A Defence of the Ramsey Test

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Abstract

According to the Ramsey Test hypothesis the conditional claim that if A then B is credible just in case it is credible that B, on the supposition that A. If true the hypothesis helps explain the way in which we evaluate and use ordinary language conditionals. But impossibility results for the Ramsey Test hypothesis in its various forms suggest that it is untenable. In this paper, I argue that these results do not in fact have this implication, on the grounds that similar results can be proved without recourse to the Ramsey test hypothesis. Instead they show that a number of well entrenched principles of rational belief and belief revision do not apply to conditionals.

1. Ramsey's Hypothesis

> If two people are arguing 'If *p* will *q*?' and are both in doubt as to *p*, they are adding p hypothetically to their stock of knowledge and arguing on that basis about *q*; so that in a sense 'If *p*, *q*' and 'If *p*, $\bar{q}$ ' are contradictories. (Ramsey 1929, p. 155)

This remark of Ramsey appears only as a footnote to his paper 'General Propositions and Causality', but it has sufficed to lend his name to a hypothesis that has figured prominently in contemporary debate in both the semantics and pragmatics of conditionals.[1] This interest in the

[1] It is not the aim of this paper to defend the view that Ramsey really did subscribe to the thesis to which he

Ramsey Test hypothesis, as it is usually called, is fuelled by widespread dissatisfaction with the material conditional as a rendition of the semantic content of ordinary language conditionals. Discontent is focused on two points: the fact that the material conditional interpretation appears to support fallacious reasoning and the fact that reasonable belief in conditionals appears to diverge from that demanded by the material conditional interpretation of them. A couple of simple examples should suffice to illustrate both. Here, and throughout the paper, upper case letters will serve as sentence variables and $\neg$, $\vee$ and & as the signs for the sentential operations of negation, disjunction and conjunction respectively.

Example 1*. The material conditional construal of ordinary language conditionals sanctions inference from the sentence 'It is not the case that if it snows tomorrow then the government will fall' to 'It will snow tomorrow', because* $\neg(\neg A \vee B)$ *implies A. But denying that the weather will have an impact on the government's fortunes surely does not commit one to any particular meteorological prognosis. Likewise, disbelieving that the government will fall if it snows does not mean believing that it will snow (and in summer, should not).*

Example 2*. On the material conditional interpretation of the claim expressed by the sentence 'If George Bush is concerned to protect the environment, then he will lower the tax on fuel' should be highly credible, because of the improbability of its antecedent. But intuitively the claim is implausible because environmentalists typically believe that fuel taxes should be raised.*

These examples suggest that material conditionals are much weaker semantically than corresponding ordinary language indicative conditionals and that their negations are much

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has lent his name. See Levi 1996 for an argument that he didn't.

stronger. The same is undoubtedly true for subjunctive or counterfactual conditionals. What is needed then is some other way of determining the content of ordinary language conditionals and the attitudes rational agents should take to the prospect of their truth. The Ramsey Test hypothesis is often advanced as a solution to both of these problems; the thought being that examination of conditions under which conditionals should be believed will, at very least, serve to constrain the semantics of conditionals. But I cannot fully evaluate the semantic project here and will concentrate on the prior issue of rational belief in conditionals.

The intuitive idea behind the Ramsey Test is, in this respect, simple and compelling. To judge whether it is credible that if P then Q, first suppose that P is true. Then adjust your beliefs no more than is necessary to accommodate this supposition. Finally observe whether your new beliefs entail that Q. If so, you should adopt the belief that if P then Q. This procedure certainly seems to give the 'right' answer in our two examples. If I suppose that it will snow tomorrow and find that this does not lead to the belief that government will fall, then the Ramsey Test does not commit me to any belief concerning tomorrow's weather. Equally the claim that George Bush will lower the tax on fuel, if he is concerned to protect the environment, does not pass the Ramsey Test because the supposition that he is concerned to protect the environment leads, if anything, to the belief that he will raise fuel taxes.

But what is meant by revising or adjusting your beliefs no more than is necessary to accommodate the supposition that P? It depends, as Joyce (1999) has convincingly argued, on the manner or mode in which we suppose that P. We might suppose that *as a matter of fact* P is true; in which case we would revise much in the way that we do when we learn of P's truth. Minimal revision in this case might require us not to give up any firm beliefs not contradicted by P. Alternatively, we might suppose or imagine that, *contrary to the facts*, P is true. A supposition of this kind may be best accommodated by giving up some of one's beliefs not contradicted by P, to allow retention of well-entrenched ideas about the way that the world works. For example when supposing what would have happened had it rained yesterday, I might have to give up my belief that I went for a walk in the park that day, even if I did in fact go for a walk (and have sore feet to prove it).

It is commonly observed that there is more than one kind of ordinary language conditional, although exactly how to classify the various kinds is a matter of some dispute.[2] One advantage of the Ramsey Test hypothesis is that it allows us to link this observation to the fact that there different kinds of suppositions or ways of supposing something true. For the Ramsey Test can be treated as a test *schema* with different types of belief revision being suitable for testing the credibility of different kinds of conditionals.

