Supplementary Information: Simulation details for the DICE model

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Emissions scenarios

The business-as-usual scenario is as standard in DICE-2009, and assumes largely unabated emissions for the next 250 years, after which a zero-emissions back-stop technology is assumed to become the competitive energy supply technology, driving energy-related anthropogenic emissions to zero. The mitigation scenario is taken from forthcoming work by Dietz and Matei (2013), and limits the median atmospheric concentration of CO_2 to 500ppm at least cost in a stochastic implementation of DICE with uncertainty about the climate sensitivity and effective heat capacity.[1] As in the business-as-usual scenario, a clean back-stop technology is assumed to operate from 2255. The two scenarios hence implement different emissions controls over the next 250 years, although the dynamics of the climate system mean that these differences will have effects beyond 2255. The emissions control rates for the two scenarios are plotted in figure 1.

Since the current atmospheric concentration of CO_2 (only) is 394ppm and the annual increase is 2.6ppm, 500ppm represents a demanding but arguably realistic goal. There have been discussions on goals for stabilizing atmospheric CO_2 as low as 350ppm, but these are infeasible without overshooting, which raises a new set of issues beyond the scope of this paper. Even targets of around 400ppm, which would be consistent with stabilizing the atmospheric concentration of all greenhouse gases at 450ppm CO_2 equivalent, may already be infeasible without overshooting (Clarke et al., 2009).

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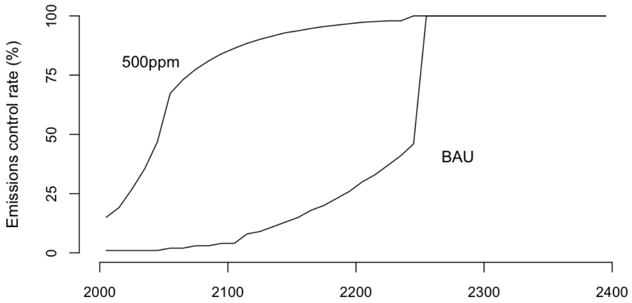
[1] Dietz and Matei make slightly different assumptions about the climate sensitivity and effective heat capacity in their work, and also explore other uncertainties in the model. Therefore, when their policy is implemented here, the outcome for atmospheric CO_2 will be somewhat different. However, since the choice of mitigation policy is arbitrary from an economic point of view, the inconsistency is not central to the analysis. The reason we draw on Dietz and Matei is that the exercise of minimizing abatement costs under uncertainty about multiple parameters in DICE is computationally highly demanding—the additional work is not justified here.

Figure 1: **Emissions control rate scenarios**



Welfare analysis

The DICE model, like virtually all IAMs, employs the constant relative risk aversion (CRRA) utility function, which takes the form $U(C) = \frac{C^{1-\eta}}{1-\eta}$, where U is utility, C is consumption, and η is the coefficient of relative risk aversion. The CRRA utility function has the property that, when the coefficient ≥ 1, utility approaches negative infinity as consumption goes to zero. As a consequence, the present discounted value of the stream of future consumption does not converge when extreme warming triggers an economic catastrophe, so defined (Weitzman, 2009). Since we are interested in the consequences of changing the probability of extreme warming, we must find a way to deal with this possibility in our numerical simulations. To make matters simple, we introduce a lower bound on output at \$1, and utility reaches a corresponding lower bound.[2] See Weitzman (2009) and Dietz (2011) for further discussion on where to place the lower bound.

Our measure of the value of mitigation is the percentage change in the expected stationary equivalent in going from the business-as-usual scenario to the mitigation scenario. The stationary equivalent is the constant level of consumption that would yield the same discounted utility as the original consumption path, over a given time horizon (400 years in our case). Formally, if the consumption path in some scenario yields a present discounted value of utility of V, the stationary equivalent is defined as the level of consumption C that solves the following equation:

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[2] Note that while the damage functions used in this study and other work with DICE rule out zero output *a priori*, the addition of positive abatement costs can result in non-positive output and consumption.

$$\sum_{t=0} \beta^t P_t U(C) = V \tag{1}$$

where β is the discount factor (defined as $\frac{1}{1-\rho}$, where ρ is the pure rate of time preference), P is population, and t is the number of the time-step. The expected utility of the stream of future consumption in this scenario can therefore be written equivalently as:

$$E\left[\sum_{t=0} \beta^t P_t U(C)\right] = E[V] \tag{2}$$

where the expectation is taken over states of the world. Our measure of the value of the mitigation policy can then be written equivalently as:

$$\frac{E[C_{500ppm}] - E[C_{BAU}]}{E[C_{BAU}]} = \left(\frac{E[V_{500ppm}]}{E[V_{BAU}]}\right)^{\frac{1}{1-\eta}} - 1 \tag{3}$$

This is a measure of net present value that works even when policy choices can cause non-marginal changes in consumption (Dietz and Hepburn, Forthcoming), something that is especially important to account for when studying extreme warming scenarios with the possibility of catastrophic economic damages.

Sampling

To calculate these expectations, our Monte Carlo simulations use a large sample of 100,000 parameter values for the climate sensitivity, drawn using Latin Hypercube sampling. This is important for getting an accurate representation of the probability of rare events in the tail of the pdfs.

Note that, when the effective heat capacity is set to $0.6\text{GJm}^{-2}\text{K}^{-1}$, numerical instabilities becoming very prevalent for lower values of the climate sensitivity. This has forced us to truncate our sample of climate sensitivities from below at 1.3°C in this case. This reduces the sample size by roughly half a percent. This truncation does not have any substantive impact on our results, however, since without this numerical instability, model runs with such low climate sensitivities make virtually no contribution to the value of the policy.

Additional results from welfare analysis

The reader may be interested to know what the results of welfare analysis look like with Nordhaus' damage function for different effective heat capacities. We include the results here for completeness. As table 1 illustrates, an effective heat capacity of $0.6\text{GJm}^{-2}\text{K}^{-1}$ leads to the value of the mitigation policy being roughly three times greater than under the baseline assumption that the effective heat capacity is $1.8\text{GJm}^{-2}\text{K}^{-1}$. The shape of the tail of the climate sensitivity distribution has a smaller effect, due to the fact that Nordhaus' damage function does not allow for catastrophic damages for the period concerned.

Table 1: **Value of 500ppm policy with varying effective heat capacity**

Climate sensitivity distribution	Increase in stationary equivalent (%)		
	$0.6\text{GJm}^{-2}\text{K}^{-1}$	$1.2\text{GJm}^{-2}\text{K}^{-1}$	$1.8\text{GJm}^{-2}\text{K}^{-1}$
IPCC AR4	0.83	0.52	0.31
Stainforth et al. (2005)	0.94	0.59	0.34
Roe and Baker (2007)	0.98	0.61	0.35

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