# MEI LT Problem Set $3^{1}$ 

Timothee Carayol

February 11, 2010
${ }^{1}$ Available on http://personal.Ise.ac.uk/carayolt/ec402.htm

## Overview

- Today's exercise covers fixed effects, Difference-in-Differences and Propensity Score Matching estimators.
- Three ways to deal with non-experimental situations (i.e. situations where we do not have a randomly assigned comparison group to which we can compare the outcome of the treatment group).


## Overview

- Widely used benchmark: the NSW data.
- Was a randomized experiment in the 70's (even though those were quite rare at the time).
- Provided work experience to recipients with weak labour force attachment.
- Several microeconometricians tried to replicate the results without using the experimental control group, but instead trying to construct another (nonexperimental) control group from national datasets.
- Lalonde (1986), Dehejia and Wahba (1999 and 2002), Smith and Todd (2001), Angrist and Pischke (2009) used this data to evaluate the performance, e.g., of propensity score matching.
- We will do the same thing today.


## Question 1

- $Y_{i t}=x_{i t}^{\prime} \beta+\epsilon_{i t}$.
- Assume $\epsilon_{i t}$ comprises a time-specific part $\delta_{t}$ and and error term $\mu_{i t}$ such that $E\left(\delta_{t}\right)=0$ and $E\left(\mu_{i} t \mid x_{i t}, \delta_{t}\right)=0$.
- Essentially:
- Random effect is GLS using $\epsilon_{i t}$ as the error term and using its decomposition to find $\Omega$. Problem: for GLS to be consistent, we need some sort of $A 3$, i.e. need to have $\delta_{t}$ uncorrelated with $x_{i t}$ (let's call this A3RE here).
- Fixed effect identifies the effect within time period (here) or individual (in most cases involving panel data). Then we do not need the time-specific error term to be uncorrelated with the observables: in effect, we control for it.
- So: FE consistency requires fewer assumptions than RE. Can be the case that FE is consistent but not RE; but not the opposite. However, if RE is consistent, then it is efficient, contrary to FE.


## Question 1 - A

- To check whether $\delta_{t}$ is indeed uncorrelated with $x_{i t}$ (A3RE), i.e. whether RE estimation is consistent: can run both RE and FE and compare estimates.
- If A3RE is correct, both RE and FE will be consistent, so estimates should not differ "too much".
- There is a formal test to see how much is "too much": the Hausman test. (This test can be used in any context where we have two estimators which are both consistent under $H_{0}$, one of which is efficient under $H_{0}$.)
- Can show that under $H_{0}: A 3 R E$, $H=\left(\beta_{R E}-\beta_{F E}\right)^{\prime}\left(a \hat{\operatorname{Var}}\left(\beta_{F E}\right)-a \hat{\operatorname{Var}}\left(\beta_{R E}\right)\right)^{-1}\left(\beta_{R E}-\beta_{F E}\right) \rightarrow_{d}$ $\chi^{2}(K)$.


## Question 1 - B

- Difference from mean: substract the averaged (by time period) equation from the original equation.
- Averaged: $\overline{Y_{t}}={\overline{x_{t}}}^{\prime} \beta+\delta_{t}+\overline{\mu_{t}}$.
- Difference from the mean: $Y_{i t}-\overline{Y_{t}}=\left(x_{i t}-\overline{x_{t}}\right)^{\prime} \beta+\left(\mu_{i t}-\overline{\mu_{t}}\right)$.
- This is then a well behaved model which we can estimate consistently with OLS.
- Keep in mind that when we use differences from the mean, we lose all the "between" variations: we only look at variations "within" a time period (here) or an individual (usually with panel data). RE, by contrast, uses all the variations.
- For it to make sense to difference from the mean, our initial specification needs to be correct; in particular:
- The $\beta^{\prime}$ 's must not differ accross individuals or time.
- The time-specific error component must take the same value for all individuals at each $t$.


