambda_{3,t},
\]

(17)

for each \( t \geq t_0 \), while the ones with respect to \( \pi_{H,t} \) and \( \pi_{F,t}^* \) are

\[
(\rho + \eta) \sigma \pi_{H,t} = -(\lambda_{1,t} - \lambda_{1,t-1}),
\]

(18)

\[
(\rho + \eta) \sigma \pi_{F,t}^* = -(\lambda_{2,t} - \lambda_{2,t-1}),
\]

(19)

for each \( t \geq t_0 \).

We can characterize some implications of the optimal cooperative solution without the need of solving this set of equations. Indeed we can use equations (15), (16), (17) and (12) to obtain

\[
y_{H,t} = -\lambda_{1,t},
\]

(20)

\[
y_{F,t}^* = -\lambda_{2,t},
\]

(21)

which imply the following relations between producer inflation rates and output gaps

\[
\sigma (\rho + \eta) \pi_{H,t} = y_{H,t} - y_{H,t-1},
\]

(22)

\[
\sigma (\rho + \eta) \pi_{F,t}^* = y_{F,t} - y_{F,t-1}.
\]

(23)

Under the optimal cooperative solution each country should adjust its domestic producer inflation in response to changes in its domestic output gap. Equations (22) and (23) are similar to the conditions that would hold in the closed-economy case under commitment.

As we show in Benigno and Benigno (2005), following terms of trade shocks, the optimal cooperative plan requires to completely offset these shocks with movements in the exchange rate and the terms of trade by setting \( \pi_{H,t} = \pi_{F,t}^* = 0 \) in each period. All the gaps are closed. Indeed, the first best is achieved and the optimal cooperative equilibrium replicates the efficient allocation.

On the other hand, under inefficient supply shocks the first best cannot be achieved and producer inflation rates are state contingent. The presence of inefficient supply
shocks creates a further distortion in the economy. Stabilizing producer inflation rates to zero is suboptimal and a sticky price allocation can improve upon the flexible price one. However, \( \pi_{H,t} \) and \( \pi_{F,t} \) should move proportionally to their respective output-gap growths.

**The Transmission Mechanism of Inefficient Supply Shocks**  We now focus on how inefficient supply shocks propagate across different countries. We start by considering the special case \( \theta = \rho^{-1} \). Under this restriction, the centralized welfare criterion can be written as a weighted average of Home and Foreign domestic targets

\[
L_t^c = nL_{H,t} + (1 - n)L_{F,t},
\]

with

\[
L_{H,t} = (\rho + \eta)^2 + \frac{\sigma(\rho + \eta)}{\kappa} \pi_{H,t}^2 \quad \text{and} \quad L_{F,t} = (\rho + \eta)^2 + \frac{\sigma(\rho + \eta)}{\kappa} \pi_{F,t}^2
\]

where \( L_{H,t} \) and \( L_{F,t} \) correspond to the loss functions that can be obtained from quadratic approximations of the welfare of the single countries, but in a closed-economy model, as shown in Woodford (2003). Moreover, the two aggregate supply equations becomes

\[
\pi_{H,t} = \kappa[(\bar{Y}_{H,t} - \bar{\bar{Y}}_{H,t}) + u_t] + \beta E_t \pi_{H,t+1},
\]

\[
\pi_{F,t}^* = \kappa^*[((\bar{Y}_{F,t}^* - \bar{\bar{Y}}_{F,t}^*) + u_t^*)] + \beta E_t \pi_{F,t+1}^*.
\]

In the optimal cooperative solution the couple of sequences \( \{\pi_{H,t}, y_{H,t}\} \) and \( \{\pi_{F,t}^*, y_{F,t}^*\} \) should only react to their respective inefficient supply shocks.

**Proposition 1**  When there are inefficient supply shocks, if \( \theta \rho = 1 \), the optimal paths for the producer inflation rates and output gaps in each country mirror the one that would arise in a closed-economy model.

