# MEI LT Problem Set $1^{1}$ 

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${ }^{1}$ Available on http://personal.Ise.ac.uk/carayolt/ec402.htm

## Question 1

## Question

- Want to estimate the effect of some treatment $T$, on the outcome $Y$. $T$ takes value 1 for the treated, 0 for the others, with $\operatorname{Pr}(T=1)=p$.
- A. Consider the regression: $Y_{i}=\alpha+\beta T_{i}+\varepsilon_{i}$. Under what conditions can you interpret $\hat{\beta}$ as the causal effect of $T$ on $Y$ ?


## Question 1

## Answer

- To interpret $\hat{\beta}$ as the causal effect of $T$, we need $A 3 R m i$, i.e. here $E(\varepsilon \mid T)=0$. This means that there are no omitted variable correlated with both the treatment assignment $T$ and the outcome $Y$.
- This plays the same role as the CIA (or more appropriately in this case UNconditional independence assumption, i.e. random assignment) here, which would imply $E\left(Y_{0 i} \mid T_{i}=1\right)=E\left(Y_{0 i} \mid T_{i}=0\right)$ implying $E\left(\varepsilon \mid T_{i}=1\right)=E\left(\varepsilon \mid T_{i}=0\right)$, which is basically A3Rmi. So the CIA also imply a causal interpretation for $\hat{\beta}$.


## Question 1

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B. Suppose the true average treatment effect $E\left(Y_{1 i}-Y_{0 i}\right)=\beta$. We estimate it with $b=E\left(Y_{i} \mid T_{i}=1\right)-E\left(Y_{i} \mid T_{i}=0\right)$ (the sample counterpart of which corresponds to OLS on $\left.Y_{i}=a+b T_{i}+e_{i}\right)$. How is $b$ related to $\beta$ ?

Answer

$$
\begin{aligned}
b & =E\left(Y_{i} \mid T_{i}=1\right)-E\left(Y_{i} \mid T_{i}=0\right) \\
& =E\left(Y_{1 i}-Y_{0 i} \mid T_{i}=1\right)+E\left(Y_{0 i} \mid T_{i}=1\right)-E\left(Y_{0 i} \mid T_{i}=0\right) \\
& =E\left(Y_{1 i}-Y_{0 i}\right) \frac{1}{p}-E\left(Y_{1 i}-Y_{0 i} \mid T=0\right) \frac{1-p}{p}+E\left(Y_{0 i} \mid T_{i}=1\right)-E\left(Y_{0 i} \mid T_{i}=0\right) \\
& =\beta \frac{1}{p}-E\left(Y_{1 i}-Y_{0 i} \mid T=0\right) \frac{1-p}{p}+E\left(Y_{0 i} \mid T_{i}=1\right)-E\left(Y_{0 i} \mid T_{i}=0\right)
\end{aligned}
$$

## Question 1

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C. Suppose we had an omitted variable W from the regression so that the true model was: $Y=\alpha+\beta T+\gamma W+\varepsilon$ but you estimated $Y=a+b T+e$. Write an expression for the bias in the OLS estimate. If you know that $\gamma>0$, is that sufficient to sign the bias?

Answer
You must remember the formula for the OVB (omitted variable bias) when there is only one regressor (+intercept) and one omitted variable: $b=\beta+\gamma \frac{\operatorname{cov}(W, T)}{\operatorname{Var}(T)}$. So the sign of $\gamma$ is not sufficient to sign the OVB: we also need to know the sign of $\operatorname{cov}(W, T)$.

## Question 1

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D. Suppose you want to know the causal effect of a treatment $T$ on the outcome of Y for a set of individuals. The treatment is randomly assigned and there is perfect compliance. How will the treatment on the treated (TOT) estimate compare to the average treatment effect (ATE).

## Answer

If the treatment is randomly assigned, then the treated do not differ from the non-treated, implying
$T O T=E\left(Y_{1 i}-Y_{0 i} \mid T=1\right)=E\left(Y_{1 i}-Y_{0 i}\right)=A T E$.

## Question 2

## Question

We would like to identify the causal effect of income on health; but for reasons we discuss in this exercise, this identification is made difficult by the existence of a number of confounding factors, some of which are unobservable. Consider the true model:
Health ${ }_{i}=\beta_{0}+\beta_{1}$ Inc $_{i}+\beta_{2} E d u_{i}+\beta_{3}$ Age $_{i}+\beta_{4}$ Occ $_{i}+\beta_{5}$ Mar $_{i}+\varepsilon_{i}$.

## Question 2

## Question

A. Want to discuss the probable sign of the omitted variable bias induced by the omission of different variables. (I omit sex and ethnicity, as predictions are unclear and unessential.)

## Answer

- In each case: need to think of the formula for the OVB, i.e. correlation between omitted variable and income; and between omitted variable and health. (Controlling for the other ones.)
- Education
- Occupation
- Age
- Marital status


## Question 2

## Question

B. Some scholars have used cross-country comparisons between the US and UK to address some of these concerns. The argument they provide is that the main excluded factor is access to medical care, which varies dramatically by education and income in the US. In the US, the NHS provides free insurance so this is not a problem. What is the assumption such a strategy would make to "identify" the causal relationship of income on health? Does this make sense?

Answer
Class discussion

