The Price Impact of Institutional Herding^{*}

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Abstract

In this paper we develop a simple theoretical model to analyze the impact of institutional herding on asset prices. A growing empirical literature has come to the intriguing conclusion that institutional herding positively predicts short-term returns but negatively predicts long-term returns. We offer a theoretical resolution to this dichotomy. In our model, career-concerned money managers interact with profit-motivated proprietary traders and security dealers endowed with market power. The reputational concerns of fund managers imply an endogenous tendency to imitate past trades, which impacts the prices of the assets they trade. We show that institutional herding positively predicts short-term returns but negatively predicts long-term returns. In addition, our paper generates several new testable predictions linking institutional herding, trade volume, and the time-series properties of stock returns.

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1 Introduction

Professional money managers are the majority owners and traders of equity in today's markets. Leading market observers commonly allege that money managers "herd" and that such herding destabilizes markets and distorts prices. For example, Jean-Claude Trichet, President of the European Central Bank, commented on the incentives and behavior of fund managers as follows: "Some operators have come to the conclusion that it is better to be wrong along with everybody else, rather than take the risk of being right, or wrong, alone... By its nature, trend following amplifies the imbalance that may at some point affect a market, potentially leading to vicious circles of price adjustments and liquidation of positions."¹

There is extensive empirical evidence of herding by institutional investors: money managers tend to trade excessively in the direction of the recent trades of other managers.² However, the literature has reached less clear conclusions regarding the impact of institutional herding on stock prices. In fact, the empirical conclusions on the price impact of institutional herding are characterized by an intriguing dichotomy. Studies examining the short-term impact of institutional trade generally find that herding has a stabilizing effect on prices. In contrast, studies focusing on longer horizons often find that herding predicts reversals in returns, thus providing empirical evidence in favor of Trichet's view.³

The theoretical literature lags behind its empirical counterpart in this area. While the well-known model of Scharfstein and Stein (1990) shows that money managers may herd due to reputational concerns, there is no systematic theoretical analysis of the effects that institutional herding may have on equilibrium prices.

In this paper we present a simple yet rigorous model of the price impact of institutional herding. Our results provide precise theoretical foundations for the dichotomous empirical conclusions with regard to the price impact of institutional herding. We analyze the interaction among three classes of traders: career-concerned fund managers, profit-motivated proprietary traders, and security dealers endowed with market power. Our results are as follows. First, we show that the reputational concerns of fund managers imply an endogenous tendency to imitate past trades, which impacts the prices of the assets they trade. Second,

¹Jean-Claude Trichet, then Governor of the Banque de France. Keynote speech delivered at the Fifth European Financial Markets Convention, Paris, 15 June 2001: "Preserving Financial Stability in an increasing globalised world."

²See, amongst others, Lakonishok, Shleifer and Vishny (1992), Grinblatt, Titman and Wermers (1995), Wermers (1999), and Sias (2004).

³For evidence on short-term return continuation following institutional herding see, for example, Wermers (1999) and Sias (2004). Dasgupta, Prat, and Verardo (forthcoming) find evidence of long-term return reversals after institutional herding. Further evidence on institutional herding and long-term reversals can be found in Gutierrez and Kelley (2009) and Brown, Wei, and Wermers (2009).

in our main set of results, we show that institutional herding positively predicts short-term returns but negatively predicts long-term returns. Our theory thus provides a simple and unified framework within which to interpret the empirical results on the price impact of institutional herding, both at short and long horizons. Finally, our theory generates several new testable predictions linking institutional herd behavior, trading volume, and the time-series properties of stock returns.

The building blocks of our theory can be traced back to Scharfstein and Stein (1990), who study a sequential choice setting with exogenous (fixed) prices in which decision makers have career concerns. We embed a related model of career concerns into a multi-period sequential trade market with endogenous price determination, in which some traders (fund managers) have career concerns, while their trading counterparties (security dealers) are endowed with market power. We describe the model below.

A number of career-concerned fund managers and profit-motivated proprietary traders trade with dealers endowed with market power over several trading rounds before uncertainty over asset valuation is resolved. Fund managers and proprietary traders receive private signals about the liquidation value of the stock and they differ in the accuracy of their signal. They are unsure about the accuracy of their own signal. Fund managers are evaluated by their investors based on their trades and the eventual liquidation value of their portfolios. The future income of a manager depends on how highly investors think of his signal accuracy. In contrast, proprietary traders are motivated purely by trading profits.

In equilibrium, if most managers have bought the asset in the recent past, a manager with a negative signal is reluctant to sell, because he realizes that: (i) his negative realization is in contradiction with the positive realizations observed by his colleagues; (ii) this is probably due to the fact that his accuracy is low; and (iii) by selling, he is likely to appear as a low-accuracy type to investors. The manager faces a tension between his desire to maximize expected profit (which induces him to follow his private information and sell) and his reputational concerns (which make him want to pretend his signal is in accordance with those of the others). This tension drives a wedge between the price at which the manager is willing to sell and the maximum price at which a profit-motivated dealer will buy from him. Thus, this pessimistic manager does not trade. Conversely, a manager with a positive signal who trades after a sequence of buys is even more willing to buy the asset, because his profit motive and his reputational incentive go in the same direction. Dealers utilize their market power to take advantage of this manager's reputational motivation and offer to trade with him at prices that are above expected liquidation values based on available information. In turn, the manager is willing to buy at such excessively high prices because buying provides him with an expected reputational reward.

In contrast, following a sequence of buy orders in the recent past, purely profit-motivated proprietary traders choose not to buy even if they receive a positive signal, because the price, which is set by dealers to extract surplus from optimistic fund managers, is higher than expected liquidation value. Proprietary traders sell if they receive the negative signal.

As the preceding discussion suggests, our model generates precise equilibrium patterns of trades and prices. We begin by describing the trading behavior of the different types of traders. In equilibrium, money managers trade in the direction of past trades or not at all (thus exhibiting herd behavior), while proprietary traders trade against the direction of past trades or not at all (thus exhibiting contrarian behavior). We relate these results to empirical evidence in subsection 2.3.

We next describe the equilibrium price patterns implied by institutional herding, and their relationship with both long-term and short-term asset values. Suppose that there has been a herd of several institutional buys up to and including time t. How does the price at which a money manager bought at time t compare to long-term asset value (the eventual liquidation value) and to short-term asset value (the price at time t + 1)? Consider first the relationship with long-run asset value. In equilibrium, the price at time t will be higher than the expected liquidation value of the asset, because the manager who bought at t did so after a sequence of buys. The endogenous reputational incentives described above imply that this manager was willing to overpay to buy the asset and dealers were happy to extract surplus by overcharging him. This implies that institutional herding up to time t is associated with long-term price reversals: buy-herds are followed by negative long-term returns, while sell-herds are followed by positive long-term returns.

The opposite relationship holds with regard to short-term asset values. When a number of managers have bought, market beliefs about the asset become quite positive. When the proportion of fund managers in the trading population is high, the next trader to face the dealer is likely to be a manager. As we have already argued, this manager's tendency to imitate past trade indicates that he will not sell, regardless of his signal. Thus, as long as there are enough fund managers in the market, the average transaction price is likely to be higher at t+1 than at t. This implies that institutional herding up to time t is associated with price continuation at horizon t + 1: buy-herds are followed by positive short-term returns, while sell-herds are followed by negative short-term returns.

To summarize, our model implies that equilibrium herding by fund managers leads to short-term price continuations and long-term price reversals. Thus, our model provides theoretical foundations to interpret the findings in the empirical literature on the price impact of institutional herding.

Our model also generates a number of other predictions. Some of these predictions find

support in existing empirical results, while others provide new testable implications. We summarize some of these results here and provide a more detailed discussion of linkages to empirical results in the body of the paper. We first define metrics for the association between institutional herding and both short and long run future returns, and demonstrate testable comparative statics. For example, we show that longer institutional buy herds are followed by larger long-term negative returns and by smaller short-term positive returns.

We then show that our model can generate return momentum in the following sense. Stocks that have been bought by institutions experience price appreciation. In turn, since institutional buying positively predicts short-term returns, the same stocks are expected to have a positive short-term return. Thus, winners remain winners in the short term. An analogous result holds for losers. This result contributes to the theoretical literature that derives momentum from a rational model rather than from a behavioral model of investors' underreaction or overreaction to news.

Our equilibrium also links together the degree of mispricing, return momentum, and the level of market acitivity, providing rich empirical predictions relating trading volume to the time series properties of returns. There are two main results in this regard. First, we show that when there are sufficient numbers of institutional traders in a market, high trading volume is associated with increasing mispricing. Reductions in mispricing, in contrast, are associated with quieter markets. This result is related to the empirical evidence that abnormally high turnover levels predict lower future returns. It is also corroborated by the extensive empirical evidence of a positive link between mispricing and volume during the internet bubble period 1998-2000. Second, we show that assets with high trading volume typically experience high return momentum. Amongst the set of assets that experience price appreciation between t - 1 and t, the ones with high institutional trade exhibit high (and positive) return continuation and high expected trade volume, while those with low institutional trade exhibit low (and even negative) return continuation and low expected trade volume. Our model thus offers a rational interpretation for the positive link between volume and momentum documented in the empirical literature.

Our core qualitative results arise from the interaction of two important ingredients. On the one hand, fund managers are career concerned. As a result, their valuation for a given asset (conditional on a given history of trades) may differ from that of traders without career concerns. On the other, the security dealers who buy and sell from fund managers have a degree of market power, which leads to some of this difference in valuations to be reflected in prices. There is extensive empirical evidence in support of both ingredients. A large empirical literature (e.g., Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997, 1999)) documents that the reward structure of portfolio managers is sensitive to their perceived ability. Furthermore, a number of studies show that OTC markets for several assets tend to be concentrated among relatively few dealers who exercise market power (see, for example, Ellis, Michaely and O'Hara (2002) and Schultz (2003) for stocks traded on the Nasdaq, and Green, Hollifield, and Schurhoff (2007) for corporate debt and municipal bonds).

It is worth pointing out that, while the first ingredient is essential for our results (and forms the backbone of our results), the second ingredient represents simply one of many possible frictions that would generate similar qualitative results. Instead of endowing the trading counterparties of fund managers with a degree of market power, we can, instead, make security dealers competitive but risk-averse, generating inventory costs in trading. We explicitly demonstrate in section 4.2 that such a modified model generates qualitatively similar results to that of the baseline model. In short, as long as the residual demand curve against which fund managers trade is not perfectly elastic, the endogenous herding we identify will give rise to similar patterns of prices and returns.

Our paper is related to the large theoretical literature on herding (e.g., Banerjee (1992), Bikhchandani, Hirshleifer, and Welch (1992), Avery and Zemsky (1998)). It is also connected to the growing theoretical literature on the financial market imperfections arising out of the delegation of portfolio management (e.g., Allen and Gorton (1993), Cuoco and Kaniel (2008), Dasgupta and Prat (2006, 2008), Guerrieri and Kondor (2009), He and Krishnamurthy (2007), Vayanos and Woolley (2009)). However, these papers do not study the effect of institutional herding on stock returns.

The rest of the paper is organized as follows. In the next section we present the model and derive equilibrium implications for trading behavior. In section 3 we describe the equilibrium implications of institutional herding for the time-series properties of stock returns and trading volume. Section 4 discusses our core assumptions in greater detail and provides an alternative model without assuming market power for security dealers. Section 5 concludes.