Inflation and output gap are orthogonal to the inefficient supply shocks of the other country.

Using the above limiting case, we now discuss the more general case in which the condition \( \theta \rho = 1 \) is not met. In the optimal cooperative solution, the transmission mechanism of inefficient supply shocks depends crucially on \( \theta \rho \) being greater or less than 1. Using (18) and (19), we obtain

\[
\pi_{H,t} = \frac{\lambda_{1,t}}{\sigma(\rho + \eta)} \
\]

\[
\pi_{F,t}^* = \frac{\lambda_{2,t}^*}{\sigma(\rho + \eta)}.
\]
Comparing with (20) and (21), it follows that the output gap in each country is negatively related to its respective producer price level\(^8\). Consider a positive (adverse) supply shock in the Home country. As in the closed-economy counterpart, this shock produces an impact inflation in the Home country and a reduction in the output gap. The increase in the producer price level in country H tends to improve the terms of trade of that country and worsen those of country F (i.e. \(T\) decreases). The impact on foreign inflation and output gap depends on \(\theta \rho\). First, assume \(\theta \rho < 1\) (what can be labelled as the ‘positive correlated scenario’). From (8), an improvement of the foreign terms of trade increases on impact foreign producer inflation and the foreign producer price level (not enough to offset the initial decrease in \(T\)). Then the foreign output gap should also decrease. Under this scenario, an adverse shock in the Home economy will produce the same pattern of producer inflation and output gap in both countries, still with different magnitude.

The intuition is related to whether the worsening in the Foreign terms of trade increases or decreases the marginal revenues in utility terms of the Foreign firms. In fact, marginal revenue depends on the factor

\[
\frac{U_C(C_t)P_{F,t}}{P_t} = \frac{U_C \left( Y_{F,t} \left( \frac{P_{F,t}}{P_t} \right)^\theta \right) P_{F,t}}{P_t},
\]

which represents firms’ marginal revenues in utility terms as if market were perfectly competitive. When the elasticity of the marginal revenue with respect to the terms of trade is positive, a worsening in the Foreign terms of trade decreases the marginal revenues since the increase in the general level of prices \(P\) offsets the increase in the marginal utility of consumption. This is so when \(\theta \rho < 1\). Given that marginal revenues fall, foreign firms should rise their prices in order to protect their profits. The expenditure switching effect captured by the parameter \(\theta\) is not so strong to sustain a positive output gap in the foreign economy. So, output gap in the foreign economy falls.

On the other hand, when \(\theta \rho > 1\) (what can be labelled as the ‘negative correlated scenario’), the marginal revenues of the firms increases in country F, so that they can

\(^8\)Conditions (27) – (28) show that the logs of the producer price levels are stationary variables since both the Lagrangian multipliers and the shocks are stationary. Prices should go back to the initial level under inefficient supply shocks. This property extends directly to the nominal exchange rate. Even if there are no direct costs in the welfare criterion (14) related to the exchange rate volatility, a necessary condition for an allocation to be optimal is that the exchange rate should be stationary following stationary disturbances of any nature (i.e. terms of trade or inefficient supply shocks).
cut their prices on impact. The foreign output gap increases since the expenditure-switching effect is stronger. In this case, the adverse supply shock produces on impact inflation in the home economy and a deflation in the foreign economy, while home output gap falls and foreign output gap rises. In later quarters, the home economy will experience periods of deflation, while the foreign economy periods of inflation.

4 Implementing the Cooperative Solution

In this section, we investigate whether it is possible that policies acting in a non-cooperative way can implement the optimal cooperative solution. Recently, Obstfeld and Rogoiff (2002) have shown that self-oriented monetary policymakers can replicate the cooperative outcome in a decentralized framework so that there is no need of international monetary policy coordination. In their case, “self-oriented” refers to policymakers that maximize the welfare of the consumers within their countries.