Example 3: *Suppose that on the basis of my knowledge of the market, I believe that a particular company will go bankrupt in the near future. For this reason I believe that if my friend were to invest in this company, he would lose a lot of money. However, I have enormous confidence in his investment savvy, to the extent that I not only believe that he won't invest in the company, but that if he does then it will not go bankrupt and he will not lose money on it. The difference in these two beliefs reflects the difference between my attitude to the possibility that he will lose money on this investment, given that he will as a matter of fact invest money in the company, and my attitude to the possibility that he would lose money, were he (contrary to my beliefs) to invest in it.*

In this paper, versions of the Ramsey Test hypothesis will be formulated for two kinds of models of belief states and associated theories of minimal revision. Firstly, for coarse-grained models of belief as an attitude with only three modalities – believe, disbelieve, or neither – and the AGM

2 See Edgington 1995 for a summary.

theory of belief-revision.[3] And secondly for more fine-grained models of degrees of belief, for which Bayesian and related forms of belief revision will be considered. The aim of the paper is to explore the connections between various 'impossibility' results for the Ramsey Test idea in its different formulations and to evaluate their significance. I shall argue that these results have been misunderstood and that, rather than undermining the Ramsey Test hypothesis, they show that belief revisions involving conditionals are markedly different from those involving non-conditional sentences.

In section 2, I present Gärdenfors's pioneering impossibility result for the Ramsey Test hypothesis as formulated in the AGM framework and show that similar reasoning to his yields an impossibility result that applies to almost any interpretation of the indicative conditional. I conclude that his result does not show that the Ramsey Test hypothesis must be abandoned. In section 3, I argue that what the impossibility results do show is that a particular principle of belief revision, the Preservation condition, does not apply to conditionals. Section 4 exams probabilistic versions of the Ramsey Test and in particular, the hypothesis (known as Adams's Thesis) that the probability of a conditional is the conditional probability of its consequent given the truth of its antecedent. In a parallel manner, 'triviality' results that are often taken to show that Adams Thesis should be rejected are generalised to a very wide class of hypothesis concerning the credibility of conditionals. But in the probabilistic case, rejecting the corresponding probabilistic preservation principle does not suffice to avoid the difficulties raised by the results, suggesting that revisions involving conditionals are quite fundamentally different from those involving factual propositions.

Throughout we will take the possibilities with respect to which agents have beliefs to be represented by sentences of some background language, **L**, closed under the sentential operations

[3] So called because of the founding contributions of Alchourrón, Gärdenfors and Makinson

$\neg$, $\vee$ and & and containing a logically true sentence $\top$ and logically false sentence $\perp$. To state the Ramsey Test hypothesis we will also assume that **L** is closed under a sentential operation, $\rightarrow$, which forms conditionals from pairs of other sentences. In view of our recognition that there is more than one kind of ordinary language conditional, there would be some advantage to treating $\rightarrow$ as a connective variable. But as this creates the need for rather tedious quantification and as the literature on belief revision is almost entirely devoted to the kind of revision appropriate to matter of fact supposition, I will not do this. Instead I will concentrate on the interpretation of $\rightarrow$ as the indicative conditional connective and generalise where necessary.

It is assumed thoughout that **L** is closed under a (compact) Boolean relation of logical consequence, $\vdash$. It follows that $\vdash$ must contain at least classical propositional logic, but further properties of the consequence relation having to do with conditionals also seem natural. In particular we assume the following principle of conditional contradiction. For any subset, *K*, of **L**:

(CC) If K, B,C $\vdash \perp$ then K, A$\rightarrow$B, A$\rightarrow$C $\vdash \perp$

This principle, apparently endorsed by Ramsey (see the opening quotation), rules out the material conditional interpretation of $\rightarrow$. But all proposals for a stronger interpretation of the conditional connective that I know of endorse CC.

2. Belief Revision and the Preservation Condition

In the AGM framework, an epistemic state of an agent is represented by a subset, *K*, of **L**, closed under the relation $\vdash$. Intuitively *K* is the set of all sentences believed or accepted by the agent in a particular context. A set of sentences *K* represents the beliefs of a maximally opinionated agent just in case *K* contains, for any **L**-sentence X, either X or $\neg$X. But while the AGM framework forces the assumption that agents are logically omniscient, there is (notably) no requirement that they be maximally opinionated. There may be some prospects about which the agent reserves judgement, has not bothered to judge, or of which the agent is simply unaware.

We denote by K_A^* the minimal revision of K induced by the supposition that A is true. Intuitively K_A^* is the set of all sentences conditionally believed true or accepted by the agent when she supposes that A, e.g. in practical deliberation or adopted 'for the sake of the argument' in conversation with others. The notion of a minimal revision of an epistemic state is formally characterised (but not completely determined) by a set of axiomatic constraints on the relation between K and K_A^*. The 'official' AGM axioms are listed in the appendix, but we will not need all of them here. In particular, we will not impose K*3 (the axiom of Inclusion), which requires that K_A^* be a subset of what is termed in the literature the expansion of K by A; namely, the closure of $K \cup \{A\}$ under the consequence relation $\vdash$. In forbidding one from adopting beliefs not deductively entailed by the combination of one's current beliefs and the sentence supposed true, the axiom of Inclusion expresses a strong principle of epistemic caution. Arguably it counsels excessive caution - in particular, in disallowing ampliative inferences - and I will not make use of it here.

What we do need to assume for the purposes of our argument is that K_A^* is itself an epistemic state (axiom K*1) and hence closed under $\vdash$, that the sentence A that is supposed true belongs to K_A^* (K*2, the axiom of success), that K_A^* is consistent whenever A is (axiom K*5), and that the following Preservation condition (axiom K*4) is satisfied:

(PRES) $\neg A \notin K, X \in K \Rightarrow X \in K_A^*$

The Preservation condition implies that whenever A is not disbelieved, revising on the supposition that A amounts to adding A to one's belief set and deriving the logical consequences.