2 A Model of Institutional Herding

2.1 Set-up

Consider a market in which trade occurs sequentially over T periods. In each period there is a large number N_F of delegated traders (fund managers) and a large number N_P of nondelegated speculators (proprietary traders), where $\eta = \frac{N_F}{N_F + N_P}$ represents the proportion of fund managers. Fund managers act on behalf of investors who cannot trade directly and must delegate trading to managers. Each trader is able to trade at most once, if he is randomly selected in one of T rounds, where $T \ll \min(N_F, N_P)$. At any time t, the probability that the trader selected to trade is a fund manager is η . There is a single asset with liquidation value v, where v = 0 or 1 with equal probability. The realized value of v is publicly revealed at time T + 1. The trader who is selected at t faces a monopolistic risk-neutral uninformed market maker (MM), who trades at t only and posts a bid (p_t^b) and an ask price (p_t^a) to buy or sell one unit of the asset.⁴ We discuss the assumption of a monopolistic market maker in Section 4. Each trader has three choices: he can buy one unit of the asset from the MM $(a_t = 1)$, sell one unit of the asset to the MM $(a_t = -1)$, or not trade $(a_t = 0)$.⁵

Regardless of whether he is a fund manager or a proprietary trader, the trader chosen to trade at t can be either good (type $\theta = g$) with probability γ or bad (type $\theta = b$), with probability $1 - \gamma$. The traders do not know their own types.⁶ The good trader observes a perfectly accurate signal: $s_t = v$ with probability 1. The bad trader observes a purely noisy signal: $s_t = v$ with probability $\frac{1}{2}$.⁷

As in many signalling games, the presence of potential out-of-equilibrium actions can result in implausible equilibria supported by arbitrary out-of-equilibrium beliefs. To ameliorate this problem, we assume that in every period t there is an exogenous probability $\rho \in (0, 1)$ that the trader is unable to trade, in which case he is immediately replaced by another trader.⁸ The parameter ρ can be as small as desired.⁹ When the investor observes a manager who does not trade, she cannot tell whether the manager was unable or unwilling to trade.

Let h_t denote the history of prices and trades up to period t (excluding the trade that

 $^{^{4}}$ Our model has features of both Glosten and Milgrom (1985) – a multi-period model with a competitive market maker – and Copeland and Galai (1983) – a single-period model with a monopolistic market maker.

⁵There is no noise trade in our set-up. Noise traders could be added without modifying the qualitative properties of our results, at the cost of substantial algebraic complexity. For a discussion of how this can be done, see Dasgupta, Prat, and Verardo (2010).

⁶This is a standard assumption in career concerns literature following from the classic papers of Holmstrom (1999) and Scharfstein and Stein (1990). Self-knowledge (signals about the precision of agents' own information) plays a nuanced role in career concerns models. For example, Avery and Chevalier (1999) show that, for any given prior, there is a threshold precision of self-knowledge above which contrarianism (instead of conformism) arises. In contrast, Dasgupta and Prat (2008) show that, if the parameters are such that the manager's reputation is helped more by showing that he received the expost correct signal about asset payoffs rather than by showing that he received a good signal about his own type, then, for any given precision of self-knowledge, for sufficiently extreme (endogenously generated) time t priors, conformism still arises at time t.

⁷We are thus implicitly assuming equal average quality of information in the population of delegated and non-delegated traders. This assumption simplifies the algebra without reducing the generality of our core message.

⁸If this replacement trader is also unable to trade, he, in turn is immediately replaced by another trader, and so on.

⁹Having $\rho > 0$ guarantees that non-trading occurs on the equilibrium path, and excludes pathological equilibria where the monopolistic market maker can extract very large surplus because non-trading is associated with large off-equilibrium reputational penalties.

occurs at t). Let $v_t = E[v|h_t]$ denote the public expectation of v. Finally, let $v_t^0 = E[v|h_t, s_t = 0]$ and $v_t^1 = E[v|h_t, s_t = 1]$ denote the private expectations of v of a trader at t who has seen signal $s_t = 0$ or $s_t = 1$ respectively.

The proprietary trader selected at t maximizes the expected value of his trading profits. Trading profit (χ_t) is given by:

$$\chi_t = \begin{cases} v - p_t^a & \text{if } a_t = 1\\ p_t^b - v & \text{if } a_t = -1\\ 0 & \text{if } a_t = 0 \end{cases}$$

Fund managers are career concerned and care about investors' opinion of their ability. Investors observe their manager's action, the history of trades, and the liquidation value of the asset, and form a posterior probability (in equilibrium) about his ability. For the manager selected at t, this posterior probability is given by:

$$\gamma_t = \Pr\left[\theta_t = g | a_t, h_{T+1}, v\right]$$

The time-t manager maximizes the expected value of the following linear combination of his trading profits (χ_t) and his reputation (γ_t):

$$\chi_t + \beta \gamma_t$$

where $\beta > 0$ measures the importance of career concerns.¹⁰ We now proceed to solve the model.

2.2 Equilibrium

As a benchmark, we first analyze the case in which $\beta = 0$, that is, there are no career concerns. In this case, it is easy to see that each trader buys if $s_t = 1$ and sells if $s_t = 0$. The MM, in turn, sets prices to extract the full surplus: bid price $p_t^b = v_t^0$ and ask price $p_t^a = v_t^1$. We summarize:

Proposition 1 When $\beta = 0$, managers and proprietary traders trade as follows:

$$a_t = \begin{cases} -1 & \text{if } s_t = 0\\ 1 & \text{if } s_t = 1 \end{cases}$$

and the market maker sets prices $p_t^b = v_t^0$ and $p_t^a = v_t^1$.

¹⁰A microfoundation for this payoff function can be found in Dasgupta, Prat, and Verardo (2010).

We now analyze the case in which $\beta > 0$. We first introduce some additional notation for useful equilibrium quantities. Let

$$w_{a_t}^{s_t} = E[\gamma_t(a_t)|s_t, h_t],$$

the expected posterior reputation of a manager who observes signal s_t and takes action a_t . This is clearly an equilibrium quantity, and turns out to be useful in summarizing prices when $\beta > 0$. The following is an equilibrium of the game with $\beta > 0$.

Proposition 2 There exists an equilibrium in which trades and prices are as follows.

If selected at t, a manager trades as follows:

$$\begin{array}{l} (1) \ \text{If } v_t \geq \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} 1 & \text{if } s_t = 1 \\ 0 & \text{otherwise} \end{array} \right. \\ (2) \ \text{If } v_t < \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} -1 & \text{if } s_t = 0 \\ 0 & \text{otherwise} \end{array} \right. \\ (3) \ \text{If } v_t \geq \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} -1 & \text{if } s_t = 0 \\ 0 & \text{otherwise} \end{array} \right. \\ (1) \ \text{If } v_t \geq \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} -1 & \text{if } s_t = 0 \\ 0 & \text{otherwise} \end{array} \right. \\ (2) \ \text{If } v_t < \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} 1 & \text{if } s_t = 1 \\ 0 & \text{otherwise} \end{array} \right. \\ (2) \ \text{If } v_t < \frac{1}{2} \ \text{then } a_t = \left\{ \begin{array}{l} 1 & \text{if } s_t = 1 \\ 0 & \text{otherwise} \end{array} \right. \\ (1) \ v_t \geq \frac{1}{2} \end{array} \right. \\ \end{array}$$

$$\begin{array}{lll} p_t^a & = & v_t^1 + \beta \left(w_1^1 - w_0^1 \right) \\ p_t^b & = & v_t^0 \end{array}$$

(2) $v_t < \frac{1}{2}$

$$\begin{array}{lll} p_t^a & = & v_t^1 \\ p_t^b & = & v_t^0 + \beta \left(w_0^0 - w_{-1}^0 \right) \end{array}$$

The proof of this result is lengthy, and is presented in full detail in the appendix. Here, we comment on the main ingredients that drive the result. We focus on the case in which $v_t > \frac{1}{2}$. The intuition for $v_t < \frac{1}{2}$ is symmetric.¹¹

When $v_t > \frac{1}{2}$, the market is optimistic about the asset payoff, and the equilibrium strategies prescribe that the manager with $s_t = 1$ should buy while the manager with $s_t = 0$ should decline to trade. The equilibrium also specifies that, in this scenario, the ask price is higher

¹¹At $v_t = \frac{1}{2}$, trades and prices specified as above for $v_t < \frac{1}{2}$ can also be sustained as an equilibrium.

than expected liquidation value conditional on a buy order, while the bid price is equal to expected liquidation value conditional on a sell order.

When $v_t > \frac{1}{2}$, it seems to fund managers that there are reputational rewards to be reaped (in equilibrium) from buying. Thus, the fund manager who receives $s_t = 1$ wishes to buy this asset due to profit motivations and for reputational reasons. Thus he is willing to pay a price above the fair informational value of the asset at t in order to own it. The monopolistic market maker sees this as an opportunity for extracting rents, and sets ask prices strictly above expected liquidation value to make positive profits. The fund manager who receives $s_t = 0$ wishes to sell for profit reasons, but to buy for reputational reasons. The price at which he would sell will be higher than v_t^0 , which is the highest price the market maker would ever be willing to pay him. Thus, this manager does not trade.

The market maker is indifferent between trading and not trading with proprietary traders, since, conditional on wishing to trade, their asset valuations coincide in equilibrium. The high willingness to pay of the fund manager with $s_t = 1$ drives up the ask price above expected liquidation value for the most optimistic trader (v_t^1) , and at such high prices proprietary traders would never wish to buy. On the other hand, as we have argued above, there is no incentive compatible price at which the market maker can buy from a fund manager, so the market maker's only trading counterparty on the bid side are proprietary traders. The market maker is indifferent between trading or not, and is thus willing to set a bid price at v_t^0 , at which point the proprietary traders who receive signal $s_t = 0$ are indifferent between selling and not trading.

Could the market maker deviate to increase his profits? It is clear that he would never wish to make fund managers with $s_t = 1$ change their behavior, since he can already extract maximal surplus from these traders. However, as long as fund managers with $s_t = 1$ buy, it is also not optimal for him to induce fund managers with $s_t = 0$ to also buy (for which he would have to lower prices). Intuitively, the market maker makes profits by "selling reputation" to fund managers. However, if he persuades all managers to always buy, there is no reputational benefit to buying. In turn, therefore, the market maker cannot extract any positive rents from his trades with fund managers, and therefore makes zero profits. It will generally be in the interest of the market maker to extract reputational rents only from a strict subset of the group of fund managers.¹²

 $^{^{12}}$ While the equilibrium analyzed here has many desirable and natural properties, it is not possible to exclude the existence of other equilibria. This is a common feature of signalling models. For example, it is possible to construct uninteresting equilibria with no trade by using suitably chosen off-equilibrium penalties for trading.

2.3 Implications for trading behavior

The equilibrium derived above has precise implications for the trading behavior of different types of traders. Fund managers never trade "against popular opinion". If their private information agrees with the public belief (for example, if $s_t = 1$ when $v_t > \frac{1}{2}$) then they trade in the direction of the public belief (e.g., buy when $v_t > \frac{1}{2}$). If their private information contradicts the public belief (for example, if $s_t = 0$ when $v_t > \frac{1}{2}$) then they choose not to trade. This implies that, immediately following a sequence of institutional purchases (sales), a fund manager will never choose to sell (buy) regardless of his private information, thus exhibiting herd behavior.

In sharp contrast, proprietary traders never trade in the direction of popular opinion. If their private information agrees with the public belief (for example, if $s_t = 1$ when $v_t > \frac{1}{2}$) then they choose not to trade. If their private information contradicts the public belief (for example, if $s_t = 0$ when $v_t > \frac{1}{2}$) then they choose to trade in a contrarian manner.

The contrasting behavior of fund managers and proprietary traders can be explained as follows. Trading in the direction of popular opinion implies buying "too high" (because $p_t^a > v_t^1$ when $v_t > \frac{1}{2}$) or selling "too low" (because $p_t^b < v_t^0$ when $v_t < \frac{1}{2}$). Fund managers are willing to do so because trading in the direction of popular opinion is, on balance, likely to enhance their reputation. Proprietary traders have pure profit-based compensation, face no career concerns, and therefore are unwilling to trade at unfavorable prices. The willingness of fund managers to trade at unfavorable prices, in turn, supports these prices.

The empirical evidence on institutional trading behavior shows that institutional investors tend to herd, i.e. they trade in the direction of recent institutional trades. Lakonishok, Shleifer and Vishny (1992) show that the trades of a sample of pension funds tend to be correlated over a given quarter, especially among small stocks. Grinblatt, Titman and Wermers (1995) and Wermers (1999) examine a larger sample of equity holdings by mutual funds and find evidence of herding in small stocks. Sias (2004) finds stronger evidence of herding behavior among institutional investors by estimating a significant positive relation between the fraction of institutions buying the same stock over adjacent quarters.

There is also evidence that non-institutional traders, i.e., individuals, tend to trade as contrarians. Kaniel, Saar, and Titman (2008), for example, examine NYSE trading data by individual investors and find that individuals buy stocks after prices decrease and sell stocks after prices increase. Griffin, Harris, and Topaloglu (2003) show evidence of short-horizon contrarian behavior by Nasdaq traders who submit orders through retail brokers. Goetzmann and Massa (2002) find that individuals who invest in an index fund are more likely to be contrarians.