In a previous works (Benigno and Benigno, 2003 and 2005) we have shown that in this model there are gains from cooperation as long as $\theta \neq 1$, so that self-oriented policymakers would not behave optimally.\textsuperscript{9} In particular in Benigno and Benigno (2005), we have discussed how it is possible to replicate the optimal cooperative allocation through the adoption of targeting rules.

Here we look at the problem from a different perspective. We try to design institutions expressed in terms of loss function (rather than targeting rules) and analyze how the assignment of these loss function to each monetary authority can implement the cooperative solution in a strategic setting. In particular we seek to design objective functions for monetary policymakers that are able to commit, but interact in strategic context.\textsuperscript{10} These objective functions do not necessarily coincide with the utility-based welfare of the country.\textsuperscript{11}

Following the recent widespread adoption of inflation-targeting regimes across several countries, we assume that each policymaker is committed to an inflation-targeting regime. In particular we assume that home and foreign policymaker are

\textsuperscript{9}Canzoneri, Cumby and Diba (2005) have explored the gains from coordination when there are important sectorial productivity differences within a single country.

\textsuperscript{10}In a closed economy, Jensen (2002) and Woodford (1999) have studied the design of the objective function of a monetary policymaker that is unable to fulfill credible commitments. In an open-economy model, Sutherland (2005) compares our flexible targeting rules with simple operational rules in a static framework.

\textsuperscript{11}See Benigno and Benigno (2005) for the analysis of the utility-based objective function of the single country.
committed to minimize their respective loss functions $L$ and $L^*$ of the form

\[
L = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\rho + \eta) \left[ \sigma_H y_{H,t}^2 + \frac{1}{\kappa} \pi_{H,t}^2 \right] \right\}, \quad L^* = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t (\rho + \eta) \left[ \sigma^* y_{F,t}^2 + \frac{1}{\kappa^*} \pi_{F,t}^2 \right] \right\},
\]

(29)

where $\sigma$ and $\sigma^*$ are non-negative parameters.

The above loss functions correspond to ‘flexible’ inflation-targeting regimes using the terminology of Svensson (2002), where each policymaker is penalized for the deviations of the inflation rate from the target (zero in this case), and for changes in the output gap. In practice, this seems a good representation of the objective function of monetary policymaker in inflation-targeting regimes, as discussed in Svensson (2002), and even in other regimes, see Blinder (1997) for the US case.

The open-economy extension of ‘inflation-targeting’ regimes requires an additional qualification in terms of choosing the proper measure of inflation rate to target, i.e. whether GDP or CPI inflation. One justification for designing our regimes in terms of GDP inflation rate follows from the works by Aoki (2001) and Benigno, P. (2004), in which the inflation coming from sectors with sticky prices is the appropriate inflation to target when trying to minimize the distortions existing in the economy. Moreover we want to restrict our analysis to policymakers that are inward-looking in the sense that their loss functions include only domestic targets. We note here that there is another dimension along which the policymakers are self-oriented: we assume that they interact strategically as in a Nash equilibrium where each policymaker sets the strategy in terms of her producer inflation and takes the strategy of the other policymaker as given.

As in Rogoff (1985), our delegation problem boils down to choosing appropriately the relative weights to the output gap and the producer inflation rate in the loss function. In our case the parameters $\sigma$ and $\sigma^*$ are chosen so that the allocation that results from the strategic interaction between the policymakers replicates the cooperative solution.

We show that there is always a positive answer to this problem. We first note that when the only disturbances are given by terms of trade shocks, the decentralized commitment to loss function $L$ and $L^*$ reproduces the cooperative solution for any $\sigma$ and $\sigma^*$. From the discussion in the previous sections, under terms of trade shocks, stabilizing producer inflation rates to zero, in each country, achieves the first-best. There are enough instruments to cope with all the distortions in the economy. Since in this case there is no trade-off between domestic output-gap and producer inflation rate, then any loss function in the class of flexible inflation-targeting rule $L$ and $L^*$
can achieve the first-best in a non-cooperative allocation.