Because it requires one to retain any beliefs not contradicted by what one has learned, PRES is best read as a principle of informational economy expressing the idea that revision is minimal when no beliefs are abandoned unless they are contradicted by what is learnt.[4]

The Ramsey Test hypothesis is conveniently formulated in the AGM framework as follows:

(RT) $X \in K_A^* \Rightarrow A \to X \in K$

Since it is not assumed that epistemic states are complete in the sense of containing every sentence or its negation, a second rule is required to determine when negations of conditionals should be adopted in one's belief set. The natural companion to RT, what may be called the negative Ramsey test, is the following:

(RT$^-$) $\neg X \in K_A^* \Rightarrow \neg(A \to X) \in K$

This rendition of the Ramsey Test differs somewhat from the standard formulation. Gärdenfors (1988), and consequently most those that he inspired, attaches the label of 'Ramsey Test' to a stronger principle; one equivalent to the conjunction of RT and the following principle, which we shall call Conditional Driven Revision:

(CDR) $A \to X \in K \Rightarrow X \in K_A^*$

CDR says that a belief in a conditional rationally commits one to a corresponding policy for revising one's beliefs. CDR has much to recommend it and is powerful enough that Gärdenfors, for instance, is able to draw very strong conclusions about the semantics of conditionals from the conjunction of it and RT. But it is clearly possible to accept the intuitive idea of the Ramsey Test without taking CDR on board - certainly Ramsey never explicitly endorsed the latter.

4 Levi (1996) argues that a weaker preservation condition is appropriate to the Ramsey test, one which requires that the belief X belongs to K_A^* only when neither A nor ¬A belong to K. Nothing in our

My construal of the negative Ramsey test also differs from some other proposals and, in particular, that of Levi (1991, 1996) and Arló-Costa and Levi (1996) who propose that ¬(A→X) should be accepted iff X does not belong to K^*_A. This condition for acceptance of a negated conditional is much too weak and as a result requires that one believe or accept either that A→X or that ¬(A→X).[5] But this is contrary to the idea of that agents should be allowed to reserve judgement when lacking the basis for making one. Suppose, for instance, that I don't know how to get to the post-office. Then I should believe neither that it is the case that if turn right at the next road that I will get to the post-office nor that it is not the case.[6]

Together with the background consequence relation, RT and RT$^-$ deliver what is required; namely sufficient conditions for belief in any sentence containing the conditional connective. So far so good. It turns out, however, that RT is not consistent with the AGM theory of belief revision. Something of this kind was first demonstrated by Peter Gärdenfors, who concluded that either the Ramsey Test or the Preservation condition should be rejected, settling on the former on the grounds that it entailed the monotonicity of belief revision (itself an undesirable property - see below).[7] Gärdenfors's impossibility result works from his much stronger formulation of the Ramsey Test and it is tempting to respond to his impossibility result by rejecting CDR and retaining RT. This move is, however, neither necessary nor sufficient to protect the Ramsey Test hypothesis. Not sufficient because CDR's contribution to the impossibility result is essentially that of ensuring that → is stronger than the material conditional; in particular, by entailing CC.

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argument would be affected by adopting the weaker condition.

[5] Arló-Costa and Levi (1996, p. 226) claim that this '… result, highly unintuitive when applied to truth-value bearing conditionals, is nevertheless quite natural for conditional that lack truth values', but fail to say why.

[6] Even worse, if one assumes that X is equivalent to ⊤→X, Levi's condition implies that if X does not belong to K then ¬X does. It thereby forces agents to be maximally opinionated, contrary to our initial assumption that this was not be required. Arló Costa and Levi do not accept this last assumption, or indeed the terms on which I have discussed the negative Ramsey test. But I will defer discussion of his for reasons for doing so until the next section.

[7] Gärdenfors 1988, p. 159

But, since we have already assumed CC, we can prove an impossibility result for RT without making use of CDR. Not necessary because, as we show below, neither RT nor CDR are the real source of the impossibility.

Let us begin by looking at Gärdenfors's own analysis of the problem. Observe that RT and CDR jointly imply that belief revision is monotonic i.e. that if $K \subseteq K'$ then $K_A^* \subseteq (K')_A^*$, for any consistent sentence A. But, Gärdenfors argues, revision should not be monotonic in cases where the input sentence contradicts what is currently believed.[8] For example, consider consistent belief set *K* that contains the sentence A and two subsets *L* and *M* of *K* respectively containing the sentencess $A \vee B$ and $A \vee \neg B$. Then by K*4 and K*2, the preservation and success conditions, $A \vee B, \neg A \in L_{\neg A}^*$ and $A \vee \neg B, \neg A \in M_{\neg A}^*$. Hence $B \in L_{\neg A}^*$ and $\neg B \in M_{\neg A}^*$. Then if belief revision is monotonic $K_{\neg A}^*$ will contain both B and ¬B. Since it should not, either RT or CDR must be rejected.

This analysis of the problem is, I think, a mistaken one, because it fails to consider whether the postulated subsets *L* and *M* really meet the conditions for epistemic states. In fact, they cannot if both RT and CDR hold. For if they are epistemic states, then RT requires both that ¬A→B belongs to *L* because B belongs to $L_{\neg A}^*$ and that ¬A→¬B belongs to *M* because ¬B belongs to $M_{\neg A}^*$. So if *L* and *M* are subsets of *K*, then *K* must be inconsistent (assuming CC). But if *L* and *M* are not epistemic states then the monotonicity condition does not apply. So although RT and CDR jointly impose monotonicity, they also suppress its problematic consequences.

