COMPLEXITY, CORRELATION AND CHOICE

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Abstract. We study an agent who chooses a profile of actions between which she may misperceive the correlation. Our agent cannot be modeled by reducing every action profile to an act, as implied by the usual monotonicity axiom. We introduce a novel framework that explicitly considers action profiles and axiomatically characterize a model that relaxes monotonicity but retains the rest of the expected utility axioms. Our agent acts as if she attaches a probability to each possible correlation structure and then maximizes expected utility using her (possibly misspecified) beliefs. This representation nests several models used in the behavioral game theory literature to consider imperfect inference, including correlation neglect.

1. Introduction

Decision problems often involve the interaction among several distinct actions, such as a portfolio of securities, a profile of strategies, or signals from distinct sources. The environment in which these variables interact is often very complex and how an agent perceives, or misperceives, the correlations among the outcomes of different actions is crucial for understanding the riskiness of her choices. We provide a decision-theoretic analysis of how environmental complexity can affect an agent's understanding and choice behavior, with the aim of providing a functional model of decisions and clarifying the connections between alternative behavioral approaches in the literature.

We depart from the standard device of modeling the decision maker (DM) as choosing individual acts, i.e. mappings from states to outcomes. Instead, we propose a framework that directly considers the choice of profiles including several distinct actions. In our framework, complexity does not affect the DM's decisions whenever she reduces profiles to individual acts, i.e. she is indifferent between any two action profiles corresponding to the same act. We model misperceptions of correlation as violations of this reduction, while retaining the remainder of the subjective expected utility axioms.

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Our axioms characterize an agent whose behavior has a *probabilistic correlation representation*, or PCR. As in standard subjective expected utility models (SEU), a PCR assigns a belief that a profile of actions yields a given outcome. Unlike SEU, it also assigns probabilities to outcomes resulting from joint realizations of actions that the modeler knows to be impossible. Given these possibly misspecified beliefs, a PCR hypothesizes expected utility maximization.

1.1. **Illustration and Plan.** We illustrate our framework, model, and results through a simple portfolio choice exercise. We then show that our trader systematically misvalues certain portfolios. In particular, she overvalues risky, complex portfolios and undervalues complex portfolios whose underlying assets hedge one another.

In Section 2, we formally define our primitives, the key ingredients of which we describe here. There are a set Ω of states of the world and a set of assets $\mathcal{A}$. Each asset $a \in \mathcal{A}$ returns a (real number) $a(\omega)$ in state ω. A *portfolio* is a finite collection assets, denoted $\langle a_1, a_2, ..., a_n \rangle$, and yields a payoff equal to the sum of the returns of all the underlying assets. The trader maximizes a preference relation $\succeq$ defined over portfolios.

Fix any two assets b and c and consider a third asset a satisfying $a(\omega) = b(\omega) + c(\omega)$ in each ω.[1] The profiles $\langle b, c \rangle$ and $\langle a \rangle$ give the same return in every state. However, if this trader misperceives the correlation between b and c - for instance, she thinks b and c are independent although they are positively correlated - then she may not be indifferent between $\langle b, c \rangle$ and $\langle a \rangle$, thus violating the typical Monotonicity axiom of choice theory. To model this behavior, we replace the standard Monotonicity axiom with a novel "Weak Monotonicity" axiom while retaining the other axioms of SEU.

In Section 3, we show that the trader's behavior satisfies our axioms if and only if her choices can be represented by a PCR. The trader's understanding is modeled via a collection of *understanding classes*. She understands the relations among the assets within an understanding class in the same way as the modeler but may incorrectly perceive the relation among different understanding classes. For this example only, there are two understanding classes, labeled B and C, and $B \cup C = \mathcal{A}$. We model the misperception of correlations by augmenting the state space to $\Omega \times \Omega$, where one copy of Ω corresponds to each understanding class. The trader's beliefs about pairwise correlations are represented by a probability measure $\pi(\cdot)$ over $\Omega \times \Omega$, and her tastes by a utility index $u(\cdot)$.

[1]While this may seem special special, many real world examples of such profiles exist. For example, a could be an exchange traded fund with one share of every stock in an industry having two firms, b and c, and Fleckenstein et al. (2014) give an example using T-Bills, TIPS, STRIPS, and inflation swaps.

Assume for simplicity that $\pi(\cdot)$ is symmetric - the probability distribution of individual assets is perceived correctly. The trader evaluates the utility of the profile $\langle b, c\rangle$ as

$$V\langle b,c\rangle = \begin{cases} \sum_{\omega} u(b(\omega)+c(\omega))\pi(\omega\times\Omega) & \text{if } b,c\in B \text{ or } b,c\in C \\ \sum_{(\omega_1,\omega_2)} u(b(\omega_1)+c(\omega_2))\pi(\omega_1,\omega_2) & \text{otherwise} \end{cases}.$$

On the one hand, if b and c belong to the *same* understanding class, i.e. $b, c \in B$ or $b, c \in C$, then the trader understands the connection between the two and evaluates the utility of the profile $\langle b, c\rangle$ according to the usual expected utility criterion. On the other hand, if b and c belong to *different* understanding classes, then the trader thinks that she gets the outcome $b(\omega_1) + c(\omega_2)$ with probability $\pi(\omega_1, \omega_2)$.

We call a collection of non-redundant understanding classes that spans all assets a *correlation cover*. In Section 4, we uniquely identify it from the preference relation under the assumption that each understanding class contains a rich enough set of acts. However, unique identification of the trader's beliefs about correlation depends critically on her risk attitude. For instance, risk-neutrality implies that any π' whose marginals agree with π represents choice, but whenever risk-neutrality fails, π can be uniquely determined. Theorem 3 characterizes the behavior necessary and sufficient to recover the DM's perceived correlation structure and relates it to the agents attitude towards (higher-order) risk.

We now turn to how misperception can affect the trader's valuation of portfolios. For the remainder of this example, we focus on a special case, the η-misperception model, inspired by Eyster and Rabin (2005), in which a single parameter, η, captures the misperception of correlation across classes. Specifically, the trader's belief $\pi(\omega_1, \omega_2)$ is given by

$$\pi(\omega_1,\omega_2) = \begin{cases} \eta q(\omega_1)q(\omega_2) & \text{if } \omega_1 \neq \omega_2 \\ \eta q(\omega_1)^2 + (1-\eta)q(\omega_1) & \text{if } \omega_1 = \omega_2 \end{cases}$$

where $q(\omega) = \pi(\omega \times \Omega)$. If $\eta = 1$, then the trader believes that any two securities in distinct understanding classes are independent, and if $\eta = 0$, then the trader understands the connection between any two securities. Thus, whenever b and c are in distinct understanding classes, $V\langle b, c\rangle$ equals

$$\eta V_{IND}(\langle b,c\rangle) + (1-\eta)V_{SEU}(\langle b,c\rangle)$$

where V_{IND} is what the trader's utility would be if she thought b and c were independent and V_{SEU} is what her utility would be if she evaluated the profile as the modeler.

Suppose now that $u(x) = x - \beta x^2$ for $\beta \geq 0$.[2] If b and c are in different understanding classes, indifference between $\langle b, c\rangle$ and a may not hold and η can be interpreted as representing the degree to which the trader misperceives correlations. Letting ρ denote the "modeler's"

[2]This specification of utility index is closely related to mean-variance risk preferences.

correlation coefficient, one can easily show that the trader's over- or under-valuation of asset depends on the sign of ρ. Specifically:

- If $\rho(b,c) > 0$, then $\langle b,c\rangle \succeq a$, and if $\rho(b,c) < 0$, then $a \succeq \langle b,c\rangle$.
- If $\rho(b,c) \neq 0$, $\langle b,c\rangle \sim a$ if and only if $b,c \in B$, $b,c \in C$, or $\eta = 0$.
- The degree of over- or under-valuation increases with η.

Misperception causes the trader to *undervalue* hedging and *overvalue* risk. In particular, the trader may fail to take advantage of arbitrage opportunities and may not realize the hedging-value of certain assets. Simultaneously, the trader may overpay for securities derived from correlated assets, underestimating their correlation. Both patterns have empirical support - for instance, Fleckenstein et al. (2014) demonstrate existence of arbitrage opportunities, and Brunnermeier (2009) appeals to misperception in their analysis of the mortgage-backed security crisis of 2008.

1.2. **Related Literature.** Our objective is to provide a general and flexible decision theoretic framework for modeling choices made by an agent who may misperceive correlation. Naturally, our analysis has several points of contact with works in decision theory, bounded rationality and behavioral economics.

Misperception of correlations is a broad concept that has been studied in various guises, ranging from the inability to infer patterns, as in Piccione and Rubinstein (2003), Eyster and Piccione (2013), and Levy and Razin (2015a), to the inability to derive some logical implications (Lipman, 1999), to attitudes towards different sources of uncertainty (French and Poterba, 1991). In particular contexts, misperception of correlation has been shown to lead to a range of behaviors, including social influence (DeMarzo et al., 2003), overconfidence (Ortoleva and Snowberg, 2015), and polarization (Levy and Razin, 2015b). A key feature of our approach is that, being based on preferences, it is neutral with respect to the psychological biases and limitations that cause agents to perceive correlations incorrectly.

Framing can also be viewed as a proximate reason for misperception. Within our framework, different framings of the same action can make understanding correlations harder; see Example 1. More fittingly, different profiles that yield the same outcomes can be viewed as different framings that affect the DM's choices. Choice theoretic works that highlight other aspects of framing include Salant and Rubinstein (2008) who study the conditions under which choice data can be rationalized as resulting from choice from that menu under different frames, and Ahn and Ergin (2010) who axiomatize a formal model where the framing of an act affects the probabilities used by the DM.

Our results also relate to a body of literature on boundedly rational choice theory. Lipman (1999) introduces a decision-theoretic model for relating an agent's logic to preferences. Al-Najjar et al. (2003) explicitly model the effects of complex environments on decision making as a preference for flexibility. Kochov (2015) develops a model of a DM with imperfect foresight, which can be interpreted as misperception of the auto-correlation between actions, where failure of Monotonicity also plays a role. Lastly, our representation can admit an interpretation in which different, endogenous sources of uncertainty (the understanding classes) determine beliefs, as in Chew and Sagi (2008) or Gul and Pesendorfer (2015).

In Section 6.2, we apply our representation to two-player incomplete information games. We show that it is closely related to, and sometimes nests, several models in behavioral game theory. The equilibrium concept that we develop allows correlation neglect, as in DeMarzo et al. (2003), and admits as special cases several models of imperfect inference in equilibrium such as Cursed Equilibrium of Eyster and Rabin (2005) and Analogy Based Expectation Equilibrium (Jehiel, 2005) in the form studied by Jehiel and Koessler (2008).

Esponda (2008) and Spiegler (2015) have developed approaches to misperception that are related to ours and are motivated by similar behavioral insights. Their models, however, do not in general admit PCR representations. Specifically, both models allow the DM's perception of correlation between two payoff-relevant variables to be affected by her choice.

Evidence for misperception of correlation in the laboratory is found by, among others, Eyster and Weizsäcker (2010), Enke and Zimmerman (2013), and Rubinstein and Salant (2015). We refer the reader to these works for additional motivation.

2. Primitives

There is a set $\mathcal{A}$ of *actions*, with typical elements a, a_i, b, b_i. Each action results in an *outcome* or consequence in a set X, with typical elements x, y, z. This outcome is determined by a *state of the world* drawn from the measurable space (Ω, Σ), with typical element ω, where Ω is a non-empty set and Σ is a σ-algebra on Ω. We primarily use the state space Ω as a convenient benchmark against which the DM's subjective perceptions of correlations are evaluated, that is, the state space is interpreted as the modeler's or an external observer's representation of how the actions correlate with one another. A map $\rho : \mathcal{A} \times \Omega \to X$ determines the relationship between actions, states, and outcomes, with the action a yielding the outcome $\rho(a, \omega)$ in state ω. Slightly abusing notation, we write $a(\omega)$ for $\rho(a, \omega)$. The map $a(\omega)$ is Σ-measurable for every $a \in \mathcal{A}$. Note that the model allows for distinct actions a and b for which $a(\omega) = b(\omega)$ for any $\omega \in \Omega$.