3 Implications for stock returns and trading volume: Time-Series Properties

We now turn to our central goal of theoretically delineating the impact of institutional herding on long-term and short-term returns. In outlining the implications of our model for the timeseries behavior of returns, we divide our results into three distinct categories. First, in section 3.1, we delineate conditions under which institutional herding is positively associated with short-term returns and negatively associated with long-term returns. Second, when these conditions are satisfied, we describe in section 3.2 how the relation between herding and future returns varies as a function of the parameters of the market and the length of the instutional herd. Finally, in section 3.3, we delineate the implications of our model for the link between trading volume, mispricing, and momentum.

We emphasize that our time series results are tightly intertwined with cross sectional predictions. The unifying theme for a majority of our results below is that the market for the asset must have sufficiently many fund managers (i.e., η must be large enough). In our model, mispricing is driven by the contractual incentives of delegated portfolio managers and is partially offset by the trading behavior of proprietary traders. Thus, for mispricing to be evident on average in the data, there must be enough fund managers trading the asset as a proportion of all traders. Therefore, for each time series prediction that requires a minimal η condition, our model yields an associated cross-sectional prediction: in a cross section of assets, the link between herding and stock returns is stronger for those assets that are traded by a higher proportion of portfolio managers.

Throughout this section we focus on the upper half of the public belief space, i.e., when we make statements about time t, we assume that $v_t > \frac{1}{2}$. All results are symmetric for the case where $v_t < \frac{1}{2}$. At this stage, it is convenient to introduce some new notation. We denote by $E_t(\cdot)$ the expected value of the argument conditional on all information available at time t, including the trade at time t. For example, to reconcile with our older notation: $E_t(v) = v_{t+1}$. In other words, v_t is the public prior on v before the period t actions are observed, while $E_t(v)$ is the public posterior immediately after the period t actions are observed.

3.1 Conditions for the link between herding and stock returns

Suppose that there has been a sequence of several institutional buys up to and including time t. The econometrician observes the buy sequence ex post in the data. She is interested in how the most recent transaction price (p_t^a) relates to:

a The asset's long-run value, measured by the expected liquidation value given the information available through time $t, E_t(v)$.

b The asset's short-run value, measured by the expected transaction price in the next period $E_t(p_{t+1})$.

Correspondingly, upon observing equilibrium data ex post:

- **a** If $E_t(v) p_t^a < 0$, the econometrician concludes that institutional herding negatively predicts long-term returns. Thus, institutional herding is associated with long-term reversals.
- **b** If $E_t(p_{t+1}) p_t^a > 0$, the econometrician concludes that institutional herding positively predicts short-term returns. Thus institutional herding is positively associated with returns in the short-term.

We outline here conditions under which the econometrician would reach each of these conclusions. Long-term reversals are immediate. Since the trade at t is a buy order (the final trade in the observed buy herd) we know that $E_t(v) = v_{t+1} = v_t^1$, and we also know that $p_t^a = v_t^1 + \beta \left(w_1^1(v_t) - w_0^1(v_t) \right)$, where $w_1^1(v_t) - w_0^1(v_t) > 0$ for any $v_t > \frac{1}{2}$. Thus, institutional herding always negatively predicts long-term returns.

The link between herding and short-term returns requires further analysis. The reason is that the next period transactions may either occur at the ask p_{t+1}^a (which is higher than p_t^a) or at the bid p_{t+1}^b (which is lower). Since, for $E_t(v) = v_{t+1} > \frac{1}{2}$, institutions buy and proprietary traders sell at time t + 1, when there are enough institutional traders in the population (i.e., η is high enough) the expected transaction price at t + 1 will be higher than the transaction price at t. To summarize:

Proposition 3 Institutional herding always negatively predicts long-term returns. For η large enough, institutional herding positively predicts short-term returns.

The empirical literature on institutional herding generally documents a positive association between herding and returns at short horizons. In particular, Wermers (1999) and Sias (2004) find that stocks that institutions herd into (and out of) exhibit positive (negative) abnormal returns at horizons of a few quarters.¹³ When examining the long-term impact of institutional herding, however, a few recent studies find evidence of a negative association between institutional trading and long-term returns. For example, Dasgupta, Prat, and Verardo (forthcoming) analyze the long-term future returns of stocks that have been persistently bought or sold by institutions over several quarters. They find that, in the long term, stocks

¹³Other papers finding evidence of a positive correlation between institutional demand and future returns include Nofsinger and Sias (1999), Grinblatt, Titman and Wermers (1995), and Cohen, Gompers and Vuolteenaho (2002).

persistently bought by institutions underperform stocks persistently sold by them. Evidence of long-term return reversals associated with institutional trading can also be found in Coval and Stafford (2007), Frazzini and Lamont (2008), Gutierrez and Kelley (2009), and Brown, Wei, and Wermers (2009).

3.2 Economic importance of the link between herding and stock returns

In this section we analyze the economic importance of the link between institutional herding behavior and future stock returns. Our starting point is an institutional buy sequence which, as shown in the previous section, has a positive impact on short-term returns and a negative impact on long-term returns. We examine here the magnitude of the positive expected return in the immediate aftermath of the buy sequence, and the magnitude of the negative return in the long run. Specifically, we ask how short-term and long-term returns change as a function of the parameters of the model, and how they vary with the length of the herd, i.e. when institutional herding becomes more persistent over time.

We begin with the long-term return, which we measure as follows:¹⁴

$$LTR_{t} = \left| \frac{E_{t}\left(v\right) - p_{t}^{a}}{p_{t}^{a}} \right|$$

Note that LTR_t is a measure for the degree of mispricing at time t. We relate the long-term return to two crucial quantities: $E_t(v)$ and β . These quantities have a natural economic interpretation. The parameter β measures the weight placed by institutional traders on their reputation. Interpreted literally, $E_t(v)$ is a measure of the market's level of optimism about the liquidation value of the asset conditional on the trade at t. It also has a second, equally instructive, alternative interpretation. Since a longer sequence of consecutive institutional purchases increases the market's level of optimism about the expected payoff of the asset, starting with any arbitrary prior ($\geq \frac{1}{2}$, so that institutions are willing to buy), $E_t(v)$ varies one-for-one with the length of the sequence of institutional purchases up to and including period t. Thus, $E_t(v)$ is also a measure of the length of the institutional buy sequence.

Proposition 4 The magnitude of the negative long-term return following an institutional buy herd is higher when institutions care more about their reputation and when herding is more persistent. Formally, LTR_t is increasing in β and $E_t(v)$.

This result indicates that the degree of reversal in long-term returns following an institutional buy sequence is higher for stocks that are traded by institutional managers with

¹⁴Note that, since $E_t(v) - p_t^a < 0$ for $v_t > \frac{1}{2}$, it is convenient to define the long-term return in terms of the absolute value of $\frac{E_t(v) - p_t^a}{p_t^a}$.

stronger career concerns. Furthermore, the degree of reversal is higher when institutional herding behavior is more persistent over time.

The first result is a new and testable prediction implied by our model. The link between career concerns and long-term return reversals associated with institutional herding has not been explored in the empirical literature. While several studies on the effects of contractual incentives in the mutual fund industry focus on the link between the performance of mutual fund managers and their risk-taking attitudes (Brown, Harlow, and Starks (1996), Chevalier and Ellison (1997, 1999)), there is no direct evidence on the price impact of career concerns for the stocks traded by career-concerned managers.

The second result finds support in the evidence of Dasgupta, Prat, and Verardo (forthcoming), who show that the degree of asset mispricing (measured by the magnitude of long-term return reversals) is larger for stocks characterized by a longer sequence of institutional buying or selling. They estimate a significantly negative relationship between future two-year stock returns and the number of consecutive quarters during which institutions buy or sell a given stock.

Next we turn to the short-term return, which we define as follows:

$$STR_t = \frac{E_t \left(p_{t+1} \right) - p_t^a}{p_t^a}$$

We can now state two relevant properties of the short-term return:

Proposition 5 The magnitude of the positive short-term return following an institutional buy herd is higher when there are more institutional traders and, for assets with sufficient institutional trade, declines as herding becomes more persistent. Formally, STR_t increases in η and, for η high enough, decreases in $E_t(v)$.

This result indicates that the positive short-term return following an institutional buy sequence is higher for stocks characterized by higher institutional trading. Moreover, as institutional buying becomes more persistent over time, the magnitude of the expected shortterm return decreases. This is the opposite of what happens with long-term reversals.

The first result can be indirectly related to the evidence on herding and short-term returns documented in Wermers (1999) and Sias (2004). Both papers find a positive correlation between the fraction of institutions buying a stock in a given quarter and the stock's returns in the following one or two quarters. Sias (2004) shows that the correlation between current herding and future short-term returns is larger when herding is measured among stocks having a minimum number of institutional traders (e.g. at least 5, 10, or 20 traders).

The second result in proposition 5, linking the positive association between herding and short-term returns to the persistence of institutional trading, represents a new testable prediction generated by our model.

3.3 Trading Volume, Mispricing, and Momentum

Our model generates several implications that allow us to analyze the link between market acivity, mispricing, and return continuation. Taken together, these additional results offer a rational "institutional" channel to further our understanding of some relevant interrelationships in financial markets. We comment on these relationships in this section.

We find that high trading volume characterizes episodes of increasing mispricing. In contrast, reductions in mispricing are associated with less active markets. We measure mispricing by the long-term return metric introduced in section 3.2, $LTR_t = \left|\frac{E_t(v) - p_t^a}{p_t^a}\right|$. Consider an asset that has been purchased at t when $v_t > \frac{1}{2}$. Following the previous analysis, it is clear that this asset is mispriced at t. We now ask how the degree of mispricing changes as a function of trading volume at t+1. Define $l_{t+1} = |a_{t+1}|$, i.e., the measure of trade volume at t+1. We can now state:

Proposition 6 In asset markets dominated by institutional traders, high trading volume is associated with increasing mispricing. Formally: (i) $\Pr(LTR_{t+1} > LTR_t | l_{t+1} \neq 0)$ is increasing in η and converges to 1 as $\eta \rightarrow 1$; (ii) For high enough η , $l_{t+1} = 0$ implies that $LTR_{t+1} < LTR_t$.¹⁵

The intuition for this result is as follows. Mispricing, as measured by the expected longterm return obtained from purchasing the stock, is increasing in the market's belief about the liquidation value. When the market is optimistic, trades can either come from optimistic fund managers or pessimistic proprietary traders. The first case reveals that the manager has positive information, making future managers even more keen to buy, and thus exacerbating mispricing. The latter case reveals that the proprietary trader has negative information, making fund managers less keen to buy in the next round, thus ameliorating mispricing. As η grows, the probability of the former event increases. By the same token, the absence of trade in a given period may imply that a pessimistic manager or an optimistic proprietary trader chose not to trade. If the market interprets no-trade as the former case, the level of optimism falls; if it interprets no-trade as the latter case, the level of optimism rises. As η grows, it is more likely that no-trade is caused by the inactivity of pessimistic managers. Thus, for assets dominated by institutional traders (high η), trade is typically associated with increasing mispricing, while the lack of trade is associated with corrections.

Evidence on the link between trading volume and mispricing is offered by studies showing that high turnover predicts future return reversals (see, for example, Brennan, Chordia

¹⁵We note that the asymmetry between parts (i) and (ii) of the proposition is due to the fact that in the latter case $l_{t+1} = 0$ uniquely pins down the action $a_t = 0$, (no-trade), whereas in the former case, $l_{t+1} \neq 0$, allows for two possible trades with different impact on the long-term return.

and Subrahmanyam (1998) and Datar, Naik and Radcliffe (1998)). There is also evidence that, on days of large market movements, stocks mostly owned by institutions are characterized by higher turnover and are followed by larger future reversals in returns (Dennis and Strickland (2002)).¹⁶ Focusing on the time-series predictability of trading volume, Baker and Stein (2004) find a negative association between NYSE turnover and market returns over the subsequent year. Finally, a number of papers document a strong cross-sectional association between abnormally high share turnover and overvaluation, particularly during the technology bubble (see, for example, Ofek and Richardson (2003), Lamont and Thaler (2003), Cochrane (2003), Mei, Scheinkman, and Xiong (2009)).¹⁷

Some other theoretical papers also derive a link between trading volume and mispricing. Scheinkman and Xiong (2003) develop a model of speculative trading in which overconfidence generates disagreement about fundamental values, and investors buy overpriced assets believing that they will be able to profitably sell them in the future, generating a link between overpricing and volume. Gervais and Odean (2001) present a model of overconfident traders in which trading volume is higher after market gains because of higher overconfidence. In contrast to these papers, our model is fully rational, and revolves around the incentives of institutional traders in linking volume and mispricing. Moreover, our framework links trading volume to mispricing in general: episodes of underpricing are also characterized by high trading volume.