The real source of the impossibility result would seem thus to lie not with RT or CDR but PRES. For the fact is that PRES alone pretty much rules out any interpretation of the conditional connective → other than the material conditional one, given the AGM framework and a rule of Modus Ponens to the effect that if K is any subset of **L**, and A any sentence consistent with K, then:

(MP) If $K \vdash$ A→B and $K \vdash$ A then $K \vdash$ B

To see this, let {A, B,C} be a set of mutually contradictory and exhaustive sentences and suppose that an agent's initial epistemic state is given by a set K containing none of A, ¬A, B, ¬B, C or ¬C, and both of the sentences A∨B→(¬A→B) and A∨C→(¬A→C). Now consider the set K_A^* that results from modifying K on learning or supposing that A. By PRES, since ¬A does not belong to K, both A∨B→(¬A→B) and A∨C→(¬A→C) belong to K_A^*. So too do A∨B and A∨C since they are logical consequences of A, which must belong to K_A^*. Then by the Modus Ponens rule, both ¬A→B and ¬A→C belong to K_A^*. But this violates our principle of conditional non-contradiction, CC.

***Example 3**: I must choose between three urns, only one of which contains a prize. I do not know in which urn it is to be found. Labelling the urns A, B and C, I might quite reasonably hold that if the prize is in urn A or urn B then if it's not in urn A it's in urn B, and that if the prize is in A or C then if it's not in A it's in C. But then if I were to learn or suppose that the prize is in urn A, I would be compelled by the PRES and Modus Ponens to infer both that if it isn't (or wasn't) in A, then it is (or would have been) in B and also that if it isn't (or wasn't) in A, then it is (or would have been) in C.*

As I see it, the fact that the principle of conditional contradiction is violated when both the Preservation condition and Modus Ponens are assumed to hold leaves the project of finding a stronger interpretation of the conditional than the material conditional with a significant

[8] Here I reconstruct the argument given in ibid, pp 59-60.

difficulty, but has no special implications for the Ramsey Test hypothesis. Adding RT to our basic assumptions would have ensured that the sentences $(A\vee B)\rightarrow(\neg A\rightarrow B)$ and $(A\vee C)\rightarrow(\neg A\rightarrow C)$ belong to every belief set not containing A, B or C or their negations. But this is also so on most accounts of conditionals. And, in any case, the argument requires no more than that it be possible for a belief set to contain these sentences without containing the sentences A, B or C or their negations. Example 3 exemplifies this possibility.

Before we can claim that the case against the Ramsey Test hypothesis is defeated, however, we need to consider some purported counter-examples to it. The counterexamples in question essentially work by illustrating the claim that someone who believes that B, but neither that A nor that $\neg A$, can consistently both believe that if A then $\neg B$ and be disposed to believe B on learning that A. Here is a version due to Lindström and Rabinowicz (1995). Suppose that Oscar believes that Tweety is a bird and that (B) Tweety can fly, but has no view as to whether (A) Tweety is a penguin or (C) penguins cannot fly. Since C is compatible with Oscar's beliefs, so too, presumably, is the possibility that $A\rightarrow\neg B$, which seems to be entailed by C. On the other hand, if Oscar were to learn that Tweety was a penguin, he might reasonably continue to believe that he can fly. But then RT seems to compel him to believe that $A\rightarrow B$, in violation of the principle of conditional contradiction.

I confess that I can see no way of construing membership of a belief set that makes sense of this example. If Oscar believes both that Tweety can fly and that if Tweety is a penguin (which he might be) then he cannot, then either his belief in the former is rather weak or he strongly believes that Tweety is not a penguin. So either B doesn't belong in the belief state or A does. The issue is much clearer when we talk in terms of probabilities: if B is very probable given that A, then it does not seem reasonable to attach much probability to the prospect that if A then $\neg B$, whatever the probability of A. The fact that C is consistent with Oscar's beliefs does not therefore rule out the possibility that he believes, to a very high degree, that if A then B. The point can be generalised. Either Oscar seriously entertains the possibility that if A then ¬B, in which case he should not be disposed to believe B on learning A (why should he, unless the preservation condition is being invoked here?). Or it is not a serious possibility, in which case there is no conflict. So I find these 'counter-examples' to the Ramsey Test hypothesis unconvincing. I conclude that the case against it fails.

3. Is Preservation Worth Preserving?

If we are to persist with the quest for a conditional stronger than the material conditional then we are free to retain the Ramsey Test but must decide whether to give up Modus Ponens, the Preservation Condition or some other feature of the AGM framework invoked here. The latter include:

(1) The axioms K*1, K*2 and K*5.

(2) The assumption that any deductively closed subset of **L** represents a potential epistemic state.

(3) The assumption that **L** is governed by a (compact) Boolean relation of logical consequence, $\vdash$.

The large literature on Gärdenfors's impossibility result contains a large number of different proposals relevant to this question. I will mention only the most salient here - see Hansson 1992 for a more comprehensive survey. Note firstly that since we did not assume the axiom of Inclusion, the way out of Gärdenfors's result endorsed by both Rott (1989) and Hansson (1992) - namely rejecting this axiom - will not carry over to our example. They note that Inclusion entails, in conjunction with the Preservation condition, that revision on the supposition of the truth of some A consistent with one's current belief set *K*, amounts to expanding *K* by A, i.e. adding A to *K* and drawing all logical consequences. Both Rott and Hansson convincingly argue that this is unacceptable for belief sets containing conditionals since the supposition that A may support belief in conditionals not logically entailed by the union of *K* and {A}. But this assumption plays no role in our arguments and so rejecting it will not solve the problem.