From the set of actions, we derive a set $\mathcal{F}$ of *action profiles* (or profiles). Each $F \in \mathcal{F}$ is a finite sequence of actions $\langle a_i \rangle_{i=1}^n$. We assume that with any permutation of F is identical to F.[3] To save notation, we sometimes write $\langle a_i \rangle$ instead of $\langle a_i \rangle_{i=1}^n$.

An agent who chooses the profile $\langle a_i \rangle_{i=1}^n$ receives the outcomes of all n actions $a_1, ..., a_n$. Outcomes combine through an operation $+ : X \times X \to X$, with $+(x, y)$ denoted by $x + y$,

[3]To define $\mathcal{F}$ more formally, let $A^* = \bigcup_{i=1}^{\infty} \mathcal{A}^i$, the set of all finite Cartesian products of $\mathcal{A}$, and define an equivalence relation R on A^* by dRd' if and only if d' is a reordering of d. The set of profiles $\mathcal{F}$ is the set of R-equivalence classes on A^*.

that is commutative and associative, i.e. $x+(y+z)=(x+y)+z$ and $x+y=y+x$. If the DM takes two actions that yield outcomes x and y, then she gets the consequence $x+y$. We assume that there exists an element $0\in X$ so that $x+0=x$. To fix ideas, we often focus on $X=\mathbb{R}$ or $X=\mathbb{R}_+$ with the standard addition operation.

We assume that each $a\in\mathcal{A}$ has a finite image and that $\mathcal{A}$ includes every constant action. Thus, for any $x\in X$ there is an $a_x\in\mathcal{A}$ such that $a_x(\omega)=x$ for every $\omega\in X$, and we denote such an action by x. We write $\sigma(a)$ ($\sigma(a,b)$) for the coarsest σ-algebra by which a is (both a and b are) measurable.

The DM chooses by maximizing a preference relation $\succeq$ defined on $\Delta\mathcal{F}$, the set of all probability distributions over $\mathcal{F}$ having finite support. Typical elements of $\Delta\mathcal{F}$ are p,q. As is customary, the symbol $\sim$ denotes indifference and $\succ$ strict preference. The set $\Delta\mathcal{F}$ contains the set of action profiles (the lottery in which profile F has probability 1, often denoted by F), the set of actions (the lottery in which profile $\langle a\rangle$ has probability 1, often denoted by a), and the set of lotteries over X (the lottery over constant actions), ΔX, with typical lottery denoted by $(p(x_i),x_i)_{i=1}^k$ or simply $(p(x_i),x_i)$.

The *basis* of $F=\langle a_i\rangle_{i=1}^n$, denoted by $\mathcal{B}(F)$, is the set of distinct actions that comprise F. That is, $\mathcal{B}(F)$ is the set $\{a_i\}_{i=1}^n$. Similarly, the basis of a lottery p, denoted by $\mathcal{B}(p)$, is the union of the bases of each action profile in its support. The complement of a set E is denoted by $\overline{E}$.

Our modeling of profiles and outcomes is novel. The overall framework builds on the reformulation by Battigalli et al. (2013) of Luce and Raiffa (1954)'s model of decsions. As in that paper, we focus exclusively on ex-ante lotteries. Note that ex-ante lotteries also play a role in, e.g. Anscombe and Aumann (1963), Seo (2009), and Saito (2015). None of these distinguish profiles from acts.

3. Foundations and Preliminary Representation

We first introduce some standard assumptions. We then move to the key axiom of *Weak Monotonicity.*

3.1. **Standard Assumptions.**

We now state our standard assumptions. Given two lotteries $p,q\in\Delta\mathcal{F}$, a mixture $\alpha p+(1-\alpha)q$, $\alpha\in[0,1]$, is the lottery in $\Delta\mathcal{F}$ in which the probability of each profile in the support of p and q is determined by compounding the probabilities in the obvious way.

Axiom 1 (Weak order)**.** The preference relation $\succeq$ is complete and transitive.

Axiom 2 (Continuity)**.** The sets $\{\alpha\in[0,1]:\alpha p+(1-\alpha)q\succeq r\}$ and $\{\alpha\in[0,1]:r\succeq\alpha p+(1-\alpha)q\}$ are closed for all $p,q,r\in\Delta F$.

Axiom 3 (Independence)**.** For any $p, q, r \in \Delta\mathcal{F}$ and any $\alpha \in (0,1]$, $p \succeq q \iff \alpha p + (1-\alpha)r \succeq \alpha q + (1-\alpha)r$.

These are the mixture space axioms of Herstein and Milnor (1953). Their interpretation is standard. While one may plausibly argue that complexity does or should cause violations of any of them, we show that complexity need not. In fact, most commonly used models featuring complexity do not violate any of them.

Note that Independence applies only to ex-ante mixtures of lotteries. Unlike the literature featuring choice of a menu, alternatives themselves are not mixed. Independence implies a behavior akin to "complexity neutrality", namely that the DM does not value the option to mix between profiles. Such mixing could plausibly reduce exposure to complexity in the same way that mixing between acts reduces exposure to ambiguity. An extension relaxing independence to imply "complexity aversion" (or seeking) is left for future work.

3.2. **Weak Monotonicity.** In the standard approach, a profile $\langle a_i \rangle_{i=1}^n$ corresponds to an act $f : \Omega \to X$, which yields the consequence $f(\omega) = \sum_{i=1}^n a_i(\omega)$ in state ω. Whenever the DM reduces action profiles to acts, the map f is a sufficient description of the profile. In particular, if $\langle a_i \rangle_{i=1}^n$ and $\langle b_i \rangle_{i=1}^m$ correspond to the same act, then $\langle a_i \rangle_{i=1}^n \sim \langle b_i \rangle_{i=1}^m$. Within the expected utility framework, reduction to acts is implied by *Monotonicity*: if for any $\omega \in \Omega$

$$\sum_{i=1}^n a_i(\omega) \succeq \sum_{i=1}^m b_i(\omega)$$

then $\langle a_i \rangle_{i=1}^n \succeq \langle b_i \rangle_{i=1}^m$. We will return to Monotonicity in Section 6.1; the following example illustrates a violation.

Example 1. A DM must choose between bets that depend on τ, tomorrow's high temperature. The DM can have either \$100 or the sum of the outcomes of bets b_C and b_F, where b_C pays \$100 if τ is less than 30 degrees Celsius (\$0 otherwise) and b_F pays \$100 if τ is at least 86 degrees Fahrenheit (\$0 otherwise). As 30° Celsius equals 86° Fahrenheit, a DM who understands and easily converts Fahrenheit to Celsius expresses indifference between the sum of b_C and b_F and \$100 for sure. However, a risk averse DM who does not know how to convert from one unit to the other and believes that the median value of $86°F$ is $30°C$ may not exhibit such indifference and reasonably prefer \$100 for sure to holding both b_C and b_F. Note this non-indifference holds even if both b_C and b_F are easy to evaluate in isolation.

The expressed preference $\langle 100 \rangle \succ \langle b_C, b_F \rangle$ contradicts Monotonicity: $\langle b_C, b_F \rangle$ and $\langle 100 \rangle$ both correspond to the act that yields 100 in every state but $\langle 100 \rangle \succ \langle b_C, b_F \rangle$. Our novel axiom, *Weak Monotonicity*, relaxes this property by also considering joint realizations of consequences possible in alternative joint distributions not implied by the primitive state space. To motivate it, consider why a DM might prefer $\langle 100 \rangle$ to $\langle b_C, b_F \rangle$. She can "plausibly" conceive four possible joint realizations of $\langle b_C, b_F\rangle$: $(100,0)$, $(0,100)$, $(0,0)$, $(100,100)$. If the DM prefers 100 to $\langle b_C, b_F\rangle$, then she must think it sufficiently likely that both bets return 0. Weak Monotonicity subsumes such considerations by strengthening the conditions under which a lottery dominates another. In particular, it requires that if the lottery over the consequences generated by p is preferred to that generated by q for *every* "plausibly" conceived joint realizations, then $p \succeq q$.

Formally, for any finite subset of actions $\{c_1, ..., c_n\} = C \subset \mathcal{A}$, the set of all *plausible realizations* of C equals

$$range(c_1) \times range(c_2) \times ... \times range(c_n).$$

Thus each plausible realization is a vector of outcomes $(x^{c_1}, x^{c_2}, ..., x^{c_n})$ such that each action c_i could, in isolation, result in x^{c_i}: for every $c_i \in C$, there exists $\omega \in \Omega$ so that $x^{c_i} = c_i(\omega)$. It should be noted that plausible realizations are defined for subsets of actions and not sequences of actions such as profiles.

In Example 1, the plausible realizations of $\{b_C, b_F\}$ are $(100,0)$, $(0,100)$, $(100,100)$, and $(0,0)$, which we interpret below as $\langle b_C, b_F\rangle$ yielding four possible aggregate outcomes, namely, 100, 100, 200, or 0. Similarly, the profile $\langle b_C, b_F, b_F\rangle$ could yield the aggregate outcomes 100, 200, 300, or 0. Naturally, the outcomes of $\langle b_C, b_F, b_F\rangle$ dominate the outcomes of $\langle b_C, b_F\rangle$ regardless of any uncertainty about the conversion of temperature. Weak Monotonicity requires that $\langle b_C, b_F, b_F\rangle \succeq \langle b_C, b_F\rangle$.

Generally, given two profiles $\langle a_i\rangle_{i=1}^n$ and $\langle b_j\rangle_{j=1}^m$, to each plausible realization

$$(x^{a'})_{a' \in \mathcal{B}(\langle a_i\rangle_{i=1}^n) \cup \mathcal{B}(\langle b_j\rangle_{j=1}^m)}$$

corresponds a joint outcome that the two profiles $\langle a_i\rangle_{i=1}^n$ and $\langle b_j\rangle_{j=1}^m$ might yield. Specifically, $\langle a_i\rangle_{i=1}^n$ yields $\sum_{i=1}^n x^{a_i}$ and $\langle b_j\rangle_{j=1}^m$ yields $\sum_{j=1}^m x^{b_j}$. It should be noted that $x^a = x^b$ whenever $a = b$ regardless of the profile in which they are listed. If

$$\sum_{i=1}^n x^{a_i} \succeq \sum_{j=1}^m x^{b_j}$$

then $\langle a_i\rangle_{i=1}^n$ has a better consequence than $\langle b_j\rangle_{j=1}^m$ in this plausible realization. If this is true for all plausible realizations of $\mathcal{B}(\langle a_i\rangle_{i=1}^n) \cup \mathcal{B}(\langle b_j\rangle_{j=1}^m)$, that is, for any perceived correlation among distinct actions, then we say that $\langle a_i\rangle_{i=1}^n$ *plausibly dominates* $\langle b_j\rangle_{j=1}^m$. We now formalize this comparison and extend it to lotteries.

Definition 1. A lottery p dominates a lottery q in a plausible realization $(x^a)_{a \in \mathcal{B}(p) \cup \mathcal{B}(q)}$ of $\mathcal{B}(p) \cup \mathcal{B}(q)$ if

$$\left(p\left(\langle a_i\rangle_{i=1}^n\right), \sum_{i=1}^n x^{a_i}\right) \succeq \left(q(\langle b_j\rangle_{j=1}^m), \sum_{j=1}^m x^{b_j}\right).$$

A lottery p *plausibly dominates* a lottery q if p dominates q in *every* plausible realization of $\mathcal{B}(p) \cup \mathcal{B}(q)$.