## Question 1 - C

- How to check whether $\beta$ does not vary with $t$ ? (i.e. the effects of the different variables do not change over the time)
- Can run one regression for each $t$ (or one big regression allowing for different coefficients for different periods) and then test whether $\beta_{1}=\beta_{2}=. .=\beta_{T}$.


## Question 2 - Difference in Differences

- A - Checking whether the experimental assignment really was random.
- We cannot reject equality of the observables between treatment and control group for most variables.
- Not so clear for the "no degree" variable. Seems to be more dropouts in the control group.
- Overall, looks acceptably good.


## Question 2 - Difference in Differences

- B - Replicating the experiment using the randomly assigned control group (from the original experiment).
- In both cases: some positive effect of the treatment, although not significant at the $5 \%$ level.
- Since it is randomly assigned, there should not be any OVB, so omitting the observables should not bias the results.
- However, it will increase the R2 (or equivalently reduce the residual variance), hence making the estimator for the treatment effect (slightly) more precise.


## Question 2 - Difference in Differences

- C,D - Difference-in-Differences
- Let us pretend we do not have a randomly assigned control group (as would typically be the case using non-experimental data).
- Then we need to find a clever identification strategy that does not rely on randomization.
- Diff-in-diff is one such strategy.
- We no longer need the individuals to look the same in the treatment and control group, nor their counterfactual $y_{0}$ and $y_{1}$ to have the same expected value.
- It only requires the following assumption: if the treatment group had not been treated, the difference in outcomes would have remained the same over the period considered. I.e., the difference in the evolution ("difference-in-difference") is due solely to the effect of the treatment.
- Stata: find a much smaller treatment effect. Why?


## Question 2 - Difference in Differences

- E - Checking our diff-in-diff assumption by looking at the pretrend ( 74 to 75 ).
- If our identifying assumption is true (i.e., that in the absence of treatment both groups have the same trend in outcomes), then we should find nothing here.
- We do find a significant difference in the pretrends across groups, i.e. diff-in-diff is probably biased.


## Question 3 - Propensity Score Matching

- Alternatives: need to rely on CIA, i.e. assignment is random conditional on a set of observables.
- We could match on all "suspicious" observables $X_{i}$ and compare outcomes for individuals with "similar" $X_{i}$ 's.
- Alternatively, can rely on propensity score, i.e. $p\left(X_{i}\right)=E\left(D_{i} \mid X_{i}\right)$.
- Fondamental theorem:
- Suppose the CIA holds, i.e $\left\{y_{0 i}, y_{1 i}\right\}$ is independent on $D_{i}$ conditional on $X_{i}$. Then it is also the case that $\left\{y_{0 i}, y_{1 i}\right\}$ is independent on $D_{i}$ conditional on $p\left(X_{i}\right)$.
- (This answers section B of this question.)
- Idea of propensity score matching: simply match individuals on the basis of the $p\left(X_{i}\right)$ instead of on the full $X_{i}$ 's.


## Question 3 - Propensity Score Matching

- A - Let $M=\{0, .1, .2, . ., .9\}$. For $m \in M$ and individual $i$, denote $p_{m, i}$ a dummy that takes value 1 iff $m \leqq p\left(X_{i}\right) \leqq m+.1$.
- Then, for example, if $p\left(X_{i}\right)=.85$, then $p_{8, i}=1$.
- Then we could run

$$
y_{i}=\sum_{m \in M} \pi_{m} p_{m, i}+\sum_{m \in M} \theta_{m}\left(p_{m, i} D_{i}\right)+\epsilon_{i}
$$

- In that case, the $\theta_{m}$ 's would identify the treatment effect for individuals with the considered propensity score.


## Question 3 - Propensity Score Matching

- C - The specification in A is quite flexible, as it allows for different treatment effects across propensity scores.


## Question 4 - Propensity Score Matching with Data

- A, B, C, D - Stata