Levi's (1996) response to Gärdenfors's impossibility result implies a rejection of assumption (2). Levi argues that conditionals are not legitimate contents of belief. Thus, while one can use the Ramsey Test hypothesis to give conditions for the acceptance of a conditional, relative to some epistemic state *K*, one should not take it as thereby giving conditions for full belief in the conditional (i.e. for adding the conditional to *K*). Levi supports this view by making two claims: that conditionals express judgements of serious possibility relative to some corpus and that modal judgements, including judgements of serious possibility, are not truth-valued. But one need accept neither of these claims in order to see the force of the objection, for there is a good deal of evidence that believing a conditional cannot be a simple matter of believing that what it says is true.[9] Perhaps it follows that we should not then speak at all of belief in conditionals. In any case, if using the term 'acceptance' causes less concern I am happy to use it.[10] This need not close off our present line of questioning, however, as we may simply give the notion of an epistemic state a different interpretation as a set of sentences accepted by an agent in the light of her beliefs, conditional or otherwise. The question remains, however: which of the assumptions used in our impossibility result is not valid for epistemic states and revisions of them?[11]

Whatever one's view about the correct interpretation of epistemic states, the axioms K*1, K*2 and K*5 appear unobjectionable. The status of Modus Ponens as a universal rule of inference, on the other hand, has been questioned before and it is notable that, just as in McGee's (1985)

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[9] See Bradley 1999 and 2000.

[10] Though frankly I find it no easier to grasp the idea of acceptance without truth conditions as I do belief.

[11] To make this move is not to deny that the distinction between belief and acceptance is important (I am in fact agnostic on this point), nor the fruitfulness of Levi's approach. It is made to allow the line of

counter-example to it, it is the application of Modus Ponens to nested conditionals that seems to cause the trouble in our example. So there are grounds for thinking that the scope of Modus Ponens should be restricted to inferences with conditionals having non-conditional consequents. Note that this would require a similar restriction on CDR, should one wish to endorse it, for it follows from CDR that if A→B belongs to *K*, then B belong to K_A^*, and from the standard axioms or revision that if A is in *K*, that $K_A^* = K$. So B must belong to *K* if A and A→B do.

I do not think however that the application of Modus Ponens is the source of the problem in our example. Whether Modus Ponens is universally valid as a form of inference or not, the inference from the supposition that the ball is not in urn C to the claim that if it's not in urn A, then it's in urn B certainly seems reasonable. If considerations of rational belief alone do not rule out this inference, then the example goes through. To strengthen the point consider this modification of our example, similar to one given by Rott (1989), which makes no use of Modus Ponens. Instead we suppose that revision is commutative in cases where the inputs are consistent with the current epistemic state and with one another. Formally:

$$\text{(COM) If } \neg A, \neg B \notin K, \neg B \notin K_A^* \text{ and } \neg A \notin K_B^* \text{ then } (K_A^*)_B^* = K_{A\&B}^*$$

COM is in fact entailed by the Preservation condition and the axiom of Inclusion, K*3, which we rejected on the grounds of counselling excessive caution. But it does not, on the face of it at least, inherit any of the objectionable features of this axiom.

As before, let {A, B, C} be a set of mutually contradictory and exhaustive sentences and suppose that an agent's initial epistemic state is given by a set *K* containing none of A, ¬A, B, ¬B, C or ¬C. Suppose also that ¬A→B belongs to $K_{A\vee B}^*$ and that ¬A→C belongs to $K_{A\vee C}^*$. Now consider

questioning to continue.

the sets $(K^*_{A\vee B})^*_A$ and $(K^*_{A\vee C})^*_A$, representing the epistemic states resulting from learning or supposing that A when in the states represented by $K^*_{A\vee B}$ and $K^*_{A\vee C}$. Since ¬A does not belong to either $K^*_{A\vee B}$ or $K^*_{A\vee C}$, it follows from PRES that ¬A→B belongs to $(K^*_{A\vee B})^*_A$ and that ¬A→C belongs to $(K^*_{A\vee C})^*_A$ and from COM that $(K^*_{A\vee B})^*_A = K^*_A$ and $(K^*_{A\vee C})^*_A = K^*_A$. Hence both ¬A→B and ¬A→C belong to K^*_A, contrary to the hypothesis that it is not permissible to believe both that ¬A→B and that ¬A→C.

Once again the only difference that the postulation of RT would make to this example would be to ensure that if K contains none of ¬A, ¬B and ¬C, then ¬A→B will belong $K^*_{A\vee B}$ and ¬A→C will belong to $K^*_{A\vee C}$, because B and C must respectively belong to K^*_B and K^*_C. But we don't need to accept RT to regard these beliefs states as reasonable ones. A modified version of the circumstances described in Example 3 would serve to illustrate this. When I learn that the prize is in either urn A or urn B (or either urn A or urn C), I might reasonably conclude that if it's not in urn A then it's in urn B (or if it's not in urn A then it's in urn C).