If p plausibly dominates q, then for any plausible realization the DM prefers the lottery generated by p better than that of q. For example, suppose that p randomizes equally between $\langle b_C, b_F\rangle$ and $\langle b_F\rangle$ and q selects $\langle b_F\rangle$ with certainty. For either of the plausible realizations $(100, 0)$ or $(100, 100)$, p randomizes equally between two consequences (100 and 0 in the first case, 200 and 100 in the second) while q selects the worse of the two with certainty (0 and 100, respectively). Similarly, for the plausible realizations $(0, 100)$ and $(0, 0)$ both lotteries yield respectively 100 and 0 with certainty. Hence, p plausibly dominates q.

Weak Monotonicity requires plausible domination to relate to preference in the natural way.

Axiom 4 (Weak Monotonicity)**.** For any $p, q \in \Delta\mathcal{F}$, if p plausibly dominates q, then $p \succeq q$.

Note that, since $\langle b_C, b_F\rangle$ does not plausibly dominate $\langle 100\rangle$, it does not restrict the ranking of these two profiles.

Weak Monotonicity has some formal similarities with the Dominance axiom of Seo (2009). It considers all possible beliefs (though here beliefs are about correlation structure) as in Seo's axiom, but requires reduction of compound lotteries as well. The Congruence axiom in Battigalli et al. (2013) is also related, but only considers the outcomes that are possible according to Ω. In our setting, Congruence is equivalent to the Monotonicity axiom of Section 6.1.

3.3. **Representation.** The violation of Monotonicity implies that the DM perceives uncertainty not captured by the state space Ω. We represent this additional uncertainty by increasing the dimension of the state space. While one can do this in many ways, we do so by considering copies of the benchmark state space. Our first result shows that, under Axioms 1-4, one can obtain an expected utility representation in which each action is assigned one copy of the benchmark state space. Naturally, the increase in the dimensionality of uncertainty for this representation may be excessive and unnecessary, as we shall see in later sections. However, the result below is instrumental for achieving a parsimonious representation. Given a collection of σ-algebras $\{\Sigma_i\}_{i\in\mathcal{I}}$ of Ω, denote the product σ-algebra of the Cartesian product $\Omega^{\mathcal{I}}$ by $\otimes_{i\in\mathcal{I}}\Sigma_i$. Let Ω^a the copy of Ω assigned to $a \in \mathcal{A}$, $\Sigma_{\mathcal{A}} = \otimes_{a\in\mathcal{A}}\sigma(a)$ be the product σ-algebra for the Cartesian product $\Omega^{\mathcal{A}} = \prod_{a\in\mathcal{A}}\Omega^a$. Given a state $\vec{\omega} \in \Omega^{\mathcal{A}}$, ω^a denotes the projection of $\vec{\omega}$ on Ω^a.

Theorem 1. *The preference relation $\succeq$ satisfies Weak Order, Continuity, Independence, and Weak Monotonicity if and only if there exist an index $u : X \to \mathbb{R}$ and a probability measure π over $\Sigma_{\mathcal{A}}$ such that $\succeq$ has an expected utility representation with utility index $V : \mathcal{F} \to \mathbb{R}$*

where

$$V(\langle a_i\rangle_{i=1}^n) = \int_{\Omega^{\mathcal{A}}} u(\sum_{i=1}^n a_i(\omega^{a_i}))d\pi.$$

Furthermore, if there exists p, q where $p \succ q$, then u is unique up to a positive affine transformation.

By increasing the dimension of uncertainty, the DM acts as if she is SEU but on a larger state space. The state space chosen allows the DM to attach positive probability to events such as "b_C yields 0 and b_F yields 0" that Ω cannot express. Naturally, uniqueness of the representation in Theorem 1 is problematic. We address this issue in the next section.

Proof of Theorem 1. Necessity is trivial. We show only the key step for sufficiency in the main text.

Assume that $\succeq$ satisfies Weak Order, Continuity, Independence, and Weak Monotonicity. Herstein and Milnor (1953) implies that when restricted to the set of finite lotteries over X, $\succeq$ has an expected utility representation with utility index u normalized such that $u(0) = 0$. The key step is to show that we can map each lottery over action profiles into a (utility valued) act on the state space $\Omega^{\mathcal{A}}$. For any $p \in \Delta\mathcal{F}$, define the mapping $f_p : \Omega^{\mathcal{A}} \to \mathbb{R}$ by

$$f_p(\vec{\omega}) = \sum_{p(\langle a_i\rangle_{i=1}^n)>0} p(\langle a_i\rangle_{i=1}^n)u(\sum_{i=1}^n a_i(\omega^{a_i}))$$

for every $\vec{\omega} \in \Omega^{\mathcal{A}}$, where ω^{a_i} is the component of $\vec{\omega}$ corresponding to action a_i.

Lemma 1. *If $f_p \geq f_q$, then $p \succeq q$.*

Proof. Fix an arbitrary plausible realization $(x^a)_{a\in B}$ of $B = \mathcal{B}(p) \cup \mathcal{B}(q)$. By definition, there exists $\omega^a \in \Omega^a$ such that $x^a = a(\omega^a)$. Then note that

$$\left(p(\langle a_i\rangle_{i=1}^n), \sum_{i=1}^n x^{a_i}\right) \succeq \left(q(\langle b_i\rangle_{i=1}^m), \sum_{i=1}^m x^{b_i}\right)$$

if and only if

$$\left(p(\langle a_i\rangle_{i=1}^n), \sum_{i=1}^n a_i(\omega^{a_i})\right) \succeq \left(q(\langle b_i\rangle_{i=1}^m), \sum_{i=1}^m b_i(\omega^{b_i})\right)$$

if and only if

$$\sum_{p(\langle a_i\rangle)>0} p(\langle a_i\rangle)u(\sum_{i=1}^n a_i(\omega^{a_i})) \geq \sum_{q(\langle b_i\rangle)>0} q(\langle b_i\rangle)u(\sum_{i=1}^m b_i(\omega^{b_i}))$$

by the above. By $f_p \geq f_q$, the last inequality is true. Since (x^a) was chosen arbitrarily, p plausibly dominates q. By Weak Monotonicity, $p \succeq q$. □

Define $U = \{f_p : p \in \Delta(\mathcal{F})\}$, noting that U is convex. For ϕ in U, define $I(\phi) = \int u(x)dr$ for some $p \in \Delta(\mathcal{F})$ s.t. $f_p = \phi$, and a lottery r over X satisfying $r \sim p$. Such an r exists for every p by Weak Monotonicity, Completeness, and Continuity, so I is well-defined. Moreover, Independence and Weak Monotonicity imply that I is positive and linear. Obviously, $I(f_p) \geq I(f_q)$ if and only if $p \succeq q$. The remainder of the proof, found in Appendix A.1, proceeds by extending the domain of $I(\cdot)$ using the Hahn-Banach theorem and then applying a Riesz representation theorem to get the desired representation. Finally, Kolmogorov's extension theorem obtains countable additivity. □

4. Identifying Understanding

The representation of Theorem 1 is sufficiently flexible to encompass a variety of subjective perceptions of correlation. However, it does not provide a tight characterization the DM's understanding of the relationship between actions. In this section, we identify the agent's degree of understanding from her preferences and consider a representation theorem in the spirit of Theorem 1 that includes an additional element, the correlation cover, that identifies the actions among which the DM perceives the relationship correctly. Under the hypothesis of "richness," namely that each action belongs to a suitably dense set of understood actions, we show that the correlation cover is unique. We then turn to the DM's beliefs about the joint distribution of outcomes from misunderstood actions. In general, the extent to which these beliefs affect the agent's preference over profiles depends on a condition closely related to her attitude towards higher order risk.

4.1. **Understanding.** We begin by providing a behavioral definition of understanding that builds on our Weak Monotonicity axiom. Intuitively, a DM understands a set of actions $\{a, b, c\}$ if she recognizes the relationship between the outcomes: she thinks that only joint realizations for which a gives $a(\omega)$, b gives $b(\omega)$, and c gives $c(\omega)$ for the same $\omega \in \Omega$ are possible. For an arbitrary $C \subset \mathcal{A}$, a DM who understands C necessarily disregards plausible realizations that do not synchronize the outcomes of the actions therein as with the state space Ω.

Definition 2. For any $B, C \subseteq \mathcal{A}$, say that a plausible realization $(x^a)_{a \in B}$ of B is C-*synchronous* if for some $\omega \in \Omega$

$$(x^a)_{a \in C \cap B} = (a(\omega))_{a \in C \cap B},$$

and that p C-*dominates* q if $(p(\langle a_i \rangle), \sum x^{a_i}) \succeq (q(\langle a_i \rangle), \sum x^{a_i})$ for every C-synchronous plausible realization of $B(p) \cup B(q)$.

A plausible realization of B is C-synchronous if it accords with the distribution of outcomes implied by Ω for the actions in C, but not necessarily for the distribution of outcomes in $B \setminus C$. That is, if $C = \{a, b\}$ and $B = \{a, b, c\}$, then a plausible realization (x^a, x^b, x^c) of B is C-synchronous if there exists ω,$\omega' \in \Omega$ so that $x^a = a(\omega)$, $x^b = b(\omega)$ and $x^c = c(\omega')$.

Furthermore, if for any such plausible realizations the lottery induced by p is preferred to the lottery induced by q, p is said to C-dominate q.

The DM understands $C \subseteq \mathcal{A}$ if C-synchronous plausible realizations suffice to determine her preference.

Definition 3. The preference relation $\succeq$ *understands* C if $p \succeq q$ whenever p C-dominates q.

Preferences that understand $\mathcal{A}$ satisfy Monotonicity. When the preference $\succeq$ understands C we sometimes say that C is understood.

Of particularly interest for the characterization of the DM's depth of understanding are *rich*, understood sets.

Definition 4. A set $B \subset \mathcal{A}$ is *rich* if, for any $a, b \in B$ and any $\sigma(a, b)$-measurable $f : \Omega \to X$, there exists $c \in B$ with $c(\omega) = f(\omega)$ for all ω.

A set is a rich if whenever it contains an action it also contains all actions with a coarser algebra and, whenever it contains two actions, it also contains an action having an algebra finer than the algebras of both actions. For instance, if $\mathcal{A}$ contains an action corresponding to every Savage act, then it is rich, but a singleton set is rich only if X is also a singleton.

4.2. **Rich Representation.** The choice of $\Omega^{\mathcal{A}}$ as a state space in the representation in Theorem 1 may be far from parsimonious. In this section, we consider a representation that preserves the product structure of the state space but greatly reduces the dimensions of uncertainty by modeling explicity the DM's understanding of the correlation structure. To this effect, we introduce a *correlation cover* that describes the subsets of actions understood by the agent. Formally, a correlation cover $\mathcal{U}$ is a collection of subsets of $\mathcal{A}$ such that (i) $\mathcal{U}$ covers $\mathcal{A}$; (ii) $\succeq$ understands each $C \in \mathcal{U}$; and (iii) no $C \in \mathcal{U}$ contains a distinct $C' \in \mathcal{U}$. The elements of $\mathcal{U}$ are referred to as *understanding classes*.

As in Theorem 1, the DM is represented as having beliefs π on a state space that is the product of copies of Ω. The state space, however, is equal to $\Omega^{\mathcal{U}} = \prod_{C \in \mathcal{U}} \Omega$ rather than $\Omega^{\mathcal{A}}$, with the C-coordinate denoted by Ω^C. Our representation requires that actions belonging to the same element C of the correlation cover depend on the same coordinate Ω^C of the product state space. Thus, if a set of actions belong to the same understanding class, the agent correctly perceives that their joint realizations are generated by a common state of Ω.

For every $C \in \mathcal{U}$, denote by Σ_C the the coarsest σ-algebra by which every $a \in C$ is measurable. Endow $\Omega^{\mathcal{U}}$ with the product σ-algebra $\Sigma_{\mathcal{U}} = \otimes_{C \in \mathcal{U}} \Sigma_C$. Given a state $\vec{\omega} \in \Omega^{\mathcal{U}}$, ω^C denotes the projection of $\vec{\omega}$ onto Ω^C.

