EC402: MEI. Spring Term: The Analysis of Time Series*

Problem Set 2

1. Hypothesis test

1. Consider the $MA(1)$ model

$$y_t = \varepsilon_t + \theta\varepsilon_{t-1}, \quad \varepsilon_t \sim iidN\left(0, \sigma^2\right), \quad t = 1, ..., T, \quad \varepsilon_0$$

   1. Derive the LM test of the null $\theta = 0$
   2. What is the advantages of the LM test over the LR test in this case?

2. Consider the model

$$y_t = \beta_1 x_{1t} + \beta_2 \frac{(x_{1t} - \gamma)^{-2}}{2} + \varepsilon_t$$

   where $\varepsilon_t \sim iidN(0, \sigma^2)$ and the regressors x_1 and x_2 are process independent of the errors.

   1. Obtain the log likelihood of the model and outline how you would obtain the NLE of the parameters $\beta_1,\ \beta_2,\ \gamma,\ \sigma^2$.
   2. Construct a LM test of the null $H_0 : \gamma = 0$.
   3. Why is the LM test easier to carry out than the Likelihood ratio test?

2. Distributed lags and transformations

1. Rewrite the distributed lag

$$B(L)\, x_t = \beta_0 x_t + \beta_1 x_{t-1} + ... + \beta_m x_{t-m}$$

in the form

$$\delta_0 x_t + \delta_1 \Delta x_{t-1} + \delta_2 \Delta x_{t-2} + ... + \delta_m \Delta x_{t-m+1}$$

   1. Show that $\delta_0 = B(1)$, the total effect. What is the relationship between the δ's and the β's?

   2. If you write the second form as

$$\delta_0^+ x_{t-1} + \delta_1^+ \Delta x_t + \delta_2^+ \Delta x_{t-1} + ... + \delta_m^+ \Delta x_{t-m+1}$$

   how are the δ^+'s related to the β's?

2. Consider the Autoregressive Distributed Lag Model

$$A(L)\, y_t = \lambda + B(L)\, x_t + \varepsilon_t$$

where

$$\begin{aligned} A(L) &= 1 - \alpha_1 L - ... - \alpha_m L^m \\ B(L) &= \beta_0 + \beta_1 L + ... + \alpha_n L^n. \end{aligned}$$

   1. Show that you can rewrite this model in the error correction form

$$\Delta y_t = \lambda + \gamma_0 y_{t-1} + \sum_{i=1}^{m-1} \gamma_i \Delta y_{t-i} + \delta_0 x_{t-1} + \sum_{i=1}^{n} \delta_i \Delta x_{t-i+1} + \varepsilon_t.$$

   What is the interpretation of γ_0 and $\delta_0$and hence of δ_0/γ_0? [Hint: use the results you derived in the first question of this section]

   2. If you estimate the original model and the model in error correction form, what is the relationship between the two sets of estimates and the two sets of residuals?