This time we need to decide between giving up PRES or giving up the commutativity of belief revision. Like Modus Ponens, commutativity is not sacred. It can, for instance, be violated when an agent revises her degrees of belief by Jeffrey conditionalisation.[12] But there is good reason for pointing the finger of suspicion at PRES. The problem with it is that it requires us to retain conclusions (such as ¬A→B) arrived at on the basis of partial information (that A∨B), but that we would not have reached had we known all that we do in the end (that A). When I learn that the prize is in either urn A or urn B, I infer that if it's not in A then it must be in B. But when it is subsequently revealed that the prize is in A, I learn that the truth of the claim that it was in either

A or B derived from that fact that it was in A and so my grounds for the inference that if it wasn't in A then it was in B is removed. Nonetheless PRES requires me to retain it.

If we drop PRES our problems will go away: the commutativity of revision will force the agent, when revising on A, to retract any conditionals in $K^*_{A\vee B}$ and $K^*_{A\vee C}$ that are not in K^*_A. Epistemic caution will be satisfied. But dropping PRES altogether seems too drastic, as it expresses a core feature of the concept of revision as learning or accumulation of information about the way that the world is. And, as we shall see, a probabilistic version of PRES is a cornerstone of most models of rational revision of degrees of belief (and, in particular, Bayesian conditioning). A less drastic move would be to qualify PRES so as to exclude sentences containing the conditional connective from its domain.

The suggestion that PRES be so-qualified is supported by the idea that discourse involving conditionals differs in important respects from factual discourse. Although our beliefs in conditionals may derive from what we learn of the facts, there may be no fact that we learn that is expressed by the conditional we come to believe. In our urn example, for instance, full belief in a conditional was acquired by inference, rather than by observation or reliable testimony as to the facts. In other cases, our belief in a conditional seems to have an inductive basis without it being possible to reduce what we believe to a set of learnt facts.

Example 4: *Suppose I am having trouble opening a lock on the front door, but know that I have the right key. After much fiddling, I conjecture that if I pull the door towards me while turning the key, the lock will open. After repeated trials, I am sure that I am right. But although I also believe that I am right in virtue of certain facts about the position of the door,*

[12] See Jeffrey 1992.

the state of the lock and so on, I do not know what these facts are. There are also some facts of which I am aware, in particular those relating to the outcomes of fiddling with the lock, that perhaps both caused and justified my belief that the lock will open if I pull the door while turning the key. But these facts do not logically entail that if that I pull the door towards me, the lock will open. Nor can they be regarded as constitutive of the content of the corresponding conditional sentence.

Those who regard discourse involving conditionals as different in nature to factual discourse will find little trouble with these arguments.[13] Indeed our urn example is a rather boring version of the sorts of cases used to motivate this 'non-factualist' position. Here is our transcription of Gibbard's (1981) famous argument. The prize is in urn A, but Zack and Jack do not know it. Jack has sneaked a look in urn C and seen that it is empty. Zack has sneaked a look in urn B and seen that it too is empty. Jack claims 'If the prize is not in A then it is in B'. Zack claims 'If the prize is not in A then it is in C'. By CC, Jack and Zack cannot both be right. But both have impeccable reasons for believing what they believe: their beliefs are rationally motivated and consistent with the facts. Their situations are completely symmetrical: any case that can be made for the truth or falsity of Zack's beliefs can be made for Jack's, and vice versa. So although their claims cannot both be true, it is implausible that one of them is true and the other false. The non-factualist concludes that both are neither true nor false; indeed that it is not generally sensible to ascribe a truth-value to a conditional.

The non-factualist position raises many questions, not least as to what it means to believe that if A then B if it is not to believe that it is true.[14] But for our purposes what is important is that from

[13] The core of non-factualism is the claim that conditionals do not bear truth-values because they do not express facts. See Gibbard 1981, Edgington 1995 and Levi 1996.

[14] Though perhaps the simplest answer is best: it is to believe that B is true on the supposition that A is. This makes the Ramsey Test an analytic truth.

a non-factualist perspective it natural to do what we have been recommending; namely restrict PRES to non-conditional beliefs. But this does not settle matters, as non-factualism also gives grounds for questioning the assumption that belief revision involving conditionals is always commutative. For if the beliefs we develop about conditionals cannot be equated with factual beliefs and if beliefs in conditionals need not be preserved then there is no reason to believe that the order in which information is acquired will not affect what we end up believing.

In this respect it is also worth noting that the conflicting conditional beliefs that PRES forces us to retain concern possibilities ruled out by what we have learnt. So it could be argued that the problems caused by PRES are without practical impact. Although I have no reason to retain my belief that if the prize is not in urn A then it's in urn B (or C, as the case may be), once I come to learn that A, I also have no reason to drop it. The facts require nothing of me one way or another. Only because belief revision is supposed to be path-independent does the retention of these conditional beliefs lead to trouble in the form of inconsistency. So even though the case against PRES seems strong, we should not ignore the possibility that the commutativity condition should be restricted as well (or perhaps even instead of it).

4. Revision of Degrees of Belief

Let us now examine these issues in the context of more fine-grained models that represent agents' epistemic states by measures of their degrees of belief for each possibility in some specified domain. In the interest of continuity with the coarse-grained AGM models we examined before, let us continue to represent the relevant possibilities by the language **L** closed under the Boolean relation of logical consequence, $\vdash$, satisfying CC. An epistemic state is then represented by a probability measure, P, on **L**. Revision of P by a sentence A of **L** is modelled as a mapping from

P to the probability measure P_A^* on **L**. It is assumed that P_A^*(A) = 1, that if P(A) = 1 then P_A^* = P, and that if A and B are logically equivalent, then $P_A^* = P_B^*$.[15]