Our equilibrium returns are characterized by momentum for stocks with high institutional trading. When the market is optimistic about the asset's future liquidation value, an asset with sufficient institutional trade that has been increasing in price between t - 1 and t (so that it is a "winner" in the short-term) is expected to continue to have a positive return between t and t+1 (i.e., on average it remains a "winner" in the next period). This is almost immediate from our previous analysis. To be precise, suppose $v_t > \frac{1}{2}$ and $r_{t,t-1} > 0$ (because the asset was bought at t). Then, Proposition 3 implies that $E_t(r_{t,t+1}) > 0$ if η is high enough.¹⁸ Our model implies higher momentum for stocks with higher institutional trading.

¹⁸We emphasize that our results on return momentum are conditional on the state of the market's beliefs about the asset's liquidation value. For example, it is not necessarily the case that, for $v_t < \frac{1}{2}$, one-period winners expect to remain one-period winners, if η is high enough. The reason is that, if $v_t < \frac{1}{2}$, buy orders

¹⁶However, Nagel (2005) finds that the link between abnormal turnover and return reversals is stronger for stocks with low institutional ownership.

¹⁷The evidence on the relation between trading volume and returns does not reach, however, uniform conclusions, as the results often vary with the measure of trading volume adopted or with the estimation frequency. For example, Avramov, Chordia and Goyal (2006) document large reversals for high turnover stocks when returns are measured at the weekly frequency, but they find that reversals are stronger for low volume stocks when considering monthly returns. Connolly and Stivers (2003) document that the weekly returns of a portfolio of large U.S. stocks exhibit reversals following a period of low abnormal turnover (see also Cooper (1999)).

This theoretical result can be viewed in light of the extensive evidence of momentum trading by institutions (see, for example, Grinblatt, Titman and Wermers (1995), Wermers (1999), and Sias (2004), among others). A recent paper that also provides an institutional theory of momentum is Vayanos and Woolley (2009).

Our model also predicts co-movement between return momentum and trading volume, because both are affected by the presence of institutional traders. Informally, consider as usual $v_t > \frac{1}{2}$, and take a stock that has been bought at t, so that $r_{t-1,t} > 0$. Proposition 3 tells us that short-term return continuation from period t to t+1 is achieved when there are enough fund managers in the population of traders. Proposition 5 tells us that the expected short-term return between t and t+1 is increasing in the proportion of fund managers. Note that trade can only occur at t+1 if a manager with signal $s_t = 1$ is selected to trade, or if a proprietary trader with signal $s_t = 0$ is selected to trade. Otherwise, there is no trade. Since $v_{t+1} > \frac{1}{2}$ (because $v_t > \frac{1}{2}$ and there was a purchase at t), the probability of any trader receiving signal $s_{t+1} = 1$ is greater than the probability of receiving signal $s_{t+1} = 0$. Thus, the overall probability of trade increases with the presence of fund managers.¹⁹ Thus, amongst the set of assets that have experienced price appreciation between t - 1 and t, the ones with high institutional trade will experience high (and positive) return continuation and high expected trade volume, while those with low institutional trade will experience low (and even negative) return continuation and low expected trade volume. Thus, we can state that:

Proposition 7 A high degree of return continuation is associated with high expected trading volume. Formally, if $r_{t-1,t} > 0$ and $v_t > \frac{1}{2}$, high (low) η implies high (low) STR_t and high (low) expected trade volume.

This result implies that, without controlling for the degree of institutional trade/ownership, it is possible for an econometrician to conclude that high volume stocks experience a high degree of return continuation. Note that, since return continuation is realized only if a fund manager is selected to trade (and in addition he observes $s_{t+1} = 1$), we could have alternatively expressed the above proposition in terms of the *probability*, rather than the degree/magnitude, of return continuation.

This result relates to the empirical evidence of a positive association between trading volume and momentum. Lee and Swaminathan (2000), for example, find that portfolios of stocks characterized by higher trading volume tend to exhibit higher momentum over a period

can come from proprietary traders. This raises the price between t-1 and t, but if η is high enough (except in the special case where $v_{t+1} > \frac{1}{2}$ even though $v_t < \frac{1}{2}$) the next trade is most likely to come from a manager, and with $v_{t+1} < \frac{1}{2}$ the manager can only sell if he trades, thus *lowering* (not raising) prices.

¹⁹ Formally, the probability of trade at t is $\eta \Pr(s_{t+1} = 1|h_{t+1}) + (1 - \eta) \Pr(s_{t+1} = 0|h_{t+1})$, which is increasing in η for $v_{t+1} > \frac{1}{2}$.

of six months in the future. Llorente et al. (2002) focus on individual stocks and show that trading volume has a positive impact on the autocorrelation of daily returns.²⁰

4 Discussion

In this section we provide further discussion of some of our crucial assumptions and describe an alternative model without monopolistic market makers that generates similar qualitative results.

4.1 Monopolistic market maker

In our baseline model, we have assumed that market makers have monopoly power. In this subsection, we discuss the content of this assumption. In a standard trading model like Glosten and Milgrom (1985), all traders pursue the same objective: they maximize expected returns. In our setting, things are very different. Some traders have career concerns and private information (fund managers) while their trading counterparties (security dealers) have no career concerns and, as is standard in microstructure models, no private information. One of our key results is that there may be a discrepancy between the willingness to pay of these two groups of traders for the same asset.

If portfolio managers and dealers value the same asset differently, what price will emerge in equilibrium? In general, we should expect the price to reflect the valuations of the two groups according to their respective price elasticities. Unfortunately, such a general approach leads quickly to intractability in the context of dynamic trading models. So we are left with two extreme alternatives: either portfolio managers have all the bargaining power (this would arise, for example, if dealers were competitive, as they are in Glosten and Milgrom 1985) or dealers have all the bargaining power (for example, the dealer is a monopolist). In the former case, the price will correspond to the valuation of dealers and our model will yield the same prices as Glosten and Milgrom. In the latter case – the interesting one to explore – prices correspond to the valuations of portfolio managers. Reality is in between these two extreme cases, and we should expect prices to partly incorporate the willingness to pay of institutions. But this means that, in a reasonable model, where the dealer and portfolio managers share the bargaining power (for example, the dealer is imperfectly competitive, but not monopolistic) we should expect prices to display the properties that we discuss here. As we have pointed out in the introduction, empirical evidence supports a degree of market power on the part of

 $^{^{20}}$ The authors develop a model in which investors trade for hedging or for speculative motives, and show that trading generated by speculative motives is characterized by return continuation. For a model of volume and momentum based on differences of opinion see Hong and Stein (2007).

security dealers.

More generally, even with a perfectly competitive market making sector, qualitatively similar price patterns can arise in the presence of alternative natural frictions. To emphasize the robustness of our results, we briefly analyze in the next subsection a model with competitive market makers who are risk averse, and thus face inventory costs of market making.

4.2 Inventory costs model

As an alternative to the monopolistic market maker, consider a setting in which the market maker is competitive, but risk-averse, so that he faces inventory costs to market making. To what extent would our qualitative results hold up in such a modified environment? This section shows that, with some caveats, we would expect to see similar qualitative predictions on the relationship between net institutional trade and both short-term and long-term return predictions in this modified setting. We consider an identical model to the baseline case above, with the following modifications. We make the market maker a competitive, linear meanvariance optimizer. We assume that the market maker myopically derives payoffs $E(W|\Upsilon) - \lambda Var(W|\Upsilon)$ where W is the market maker's terminal wealth, and Υ represents his information set.²¹

We can show that as long as the importance of career concerns is sufficiently high, and when public beliefs are not concentrated close to 0 or 1, the trading of fund managers in this modified model is identical to that in the baseline model. To state the formal result, we need to introduce some additional notation. Denote the inventory owned by the market maker after history h_t by I_{h_t} . Denote the history induced by a buy (sell) order following h_t by $h_{tb}(h_{ts})$.

Proposition 8 For any $v^* < \bar{v}(\gamma, \rho)$,²² there exists a $\beta^* > 0$ such that for $\beta > \beta^*$ the following strategies constitute an equilibrium: The fund manager trades as follows:

(1) if $v_t \in \left[\frac{1}{2}, v^*\right)$, then $a_t = \left\{$	1	if $s_t = 1$	
(2) If $v_t \in (1 - v^*, \frac{1}{2})$, then $a_t =$	— .	$\int -1$	if $s_t = 0$
	. — .	0	otherwise
(3) if $v_t \notin (1 - v^*, v^*)$, then a_t	t = 0	0 <i>for</i> a	all s_t .

²¹The use of myopic mean-variance optimization as a modelling tool is quite common in the literature (see, for example, Acharya and Pedersen 2005, Hong, Scheinkman and Xiong, 2006 amongst many others). At substantial algebraic cost, which would distract us from the main purpose of our model, we could instead work with quadratic utility, which would deliver non-linear mean-variance preferences with qualitatively similar implications.

 $^{22}\overline{v}(\gamma,\rho)$ is the unique solution to: $w_0^0(v_t) - w_1^0(v_t) = 0$, where $w_0^0(v_t)$ and $w_1^0(v_t)$ are as in Proposition 2 (see the proof in the appendix for more detail).

The market maker quotes the following prices following any history h_t :

$$p^{a}(h_{t}) = v_{t}^{1} + \lambda (1 - 2I_{h_{t}}) Var(v|h_{tb})$$
$$p^{b}(h_{t}) = v_{t}^{0} - \lambda (1 + 2I_{h_{t}}) Var(v|h_{ts})$$

Note that when $I_{h_t} < 0$ ($I_{h_t} > 0$), i.e., when net trades to the market maker have been positive (negative), both bid and ask prices are above (below) expected liquidation value. The basic mechanism is intuitive: as fund managers with positive information buy from the market maker, he faces a risky negative inventory, and thus raises the prices at which he is willing to sell to fund managers above the informationally fair value. Recent purchases make fund managers optimistic, and thus via the reputational mechanism of the baseline model, raise their valuation for the asset, and thus those with positive information are willing to purchase the asset at these high prices. The two caveats above, that β must be sufficiently high and that v_t must not be too close to 0 or 1, are very intuitive consequences of our modifications. Since the market maker is competitive, prices reflect his, and not the fund managers' valuations. Thus, when the market maker overcharges, the premium reflects his own preference parameter λ . To ensure that optimistic fund managers are willing to buy at such premia, their reputational concerns must be sufficiently strong. Similarly, consider the case of the fund manager with signal 0. As in the baseline model, when $v_t > \frac{1}{2}$, selling is reputationally costly to him. He is willing to sell only if the price at which he sells is high enough to offset this reputational cost. Unlike in the baseline model, in this modification, the market maker is indeed willing to bid a premium price for the asset, since he wishes to balance his net short inventory. However, as long as the reputational concerns are sufficiently important, the premium offered by the market maker is insufficient to offset the reputational cost and this fund manager prefers not to sell, and prefers, as in the baseline model nontrading to selling or buying. However, for extremely high v_t the nature of the equilibrium changes. At this point, a fund manager with s = 0 may actually prefer to buy, because, unlike in the baseline model, the ask price of the market maker does not vary one-to-one with the expected reputational benefit received by the fund managers. But, if both s = 0 managers and s = 1 managers wish to buy, then there is no reputational benefit from buying, and thus no reason for the managers to trade with the market maker. Thus, for sufficiently high v_t , a natural continuation equilibrium has fund managers not trading.²³

²³On a more technical note, it is easy to see that β^* is increasing in v^* ; i.e., in order to support conformist behavior over larger ranges of public belief v_t it is necessary to have larger β . Numerical computations show that the range of beliefs over which conformist trading behavior can hold is quite large for reasonable parameter values. For example, with $\gamma = 0.5$ and $\rho = 0.01$, $\bar{v} = 0.76$, i.e., the behavior of the baseline model is replicated for public beliefs between 0.24 and 0.76.