Definition 5. The preference relation $\succeq$ has a *probabilistic correlation representation*, or *PCR*, $(\mathcal{U}, \pi, u)$ if there exist a correlation cover $\mathcal{U}$, a finitely-additive probability measure π

on the probability space $(\Omega^{\mathcal{U}}, \Sigma_{\mathcal{U}})$, and utility function $u : X \to \mathbb{R}$ such that the preference $\succeq$ has an expected utility representation with utility index $V : \mathcal{F} \to \mathbb{R}$ where for any $\langle a_i \rangle_{i=1}^n$,

$$V(\langle a_j \rangle_{j=1}^n) = \int_{\Omega^{\mathcal{U}}} u\left(\sum_{j=1}^n a_j(\omega^{C_j})\right) d\pi$$

for any vector $(C_1, ..., C_n)$ with $C_j \in \mathcal{U}$ and $a_j \in C_j$, $j = 1, ..., n$. A PCR $(\mathcal{U}, \pi, u)$ is *rich* if every $C \in \mathcal{U}$ is rich.

To illustrate the PCR, recall Example 1. Supposing that $\mathcal{U} = \{B_C, B_F\}$ where $b_C \in B_C$ and $b_F \in B_F$, we can interpret B_C and B_F as corresponding to actions expressed in Celsius and actions expressed in Fahrenheit, respectively. Thus each $\vec{\tau} \in \Omega^{\{B_C, B_F\}}$ can be thought of as a pair of temperatures, one in Celsius and the other in Fahrenheit. A DM for whom $100 \succ \langle b_C, b_F \rangle$ must attach positive probability to $\tau^{B_C} > 30°C$ and $\tau^{B_F} \leq 86°F$. While such an event cannot occur if all uncertainty is captured by Ω, our DM can express such a preference if she does not think that the temperature in Fahrenheit equals that in Celsius, i.e. that $\pi(\tau^{B_C} \neq \frac{5}{9}\left(\tau^{B_F} - 32\right)) > 0$.

The representation in Theorem 1 is a PCR in which each action is assigned to a distinct understanding class, i.e. $U = \{\{a\} : a \in A\}$. Clearly, this PCR is not rich. However, rich PCRs arise in many natural contexts.

Example 2. The following are some examples of rich PCRs.

(1) Asset pricing: An asset market consists of a set $\mathcal{A}^o$ of assets and the derivatives thereof. We model a derivative as a pair (γ, a^o), where γ is a function from X to itself and an asset $a^o \in \mathcal{A}^o$, that yields $\gamma(x)$ when a^o yields x. If the DM understands the set of all derivatives that depend on the same underlying asset, then she has a rich PCR when $\mathcal{A} = \mathcal{G} \times \mathcal{A}^o$, for $\mathcal{G}$ equal to all functions from X to X, and the correlation cover consists of the sets $B_{a^o} = \{(\gamma, a^o) : \gamma \in \mathcal{G}\}$.
(2) Framing: Each action consists of a (Savage) act a and a frame $\mathbf{f} \in \mathbb{F}$, such as Celsius or Fahrenheit. Let $\mathcal{H}$ be the set of measurable functions with finite range for (Ω, Σ). The DM understands the connection between any acts framed in the same way. We can model this as a rich PCR where the correlation cover consists of the sets $B_{\mathbf{f}} = \{(a, \mathbf{f}) : a \in \mathcal{H}\}$.
(3) Source preference: Each action is associated with a source S_i from a set $\mathcal{S}$. Each S_i is a sub-sigma algebra of Σ. Let B_i be the set of functions with finite range that are measurable with respect to S_i. The correlation cover consists of all sets B_i, one for each source. Thus, the DM reduces any profile whose contents depend on the same source to one act but fails to do so when it depends on more than one source.

Note that a correlation cover need not be a partition.[4] While this may seem surprising at first, we argue below that a parsimonious correlation cover in our model is partitional only if the DM understands the relation among all actions. In particular, the "largest" correlation covers are not partitional since each understanding class can always be enlarged to include all constant actions. More interestingly, consider Example 1 where the DM knows that $0°C = 32°F$ but is unsure about the scaling factor. Thus, she understands the connection between any actions that only depend on whether or not the temperature is below freezing, and every such actions belongs to both understanding classes. In the next section, we discuss the structure of $\mathcal{U}$ and show that if $\mathcal{U}$ is "universal" in a sense to be made precise, then $\mathcal{U}$ is unique.

4.3. **Existence and Uniqueness of Rich PCR.** The main theorem in this subsection shows that, under mild assumptions, a rich PCR exists. Moreover, the correlation cover is unique, under an appropriate normalization. We begin by defining this normalization.

Definition 6. A rich correlation cover $\mathcal{U}$ is *universal* if the preference $\succeq$ has a rich PCR $(\mathcal{U}, \pi, u)$ and for any other rich PCR $(\mathcal{U}', \pi', u')$, whenever $B' \in \mathcal{U}'$ there exists $B \in \mathcal{U}$ such that $B' \subseteq B$.

For any understanding class in $\mathcal{U}'$, there is a larger understanding class in $\mathcal{U}$. Therefore, any understanding relationship captured by $\mathcal{U}'$ is also captured by $\mathcal{U}$. Obviously, a rich, universal correlation cover is unique.

Theorem 2 shows that under two weak assumptions, a rich PCR with a universal correlation cover exists.

Assumption 1 (Strict Concavity)**.** The set X is a convex subset of a linear space and for any $x \neq y \in X$ and $\lambda \in (0,1)$, $(1, \lambda x + (1-\lambda)y) \succ (\lambda, x; (1-\lambda), y)$.

Strict Concavity is ubiquitous in the economic literature and has well-understood behavioral content, specifically its connection to risk aversion. At the end of this subsection, we discuss significantly weaker alternative assumptions that suffice for Theorem 2.

Assumption 2 (Non-Singularity)**.** Each action $a \in \mathcal{A}$ belongs to a rich, understood subset of actions.

Non-singularity is a joint assumption on both $\succeq$ and $\mathcal{A}$. It is an assumption in the spirit of the Savage (1954) assumption that the domain of preference contains all possible acts. It is clearly necessary for existence of a rich PCR. More surprisingly, it is also sufficient when paired with Strict Concavity and Axioms 1-4.

[4]Because of this, an action that appears twice in the same profile may be modeled as depending on different components of the product state space. This is purely for notational convenience. The requirement that every copy of the same action depends on the same coordinate can be imposed without changing any results.

Theorem 2. *The preference relation $\succeq$ satisfies Axioms 1-4, Non-Singularity, and Strict Concavity if and only if it has a rich PCR $(\mathcal{U}, \pi, u)$ with a strictly concave u and a universal $\mathcal{U}$. Furthermore, if there exists p, q where $p \succ q$, then u is unique up to a positive affine transformation.*

Proof. See Appendix A.2. □

The main difficulty of Theorem 2 is showing that if several sets of actions are understood individually (as per Definition 3), then plausible realizations that do not synchronize states within each of these sets simultaneously can be ignored. Strict concavity is one of several assumptions sufficient for our proof technique to establish this.

Remark 1. Theorem 2 remains true, albeit with different restrictions on u, in any of the following settings: (i) u is twice differentiable, (ii) strict concavity is replaced by either additivity or strict convexity, (iii) for all $x, y \in X$ $(\frac{1}{2}, x; \frac{1}{2}, y) \succeq (\frac{1}{2}, x + y; \frac{1}{2}, 0)$ without indifference when $x, y \neq 0$, or (iv) for for any $x, y \in X \setminus 0$, there exists $z \in X$ such that $(\frac{1}{2}, x + z; \frac{1}{2}, y + z) \not\sim (\frac{1}{2}, x + y + z; \frac{1}{2}, z)$.[5]

4.4. **Uniqueness of π.** In general, a rich PCR may fail to have unique beliefs. Our next set of results describes the relationship between the beliefs associated with PCRs that are derived from the same preferences and have a universal correlation cover. Of particular interest is whether such beliefs necessarily agree on the joint distribution of outcomes from a given set of actions. If so, then the DM's perceptions of all orders of correlation among the actions matter for the choice of the optimal profile. Conversely, when beliefs are not uniquely pinned down, some of the DM's perceptions of correlation are inessential for her choices.

A linear utility provides the most obvious case of non-unique representation of beliefs. Since maximizing utility is equivalent to maximizing the expected value of the actions, their hedging properties do not enter her decision. In this case, the DM's beliefs are pinned down only if $\mathcal{U} = \mathcal{A}$. Thus, non-linearity of the utility index plays a role in identifying the agent's beliefs; in fact, any non-linearity of the utility index suffices to identify pairwise joint distributions. However in general, even strict risk-aversion does not suffice on its own to identify uniquely the DM's perceived joint distribution of larger sets of actions. As the following example shows, conditions related to her higher-order risk attitudes are also needed.

Example 3. Suppose that $\Omega = \{H, T\}$ and that there is a rich PCR with three understanding classes $\mathcal{U} = \{C_1, C_2, C_3\}$ with $\Sigma_{C_i} = 2^{\Omega}$ for all i. Consider the probability measure π where $\pi(HHT) = \pi(TTT) = \pi(THH) = \pi(HTH) = \frac{1}{4}$ and the probability measure μ

[5]In (iv), "$x, y \in X \setminus 0$" can be replaced by "$x, y \in Q \setminus 0$" where Q is an absorbing subset of X.

where $\mu((\omega_1,\omega_2,\omega_3)) = \frac{1}{8}$ for every $(\omega_1,\omega_2,\omega_3) \in \Omega^{\{C_1,C_2,C_3\}}$. Note that

$$\mu(\omega_i|\omega_j) = \mu(\omega_i) = \pi(\omega_i|\omega_j) = \pi(\omega_i)$$

whenever $i \neq j$ but that $\{\omega_1,\omega_2,\omega_3\}$ are mutually independent according to μ but not π:

$$\pi(H_1|H_2T_3) = 1 \neq \pi(H_1) = \frac{1}{2};$$

Then, if $u(x) = x - \beta x^2$, both μ and π represent the DM's beliefs. For instance, for any profile $\langle a_i \rangle$, the expectation of a_i, a_i^2 and $a_i \cdot a_j$ are identical since μ and π agree on pairwise correlation. However, no higher degrees of correlation enter the valuation of any profile. Hence, the beliefs μ and π have the same expected utility.

We now define a condition that is closely related to higher-order risk attitudes. Theorem 3 shows that it is necessary and sufficient for unique identification of the DM's belief about the joint distributions of actions. We first introduce the *n-order apportioned lotteries for* $x_1,...,x_n \in X$, denoted p_n and p'_n, recursively as follows. For $n = 1$, $p_1 = (1, x_1)$ and $p'_1 = (1, 0)$, and for any $n > 1$,

$$p_n = \frac{1}{2}p_{n-1} + \frac{1}{2}(p'_{n-1}(x), x + x_n)_{p'_{n-1}(x)>0}$$

and

$$p'_n = \frac{1}{2}p'_{n-1} + \frac{1}{2}(p_{n-1}(x), x + x_n)_{p_{n-1}(x)>0}.$$

Similar lotteries are used by Eeckhoudt et al. (2009) in their study of higher order risk. An implication of their results is that if $x_i > 0$ for $i > 0$ then p_n nth order stochastically dominates p'_n.

Condition (n)**.** There exists $x_1,...,x_n \in X$ for which the n-order apportioned lotteries p_n and p'_n satisfy $p_n \not\sim p'_n$.

This conditions is related to risk attitudes when $X \subseteq \mathbb{R}$**.** Specifically, Eeckhoudt et al. (2009) show that if $p_n \succ p'_n$ for any n-order apportioned lotteries, then the DM's lottery preference respects n-order stochastic dominance. For example, Condition (2) fails for a risk-neutral DM and Condition (3) fails for the DM described in Example 3.

We now show that Condition (N) suffices to uniquely identify the DM's beliefs about the correlation between any N actions. Given a correlation cover $\mathcal{U}$, define an event $E \in \Sigma_{\mathcal{U}}$ to be *an n-dimensional rectangle* if

$$\mathcal{E} = \{\vec{\omega} : \omega^{C_i} \in E_i \text{ for } i = 1, ..., n\}$$

for distinct $C_1,...,C_n \in \mathcal{U}$ and $E_i \in \sigma(a_i)$, $a_i \in C_i$ for $i = 1,...,n$. The utility of any profile of n actions is measurable with respect to n-dimensional rectangles.