A number of different formulations of the Ramsey Test idea are possible in this framework, including a conservative transcription of RT and RT$^-$ that takes belief in a sentence X to be a matter of assigning probability one to its truth (and disbelief a matter of assignment of probability zero). This gives:

(RT) $P_A^*(X) = 1 \Rightarrow P(A \to X) = 1$

(RT$^-$) $P_A^*(X) = 0 \Rightarrow P(A \to X) = 0$

RT and RT$^-$ are rather weak and seemingly uncontroversial, as long as the mode in which A is supposed true is appropriate for the kind of conditional being tested. A much more interesting probabilistic version of the Ramsey Test hypothesis, which implies both RT and RT$^-$, is the following:

(PRT) $P(A \to X) = P_A^*(X)$

PRT is no mere transcription of RT: it considerably tightens the postulated connection between the credibility of conditionals and the outcomes of belief revision. Like RT, however, PRT is consistent with recognition of the existence of more than one kind of conditional in ordinary language and more than one way of revising beliefs. Once again, we might read PRT as a *schema* for relating conditionals to belief revision: perhaps indicative conditionals to the kind of revision of degrees of belief characteristic of matter-of-fact supposition and subjunctive conditionals to that involved in contrary-to-fact supposition.

[15] I will not assume, however, that probabilistic revision satisfies the other axioms proposed by Gärdenfors (1998).

Let us turn now to the formulation of preservation conditions in this environment. A weak probabilistic analogue of our preservation condition, PRES, is the following principle of preserving certainties:

(*Pres-Cert*) $P(A) > 0,\ P(X) = 1 \Rightarrow P_A^*(X) = 1$.

Not all notions of minimal probabilistic revision satisfy *Pres-Cert*, but we will confine our attention to those that do.[16] *Pres-Cert* is a hallmark of revision by accumulation of information and it is essential to the sort of matter-of-fact supposition we appear to perform when evaluating indicative conditionals. It would also seem to apply to the sort of supposition involved in evaluating future-orientated subjunctive conditionals, like the one in Example 3. For were it certain that the company will lose money, then it would both be certain that it will lose money, *given* that my friend will buy shares in it, and that it would lose money *were* he to buy shares in it.

The most common account of minimal revision in the probabilistic context is that implicit in Bayesian theories of conditioning. Bayesian revision on a sentence A not only satisfies *Pres-Cert*, it also leaves all the probabilities conditional on A unchanged. Indeed, as Richard Jeffrey (1992) points out, satisfaction of the following stronger preservation condition is both necessary and sufficient for the validity of Bayesian conditioning:

(*Pres-CP*) $P(A) > 0 \Rightarrow P_A^*(X \mid A) = P(X \mid A)$.

When revision is a matter of Bayesian conditioning the probabilistic Ramsey test, PRT, implies the famous thesis of Ernest Adams (1975) that the probability of a conditional is the conditional probability of its consequent given its antecedent (though, of course, Adams's claim does not depend on the assumption that conditioning is the correct way to revise beliefs). Adams's thesis is massively supported by the empirical evidence relating to our actual use of indicative

16 In particular, Lewis's (1976) notion of imaging does not. See Gärdenfors 1988 and Levi 1996 for a

conditionals. On the other hand, it has also been the subject of a host of 'triviality' results, many of which may be viewed as probabilistic analogues of Gärdenfors's impossibility result for RT. But although the consensus seems to be that there are 'few philosophical theses that have been more decisively refuted' (Joyce 1999, p. 191) than Adams's Thesis, I think most interpreters of these results miss the essential point. For it is possible to produce a triviality result for any proposed theory of conditionals that implies satisfaction of the following principle:

(Cond-Cert) If $P(A) > 0$ then:

(a) $P(X) = 1 \Rightarrow P(A \to X) = 1$

(b) $P(X) = 0 \Rightarrow P(A \to X) = 0$

Triviality Theorem*: Assume Cond-Cert. Then:*

$P(A|X) > 0, P(A|\neg X) > 0 \Rightarrow P(A \to X) = P(X)$

Proof*: By the additivity of probabilities,* $P(A \to X) = P(X(A \to X)) + P(\neg X(A \to X)) = P(A \to X|X).P(X) + P(A \to X|\neg X).P(\neg X)$*. But* $P(\ |X)$ *is an epistemic state and* $P(X|X) = 1$ *and* $P(X|\neg X) = 0$*. So by Cond-Cert, if* $P(A|X) > 0$ *and* $P(A|\neg X) > 0$*, then* $P(A \to X|X) = 1$ *and* $P(A \to X|\neg X) = 0$ *. Hence* $P(A \to X) = P(X)$*.*

Our theorem shows that there are no non-trivial interpretations of → consistent with *Cond-Cert*. But while *Cond-Cert* may not hold for counterfactual conditionals, its validity with regard to indicative conditionals seems very hard to deny. The only alternative, however, is to question the assumption that degrees of belief in conditionals are classical probabilities – in particular that, for all Y, $P(A \to X) = P((A \to X)\&Y) + P((A \to X)\&\neg Y)$. So let us consider where *Cond-Cert* might go astray. Notice firstly that *Cond-Cert* is an immediate consequence of RT and *Pres-Cert*, so that one of these must be given up if the triviality result is to be avoided. But any temptation to attribute the source of the triviality to RT should, I hope, have been dispelled by our earlier

discussion.

discussion of the non-probabilistic versions of Ramsey test. At the risk of labouring the point, however, let me rehearse the argument in the probabilistic context using the urn example.