As the above discussion makes clear, assets are overpriced when the market maker holds negative inventory and underpriced when he holds positive inventory. Since the market maker's inventory reflects the trades of fund managers, it is, on average, only when fund managers buy (sell), that his inventory becomes negative (positive).²⁴ Thus, persistent institutional buying or selling will, on average, be associated with return reversals at horizon T + 1. By the same token, persistent institutional trade will, on average, be associated with short-term return continuation (since the only traders are fund managers, and they are conformist).²⁵

5 Conclusion

This paper presents a simple yet rigorous model of the price impact of institutional herding. While the well-known model of Scharfstein and Stein (1990) shows that money managers may herd due to reputational concerns, there is no prior systematic theoretical analysis of the price impact of institutional herding. In contrast, there is a significant empirical literature on the price impact of institutional herding. This literature concludes that institutional herding positively predicts short term returns but negatively predicts long term returns. Thus, the empirical literature suggests, intriguingly, that institutional herding is stabilizing in the short-term but destabilizing in the long-term.

Our paper provides a theoretical resolution to this empirical dichotomy. We analyze the interaction among three classes of traders: career-concerned money managers, profitmotivated proprietary traders, and security dealers endowed with market power. The interaction among these traders generates rich implications. First, we show theoretically that money managers tend to imitate past trades (i.e., herd) due to their reputational concerns, despite the fact that such herding behavior has a first-order impact on the prices of assets that they trade.

Second, in our main set of results, we formalize the relationship between institutional

²⁴The qualification "on average" is inserted for the following reason: since non-trades are informative as well, with low probability (since $s_t = 0$ is more likely than $s_t = 1$ when $v_t < \frac{1}{2}$) a situation can arise where $I_{h_t} > 0$ (that is, there have been more sales than buys) but $v_t > \frac{1}{2}$ (because there is a long-enough sequence of no-trades, which do not affect the market maker's inventory). However, on average, institutional buying will be associated with $I_{h_t} < 0$.

²⁵For brevity, we have stated the inventory costs model without including proprietary traders (implicitly setting $\eta = 1$). As a result, we did not need to require η to be high enough to generate short-term return continuation. It would be notationally complex but conceptually straightforward to add in proprietary traders. They would be contrarians as in the baseline model (since they never would buy above or sell below expected liquidation value), and thus in their presence we would need η to be high enough to generate short-term return continuation.

herding and returns. We show that assets persistently bought (sold) by money managers trade at prices that are too high (low), thus generating return reversals in the long-term. We also show that, when there are enough institutional traders, our equilibrium generates a positive correlation between institutional herding and short-term returns. Our analysis, therefore, provides a simple and stylized framework to interpret the empirical evidence on the price impact of institutional herding, which finds a stabilizing effect of herding in the short term and a destabilizing effect in the long term.

Finally, our model generates a number of new empirical predictions that link herding behavior, trading volume, and the time-series of stock returns. We show that, in markets dominated by institutional traders, increasing mispricing is associated with high trading volume. Furthermore, conditional on institutional herding, our model can generate momentum. Finally, momentum in stock returns is associated with high trading volume. Some of these predictions are supported by existing empirical findings. Others represent potential directions for future empirical analysis.

6 Appendix

Proof of Proposition 2: We first demonstrate the proof for the case in which $v_t \ge \frac{1}{2}$. The case for $v_t < \frac{1}{2}$ is symmetric. **Case:** $v_t > \frac{1}{2}$

Fund manager's strategy: We begin by computing some equilibrium posteriors.

$$\begin{split} w_1^1 &= & \Pr\left(g|v=1, a=1\right) \Pr\left(v=1|s=1\right) + \Pr\left(g|v=0, a=1\right) \Pr\left(v=0|s=1\right) \\ &= & \Pr\left(g|v=1, s=1\right) v_t^1 + \Pr\left(g|v=0, s=1\right) \left(1 - v_t^1\right) \\ &= & \frac{2\gamma}{1+\gamma} v_t^1 \end{split}$$

$$w_0^1 = \Pr(g|v=1, a=0) \Pr(v=1|s=1) + \Pr(g|v=0, a=0) \Pr(v=0|s=1)$$

= $\frac{2\rho\gamma}{2\rho\gamma + (1+\rho)(1-\gamma)} v_t^1 + \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)} (1-v_t^1)$

because

$$\begin{aligned} \Pr(g|v, a = 0) &= \frac{\Pr(a = 0|g, v) \Pr(g|v)}{\Pr(a = 0|g, v) \Pr(g|v) + \Pr(a = 0|b, v) \Pr(b|v)} \\ &= \frac{\Pr(a = 0|g, v) \gamma}{\Pr(a = 0|g, v) \gamma + \Pr(a = 0|b, v) (1 - \gamma)} \\ &= \frac{(\rho + (1 - \rho) \Pr(s = 0|g, v)) \gamma}{(\rho + (1 - \rho) \Pr(s = 0|g, v)) \gamma + (\rho + (1 - \rho) \Pr(s = 0|b, v)) (1 - \gamma)} \\ &= \begin{cases} \frac{\rho\gamma}{\rho\gamma + (\rho + \frac{1}{2}(1 - \rho))(1 - \gamma)} & \text{if } v = 1 \\ \frac{(\rho + (1 - \rho))\gamma}{(\rho + (1 - \rho))\gamma + (\rho + (1 - \rho)\frac{1}{2})(1 - \gamma)} & \text{if } v = 0 \end{cases} \\ &= \begin{cases} \frac{2\rho\gamma}{2\gamma + (1 + \rho)(1 - \gamma)} & \text{if } v = 1 \\ \frac{2\gamma}{2\gamma + (1 + \rho)(1 - \gamma)} & \text{if } v = 0 \end{cases} \end{aligned}$$

The expressions for w_0^0 and w_{-1}^0 are analogous

Suppose the fund manager has received signal $s_t = 1$. If he buys, he receives:

$$v_t^1 - p_t^a + \beta w_1^1 = \beta w_0^1$$

If he does not trade, he also receives βw_0^1 . Finally, if he sells (an off equilibrium action) we assume that the investor believes that it was because he received signal $s_t = 0$, so that $w_{-1}^1 = (1 - v_t^1) \frac{2\gamma}{1+\gamma}$.²⁶ Thus, the manager's expected payoff from selling is

$$p_t^b - v_t^1 + \beta w_{-1}^1 = v_t^0 - v_t^1 + \beta w_{-1}^1 < \beta w_{-1}^1$$

²⁶This is the "natural" off-equilibrium belief, that is robust to the presence of a small number of "naive" fund managers who always trade sincerely.

We show next that $w_{-1}^1 < w_0^1$, which will imply that the expected (deviation) payoff from selling is strictly smaller than the expected (equilibrium) payoff from buying. Recall that

$$w_0^1 = \frac{2\gamma}{2\gamma + \left(\frac{1}{\rho} + 1\right)\left(1 - \gamma\right)} v_t^1 + \frac{2\gamma}{2\gamma + \left(\rho + 1\right)\left(1 - \gamma\right)} \left(1 - v_t^1\right)$$

It is clear that at $\rho = 0$, $w_0^1 = w_{-1}^1$. We shall demonstrate that, for $v_t > \frac{1}{2}$, w_0^1 is increasing in ρ , which implies that for $v_t > \frac{1}{2}$ and $\rho > 0$ it must be the case that $w_0^1 > w_{-1}^1$. To do so, we take the derivative of w_0^1 with respect to ρ :

$$\begin{aligned} \frac{\partial w_0^1}{\partial \rho} &= \frac{\partial}{\partial \rho} \left(\frac{2\gamma}{2\gamma + \left(\frac{1}{\rho} + 1\right)(1 - \gamma)} v_t^1 + \frac{2\gamma}{2\gamma + (\rho + 1)(1 - \gamma)} \left(1 - v_t^1\right) \right) \\ &= 2\gamma \left(1 - \gamma\right) \left(\frac{\frac{1}{\rho^2}}{\left(2\gamma + \left(\frac{1}{\rho} + 1\right)(1 - \gamma)\right)^2} v_t^1 - \frac{1}{\left(2\gamma + (\rho + 1)(1 - \gamma)\right)^2} \left(1 - v_t^1\right) \right) \end{aligned}$$

This expression is increasing in v_t^1 . Whenever $v_t > \frac{1}{2}$, it is clear that $v_t^1 > \frac{1}{2}$. Evaluating this expression at $v_t^1 = \frac{1}{2}$ gives

$$\gamma (1 - \gamma) \left(\frac{\frac{1}{\rho^2}}{\left(2\gamma + \left(\frac{1}{\rho} + 1\right)(1 - \gamma) \right)^2} - \frac{1}{\left(2\gamma + (\rho + 1)(1 - \gamma) \right)^2} \right)$$

= $\gamma (1 - \gamma) \left(\frac{1}{\left(2\gamma\rho + (1 + \rho)(1 - \gamma) \right)^2} - \frac{1}{\left(2\gamma + (\rho + 1)(1 - \gamma) \right)^2} \right) > 0$

Which then establishes that $w_{-1}^1 < w_0^1$, and thus selling is dominated for the manager with $s_t = 1$.

Suppose instead that the fund manager has received signal $s_t = 0$. His payoff from buying is:

$$\begin{aligned} v_t^0 - p_t^a + \beta w_1^0 &= v_t^0 - v_t^1 - \beta \left(w_1^1 - w_0^1 \right) + \beta w_1^0 \\ &= \left(v_t^0 - v_t^1 \right) + \beta \left(w_1^0 - w_1^1 \right) + \beta w_0^1 \\ &< \beta w_0^1 < \beta w_0^0 \end{aligned}$$

The penultimate inequality follows from the fact that $v_t^0 - v_t^1 < 0$ and $w_1^0 - w_1^1 < 0$. To see why the latter is true, note that $w_1^0 = \frac{2\gamma}{1+\gamma}v_t^0 < \frac{2\gamma}{1+\gamma}v_t^0 = w_1^1$. The final inequality follows from the fact that

$$w_0^0 = \frac{2\gamma}{2\gamma + \left(\frac{1}{\rho} + 1\right)(1-\gamma)} v_t^0 + \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)} \left(1 - v_t^0\right)$$

while

$$w_0^1 = \frac{2\gamma}{2\gamma + \left(\frac{1}{\rho} + 1\right)(1-\gamma)}v_t^1 + \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)}\left(1 - v_t^1\right)$$

and, clearly $\frac{2\gamma}{2\gamma + (\frac{1}{\rho} + 1)(1-\gamma)} < \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)}$ and $v_t^0 < v_t^1$. If he does not trade, his payoff is βw_0^0 . Thus, buying is dominated.