Theorem 3. *If the preference* $\succeq$ *has a rich PCR* $(\mathcal{U}, \pi, u)$ *and*

$$N^* = \inf\{N \in \mathbb{N} : \textit{Condition (N) fails}\},$$

then the PCR $(\mathcal{U}, \mu, u)$ *represents* $\succeq$ *if and only if* $\pi(\mathcal{E}) = \mu(\mathcal{E})$ *for all* n*-dimensional rectangles* $\mathcal{E}$ *with* $n < N^*$*.*

Proof. See Appendix A.3. □

For most standard utility indexes, including CARA and CRRA, Condition (n) holds for every (n); see Corollary 1. By convention, $\inf \emptyset = \infty$, so if Condition (n) holds for every n, then $\mu(E) = \pi(E)$ for any n-dimensional rectangle. Moreover, if π and μ are countably additive, then Kolmogorov's extension theorem implies that $\pi = \mu$; see Section 5. Thus, non-uniqueness is not problematic for widely used utility indexes. There exist examples of k non-independent random variables where any $k-1$ element subset are independent. Hence, Example 3 generalizes. In particular, for any $u(\cdot)$ for which Condition (n) fails, one can construct a correlation cover, state spaces, and two distinct probability measures that represent the same preference relation.

Corollary 1. *Suppose that* X *is either* $\mathbb{R}$ *or* $\mathbb{R}_+$ *and that* $\succeq$ *has rich PCR* $(\mathcal{U}, \pi, u)$*, with* $u \in C^\infty$*. For some point* $x \in int(X)$*, let*

$$N^* = \inf\{N \in \mathbb{N} : \textit{ for all } n \geq N, \ \frac{\partial^n u(x)}{\partial x^n} = 0\}.$$

Then, the preference $\succeq$ *has a PCR* $(\mathcal{U}, \mu, u)$ *if and only if* $\mu(E) = \pi(E)$ *for every* n*-dimensional rectangle* E *with* $n < N^*$*.*

Proof. See Appendix A.4. □

5. Interpreting the Correlation Cover

In this section we explore and interpret further the properties of the correlation cover.

5.1. **Understanding as Simplification.**

We define which actions are "revealed" as simplifications of other actions from the DM's preference. Roughly, action b is a simplification of action a if b depends on a subset of the events on which a does and, for any rich set of actions that contains a, adding b to this set preserves understanding. Formally, we define our simplification relation as follows.

Definition 7. Action b is *a rich simplification of* a, denoted by aKb, if (i) for any rich, understood B with $a \in B$, $\succeq$ understands $B \cup \{b\}$; (ii) there exists a rich, understood C with $a \in C$; and (iii) $\sigma(b) \subseteq \sigma(a)$.

The first and key requirement for b to be a rich simplification of a is that understanding the connection between a and a rich set of actions is sufficient for understanding the connection between b and that set. That is, if $a \in B$ is understood, the DM recognises the relation between b and B. This comparison is meaningful only if some rich, understood set containing a exists, which is the second requirement of our definition. Finally, the outcome of b is determined by a subset of the events that determine the consequences of a. This ensures that inserting b into the set does not introduce new relationships between events that must be understood.

The rich simplification relation has a tight connection to existence of a rich PCR.

Proposition 1. *If the preference $\succeq$ satisfies Axioms 1-4 and Strict Concavity holds, then the set $\{b : aKb\}$ is non-empty for every a if and only if $\succeq$ has a rich PCR.*

Proof. If $\{b : aKb\}$ is non-empty for all a, then for every a, aKb for some b, there must exists a rich, understood subset containing a. Theorem 2 implies existence of a rich PCR. Conversely, if $\succeq$ has a rich PCR, then there must exists a rich, understood subset containing a for each a, so clearly aKa and $a \in \{b : aKb\}$. □

We now construct some properties of K under the assumption that a rich PCR exists. Here, we view K as a binary relation.

Theorem 4. *If the preference $\succeq$ has a rich PCR and Strict Concavity holds, then K is reflexive, transitive, and the set $\{b : aKb\}$ is rich and understood.*

The properties of the binary relation K in Theorem 4 are quite natural and need no additional remarks. The role of Strict Concavity is again to extend the individual understanding of some sets to their simultaneous understanding as shown in following Lemma, which is an important tool to prove Theorem 4 and later 5.

Lemma 2. *Consider rich $B, C \subseteq \mathcal{A}$ and for $a \in B \cap C$ let $C_a = \{c \in C : \sigma(c) \subseteq \sigma(a)\}$. If the DM is strictly risk averse and $\succsim$ understands B and C, then $\succeq$ understands $C_a \cup B$.*

Proof. Fix any a. Take $\mathcal{U} = \{B, C_a, \{a'\}_{a' \in \mathcal{A} \setminus (B \cup C_a)}\}$. An argument analogous to the proof of Theorem 2 establishes that there exists a PCR $(\mathcal{U}, \pi, u)$ where u is strictly concave. Now, consider the profile $\langle a, a \rangle$. Then,

$$\begin{aligned} V(\langle a, a \rangle) &= \int_{\Omega^{\mathcal{U}}} u\left(a(\omega^B) + a(\omega^{C_a})\right) d\pi \\ &= \int_{\Omega^{\mathcal{U}}} u\left(2a(\omega^B)\right) d\pi = \int_{\Omega^{\mathcal{U}}} u\left(2a(\omega^{C_a})\right) d\pi. \end{aligned}$$

Thus,

$$\int_{\Omega^{\mathcal{U}}} u\left(a(\omega^B) + a(\omega^{C_a})\right) d\pi = \int_{\Omega^{\mathcal{U}}} \frac{1}{2} u\left(2a(\omega^B)\right) + \frac{1}{2} u\left(2a(\omega^{C_a})\right) d\pi.$$

It easily follows that, by strict concavity, if x and y are in the image of a, $x \neq y$,

$$\pi\left(a^{-1}(x) \times a^{-1}(y) \times \Omega^{\mathcal{A} \setminus (B \cup C_a)}\right) = 0.$$

One can easily verify that since $\sigma(c) \subseteq \sigma(a)$ for every $c \in C_a$ if p $C_a \cup B$-dominates q, then $V(p) \geqslant V(q)$. □

Proof of Theorem 4. K is reflexive since there exists a rich, understood B with $a \in B$ for all a. If aKb and bKc, and B is rich and understood with $a \in B$, then $B \cup \{b\}$ is rich and understood. Since $b \in B \cup \{b\}$, $B \cup \{b, c\}$ and thus $B \cup \{c\}$ are understood, so aKc. To see that $\{b : aKb\}$ is rich, fix any rich and understood B with $a \in B$. Consider an understanding class C with $a \in C$. Now, define C_a as in Lemma 2 and pick any $c \in C_a$. Then $B \cup C_a$ is understood, implying that $B \cup \{c\}$ is also understood. Hence aKc for all $c \in C_a$, which is rich, implying that $\{b : aKb\}$ is rich. □

The next result describes how K characterizes the universal correlation cover for a PCR.

Theorem 5. *Suppose that the preference $\succeq$ has a rich PCR $(\mathcal{U}, \pi, u)$ with universal $\mathcal{U}$ and strictly concave u. Consider any $a, b \in \mathcal{A}$ with $\sigma(b) \subseteq \sigma(a)$. The following are equivalent: (i) aKb; (ii) $b \in C$ whenever $a \in C \in \mathcal{U}$; (iii) $a, b \in C$ for some $C \in \mathcal{U}$.*

Theorem 5 characterizes the relationship between K and $\mathcal{U}$. The action b is a simplification of a if and only if there exists $C \in \mathcal{U}$ with $a, b \in C$. Moreover, *any* understanding class containing a also contains b. Theorem 5 thus characterizes the structure of the universal correlation cover: if $\sigma(b) \subseteq \sigma(a)$, then a and b belong to some understanding class if and only if whenever a belongs to an understanding class, so does b.

Proof of Theorem 5. $[(i) \to (ii)]$ Suppose that aKb and let $a \in C \in \mathcal{U}$. By definition, aKb implies $C \cup \{b\}$ is understood and rich. Since $\mathcal{U}$ is universal, $b \in C$. $[(ii) \to (iii)]$ This implication is trivial. $[(iii) \to (i)]$ Suppose $a, b \in C$ for some $C \in \mathcal{U}$, and $a \in B$ is rich and understood. Lemma 2 implies $B \cup C_a$ is understood. Since $b \in C_a$, $B \cup \{b\}$ is also understood, and so aKb. □

Theorem 5 suggests the following "as if" interpretation of the DM's evaluation of a profile $\langle a_1, a_2, ..., a_n \rangle$. The DM consider which pairs of actions are simpler than a common third action and reduces them to single action. For instance, if a_1, a_2 belong to the understanding class C, there exists b such that bKa_1, bKa_2, and $b(\omega) = a_1(\omega) + a_2(\omega)$. Therefore, the DM recognizes that $\langle a_1, a_2, ..., a_n \rangle$ is the same as $\langle b, a_3, ..., a_n \rangle$. The DM can then apply the same procedure to the profile $\langle b, a_3, ..., a_n \rangle$, and then to the resulting profile $\langle b, c, a_4, ..., a_n \rangle$, etc. Eventually, she ends up with a profile $\langle b_1, ..., b_m \rangle$ with fewer actions than before and where no actions further reduce. She then evaluates this profile using her best estimate, i.e π, of their joint distribution.

5.2. **Comparing Understanding.** Consider two DMs, DM1 and DM2, having the preference relations $\succeq_1$ and $\succeq_2$, respectively.

Definition 8. The preference $\succeq_1$ understands more connections than the preference $\succeq_2$ whenever for any rich $B \subseteq \mathcal{A}$, if $\succeq_2$ understands B, then $\succeq_1$ understands B.

In words, whenever DM2 understands a rich subset of actions, so does DM1. This comparison corresponds to the following relationship between the universal correlation covers representing the two DMs.

Theorem 6. *If each preference $\succeq_i$ has a universal and rich PCR $(\mathcal{U}_i, \pi_i, u_i)$, $i = 1, 2$, then $\succeq_1$ understands more connections than $\succeq_2$ if and only if for any $B \in \mathcal{U}_2$, there exists $B' \in \mathcal{U}_1$ with $B \subseteq B'$.*

When $\succeq_1$ understands more connections than $\succeq_2$, there is a sense in which $\mathcal{U}_2$ is a subset of $\mathcal{U}_1$. In fact, Theorem 2 implies that $\succeq_1$ can be represented with the correlation cover $\mathcal{U}_2$.

Proof. Suppose $\succeq_1$ understands more connections than $\succeq_2$ and pick any rich $B \in \mathcal{U}_2$. The set B is rich and $\succeq_1$-understood. Hence, it is contained in a maximal, rich, $\succeq_1$-understood subset B'. By the construction of $\mathcal{U}_1$ in Theorem 2, $B' \in \mathcal{U}_1$. Conversely, suppose for any $B \in \mathcal{U}_2$, there exists $B' \in \mathcal{U}_1$ with $B \subseteq B'$, that B is rich, and that $\succeq_2$ understands B. Then B is contained in a maximal element $C \in \mathcal{U}_2$, which is in turn contained in $C' \in \mathcal{U}_1$. Since C' is $\succeq_1$-understood and $B \subseteq C'$, B is $\succeq_1$-understood, completing the proof. □

Other comparisons of understanding relate to the DMs' beliefs rather than correlation covers. For instance, Subsection 1.1 offers one case in which the correlation coefficient characterizes the DM's perception of correlation. In the interest of space, we explore characterize this comparison fully in the appendix. Roughly, we show that DM1 overvalues (undervalues) positively (negatively) correlated profiles relative to DM2 if and only if the absolute value of her subjective correlation coefficient is smaller than DM2's for every pair of actions.[6]

5.3. **The Minimal Correlation Cover.** A rich correlation cover $\mathcal{U}$ is *minimal* if $\mathcal{U}$ fails to cover $\mathcal{A}$ whenever any one of its understanding classes is removed. Note that if a universal correlation cover $\mathcal{U}$ is minimal, then for any PCR rich $(\mathcal{U}', \mu', u)$, the cardinality of $\mathcal{U}$ cannot exceed that of $\mathcal{U}'$. In particular, exists a surjective map $f : \mathcal{U}' \to \mathcal{U}$ such that $C \subseteq f(C)$ for any $C \in \mathcal{U}'$.