Suppose that probability function P measures my initial degrees of belief and that probabilities Q and R respectively measure my degrees of belief after learning that the prize is in either urn A or urn B, and after learning that the prize is in either urn A or urn C, i.e that $Q = P^*_{A \vee B}$ and $R = P^*_{A \vee C}$. Suppose that $Q(\neg A \to B) = 1$ and $R(\neg A \to C) = 1$. If I then learn that the prize in urn A, it will follow from *Pres-Cert* that $Q^*_A(\neg A \to B) = 1$ and that $R^*_A(\neg A \to C) = 1$. Assuming that probabilistic revision is commutative, it then follows that $Q^*_A = (P^*_{A \vee B})^*_A = P^*_A = (P^*_{A \vee C})^*_A = R^*_A$ and hence both that $P^*_A(\neg A \to B) = 1$ and that $P^*_A(\neg A \to C) = 1$. But this is in violation of CC, the principle of conditional contradiction.

As with the non-probabilistic version of this example, the crucial assumption is that it is reasonable to infer from the fact that the prize is in either urn A or urn B, to the conclusion that if it is not in urn A then it is in urn B. This is what allows us to dispense with RT in deriving a violation of CC. It is true that RT obliges us to reason this way in this case, since $Q^*_{\neg A}(B) = 1$. But so does *Cond-Cert* alone, on the natural assumption that A→B is equivalent to A→AB. For then if you learn that A∨B, it follows from *Cond-Cert* that *Q*(¬A→(A∨B)) = 1, and hence that *Q*(¬A→¬A(A∨B)) = *Q*(¬A→B) = 1. In any case, our argument requires only that such reasoning is not unreasonable and not that it is obligatory. This strongly suggests that the probabilistic Ramsey Test hypothesis is not the source of our difficulties.

In the non-probabilistic case we were inclined to conclude that the preservation condition PRES needed qualification so as to exclude conditionals from its domain of application. But this move seems insufficient in the probabilistic case. For notice that in our urn example, Q(B|¬A) = $P^*_{A\vee B}$(B|¬A) = 1 and R(B|¬A) = $P^*_{A\vee c}$(B|¬A) = 0. Now consider what happens in the two cases as the probability of the prize being in urn A rises. Here a natural generalisation of the principle of preserving certainties (even one restricted to factual beliefs) will require that the probabilities of A∨B and A∨C remain at one. Hence my conditional probabilities for B given ¬A remain unchanged (at one and zero in the two cases) as the probability of A goes to one. So assuming the quantities in question are defined, Q^*_A(B|¬A) should equal Q(B|¬A) and R^*_A(B|¬A) should equal R(B|¬A). But $Q^*_A = (P^*_{A\vee B})^*_A = P^*_A = R^*_A$, and so we are forced to conclude that P^*_A(B|¬A) = 1 and that P^*_A(B|¬A) = 0. A straightforward contradiction! Even if one wishes to deny that Q^*_A(B|¬A) and R^*_A(B|¬A) are well-defined, one is left with a clear tension between the idea that the two beliefs states initially measured by the functions Q and R should converge as the probability of A goes to one and the fact that the two corresponding conditional probabilities for B given ¬A should fail to converge at all.

To avoid these unpalatable consequences for the revision of conditional degrees of belief, we would need to not just qualify *Pres-Cert* but abandon it altogether. A similar conclusion can be drawn from our triviality theorem. For notice that to derive *Cond-Cert* from RT we only require that *Pres-Cert* applies to non-conditional sentences. So the triviality theorem would be left untouched by a mere qualification of the probabilistic preservation condition so as to exclude application to sentences containing the conditional connective. But jettisoning *Pres-Cert* altogether is a very heavy price to pay: it is at the heart of our best theories of how to revise our beliefs in the light of evidence. It seems that we are forced to direct our attention at the two other crucial assumptions at work in our arguments: that the logic of conditionals is classical and that belief revision is commutative. The former was invoked in our triviality result; the latter in our urn example. That both of these might prove to be untenable, irrespective of the status of the Ramsey Test hypothesis is a rather surprising outcome of our investigations.

5. Conclusion

I have argued that the various impossibility and triviality results directed at the Ramsey Test hypothesis in its different manifestations have, in fact, very little significance for its validity. For all of the results are reproducible to a large extent without invoking it. This leaves us free to adopt the Ramsey Test hypothesis as a guide to our investigations of rational belief in conditionals. On the other hand, these results do have significant implications for accepted theories of rational belief and belief revision. For they show that many of the properties postulated by these theories are not displayed by revisions involving conditionals or indeed by revisions of conditional beliefs.[17]

6. Appendix

Let K_A^+ denote the closure of $K \cup \{A\}$ under the consequence relation. Then:

(K*1) If $A \in \mathbf{L}$, then K_A^* is an epistemic state.

(K*2) $A \in K_A^*$ *(Success)*

(K*3) $K_A^* \subseteq K_A^+$ *(Inclusion)*

(K*4) If $\neg A \notin K$, then $K_A^+ \subseteq K_A^*$ *(Preservation)*

(K*5) $K_A^* = K_\perp^* \Leftrightarrow \vdash \neg A$

(K*6) If $A \vdash B$ and $B \vdash A$, then $K_A^* = K_B^*$

(K*7) $K^*_{AB} \subseteq (K^*_A)^+_B$

(K*8) If $\neg B \notin K^*_A$, then $(K^*_A)^+_B \subseteq K^*_{AB}$ *(Restricted Weakening)*

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[17] This paper has benefited from comments from Frank Döring and Wlodek Rabinowicz on earlier drafts.

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