MEI LT Problem Set 2[1]

Timothee Carayol

February 5, 2010

[1] Available on http://personal.lse.ac.uk/carayolt/ec402.htm

Question 1 - A

Intro

- True model: $y_i = \beta + R_i \alpha_i + u_i$ where:
  - y_i: outcome for individual i (here: probability of reoffense)
  - R_i: whether individual i is treated (here: treated = "coddled"
  - α_i: treatment effect for individual i
  - u_i: other individual specific characteristics
- TOT = Treatment effect on the treated, i.e. we are interested in $\bar{\alpha}_{TOT} = E(\alpha_i | R_i = 1)$
- Denoting $\bar{\alpha}_{ATE} = E(\alpha_i)$ and defining ϵ_i such that $E(\epsilon_i) = 0$ and $\alpha_i = \bar{\alpha}_{ATE} + \epsilon_i$ (i.e., ϵ_i is the individual deviation from the average treatment effect), we therefore obtain $\bar{\alpha}_{TOT} = \bar{\alpha}_{ATE} + E(\epsilon_i | R_i = 1)$.

Question 1 - A

Intro

- Going back to model: $y_i = \beta + R_i\bar{\alpha}_{ATE} + u_i + R_i\epsilon_i$.
- $E(y_i|R_i = 1) = \beta + \bar{\alpha}_{ATE} + E(u_i|R_i = 1) + E(\epsilon_i|R_i = 1)$
- $E(y_i|R_i = 0) = \beta + E(u_i|R_i = 0)$
- Can show that $E(\hat{\alpha}_OLS) = E(y_i|R_i = 1) - E(y_i|R_i = 0) = \bar{\alpha}_{ATE} + (E(u_i|R_i = 1) - E(u_i|R_i = 0)) + E(\epsilon_i|R_i = 1)$
- Two sources of bias if what we want was ATE: from ϵ (heterogenous treatment: what we get is TOT rather than ATE); and from u_i: maybe treatment is correlated with some unobserved characteristics. (If what we want is TOT, or if treatment is homogenous, the latter is the only bias).

Question

Do you think we have a problem here if some policemen forget their notepads?

Question 1 - B and C

Answer

- A bit of Stata
- Vocabulary: what are always takers, compliers, never takers, defiers?
- We see almost no always takers (coddled when should have been arrested), but quite a few never takers.
- In what case would this be a concern?
- We see that reasons for non-compliance are often correlated with the seriousness of the situation.
- Introduces an (omitted variable) bias in our OLS: less violent individuals will be over-represented in the treatment group (coddle), and will have a lower probability of reoffense. Hence our estimate will under-estimate the effect of the treatment (coddling).

Question 2

Intro

- A quick note on the Wald estimator.
- The Wald estimator is a special case of instrumental variable estimator, where the instrument we use is binary.
- A quick example to show how this can solve our compliance problem in our context, supposing an homogenous treatment effect α: $y_i = \beta + \alpha R_i + \epsilon_i$.
- Problem: for non-compliance reasons, $E(\epsilon_i|R_i) \neq 0$; however, for randomization reasons, $E(\epsilon_i|T_i) = 0$.

Question 2

Intro

- Then we have: $E(Y_i|T_i = 1) = \beta + \alpha E(R_i|T_i = 1) + E(\epsilon_i|T_i = 1) = \beta + \alpha E(R_i|T_i = 1)$; likewise $E(Y_i|T_i = 0) = \beta + \alpha E(R_i|T_i = 0) + E(\epsilon_i|T_i = 0) = \beta + \alpha E(R_i|T_i = 0)$.
- Hence: $E(Y_i|T_i = 1) - E(Y_i|T_i = 0) = \alpha\left(E(R_i|T_i = 1) - E(R_i|T_i = 0)\right)$, or $\alpha = \frac{E(Y_i|T_i=1)-E(Y_i|T_i=0)}{E(R_i|T_i=1)-E(R_i|T_i=0)} = \frac{ITT}{Compliance}$. This is the Wald estimator, which coincides with the IV estimator where we instrument the endogenous variable R by the exogenous instrument T. Using IV vocabulary: ITT, the numerator, is the reduced form, and the denominator is the first stage.
- Solves selective non-compliance problem. If there is heterogeneity in the treatment effect, then this retrieves, instead of α, $E(\alpha_i|R_{1i} > R_{0i}$.

Question 2 - A

Question

Show that in presence of non-compliance, the ITT is smaller (in absolute value) than ATE.

Answer

- Start with intuition:
  - Extreme case: complete non-compliance. Same likelihood of receiving treatment whichever group you have been assigned to. Intention-to-treat has no effect at all.
  - Milder cases: some selected people are never takers (arrested no matter what). They are likely to have higher probability of reoffense than average. An ITT estimator will mistake some of those for "coddle" individuals, thus diluting the estimated effect, and will therefore underestimate the effect of the treatment.

Question 2 - A

Answer

- Define $p_a = P(Always\ taker) = P(R = 1|T = 0)$ and $p_n = P(Never\ taker) = P(R = 0|T = 1)$, implying $1 - p_n = P(R = 1|T = 1)$ and $1 - p_a = P(R = 0|T = 0)$

$$\begin{aligned} ITT &= E(y_i|T_i = 1) - E(y_i|T_i = 0) \\ &= E(y_i|T_i = 1 \cap R_i = 1)(1 - p_n) + E(y_i|T_i = 1 \cap R_i = 0)p_n \\ &\ - E(y_i|T_i = 0 \cap R_i = 1)p_a - E(y_i|T_i = 0 \cap R_i = 0)(1 - p_a) \\ &= E(y_i|T_i = 1 \cap R_i = 1) - E(y_i|T_i = 0 \cap R_i = 0) - p_n N - p_a A \end{aligned}$$

- where $N = E(y_i|T_i = 1 \cap R_i = 1) - E(y_i|T_i = 1 \cap R_i = 0)$ and $A = E(y_i|T_i = 0 \cap R_i = 1) - E(y_i|T_i = 0 \cap R_i = 0)$. What do you think?

Question 2 - B

Question

Show that TOT is a weighted average of two effects: one on always-takers and one on compliers.

Answer

- By definition: $TOT = E(y_1 - y_0|R = 1)$ where y_1 and y_0 are the (counterfactual) possible outcomes. I omit i index for convenience.
- Pose $p = P(T = 1|R = 1)$, so that $1 - p = P(T = 0|R = 1)$. Conditional on $R = 1$, fraction $1 - p$ of always-takers.

Question 2 - B

Answer

- $TOT = E(y_1 - y_0 | T = 1 \cap R = 1)p + E(y_1 - y_0 | T = 0 \cap R = 1)(1 - p)$
- First term: effect of treatment on people who are treated and were assigned to the treatment group; second term: effect of treatment on those who are treated but were NOT assigned to the treatment group (always-takers).
- So TOT includes some effect from always-takers, contrary to $LATE$ which only estimates the effect on compliers. (If $p = 1$, i.e. no always-taker, then they coincide.)

Question 2 - C

Question

In our case there are only few always-takers. What does it imply for TOT and LATE?

Answer

We had found that there were almost no always-takers: that means that TOT and $LATE$ should be very close. Our IV estimate for $LATE$ can therefore be expected to be safely interpreted as a TOT effect.

Question 3 - A B C D

- Stata.

Question 4 - A B C

- Class discussion.