Theorem 7. *Suppose that preference $\succeq$ has a PCR $(\mathcal{U}, \pi, u)$ where $\mathcal{U}$ is a universal correlation cover and u is strictly concave. If for each $C \in \mathcal{U}$ there exists $a \in C$ such that $\sigma(a) = \Sigma_C$, then:*

(i) π can be taken to be countably additive,

(ii) $\mathcal{U}$ is a minimal correlation cover, and

(iii) for every $C \in \mathcal{U}^*$, there exists a so that $C = \{b : aKb\}$.

---

[6]We thank Davin Ahn for drawing our attention to this characterization.

Proof. See Appendix A.5 □

Of course, the hypothesis of Theorem 7 is satisfied whenever Ω or Σ is finite. Arguments in the proof also establish that the universal $\mathcal{U}$ is minimal whenever $\mathcal{U}$ itself is finite, regardless of the cardinality of Ω. Finally, it should be noted that counter-examples exist for each of the three claims when the hypothesis of Theorem 7 fails.

6. Discussion

In this section, we illustrate some applications of our approach and relate them to the existing literature.

6.1. Special Cases of PCRs.

For simplicity of exposition, we maintain throughout this subsection that Ω is finite, that $\Sigma = 2^{\Omega}$, and that for all $C \in \mathcal{U}$, $\Sigma_C = \Sigma$. For $E \in \Sigma$, write E_C for $\{\vec{\omega} : \omega_C \in E\}$. It is easy to adapt Theorem 2 to show existence of such a representation by strengthening Non-Singularity in the natural way.

The first is the model where the DM understands all correlations. This is simply the usual Monotonicity condition.

Axiom 5 (Monotonicity)**.** If

$$\left(p(\langle a_i \rangle_{i=1}^n), \sum_{i=1}^n a_i(\omega)\right) \succeq \left(q(\langle b_i \rangle_{i=1}^m), \sum_{i=1}^m b_i(\omega)\right)$$

for every $\omega \in \Omega$, then $p \succeq q$.

See also Battigalli et al. (2013) who call their analog of this condition congruence. Axioms 1, 2, 3, and 5 imply SEU.

Proposition 2. *Suppose $\succeq$ has a rich PCR $(\mathcal{U}, \pi, u)$ with universal $\mathcal{U}$. the preference $\succeq$ satisfies Monotonicity if and only if $\mathcal{U} = \{\mathcal{A}\}$.*

Proof. By Monotonicity, the DM understands any finite subset of $\mathcal{A}$. This implies the DM understands $\mathcal{A}$ (which is rich), and $\mathcal{U} = \{\mathcal{A}\}$ if $\mathcal{U}$ is universal. □

In some models of imperfect inference, agents have the same beliefs about the distribution of individual, equivalent actions. That is, an agent's beliefs about the distribution of Ω do not depend on the action being evaluated. In particular, if $a(\omega) = b(\omega)$ for all ω, then $a \sim b$. Formally, the preference $\succeq$ has a representation in the following class.

Definition 9. A rich PCR $(\mathcal{U}, \pi, u)$ has *consistent marginal beliefs* if $\pi(E_C) = \pi(E_{C'})$ for all $C, C' \in \mathcal{U}$ and all $E \in \Sigma$.

The following axiom characterizes marginal belief consistency. It requires that Monotonicity holds when comparing any two lotteries whose support includes only "simple" profiles that consist of a single action.

Axiom 6 (Simple Monotonicity)**.** If $p, q \in \Delta(\mathcal{A})$ and

$$(p(a), a(\omega)) \succeq (q(b), b(\omega))$$

for all $\omega \in \Omega$, then $p \succeq q$.

Proposition 3. *Let the preference $\succeq$ have a rich PCR $(\mathcal{U}, \pi, u)$ with non-constant u. Then, the preference $\succeq$ satisfies Simple Monotonicity if and only if it has consistent marginal beliefs.*

Proof. Apply Anscombe and Aumann (1963) to $\Delta(\mathcal{A})$ to get a measure $q : \Sigma \to [0, 1]$ representing beliefs and a utility index v. As $\Delta(X)$ belongs to both $\Delta(\mathcal{A})$ and $\Delta(\mathcal{F})$, u must be an affine transformation of v. By Theorem 3, $q(E) = \pi(E_C)$, establishing the result. $\square$

Note that a risk neutral DM who satisfies simple montonicity acts as if she understands any subset of actions (provided that $\mathcal{A}$ is rich). Under risk neutrality, the representation in Theorem 1 yields

$$V(\langle a_i \rangle_{i=1}^n) = \sum_{i=1}^n V(\langle a_i \rangle)$$

where each $V(\langle a_i \rangle)$ can be obtained by restricting the representation to individual actions in the standard way and thus with one copy of the state space. Given a linear u, simple montonicity is equivalent to monotonicity, and non-linearity of u is thus a necessary condition for this equivalence to fail.

Within the class of models that satisfy simple monotonicity, *full correlation neglect* has received a great deal of attention, see DeMarzo et al. (2003), Eyster and Weizsäcker (2010), and Levy and Razin (2015a). In these models either the DM perfectly understands the connection between a and b or believes that a and b are independent. In our model, this can be obtained by having

$$\pi(E_C \cap E'_{C'}) = \pi(E_C)\pi(E'_{C'}) \tag{1}$$

for all $C \neq C' \in \mathcal{U}$ and all $E, E' \in \Sigma$. An axiomatization of full correlation neglect is provided in the Appendix. One can of course envisage intermediate cases of correlation neglect as in Subsection 1.1, where full correlation neglect corresponds to the case $\eta = 1$.

6.2. **Misperceived Incomplete Information Games.** In this section, we adapt and apply our framework and model to strategic environments. In particular, we consider games of incomplete information played by players represented by PCRs. We formulate a general solution concept, which is shown to be equivalent to a game played between two players who misperceive the opponent's strategies in a systematic way. We generalize some existing solution concepts and suggests new ones, such as *self-similar* equilibrium where each player overweighs the probability that the opponent's type is the same as hers.

An two-player *game of incomplete information* Γ is a tuple

$$(T_1, T_2, q, \mathcal{A}_1, \mathcal{A}_2, u_1, u_2)$$

where T_i is the set of player i's types, which we assume to be finite for simplicity, q is a probability distribution over $T_1 \times T_2$ assigning positive probability to every $\{t_1\} \times T_2$ and $T_1 \times \{t_2\}$, $\mathcal{A}_i$ is a finite set of strategies for player i, and $u_i : \mathcal{A}_1 \times \mathcal{A}_2 \times T_1 \times T_2 \to \mathbb{R}$ is the payoff for player i. The set of Bayesian strategies S_i of player i maps T_i to $\Delta(\mathcal{A}_i)$, the set of probability distributions on $\mathcal{A}_i$. The probability with which type $t_i \in T_i$ plays $a_i \in \mathcal{A}_i$ is denoted by $\sigma_i(t_i)(a_i)$. For any $\sigma_i, \sigma_i' \in S_i$ and $\alpha \in [0,1]$, we write $\alpha\sigma_i + (1-\alpha)\sigma_i'$ for the strategy that plays $a_i \in \mathcal{A}_i$ with probability $\alpha\sigma_i(t_i)(a_i) + (1-\alpha)\sigma_i'(t_i)(a_i)$ when player i's type is t_i.

We generalize the PCR to allow state-dependent preferences and adapt it to a strategic setting. For the exercise, we are only interested in how players evaluate profiles consisting of pairs of strategies (σ_1, σ_2) in $S_1 \times S_2$. The state space is $T_1 \times T_2$, and we assume player i's correlation cover of $S_1 \cup S_2$ is $\mathcal{U}_i = \{S_1, S_2\}$. In what follows, with a slight abuse we adopt a symmetric notation where player i denotes a player and player j her opponent. Thus, player i's utility index is $v_i(a_i, a_j, t_i, t_j)$ and her beliefs are $\pi_i : (T_i \times T_j)^{\{S_i, S_j\}} \to [0,1]$.

Definition 10. Player i has *a state-dependent PCR* $(\mathcal{U}_i, \pi_i, v_i)$ if her utility from the strategy profile (σ_i, σ_j) is given by

$$V_i(\sigma_i, \sigma_j) = \sum_{t_i, t_j, \hat{t}_j} \pi_i\left(t_i, t_j, T_i, \hat{t}_j\right) \sum_{a_j, a_i} \sigma_i(t_i)(a_i)\sigma_j(\hat{t}_j)(a_j) v_i(a_i, a_j, t_i, t_j).$$

All components of the state-dependent PCR have similar interpretation as in PCR.[7] The key difference is that the utility index, v_i, depends on the S_i-copy of the state space. Hence player i understands the connection between the payoff relevant state and her own strategy but may not understand its connection to the strategy of player j.

The definition below defines equilibrium when each player has a state-dependent PCR over strategy profiles.

Definition 11. A *PCR equilibrium* is a pair of strategies (σ_1^*, σ_2^*) and a pair of state-dependent PCR's $(\mathcal{U}_1, \pi_1, u_1)$ and $(\mathcal{U}_2, \pi_2, u_2)$ where, for $i = 1, 2$,

$$\pi_i\left(t_i, t_j, T_i, T_j\right) = q\left(t_i, t_j\right) \text{ and } \pi_i\left(T_i, T_j, T_i, \hat{t}_j\right) = q\left(T_i, \hat{t}_j\right)$$

and

$$V_i(\sigma_i^*, \sigma_j^*) \geq V_i(\sigma_i, \sigma_j^*)$$

for all $\sigma_i \in S_i$.

[7]Indeed, a PCR generates a family of state dependent PCRs for appropriately chosen X, $+$, and $\mathcal{U}$; see supplementary material.

Remark 2. Given our assumptions that T_i and $\mathcal{A}_i$ are finite, standard arguments show that a PCR equilibrium exists.

To compare our approach to other approaches in the literature, we first introduce an alternative definition of equilibrium with misperceptions and show its equivalence with PCR equilibria. A *family of misperceptions* associates to each strategy $\sigma_j \in S_j$ a misperception $\bar{\sigma}_j : T_i \times T_j \to \Delta(\mathcal{A}_j)$. Thus, $\bar{\sigma}_j$ denotes player i's misperception of player j's strategy σ_j. Note that we allow player i's misperception of player j's strategy to depend on player i's type.

Definition 12. A family of misperceptions $\{\bar{\sigma}_j\}_{\sigma_j \in S_j}$ is *Markov* if:

(1) For any $\sigma_j \in S_j$ and any $a_j \in \mathcal{A}_j$,

$$\sum_{(\tau_i,\tau_j)\in T_i\times T_j} q(\tau_i,\tau_j)\overline{\sigma_j}(\tau_i,\tau_j)(a_j) = \sum_{t_j\in T_j} q(t_j, T_{-j})\sigma_j(t_j)(a_j),$$

and

(2) For any $\sigma_j, \upsilon_j \in S_j$ and any $\alpha \in [0,1]$,

$$\overline{\alpha\sigma_j + (1-\alpha)\upsilon_j} = \alpha\overline{\sigma_j} + (1-\alpha)\overline{\upsilon_j}.$$

Property 1 says that the ex-ante probability for a strategy of playing any action is the same as for its misperception. Property 2 says that the misperception of a mixture of two strategies is exactly the mixture of the misperceptions. Critically, $\bar{\sigma}_j$ does not depend on σ_i, and the support of $\bar{\sigma}_j$ is the same as that of σ_j. This rules out some interesting deviations, including Esponda (2008), self confirming equilibria of Fudenberg and Levine (1993), and Madarasz (2014).