Finally, if he chooses to sell (an off-equilibrium action), then, as before, the investor assumes (correctly in this case) that the signal received was $s_t = 0$, and thus the expected reputational payoff associated with selling is $w_{-1}^0 = (1 - v_t^0) \frac{2\gamma}{1+\gamma}$. His total payoff from selling is

$$p_t^b - v_t^0 + \beta w_{-1}^0 = \beta w_{-1}^0$$

To show that selling is dominated by non-trading, we need to show that $w_{-1}^0 < w_0^0$ for $v_t > \frac{1}{2}$. For this note that w_{-1}^0 and w_0^0 are both decreasing in v_t . We shall show that $w_{-1}^0 < w_0^0$ at $v_t = \frac{1}{2}$ for $\rho > 0$, and that w_0^0 decreases at a slower rate than w_{-1}^0 , which will establish the required claim. For the first part, note that at $v_t = \frac{1}{2}$ and $\rho = 0$, $w_0^0 = w_{-1}^0 = \gamma$ and for $v_t = \frac{1}{2}$ and $\rho = 1$, $w_0^0 = w_{-1}^0 = \gamma$. Then note that

$$\frac{\partial w_0^0}{\partial \rho} = 2\gamma \left(1 - \gamma\right) \left(\frac{\frac{1}{\rho^2}}{\left(2\gamma + \left(\frac{1}{\rho} + 1\right)\left(1 - \gamma\right)\right)^2} v_t^0 - \frac{\frac{1}{\rho^2}}{\left(2\gamma + \left(\frac{1}{\rho} + 1\right)\left(1 - \gamma\right)\right)^2} \left(1 - v_t^0\right) \right)$$

Solving this for an optimum at $v_t = \frac{1}{2}$ gives the following first order condition:

$$\frac{1}{(2\rho\gamma + (1+\rho)(1-\gamma))^2} \frac{1-\gamma}{2} - \frac{1}{(2\gamma + (\rho+1)(1-\gamma))^2} \left(1 - \frac{1-\gamma}{2}\right) = 0.$$

There is clearly only one positive solution $\frac{1}{\gamma^2+3}\left(\gamma^2+2\sqrt{-\gamma^2+1}-1\right)$. Finally, note that evaluating the derivative at $v_t = \frac{1}{2}$, so that $v_t^0 = \frac{1-\gamma}{2}$, and $\rho = 0$ gives $2\frac{\gamma^2}{\gamma+1} > 0$. Finally, evaluating the derivative at at $v_t = \frac{1}{2}$, so that $v_t^0 = \frac{1-\gamma}{2}$, and $\rho = 1$ gives: $-\frac{1}{2}\gamma^2(1-\gamma) < 0$. Now we shall show that w_0^0 decreases more slowly than w_{-1}^0 . For this note that $\frac{\partial w_{-1}^0}{\partial v_t^0} = -\frac{2\gamma}{1+\gamma}$, while

$$\frac{\partial w_0^0}{\partial v_t^0} = \frac{2\gamma}{2\gamma + \left(\frac{1}{\rho} + 1\right)(1-\gamma)} - \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)}$$

This expression is increasing in ρ , so the smallest it can be is at $\rho = 0$, when it coincides with $\frac{\partial w_{-1}^0}{\partial v_t^0} = -\frac{2\gamma}{1+\gamma}$. Thus, the claim is proved. Therefore, it is optimal for the manager with $s_t = 0$ not to trade.

Proprietary trader's strategy: Consider the proprietary trader who observes $s_t = 1$. If he buys his payoff is

$$v_t^1 - p_t^a = v_t^1 - v_t^1 - \beta \left(w_1^1 - w_0^1 \right) = -\beta \left(w_1^1 - w_0^1 \right) < 0$$

where the inequality follows from three observations: (i) as we have established above, w_0^1 is increasing in ρ for $v_t > \frac{1}{2}$; (ii) for $\rho = 1$, $w_0^1 = \gamma$; and finally (iii) for $v_t > \frac{1}{2}$, it is easy to see that $w_1^1 > \gamma$. If the trader does not trade his payoff is 0. If, instead, he sells, his payoff is

$$p_t^b - v_t^1 = v_t^0 - v_t^1 < 0$$

Thus, it is optimal for the proprietary trader not to trade.

Next consider the proprietary trader who observes $s_t = 0$. If he buys, his expected payoff is strictly smaller than that of the proprietary trader who observed $s_t = 1$, which itself was negative. If he does not trade, his payoff is 0. If he sells, his expected payoff is

$$p_t^b - v_t^0 = v_t^0 - v_t^0 = 0$$

Thus, it is a best response for this proprietary trader to sell.

Market maker's strategy: Since the market maker trades with proprietary traders at fair value, he is indifferent between trading with them or not. So, the only question is whether the market maker can improve terms of trade with fund managers.

By using the equilibrium strategies, the MM can extract positive (maximal) surplus from $s_t = 1$ fund managers, but gets zero surplus from interacting with $s_t = 0$ fund managers. It is clear that he will not wish to change the behavior of $s_t = 1$ managers.

We first show that as long as $s_t = 1$ managers buy, the MM will never wish to have $s_t = 0$ managers sell with positive probability. In any putative equilibrium in which the $s_t = 1$ managers buy, and the $s_t = 0$ sell with positive probability, the posterior for non-trading is identical to the original equilibrium posterior w_0^0 (because non-trading reveals that the manager either got signal $s_t = 0$ or did not receive a trading opportunity. Similarly, the putative equilibrium posterior for selling is identical to the "sincere" off-equilibrium belief used above: w_{-1}^0 (because sales in the putative equilibrium identify the manager as having received signal $s_t = 0$). In order to sell with positive probability, the manager with $s_t = 0$ must at least weakly prefer selling to non-trading. Denoting the bid price in this putative equilibrium by \tilde{p}_t^b , we now can write down:

$$\widetilde{p}_t^b - v_t^0 + \beta w_{-1}^0 \geq \beta w_0^0$$

This, in turn, implies that

$$\tilde{p}_t^b \ge v_t^0 + \beta \left(w_0^0 - w_{-1}^0 \right) > v_t^0$$

since we have shown earlier that $w_0^0 > w_{-1}^0$. But bidding such a price can never be incentive compatible for the MM, which rules out this possible deviation.

The only remaining alternative is that the market maker prices to induce both $s_t = 1$ and $s_t = 0$ managers to buy. We need to check that his profits in this potential deviation are smaller than his (strictly positive) equilibrium profits. Suppose that the market maker prices to induce the $s_t = 0$ manager to buy with probability $\alpha \in (0, 1]$, and to not trade with probability $1 - \alpha$. The expected reputational payoffs from buying in this putative equilibrium are as follows:

$$\widehat{w}_{1}^{1} = \left(v_{t}^{1} \frac{\gamma}{\gamma + \frac{1}{2} (1 - \gamma) (1 + \alpha)} + (1 - v_{t}^{1}) \frac{\gamma \alpha}{\gamma \alpha + \frac{1}{2} (1 - \gamma) (1 + \alpha)} \right)$$

$$\widehat{w}_{1}^{0} = \left(v_{t}^{0} \frac{\gamma}{\gamma + \frac{1}{2} (1 - \gamma) (1 + \alpha)} + (1 - v_{t}^{0}) \frac{\gamma \alpha}{\gamma \alpha + \frac{1}{2} (1 - \gamma) (1 + \alpha)} \right)$$

It is easy to see that $\hat{w}_1^1 > \hat{w}_1^0$. By a similar set of computations, the reputational payoffs from not trading in this putative equilibrium are as follows:

$$\widehat{w}_{0}^{1} = v_{t}^{1} \frac{\rho\gamma}{\rho\gamma + \left(\rho + (1-\rho)\left(1-\alpha\right)\frac{1}{2}\right)(1-\gamma)} + (1-v_{t}^{1})\frac{\left(\rho + (1-\rho)\left(1-\alpha\right)\right)\gamma}{\left(\rho + (1-\rho)\left(1-\alpha\right)\right)\gamma + \left(\rho + (1-\rho)\left(1-\alpha\right)\frac{1}{2}\right)(1-\gamma)}$$

$$\widehat{w}_{0}^{0} = v_{t}^{0} \frac{\rho\gamma}{\rho\gamma + \left(\rho + (1-\rho)\left(1-\alpha\right)\frac{1}{2}\right)(1-\gamma)} + (1-v_{t}^{0})\frac{\left(\rho + (1-\rho)\left(1-\alpha\right)\right)\gamma}{\left(\rho + (1-\rho)\left(1-\alpha\right)\right)\gamma + \left(\rho + (1-\rho)\left(1-\alpha\right)\frac{1}{2}\right)(1-\gamma)}$$

It is easy to see that $\hat{w}_0^1 < \hat{w}_0^0$. Denote the revised ask price in such a putative equilibrium by \hat{p}_t^a . Since the fund manager with $s_t = 0$ must weakly prefer buying to not trading, it must be the case that

$$\begin{aligned} \beta \widehat{w}_0^0 &\leq v_t^0 - \widehat{p}_t^a + \beta \widehat{w}_1^0 \\ \text{i.e., } \widehat{p}_t^a &\leq v_t^0 + \beta \left(\widehat{w}_1^0 - \widehat{w}_0^0 \right) \end{aligned}$$

The MM's expected profit under the equilibrium strategy is:

$$\eta \Pr(s_t = 1)(p_t^a - v_t^1) = \eta \Pr(s_t = 1)\beta \left(w_1^1 - w_0^1\right)$$

Define

$$\pi_E \equiv \Pr(s_t = 1)\beta \left(w_1^1 - w_0^1\right)$$

The MM's expected profit under the putative deviation is:

$$\eta \Pr(s_t = 1)(\hat{p}_t^a - v_t^1) + \eta \Pr(s_t = 0)(\hat{p}_t^a - v_t^0)\alpha.$$

Define

$$\pi_D \equiv \Pr(s_t = 1)(\hat{p}_t^a - v_t^1) + \Pr(s_t = 0)(\hat{p}_t^a - v_t^0)\alpha$$

Since $\widehat{p}_t^a \leq v_t^0 + \beta \left(\widehat{w}_1^0 - \widehat{w}_0^0 \right)$

$$\begin{aligned} \pi^{D} &\leq \Pr(s_{t} = 1)(v_{t}^{0} + \beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) - v_{t}^{1}) + \Pr(s_{t} = 0)(v_{t}^{0} + \beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) - v_{t}^{0})\alpha \\ &= \Pr(s_{t} = 1) \left(v_{t}^{0} - v_{t}^{1}\right) + \Pr(s_{t} = 1)\beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) + \Pr(s_{t} = 0)(\beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right))\alpha \\ &< \Pr(s_{t} = 1)\beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) + \Pr(s_{t} = 0)(\beta \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right))\alpha \\ &= \beta \left(\Pr(s_{t} = 1) + \Pr(s_{t} = 0)\alpha\right) \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) \\ &= \beta \Pr(a_{t} = 1) \left(\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0}\right) \equiv UB_{D}(\alpha) \end{aligned}$$

We show below that $UB_D(\alpha) < \pi_E$ for all $\alpha \in [0,1]$, which implies that the deviation is unprofitable for the MM. First, note that

$$\widehat{w}_{1}^{0} - \widehat{w}_{0}^{0} = \begin{pmatrix} \Pr(v = 1 | s = 0) \left(\Pr\left(\theta = g | a_{t} = 1, v = 1; \alpha\right) - \Pr\left(\theta = g | a_{t} = 0, v = 1; \alpha\right) \right) \\ + \Pr(v = 0 | s = 0) \left(\Pr\left(\theta = g | a_{t} = 1, v = 0; \alpha\right) - \Pr\left(\theta = g | a_{t} = 0, v = 0; \alpha\right) \right) \end{pmatrix} \\ < \begin{pmatrix} \Pr(v = 1 | a_{t} = 1) \left(\Pr\left(\theta = g | a_{t} = 1, v = 1; \alpha\right) - \Pr\left(\theta = g | a_{t} = 0, v = 0; \alpha\right) \right) \\ + \Pr(v = 0 | a_{t} = 1) \left(\Pr\left(\theta = g | a_{t} = 1, v = 0; \alpha\right) - \Pr\left(\theta = g | a_{t} = 0, v = 0; \alpha\right) \right) \end{pmatrix}$$

where the inequality follows from the fact that, since managers with $s_t = 1$ also buy, $\Pr(v = 1 | a_t = 1) > \Pr(v = 1 | s = 0)$. The expression to the right of the inequality can, in turn, be written as follows:

$$\begin{pmatrix} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 1, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 1, v = 0; \alpha) \\ - \begin{pmatrix} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \end{pmatrix} \end{pmatrix}$$

= $\Pr(\theta = g | a_t = 1; \alpha) - \begin{pmatrix} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \end{pmatrix}$

Thus, $\frac{1}{\beta}UB_D(\alpha)$ can be written as:

$$\Pr(a_t = 1) \left(\Pr(\theta = g | a_t = 1; \alpha) - \left(\begin{array}{c} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \end{array} \right) \right)$$

which, by adding and subtracting $Pr(a_t = 0) Pr(\theta = g | a_t = 0; \alpha)$, can be further written as:

$$\begin{cases} \Pr(a_t = 1) \Pr(\theta = g | a_t = 1; \alpha) \\ + (\Pr(a_t = 0) \Pr(\theta = g | a_t = 0; \alpha) - \Pr(a_t = 0) \Pr(\theta = g | a_t = 0; \alpha)) \\ - \Pr(a_t = 1) \begin{pmatrix} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \end{pmatrix} \end{pmatrix} \\ = \gamma - \begin{pmatrix} \Pr(a_t = 0) \begin{pmatrix} \Pr(v = 1 | a_t = 0) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 0) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \end{pmatrix} \\ + \Pr(a_t = 1) \begin{pmatrix} \Pr(v = 1 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 0; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) \\ + \Pr(v = 0 | a_t = 1) \Pr(v = 1 | a_t = 0) \\ + \Pr(a_t = 1) \Pr(v = 1 | a_t = 1) \Pr(v = 1 | a_t = 0) \\ + \Pr(a_t = 1) \Pr(v = 1 | a_t = 1) \Pr(v = 1 | a_t = 1) \end{pmatrix} \\ - \Pr(\theta = g | a_t = 0, v = 1; \alpha) \begin{pmatrix} \Pr(a_t = 0) \Pr(v = 0 | a_t = 0) \\ + \Pr(a_t = 1) \Pr(v = 0 | a_t = 0) \\ + \Pr(a_t = 1) \Pr(v = 0 | a_t = 1) \end{pmatrix} \\ = \gamma - (\Pr(v = 1) \Pr(\theta = g | a_t = 0, v = 1; \alpha) + \Pr(v = 0) \Pr(\theta = g | a_t = 0, v = 0; \alpha)) \end{cases}$$