When instead there are inefficient supply shocks, the following proposition holds:

**Proposition 2** In a Nash equilibrium in which each policymaker is committed to a flexible inflation targeting policy of the form (29), following inefficient supply shocks, the cooperative allocation can be achieved if and only if
\[
\bar{\omega} = \frac{\theta(\rho + \eta) + (1 - \rho\theta)}{\theta(\rho + \eta) + n(1 - \rho\theta)}, \quad \bar{\omega}^{*} = \frac{\theta(\rho + \eta) + (1 - \rho\theta)}{\theta(\rho + \eta) + (1 - n)(1 - \rho\theta)},
\]
In particular \( \bar{\omega} > 1 \) and \( \bar{\omega}^{*} > 1 \) if \( \rho\theta < 1 \) and \( \bar{\omega} < 1 \) and \( \bar{\omega}^{*} < 1 \) if \( \rho\theta > 1 \). Note that when \( \rho\theta = 1 \), then the cooperative allocation can be replicated if and only if \( \bar{\omega} = \bar{\omega}^{*} = 1 \).

When \( \rho\theta = 1 \) there is complete separation in the optimal stabilization plan between Home and Foreign inefficient supply shocks. Indeed, domestic output gap and inflation rate should react only to domestic inefficient shocks and not to foreign ones. An inspection of the centralized welfare criterion (24) can help to identify the appropriate design of the decentralized solution. It is sufficient that each policymaker commits to maximize its appropriate targets, producer inflation and output gap, in the exact combination as they are in (24). It is again worth noting that when \( \bar{\omega} = \bar{\omega}^{*} = 1 \) the loss functions \( L \) and \( L^{*} \) correspond to the losses that one would get as an approximation of the welfare in the closed-economy version of this model. Since there is no strategic interdependence in the stabilization problem, policymakers can implement the cooperative allocation by minimizing a ‘closed-economy’ loss function.\(^{12}\)

We now discuss the general case, \( \rho\theta \neq 1 \). We can write equations (7) and (8) using condition (12) as
\[
\pi_{H,t} = \kappa[y_{H,t} + (1 - n)\psi^{-1}(y_{H,t} - y_{F,t}^{*}) + u_{t}] + \beta E_{t}\pi_{H,t+1}, \quad (30)
\pi_{F,t}^{*} = \kappa^{*}[y_{F,t}^{*} - n\psi^{-1}(y_{H,t} - y_{F,t}^{*}) + u_{t}^{*}] + \beta E_{t}\pi_{F,t+1}^{*}, \quad (31)
\]
from which we can define
\[
p_{1} \equiv \kappa[1 + (1 - n)\psi^{-1}], \quad p_{2} \equiv -\kappa(1 - n)\psi^{-1},
\]
\[
p_{3} \equiv \kappa^{*}[1 + n\psi^{-1}], \quad p_{4} \equiv -\kappa^{*}n\psi^{-1}.
\]
\(^{12}\)However, in the open-economy model, these loss functions do not corresponds to the second-order approximation of the welfare of the single countries, since the welfare of each country has an higher weight on the disutility of output relative to the utility consumption than the centralized welfare criterion. Indeed, the disutility of output in the centralized criterion is weighed by the size of the country. See Benigno and Benigno (2005) for the individual welfare functions in this case.
Combining the above equations we get

\[ \pi_{H,t} = p_5 y_{H,t} + \kappa u_t + p_6 (\beta E_t \pi^*_{F,t+1} - \pi^*_{H,t}) + p_6 \kappa^* u_t^* + \beta E_t \pi_{H,t+1}, \]  
\[ \pi^*_{F,t} = p_7 y^*_{F,t} + \kappa^* u_t^* + p_8 (\beta E_t \pi_{H,t+1} - \pi_{H,t}) + p_8 \kappa u_t + \beta E_t \pi^*_{F,t+1}, \]  
(32)
(33)

where

\[ p_5 = \frac{p_3 p_4 - p_2 p_4}{p_3}, \quad p_6 = \frac{p_2}{p_4}, \quad p_7 = \frac{p_3 p_4 - p_2 p_4}{p_3}, \quad p_8 = \frac{p_4}{p_1}. \]

In the Nash allocation, the monetary policymaker in country \( H \) maximizes \( L \), under the constraint (32) taken as given the sequence \( \{\pi^*_{F,t}\}_{t=0}^\infty \) chosen by the policymaker in country \( F \) and the initial inflation rate given by the timeless-perspective commitment \( \pi_{H,t_0} = \tilde{\pi}_{H,t_0} \).

