EC402: MEI. Spring Term: The Analysis of Time Series*

Problem Set 1

1. Regression with $MA(1)$ errors

Consider a regression model with $MA\,(1)$ distrurbance term

$$\begin{aligned} y_t &= x_t'\beta + u_t \\ u_t &= \varepsilon_t + \theta\varepsilon_{t-1}, \quad t = 1, ..., T \end{aligned} \tag{1.1}$$

where $\varepsilon_t \sim iid\,(0, \sigma^2)$ with $\varepsilon_0 = 0$ and x_t non-stochastic.

1. Derive an expression for the covariance matrix, $\sigma^2\Omega$, of the vector of disturbances $u = (u_1, ..., u_T)'$ in terms of θ.

2. By using $\varepsilon_t = u_t - \theta\varepsilon_{t-1}$ recursively, starting from $\varepsilon_0 = 0$, $\varepsilon_1 = u_1$, find the lower triangular matrix L such that $\varepsilon = Lu$ where $\varepsilon = (\varepsilon_1, ..., \varepsilon_T)'$.

3. Assume θ is known. Noting that $Var\,(Lu) = \sigma^2 I$, determine a method for computing the best linear unbiased estimator of β which does not require the construction and inversion of Ω.

2. $F-GLS$ estimation of the regression model with $AR(1)$ errors

Consider the model

$$\begin{aligned} y_t &= x_t'\beta + u_t \\ u_t &= \phi u_{t-1} + \varepsilon_t \quad t=1,...,T \end{aligned} \tag{2.1}$$

where $|\phi|<1$, $\varepsilon_t \sim iid\left(0,\sigma^2\right)$, x_t' and ε_t are process independent and

$$p\lim \frac{1}{T}\sum_{t=1}^{T} x_t x_t' = \Sigma_{xx}$$

and Σ_{xx} is non-singular.

1. You estimate (2.1) by OLS and obtain the residual $\hat{u}_t$. You estimate ϕ by
$$\hat{\phi} = \frac{\sum_{t=2}^{T} \hat{u}_t \hat{u}_{t-1}}{\sum_{t=2}^{T} \hat{u}_{t-1}^2}.$$
Assuming that $\hat{\beta}_{OLS}$ is consistent, show that $\hat{\phi}$ is a consistent estimator of ϕ.

3. Maximum Likelihood of the ARMA(1,1)

Consider the $ARMA(1,1)$ process

$$y_t = \phi y_{t-1} + \varepsilon_t + \theta\varepsilon_{t-1} \quad t=1,...,T$$

where $|\phi|<1$, $\varepsilon_t \sim iidN\left(0,\sigma^2\right)$

1. Assuming, $y_1 = \varepsilon_1 = 0$ write down the log likelihood.
2. Obtain the FOCs wrt ϕ and θ.
3. Obtain
$$\frac{1}{T} I(\psi)$$
where $\psi := \left[\phi,\theta,\sigma^2\right]'$.
4. In finite sample we can approximate the distribution of $\left(\hat{\psi}-\psi_0\right)$ as mean zero normal with variance depending on $I\left(\hat{\psi}\right)$. How would you estimate the covariance matrix?