Claim 9

$$\frac{\partial}{\partial \alpha} \left[\Pr(v=1) \Pr\left(\theta = g | a_t = 0, v = 1; \alpha\right) + \Pr(v=0) \Pr\left(\theta = g | a_t = 0, v = 0; \alpha\right) \right] > 0$$

Proof of claim: From the expressions delineated above we know that:

$$\Pr(\theta = g | a_t = 0, v = 1; \alpha) = \frac{\rho \gamma}{\rho \gamma + \left(\rho + (1 - \rho) \left(1 - \alpha\right) \frac{1}{2}\right) (1 - \gamma)}$$

and

$$\Pr\left(\theta = g | a_t = 0, v = 0; \alpha\right) = \frac{\left(\rho + (1 - \rho) (1 - \alpha)\right)\gamma}{\left(\rho + (1 - \rho) (1 - \alpha)\right)\gamma + \left(\rho + (1 - \rho) (1 - \alpha)\frac{1}{2}\right)(1 - \gamma)}$$

Direct computation shows that

$$\frac{\partial}{\partial \alpha} \Pr\left(\theta = g | a_t = 0, v = 1; \alpha\right) > 0, \ \frac{\partial}{\partial \alpha} \Pr\left(\theta = g | a_t = 0, v = 0; \alpha\right) < 0$$

and that

$$\frac{\partial}{\partial \alpha} \Pr\left(\theta = g | a_t = 0, v = 1; \alpha\right) + \frac{\partial}{\partial \alpha} \Pr\left(\theta = g | a_t = 0, v = 0; \alpha\right) > 0.$$

Since, for $v_t > \frac{1}{2}$, by definition, $\Pr(v=1) > \frac{1}{2} > \Pr(v=0)$, the claim is proved.

From Claim 9 it follows that $UB_D(\alpha)$ is decreasing in α , and thus is maximized for $\alpha = 0$. But, since at $\alpha = 0$, $\widehat{w}_1^0 = v_t^0 \frac{2\gamma}{1+\gamma}$, and $\widehat{w}_0^0 = w_0^0$ (the equilibrium expected posterior for a manager who does not trade when he receives signal $s_t = 0$).

$$UB_D(0) = \beta \Pr(s_t = 1) \left(v_t^0 \frac{2\gamma}{1+\gamma} - w_0^0 \right) < \beta \Pr(s_t = 1) \left(w_1^1 - w_0^1 \right) = \pi_E$$

because, it is clear that $w_1^1 = v_t^1 \frac{2\gamma}{1+\gamma} > v_t^0 \frac{2\gamma}{1+\gamma}$, and we have shown earlier than $w_0^1 < w_0^0$. Thus the deviation is unprofitable. This completes the proof for $v_t > \frac{1}{2}$.

Proof of Proposition 3: To check whether $E_t(p_{t+1}) - p_t^a > 0$, we first restate the definition of $E_t(p_{t+1})$ below:

$$E_t(p_{t+1}) = \frac{1}{1 + \frac{1 - \eta}{\eta} \frac{\Pr(s_{t+1} = 0|h_{t+1})}{\Pr(s_{t+1} = 1|h_{t+1})}} \left(v_{t+1}^1 + \beta \left(w_1^1(v_{t+1}) - w_0^1(v_{t+1}) \right) \right) + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + 1} v_{t+1}^0 + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + 1 + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})}}} + \frac{1}{\frac{\eta}{1 - \eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}$$

Since there was a buy order at $t, v_{t+1} > v_t$, and thus $v_{t+1}^1 + \beta \left(w_1^1(v_{t+1}) - w_0^1(v_{t+1}) \right) > p_t^a$. However, $v_{t+1}^0 < p_t^a$ (because $v_{t+1} = v_t^1$, and thus $p_t^a > v_{t+1}$, and $v_{t+1} > v_{t+1}^0$). Now note that $\frac{1}{1 + \frac{1-\eta}{\eta} \frac{\Pr(s_{t+1} = 0|h_{t+1})}{\Pr(s_{t+1} = 1|h_{t+1})}}$ is increasing in η and converges to 1 as $\eta \to 1$, and $\frac{1}{\frac{\eta}{1-\eta} \frac{\Pr(s_{t+1} = 1|h_{t+1})}{\Pr(s_{t+1} = 0|h_{t+1})} + 1}$ is decreasing in η and converges to 1 as $\eta \to 0$. Thus, there exists $\bar{\eta} \in (0, 1)$, such that for $\eta > \bar{\eta}, E_t(p_{t+1}) - p_t^a > 0$.

Proof of Proposition 4: For the case where $v_t > \frac{1}{2}$, $LTR_t = -\frac{v_{t+1}-p_t^a}{p_t^a} = 1 - \frac{v_{t+1}}{p_t^a}$.

The comparative static with respect to β is immediate, since p_t^a increases in β while v_{t+1} is unaffected by β .

The remaining goal is to show that LTR_t is increasing in v_{t+1} .

$$\begin{split} LTR_t &= -\frac{v_{t+1} - p_t^a}{p_t^a} = -\frac{v_t^1 - v_t^1 - \beta \left(w_1^1(v_t) - w_0^1(v_t)\right)}{v_t^1 + \beta \left(w_1^1(v_t) - w_0^1(v_t)\right)} = \frac{\beta \left(w_1^1(v_t) - w_0^1(v_t)\right)}{v_t^1 + \beta \left(w_1^1(v_t) - w_0^1(v_t)\right)} \\ \frac{1}{LTR_t} &= 1 + \frac{1}{\beta} \frac{v_t^1}{\left(w_1^1(v_t) - w_0^1(v_t)\right)} = 1 + \frac{1}{\beta}f, \text{ where } f = \frac{v_t^1}{\left(w_1^1(v_t) - w_0^1(v_t)\right)}. \\ \frac{1}{f} &= \frac{w_1^1(v_t) - w_0^1(v_t)}{v_t^1} = \frac{\frac{2\gamma}{1+\gamma}v_t^1 - \left(\frac{2\rho\gamma}{2\rho\gamma + (1+\rho)(1-\gamma)}v_t^1 + \frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)}\left(1 - v_t^1\right)\right)}{v_t^1} \\ &= \frac{2\gamma}{1+\gamma} - \frac{2\rho\gamma}{2\rho\gamma + (1+\rho)\left(1-\gamma\right)} + \frac{2\gamma}{2\gamma + (1+\rho)\left(1-\gamma\right)} - \frac{\frac{2\gamma}{2\gamma + (1+\rho)(1-\gamma)}}{v_t^1} \end{split}$$

so that $\frac{1}{f}$ is increasing in v_t^1 , so that f is decreasing in v_t^1 , so that $\frac{1}{LTR_t}$ is decreasing in v_t^1 , so that LTR_t is increasing in $v_t^1 = v_{t+1}$, which establishes the desired result.

Proof of proposition 5: The comparative static relative to η is immediate. Increasing η increases $E_t(p_{t+1})$ without affecting p_t^a .

For the remainder, we are trying to show that $\frac{E_t(p_{t+1})-p_t^a}{p_t^a}$ is decreasing in $v_{t+1} = v_t^1$. Since p_t^a is increasing in v_t^1 , a sufficient condition is that $E_t(p_{t+1}) - p_t^a$ is decreasing in v_t^1 . We prove that this is true for η large enough.

First, note that $v_{t+1}^1 - v_t^1$ is decreasing in v_t^1 :

$$v_{t+1}^{1} - v_{t}^{1} = \frac{(1+\gamma)v_{t}^{1}}{2\gamma v_{t}^{1} + 1 - \gamma} - v_{t}^{1} = 2\gamma \frac{v_{t}^{1}\left(1 - v_{t}^{1}\right)}{1 - \gamma + 2\gamma v_{t}^{1}}$$

This is clearly decreasing for $v_t^1 > \frac{1}{2}$ since the numerator is decreasing in this range and the denominator is always increasing. Let

$$f(v_{t+1},\eta) \equiv \frac{1}{1 + \frac{1-\eta}{\eta} \frac{\Pr(s_{t+1}=0|h_{t+1})}{\Pr(s_{t+1}=1|h_{t+1})}}$$

Then,

$$E_{t}(p_{t+1}) - p_{t}^{a}$$

$$= \frac{1}{1 + \frac{1 - \eta}{\eta} \frac{\Pr(s_{t+1} = 0|h_{t+1})}{\Pr(s_{t+1} = 1|h_{t+1})}} p_{t+1}^{a} + \left(1 - \frac{1}{1 + \frac{1 - \eta}{\eta} \frac{\Pr(s_{t+1} = 0|h_{t+1})}{\Pr(s_{t+1} = 1|h_{t+1})}}\right) v_{t+1}^{0} - p_{t}^{a}$$

$$= f(v_{t+1}, \eta) p_{t+1}^{a} + (1 - f(v_{t+1}, \eta)) v_{t+1}^{0} - p_{t}^{a}$$

$$= f(v_{t+1}, \eta) \left(p_{t+1}^{a} - p_{t}^{a}\right) + (1 - f(v_{t+1}, \eta)) \left(v_{t+1}^{0} - p_{t}^{a}\right)$$

$$= f(v_{t+1}, \eta) a (v_{t+1}) + (1 - f(v_{t+1}, \eta)) b (v_{t+1})$$

where $a(v_{t+1}) = p_{t+1}^a - p_t^a$ and $b(v_{t+1}) = v_{t+1}^0 - p_t^a$. Now,

$$\begin{aligned} &\frac{\partial}{\partial v_{t+1}} \left(f(v_{t+1},\eta) a\left(v_{t+1}\right) + \left(1 - f(v_{t+1},\eta)\right) b\left(v_{t+1}\right) \right) \\ &= f_{v_{t+1}}(v_{t+1},\eta) a\left(v_{t+1}\right) + f(v_{t+1},\eta) a'\left(v_{t+1}\right) - f_{v_{t+1}}(v_{t+1},\eta) b\left(v_{t+1}\right) + \left(1 - f(v_{t+1},\eta)\right) b'\left(v_{t+1}\right) \\ &= f(v_{t+1},\eta) a'\left(v_{t+1}\right) + f_{v_{t+1}}(v_{t+1},\eta) \left(a\left(v_{t+1}\right) - b\left(v_{t+1}\right)\right) + \left(1 - f(v_{t+1},\eta)\right) b'\left(v_{t+1}\right). \end{aligned}$$

It is obvious that $f(v_{t+1}, \eta) > 0$, $f(v_{t+1}, \eta) \to 1$ as $\eta \to 1$, and that $a(v_{t+1}) - b(v_{t+1})$ and $b'(v_{t+1})$ are bounded. We'll show below that (i) $a'(v_{t+1}) < 0$, and (ii) that $f_{v_{t+1}}(v_{t+1}, \eta) \to 0$ as $\eta \to 1$. Thus, for large enough η , the second and third terms become arbitrarily small, and the first term is negative and becomes large in absolute value, meaning that $E_t(p_{t+1}) - p_t^a$ decreases in v_{t+1} .