The first-order necessary conditions with respect to \( y_{H,t} \) and \( \pi_{H,t} \) are

\[ \omega y_{H,t} = -p_5 \psi_t, \]  
\[ \sigma \pi_{H,t} = \psi_t - \psi_{t-1}, \]  
(34)
(35)
at each date \( t \geq 0 \) and for each possible state, where \( \psi_t \) is the Lagrangian multiplier associated with the constraint (32). Looking at the problem of policymaker \( F \) and maximizing \( L^* \) with respect to \( y^*_{F,t} \) and \( \pi^*_{F,t} \), we obtain that

\[ \omega^* y^*_{F,t} = -p_7 \psi^*_t, \]  
\[ \sigma^* \pi^*_{F,t} = \psi^*_t - \psi^*_{t-1}, \]  
(36)
(37)
at each date \( t \geq 0 \) and for each possible state, where \( \psi^*_t \) is the Lagrangian multiplier associated with the constraint (32).

The set of equilibrium conditions (7), (8), (34), (35), (36), (37) are equivalent to the set of equilibrium conditions that characterize the optimal cooperative solution if and only if \( \omega = p_5 \) and \( \omega^* = p_7 \).

Even if there is a need of monetary coordination at an international level, this need can be satisfied by delegating monetary policy to inward-looking institutions that care only about domestic objectives. In particular in the ‘positive-correlated scenario’, a relative higher weight should be given to the stabilization of the domestic output gap in comparison to the scenario in which there is no strategic interdependence across countries (\( \rho \theta = 1 \)). The intuition for this result can be understood by inspecting equations (30) and (31). In fact, an adverse inefficient supply shock can be absorbed by an increase in home producer inflation, a decrease in home output gap and in the one-period ahead expectations on the producer inflation rate. When \( \rho \theta < 1 \) and
while acting strategically, each policymaker has an incentive to lower her output gap below the foreign one in order to export the shock to the other country, through an improvement of her own terms of trade. This incentive creates a ‘contractionary’ bias that can be corrected by assigning a relatively higher weight to the output gap in the flexible inflation-targeting regime. On the contrary, when $\rho \theta > 1$, each country would tend to be more expansionary reducing less the output gap and increasing more the response of the producer inflation rate. This ‘expansionary bias’ can be corrected by giving higher weight to the inflation rate than to the output gap in the loss function.

A final result is related to the degree of openness. The more open is a country, i.e. the more the country is affected by terms of trade movements, the higher is the conflict in the stabilization problem. In this case the correction should be greater. Indeed, if for example $n < 1/2$, the home country will be more open (and the foreign country more closed), then $\varpi > \varpi^* > 1$ if $\rho \theta < 1$ and $\varpi < \varpi^* < 1$ if $\rho \theta > 1$.

5 Conclusions

In this work we proposed a standard two-country dynamic general equilibrium model with imperfect competition and nominal price rigidities. Despite goods and financial market integration, there is scope for international monetary policy coordination arising from the interaction between non-unitary intratemporal elasticity of substitution and terms of trade shocks and the presence of inefficient supply shocks.

We show that by committing to a properly designed flexible inflation targeting regimes our monetary authorities can replicate the optimal outcome independently of the type of shock that hits the economy even in a strategic setting. Importantly these institutions are “inward-looking” in the sense that each policymaker cares only about domestic targets –the output gap and the domestic producer inflation rate.
References