The following definition extends the notion of Bayesian Nash equilibrium by incorporating Markov misperceptions.

Definition 13. Given Markov families of misperceptions, $\{\bar{\sigma}_1\}_{\sigma_1\in S_1}$ and $\{\bar{\sigma}_2\}_{\sigma_2\in S_2}$, a *misperception equilibrium* is pair of strategies (σ_1^*, σ_2^*) such that

$$\sigma_i^*(t_i) \in \arg\max_{a_i\in\mathcal{A}_i} \sum_{t_j\in T_j} q(t_i,t_j) \sum_{a_j\in\mathcal{A}_j} \bar{\sigma}_j^*(t_i,t_j)(a_j)u_i(a_i,a_j,t_i,t_j).$$

for every $t_i \in T_i$, $i = 1, 2$.

We now turn to the equivalence of PCR and misperception equilibria. To establish this, we characterize the structure of Markov families of misperceptions.

Lemma 3. *The family of misperceptions $\{\bar{\sigma}_j : \sigma_j \in S_j\}$ is Markov if and only if there exists a Kernel $K_j : T_i \times T_j \to \Delta T_j$ such that*

$$\sum_{(\tau_i,\tau_j)\in T_i\times T_j} q(\tau_i,\tau_j)K_j(\tau_i,\tau_j)(t_j) = q(t_j, T_{-j})$$

and, for any $a_j \in \mathcal{A}_j$*,*

$$\bar{\sigma}_j(\tau_i, \tau_j)(a_j) = \sum_{t_j \in T_j} K_j(\tau_i, \tau_j)(t_j)\sigma_j(t_j)(a_j).$$

Proof. See Appendix A.6. □

Proposition 4. *For any* Γ*, the set of misperception equilibria equals the set of PCR equilibria when*

$$\pi_i(\tau_i, \tau_j, T_i, t_j) = q(\tau_i, \tau_j) K_j(\tau_i, \tau_j)(t_j)$$

for all (τ_i, τ_j) *with* $q(\tau_i, \tau_j) > 0$*.*

Proof. Fixing either π_1 and π_2 or K_1 and K_2, and defining the other pair from the equation in the Proposition, note that

$$\begin{aligned}
&\sum_{t_j, \tau_j} \pi_i(\tau_i, \tau_j, T_i, t_j) \sum_{a_j} \sigma_j(t_j)(a_j) u_i(a_i, a_j, \tau_i, \tau_j) \\
&= \sum_{t_j, \tau_j} q(\tau_i, \tau_j) K_j(\tau_i, \tau_j)(t_j) \sum_{a_j} \sigma_j(t_j)(a_j) u_i(a_i, a_j, \tau_i, \tau_j) \\
&= \sum_{\tau_j} q(\tau_i, \tau_j) \sum_{a_j} \overline{\sigma}_j(\tau_i, \tau_j)(a_j) u_i(a_i, a_j, \tau_i, \tau_j)
\end{aligned}$$

so the evaluations of every pair of strategies coincide. □

We can thus easily establish that many equilibrium notions of imperfect inference are special cases of misperception equilibria. Cursed equilibrium (Eyster and Rabin, 2005) is such that

$$K_j(\tau_i, \tau_j)(t_j) = \chi q(t_j|\tau_i) + (1 - \chi)\delta_{\tau_j}(t_j)$$

where $\delta_{\tau_j}(t_j) = 1$ when $\tau_j = t_j$ and zero otherwise. Thus,

$$\bar{\sigma}_j(\tau_i, \tau_j)(a_j) = \chi \sum_{t_j \in T_j} q(t_j|\tau_i)\sigma_j(t_j)(a_j) + (1 - \chi)\sigma_j(\tau_j)(a_j),$$

In ABEE with analogy partition of $T_i \times T_j$ for player i being Q^i(Jehiel and Koessler, 2008)

$$K_j(\tau_i, \tau_j)(t_j) = q(t_j|Q^i(\tau_i, \tau_j))$$

and thus

$$\bar{\sigma}_j(\tau_i, \tau_j)(a_j) = \sum_{t_j \in T_j} \sigma_j(t_j)(a_j) q(\tau_j|Q^i(\tau_i, \tau_j))$$

where $Q^i(\tau_i, \tau_j)$ is the element of Q^i containing (τ_i, τ_j). Of course, Bayesian Nash equilibrium (Harsanyi, 1967-1968) is such that

$$K(\tau_i, \tau_j)(t_j) = \delta_{\tau_j}(t_j).$$

Note that, if neither $\mathcal{A}_1$ nor $\mathcal{A}_2$ is a singleton, then Kernels are (q almost surely) unique given a family of misperceptions. To see that the Kernel, if it exists, is unique, let σ_i be a strategy of player i where type τ_i plays b_i and all other types play $a_i \neq b_i$. Any Kernel must satisfy $K_j(t_j, t_i)(\tau_i) = \bar{\sigma}(t_j, t_i)(b_i)$, establishing uniqueness. Although the Kernel is a high-dimensional parameter, the unique correspondence between it and the misperceived strategy makes interpreting changes in the Kernel straightforward. In contrast, for instance, two distinct analogy partitions in ABEE (Jehiel and Koessler, 2008) may generate the same families of misperceptions.

Of course, our approach to misperception is sufficiently general to incorporate new and intuitive concepts. Rubinstein and Salant (2015) suggest, albeit in a perfect information game, that players believe others are more likely to act like them. To incorporate this insight, we introduce a special case of misperception/PCR equilibrium, the χ-*self-similar equilibrium*, that applies to symmetric games, or more generally those where $T_1 = T_2$. We characterize the misperception strategies by the Kernel

$$K_j(\tau_i, \tau_j)(t_j) = \chi \delta_{\tau_i}(t_j) + (1-\chi)\delta_{\tau_j}(t_j)$$

so that

$$\bar{\sigma}_j(\tau_i, \tau_j)(a_j) = \chi \sigma_j(\tau_i)(a_j) + (1-\chi)\sigma_j(\tau_j)(a_j)$$

for all j. In any symmetric, self-similar equilibrium, player i overweighs the probability that player j takes the same action as hers.

Appendix A. Proof not in main text

A.1. **Proof of Theorem 1.**

Lemma 4. *I has a positive linear extension F to the smallest subspace U^* that contains U.*

Proof. Define $U^* = \{\lambda_1 x_1 - \lambda_2 x_2 : x_1, x_2 \in U, \lambda_1, \lambda_2 \in \mathbb{R}_+\}$. U^* is clearly a subspace and contains U. Let U' be any other subspace containing U. Pick any $y \in U^*$. Then $y = \lambda_1 x_1 - \lambda_2 x_2$, and since $x_1, x_2 \in U \subset U'$, $y \in U'$; hence $U^* \subseteq U'$. Suppose that $\lambda_1 x_1 - \lambda_2 x_2 = y$ and $y \in U$. Then

$$\frac{\lambda_1}{1+\lambda_1+\lambda_2} x_1 = \frac{\lambda_2}{1+\lambda_1+\lambda_2} x_2 + \frac{1}{1+\lambda_1+\lambda_2} y.$$

Since $x_1, x_2, y, 0 \in U$, so are the LHS and RHS above. Linearity of I on U gives that $I(y) = \lambda_1 I(x_1) - \lambda_2 I(x_2)$. So the function $F = y \mapsto \lambda_1 I(x_1) - \lambda_2 I(x_2)$ whenever $y = \lambda_1 x_1 - \lambda_2 x_2$ is well-defined and extends I. Linearity of F follows from linearity of I. To see that F is positive, fix $\phi \in U^*$ with $\phi \geq 0$. Then $\phi = \lambda_1 x_1 - \lambda_2 x_2$; if $\lambda_1 = \lambda_2 = 0$, $\phi = 0$ so $F(\phi) = F(0) = 0$. Otherwise,

$$\frac{\lambda_1}{\lambda_1+\lambda_2} x_1 \geq \frac{\lambda_2}{\lambda_1+\lambda_2} x_2$$

and since both the LHS and RHS are in U, $F(\frac{\lambda_1}{\lambda_1+\lambda_2} x_1) \geq F(\frac{\lambda_2}{\lambda_1+\lambda_2} x_2)$. The remainder follows from linearity of F. $\square$

For any $J \subseteq \mathcal{A}$, define $\Sigma_J = \otimes_{a \in J} \sigma(a)$, the product $\sigma-$algebra on Ω^J and $B_0(\Sigma_{\mathcal{A}})$ the set of simple $\Sigma_{\mathcal{A}}$-measurable functions. Note that the set U^* is a vector subspace of $B_0(\Sigma_{\mathcal{A}})$.

Lemma 5. *There is a positive linear extension $\hat{F}$ of F to all of $B_0(\Sigma_{\mathcal{A}})$ such that*

$$\hat{F}(\phi) = \int \phi d\pi^o,$$

for a finitely additive probability measure π^o on $\Sigma_{\mathcal{A}}$.

Proof. The function F is linear on U^* and $x \geq 0$ implies $F(x) \geq 0$. Pick any $y \in B_0(\Sigma_{\mathcal{A}})$. Since y is bounded, let z be an upper bound for y. z is a constant so $z \in U^*$. Hence U^* majorizes $B_0(\Sigma_{\mathcal{A}})$. By Theorem 8.32 of Aliprantis and Border (2006), F extends to a positive linear function on $B_0(\Sigma_{\mathcal{A}})$. By Theorem 14.4 of Aliprantis and Border (2006), there is a finitely additive signed measure of bounded variation, $\pi^o : \Sigma_{\mathcal{A}} \to R$, such that

$$\hat{F}(\phi) = \int \phi d\pi^o,$$

and the result follows from showing that we can replace π^o with a finitely additive probability measure. Consider the constant function 1. Since $\hat{F}(1) = 1$, $\pi^o(\Omega^{\mathcal{A}}) = 1$. Now, for any $E \in \Sigma_{\mathcal{A}}$, consider the indicator function χ_E. Since $\chi_E \geq 0$, $\hat{F}(\chi_E) \geq \hat{F}(0) = 0$. Since $\hat{F}(\chi_E) = \pi^o(E)$, $\pi^o(E) \geq 0$ for all E. Consequently, π^o is a finitely additive probability measure. □

Proof of Theorem 1. Necessity is easily verified. Lemma 5 yields a finitely additive probability π^o. To construct a countably additive probability, for every finite $J \subseteq \mathcal{A}$ define a set function π_J on (Ω^J, Σ_J) using the formula

$$\pi_J(E) = \pi^o(E \times \Omega^{\mathcal{A} \setminus J})$$

for every $E \in \Sigma_J$. Each π_J inherits finite additivity from π^o; in fact, since Σ_J has a finite number of members, π_J is countably additive and so a probability measure. By construction the family $\{\pi_J\}$ is Kolmogorov consistent. As a finite set, each Σ_J is a compact class, and trivially

$$\pi_J(E) = \sup\{\pi_J(E') : E \supseteq E' \in \Sigma_J\}.$$

By Kolmogorov's extension theorem (Theorem 15.26, Aliprantis and Border (2006)), there is a unique, countably additive $\pi : \Sigma_{\mathcal{A}} \to [0, 1]$ that extends each π_J. For any p, there is a finite $J_p \subset \mathcal{A}$ such that f_p is Σ_{J_p} measurable. Letting $\hat{f}_p$ be the natural projection of f_p onto Ω_{J_p},

$$\int_{\Omega^{\mathcal{A}}} f_p d\pi^o = \int_{\Omega_{J_p}} \hat{f}_p d\pi_{J_p} = \int_{\Omega^{\mathcal{A}}} f_p d\pi.$$