To see that (ii) is true, observe that since $\Pr(s_{t+1} = 1 | h_{t+1}) = \gamma v_{t+1} + \frac{1-\gamma}{2}$,

$$f_{v_{t+1}}(v_{t+1},\eta) = \frac{\partial}{\partial v_{t+1}} \frac{1}{1 + \frac{1-\eta}{\eta} \frac{1-\gamma v_{t+1} - \frac{1-\gamma}{2}}{\gamma v_{t+1} + \frac{1-\gamma}{2}}} = 4\gamma \frac{\eta \left(1-\eta\right)}{\left(\gamma - 2v\gamma - 2\gamma\eta + 4v\gamma\eta + 1\right)^2} \to 0 \text{ as } \eta \to 1$$

To establish (i), we observe that

$$\begin{aligned} a\left(v_{t+1}\right) &= p_{t+1}^{a} - p_{t}^{a} \\ &= v_{t+1}^{1} + \beta\left(w_{1}^{1}(v_{t+1}) - w_{0}^{1}\left(v_{t+1}\right)\right) - v_{t}^{1} - \beta\left(w_{1}^{1}(v_{t}) - w_{0}^{1}\left(v_{t}\right)\right) \\ &= \left(v_{t+1}^{1} - v_{t}^{1}\right) + \beta\left(w_{1}^{1}(v_{t+1}) - w_{1}^{1}(v_{t})\right) + \beta\left(w_{0}^{1}(v_{t}) - w_{0}^{1}(v_{t+1})\right) \\ &= \left(v_{t+1}^{1} - v_{t}^{1}\right) + \beta\frac{2\gamma}{1+\gamma}\left(v_{t+1}^{1} - v_{t}^{1}\right) + \\ &\beta\left(\begin{array}{c} \Pr\left(g|v=1, a=0\right)v_{t}^{1} + \Pr\left(g|v=0, a=0\right)\left(1 - v_{t}^{1}\right) \\ -\Pr\left(g|v=1, a=0\right)v_{t+1}^{1} - \Pr\left(g|v=0, a=0\right)\left(1 - v_{t+1}^{1}\right) \end{array}\right) \\ &= \left(v_{t+1}^{1} - v_{t}^{1}\right)\left[1 + \beta\frac{2\gamma}{1+\gamma} + \beta\left(\frac{2\gamma}{2\gamma + (1+\rho)\left(1-\gamma\right)} - \frac{2\rho\gamma}{2\rho\gamma + (1+\rho)\left(1-\gamma\right)}\right)\right], \end{aligned}$$

which is clearly decreasing in v_{t+1} since $v_{t+1}^1 - v_t^1$ is decreasing in v_{t+1} , and $1 + \beta \frac{2\gamma}{1+\gamma} + \beta \left(\frac{2\gamma}{2\gamma+(1+\rho)(1-\gamma)} - \frac{2\rho\gamma}{2\rho\gamma+(1+\rho)(1-\gamma)}\right) > 0$. This concludes the proof.

Proof of Proposition 6: Since $v_{t+1} > \frac{1}{2}$, if there is a trade, there are two possibilities: Either a manager was selected to trade and $s_{t+1} = 1$, in which case $a_{t+1} = 1$ and so $v_{t+2} > v_{t+1}$ and thus $LRT_{t+1} > LRT_t$, or a proprietary trader was selected to trade and $s_{t+1} = 0$, in which case $a_{t+1} = -1$ and so $v_{t+2} < v_{t+1}$ and thus $LRT_{t+1} < LRT_t$. Conditional on a trade taking place, the probability of the former event is $\frac{\eta \Pr(s_{t+1}=1|h_{t+1})}{\eta \Pr(s_{t+1}=1|h_{t+1})+(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}$ which is increasing in η (and converges to 1 as $\eta \to 1$); in contrast, the probability of the latter event is $\frac{(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}{\eta\Pr(s_{t+1}=1|h_{t+1})+(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}$ which is decreasing in η (and converges to 0 as $\eta \to 1$). In other words, conditional on $l_t \neq 0$, as η increases the probability of the former event is $\frac{\eta\Pr(s_{t+1}=0|h_{t+1})}{\eta\Pr(s_{t+1}=0|h_{t+1})+(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}$ which is increasing in η (and converges to 1 as $\eta \to 1$); in contrast, the probability of the former event is $\frac{\eta\Pr(s_{t+1}=0|h_{t+1})}{\eta\Pr(s_{t+1}=0|h_{t+1})+(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}$ which is increasing in η (and converges to 1 as $\eta \to 1$); in contrast, the probability of the latter event is $\frac{(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}{\eta\Pr(s_{t+1}=0|h_{t+1})+(1-\eta)\Pr(s_{t+1}=1|h_{t+1})}$ which is increasing in η (and converges to 1 as $\eta \to 1$); in contrast, the probability of the latter event is $\frac{(1-\eta)\Pr(s_{t+1}=0|h_{t+1})}{\eta\Pr(s_{t+1}=0|h_{t+1})+(1-\eta)\Pr(s_{t+1}=1|h_{t+1})}}$ which is increasing in η (and converges to 1 as $\eta \to 1$); in contrast, the probability of the latter event is $\frac{(1-\eta)\Pr(s_{t+1}=1|h_{t+1})}{\eta\Pr(s_{t+1}=0|h_{t+1})+(1-\eta)\Pr(s_{t+1}=1|h_{t+1})}}$ which is decreasing in η (and converges to 0 as $\eta \to 1$). Thus, conditional on $l_t = 0, v_{t+2}$ is decreasing in η . Conditional on no-trade, therefore, for η large enough, $v_{t+2} < v_{t+1}$ and thus $LRT_{t+1} < LRT_t$.

Proof of Proposition 8: We write the proof for $v_t \in \left[\frac{1}{2}, v^*\right]$ and $v_t > v^*$. The cases for $v_t \in \left[1 - v^*, \frac{1}{2}\right)$ and $v_t < 1 - v^*$ are symmetric.

Consider w_0^0 and w_1^0 as defined for $v_t \ge \frac{1}{2}$ in the proof of Proposition 2. Note that for $v_t = \frac{1}{2}$, $w_0^0 > w_1^0$, for $v_t = 1$, $w_0^0 < w_1^0$, w_0^0 is strictly decreasing in v_t and w_1^0 is strictly increasing in v_t . Define $\bar{v}(\gamma, \rho)$ as the unique solution to $w_0^0(v_t) - w_1^0(v_t) = 0$.

Consider $v_t \in \left[\frac{1}{2}, v^*\right]$. Note that since equilibrium strategies for fund managers are identical in this region to those in the main proposition for $v_t \geq \frac{1}{2}$, all expected reputation terms w_a^s in the proof of Proposition 2 are unchanged and we can re-use their properties.

First consider the manager with $s_t = 1$. The manager's payoff from buying is $v_t^1 - p^a(h_t) + \beta w_1^1$. From not trading, the manager gets βw_0^1 . From selling, he gets $p^b(h_t) - v_t^1 + \beta w_{-1}^1$. Note also that $w_1^1 > w_0^1 > w_{-1}^1$. The incremental payoff from buying vs not trading is

$$v_t^1 - p^a(h_t) + \beta w_1^1 - \beta w_0^1$$

= $v_t^1 - (v_t^1 + \lambda (1 - 2I_{h_t}) Var(v|h_{tb})) + \beta (w_1^1 - w_0^1)$
= $-\lambda (1 - 2I_{h_t}) Var(v|h_{tb}) + \beta (w_1^1 - w_0^1)$

If $I_{h_t} < 0$ the first term is negative, and the second term is positive since $w_1^1 - w_0^1 > 0$. Clearly, as long as β is large enough (say, $\beta > \beta_1$), the second term will dominate, and the manager will buy rather than not trade. If $I_{h_t} > 0$, then the first term is positive, thus making buying even more desirable for a manager with $s_t = 1$. The incremental payoff from buying vs selling is

$$v_t^1 - p^a(h_t) + \beta w_1^1 - \left(p^b(h_t) - v_t^1 + \beta w_{-1}^1\right)$$

= $v_t^1 - \left(v_t^1 + \lambda \left(1 - 2I_{h_t}\right) Var(v|h_{tb})\right) + \beta w_1^1 - \left(v_t^0 - \lambda \left(1 + 2I_{h_t}\right) Var(v|h_{ts}) - v_t^1 + \beta w_{-1}^1\right)$
= $\left(v_t^1 - v_t^0\right) + \left[-\lambda \left(1 - 2I_{h_t}\right) Var(v|h_{tb}) + \lambda \left(1 + 2I_{h_t}\right) Var(v|h_{ts})\right] + \beta \left(w_1^1 - w_{-1}^1\right)$

The first and third terms are positive, while the second term is of ambigious sign if $I_{h_t} < 0$. However, again, if β is large enough (say $\beta > \beta_2$), the positive terms will dominate, and the manager will buy rather than sell. If $I_{h_t} > 0$, the middle term is also positive, so the conclusion is reinforced. Thus, the manager with $s_t = 1$ will always buy.

Now consider the manager with $s_t = 0$. The manager's payoff from buying is $v_t^0 - p^a(h_t) + \beta w_1^0$. From not trading, the manager gets βw_0^0 . From selling, he gets $p^b(h_t) - v_t^0 + \beta w_{-1}^0$. The incremental payoff from not trading instead of selling is as follows:

$$\beta w_0^0 - \left(p^b \left(h_t \right) - v_t^0 + \beta w_{-1}^0 \right)$$

= $\beta \left(w_0^0 - w_{-1}^0 \right) + v_t^0 - v_t^0 + \lambda \left(1 + 2I_{h_t} \right) Var \left(v | h_{ts} \right)$
= $\beta \left(w_0^0 - w_{-1}^0 \right) + \lambda \left(1 + 2I_{h_t} \right) Var \left(v | h_{ts} \right)$

Note that, as shown in the proof of Proposition 2, $w_0^0 > w_{-1}^0$ for $v_t \ge \frac{1}{2}$. Thus, the first term is positive. The second term is negative if $I_{h_t} < 0$. However, for β large enough (say $\beta > \beta_3$), the positive term dominates even if $I_{h_t} < 0$. For $I_{h_t} > 0$, the whole term is always positive. Thus, the manager always prefers not to trade rather than sell. The incremental payoff from not trading instead of buying is as follows:

$$\beta w_0^0 - \left(v_t^0 - p^a(h_t) + \beta w_1^0\right) \\ = \beta \left(w_0^0 - w_1^0\right) + \left(v_t^1 - v_t^0\right) + \lambda \left(1 - 2I_{h_t}\right) Var\left(v|h_{tb}\right)$$

By definition, since $v^* < \bar{v}(\gamma, \rho)$, there exists an $\epsilon > 0$, such that for $v_t \le v^*$, $w_0^0 - w_1^0 \ge \varepsilon > 0$. Thus, if $I_{h_t} < 0$, this expression is positive. If $I_{h_t} > 0$, the final term is negative, but nevertheless, for β large enough (say $\beta > \beta_4$), the positive terms dominate and thus not trading dominates buying.

Now consider $v_t > v^*$. In this region, equilibrium strategies prescribe non-trading, which come with a reputational payoff of γ . Specify off equilibrium beliefs that give the manager a posterior of 0 if he trades in either direction. Then, since profits are bounded, for β large enough (say $\beta > \beta_5$), he will not trade, regardless of the profits that may be associated with such a trade. Now, set $\beta^* = \max(\beta_i \text{ for } i = 1, 2, 3, 4, 5)$, and let $\beta > \beta^*$.

Finally, we complete the proof by writing down the market maker's pricing rule. Suppose the market maker has inventory I_{h_t} and cash position C_{h_t} . If a trader offers to buy from him following h_t (inducing history $h_{t+1} = h_{tb}$), his invfentory will change to $I_{h_t} - 1$. If he accepts this trade, at price p, his utility will be given by

$$C_{h_t} + p + E[(I_{h_t} - 1)v|h_{tb}] - \lambda Var[(I_{h_t} - 1)v|h_{tb}]$$

whereas if he does not trade, his utility will be

$$C_{h_t} + E\left[(I_{h_t})v|h_{tb}\right] - \lambda Var\left[(I_{h_t})v|h_{tb}\right]$$

Competition implies that he will trade at a price that makes him indifferent between trading and not trading, so that the ask price is defined by

$$p^{a}(h_{t}) = E[v|h_{tb}] + \lambda \left(Var\left[(I_{h_{t}} - 1)v|h_{tb} \right] - Var\left[(I_{h_{t}})v|h_{tb} \right] \right)$$

= $v_{t}^{1} + \lambda \left(1 - 2I_{h_{t}} \right) Var(v|h_{tb})$

because buy orders are generated by traders with $s_t = 1$. The bid price is computed similarly.

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