Therefore the function $U : \Delta\mathcal{F} \to \mathbb{R}$ defined by

$$U(p) = \int f_p d\pi$$

represents the DM's preference. To conclude, rewrite $U(p)$ as

$$\int_{\Omega^{\mathcal{A}}} f_p d\pi = \int_{\Omega^{\mathcal{A}}} \sum_{p(\langle a_i \rangle)>0} p(\langle a_i \rangle_{i=1}^n) u(\sum_{i=1}^n a_i(\omega^{a_i})) d\pi$$

$$= \sum_{p(\langle a_i \rangle)>0} p(\langle a_i \rangle_{i=1}^n) \int_{\Omega^{\mathcal{A}}} u(\sum_{i=1}^n a_i(\omega^{a_i})) d\pi,$$

the desired representation. □

A.2. **Proof of Theorem 2.**

Proof of Theorem 2. Necessity is straightforward. For sufficiency, we first show the existence of a universal, rich correlation cover $\mathcal{U}$.[8] Let $\{B_t\}_{t\in T}$ be a chain of rich understood subsets of $\mathcal{A}$. We claim that $B^* = \cup_{t\in T} B_t$ is rich and understood and thus an upperbound by set inclusion. The set B^* is understood since for any p, q, $\mathcal{B}(p) \cup \mathcal{B}(q)$ is finite. Richness follows since, if a, b in B^*, $a, b \in B_t$ for some t so for any $f : \Omega \to X$ that is $\sigma(a, b)$-measurable, there exists $c \in B_t \subseteq B^*$ with $c(\omega) = f(\omega)$ for any $\omega \in \Omega$. Hence, by Zorn's lemma, there exists at least one maximal element. Let $\mathcal{U}$ be the set of all maximal, rich and understood subsets. The set $\mathcal{U}$ covers $\mathcal{A}$ by assumption and, by definition, no element is contained in any other. By Theorem 1, the preference relation $\succeq$ has a PCR $(\{a\}_{a\in\mathcal{A}}, \pi_0, u_0)$. We show it also has rich PCR with $(\mathcal{U}, \pi, u)$ with $u = u_0$. Consider a lottery r. For each profile $F = \langle a_j \rangle_{j=1}^n$ in the support select a vector $C^F = \left(C_1^F, ..., C_n^F\right)$ such that $C_j^F \in \mathcal{U}$ and $a_j \in C_j^F$, $j = 1, ..., n$. For every $\vec{\omega} \in \Omega^{\mathcal{U}}$, construct a map

$$f_r(\vec{\omega}) = \sum_{r(F)>0} r(F) u \left(\sum_{j=1}^n a_j(\omega^{C_j^F}) \right)$$

Each vector C^F, which we call a *profile allocation*, assigns actions in each profile that has a positive probability to understanding classes. Note that many profile allocations may be associated with the same lottery if its actions belongs to several understanding classes and that many f_r's are generated for the same lottery r depending on the profile allocations chosen. To save notation, we omit the dependence of the f_r's generated for the same lottery on the profile allocations. In particular, the same action can be assigned to different classes even within the same profile should the action appear more than once. Construct all such maps for all lotteries and profile allocations. The remainder of the proof follows from the arguments in Lemmas 1-5 if we show that, for any such maps, if for some choice of profile allocations $f_p \geq f_q$ then $p \succeq q$. Pick arbitrary p, q satisfying $f_p \geq f_q$ for some selection of profile allocations $\{C^F\}_{p(F)>0}$ and $\{(C')^F\}_{q(F)>0}$. Define $\{B_1, ..., B_n\} = \{C^F\}_{p(F)>0} \cup \{(C')^F\}_{q(F)>0}$; note that the corresponding f_p and f_q are both measurable with respect to

[8]The proof below works (but does not deliver maximality) for any rich understanding structure $\mathcal{U}$.

the product σ-algebra

$$\otimes_{i=1}^{n}\Sigma_{B_i}\otimes_{B\in\mathcal{U}\setminus\{B_i\}_{i=1}^{n}}\{\Omega,\emptyset\}.$$

For every $F=\langle a_j\rangle_{j=1}^{n}$ with $p(F)>0$, define $a_1^F,...,a_m^F\in\mathcal{A}$ by aggregating the actions assigned to the same understanding class, that is,

$$a_i^F(\omega)=\sum_{\{a_j:C_j^F=B_i\}}a_j(\omega)$$

By richness, $a_i^F\in B_i$. Since the preference $\succeq$ understands every B_i, $\langle a_i^F\rangle_{i=1}^{m}\sim F$. Repeat this procedure for all profiles in the support of p, and similarly for q. Consider an understanding class B in a profile allocation. Let $\{E^k\}_{k=1}^{K}$ be the finest partition of Ω for which every action in B that is assigned positive probability by p or q is measurable. For $x\in X$, define the action $\beta_x^k\in B$ such that

$$\beta_x^k(\omega)=xE^k0(\omega).$$

for all ω. Since $\mathcal{U}$ is rich such actions exists in B. Now, consider $\{x_k\}_{k=1}^{K}$, $x_k\neq 0$ for all k. For any x_i,x_j, $i\neq j$,

$$\frac{1}{2}\beta_{x_i}^i+\frac{1}{2}\beta_{x_j}^j\sim\frac{1}{2}\langle\beta_{x_i}^i,\beta_{x_j}^j\rangle+\frac{1}{2}0.$$

Then, letting $\mathcal{E}^k=\{\vec{\omega}\in\Omega^{\mathcal{A}}:\omega^{\beta_{x_i}^k}\in E^k\}$ and $\mathcal{E}=\mathcal{E}^i\cap\mathcal{E}^j$, we have

$$\begin{aligned}&\pi_0(\mathcal{E}^i)u(x_i)+\pi_0(\mathcal{E}^j)u(x_j)\\&=[\pi_0(\mathcal{E}^i)-\pi_0(\mathcal{E})]u(x_i)+[\pi_0(\mathcal{E}^j)-\pi_0(\mathcal{E})]u(x_j)+\pi_0(\mathcal{E})u(x_i+x_j)\end{aligned}$$

after normalizing so that $u(0)=0$. Since $u(x_i+x_j)<u(x_i)+u(x_j)$ by Strict Concavity, the two are equal if and only if

$$\pi_0(\mathcal{E})=0. \tag{2}$$

Equation 2 implies that

$$\pi_0\left(\bigcup_{k=1}^{K}\mathcal{E}^k\right)=\sum_{k=1}^{K}\pi_0\left(\mathcal{E}^k\right)$$

Now consider $\beta_x^k,\beta_y^k\in B$ and $b\in B$ that is equal to $x\overline{E^k}0$. Define

$$\mathcal{E}^{k,x}=\{\vec{\omega}\in\Omega^{\mathcal{A}}:\omega^{\beta_x^k}\in E^k\}$$

$$\mathcal{E}^{k,y}=\{\vec{\omega}\in\Omega^{\mathcal{A}}:\omega^{\beta_y^k}\in E^k\}$$

$$\mathcal{E}^{b}=\{\vec{\omega}\in\Omega^{\mathcal{A}}:\omega^{b}\in\overline{E^k}\}.$$

By arguments analogous to the preceding ones,

$$\pi_0\left(\mathcal{E}^{k,x}\cap\mathcal{E}^b\right)=\pi_0\left(\mathcal{E}^{k,y}\cap\mathcal{E}^b\right)=0.$$

The above equality yields

$$\pi_0\left(\mathcal{E}^{k,x}\cap(\overline{\mathcal{E}^b}\right)=\pi_0\left(\mathcal{E}^{k,x}\right)$$

Since $\succeq$ understands $\{\beta_x^k, b\}$, $\langle\beta_x^k, b\rangle \sim x$ and $\pi_0(\mathcal{E}^{k,x}) + \pi_0(\mathcal{E}^b) = 1$. Then

$$\pi_0\left(\overline{\mathcal{E}^{k,x}} \cap \overline{\mathcal{E}^b}\right) + \pi_0\left(\mathcal{E}^{k,x}\right) = 1 - \pi_0(\mathcal{E}^b) = \pi_0(\mathcal{E}^{k,x})$$

and thus

$$\pi_0\left(\overline{\mathcal{E}^{k,x}} \cap \overline{\mathcal{E}^b}\right) = 0.$$

Now,

$$\pi_0(\overline{\mathcal{E}^{k,x}} \cap \mathcal{E}^{k,y} \cap \overline{\mathcal{E}^b}) + \pi_0(\overline{\mathcal{E}^{k,x}} \cap \mathcal{E}^{k,y} \cap \mathcal{E}^b) = 0$$

implying that

$$\pi_0\left(\overline{\mathcal{E}^{k,x}} \cap \mathcal{E}^{k,y}\right) = 0$$

and that

$$\pi_0\left(\mathcal{E}^{k,y} \cap \mathcal{E}^{k,x}\right) = \pi_0\left(\mathcal{E}^{k,y}\right)$$

By a symmetric argument with $b' = y\overline{E^k}0$, we conclude

(3)
$$\begin{aligned}\pi_0\left(\overline{\mathcal{E}^{k,x}} \cap \mathcal{E}^{k,y}\right) &= \pi_0\left(\mathcal{E}^{k,x} \cap \overline{\mathcal{E}^{k,y}}\right) = 0\\ \pi_0\left(\mathcal{E}^{k,y} \cap \mathcal{E}^{k,x}\right) &= \pi_0\left(\mathcal{E}^{k,y}\right) = \pi_0(\mathcal{E}^{k,x})\end{aligned}$$

Take $y \neq 0$ and note

$$(\frac{1}{K}, \langle\beta_y^i\rangle)_{i=1}^n \sim (\frac{1}{K}, y; \frac{K-1}{K}, 0).$$

since $\beta_y^i \in B$ for all i. Then, by Equations 2 and 3

$$\sum_{k=1}^K \pi_0\left(\mathcal{E}^{k,y}\right) u\left(y\right) = \sum_{k=1}^K \pi_0\left(\mathcal{E}^{k,x_k}\right) u\left(y\right) = u\left(y\right).$$

Hence,

$$\pi_0\left(\bigcup_{k=1}^K \mathcal{E}^{k,x_k}\right) = \sum_{k=1}^K \pi_0\left(\mathcal{E}^{k,x_k}\right) = 1.$$

and

(4)
$$\pi_0\left(\bigcap_{k=1}^K \overline{\mathcal{E}^k}\right) = 0.$$

Thus, π_0 assigns zero probability to any event where the outcomes of two distinct $\beta_{x_i}^i$ and $\beta_{x_j}^j$ are "misaligned" with respect to $\{E^k\}_{k=1}^K$. Consider again an action $a_i^F \in B_i$ and let $\{E^{i,k}\}_{k=1}^{K_i}$ be the finest partition of Ω for which every action allocated to B_i and assigned positive probability by p or q is measurable. Define a profile $\langle\beta_{x_k}^{i,F}\rangle_{k\in I^{i,F}}$ where $I^{i,F}$ is a subsequence of $\{1, ..., K_i\}$, each $\beta_{x_k}^{i,F}$ is such that

$$\beta_{x_k}^{i,F} = x_k E^{i,k} 0,\ x_k \neq 0,$$

and

$$a_i^F\left(\omega\right) = \sum_{k\in I^{i,F}} \beta_{x_k}^{i,F}\left(\omega\right)$$

for every $\omega \in \Omega$. Note that the subsequence $I^{i,F}$ excludes actions for which $x_k = 0$ and that are thus constant actions. By replacing each a_i^F in the profile F with the corresponding profile $\langle \beta_{x_k}^{i,F} \rangle_{k \in I^{i,F}}$, we obtain a profile F' that is such that $F \sim F'$ since all substitutions are made within the same understanding class. Repeat this procedure until every F is replaced by an F'. Call the resulting lotteries over profiles p' and q'. Note $p' \sim p$ and $q' \sim q$ by Independence. For notational covenience, relabel all the actions $\beta_{x_k}^{i,F}$ in p' and q' as $\{\beta_i\}_{i=1}^T$ and the corresponding events $E^{i,k}$ as $\{E_i\}_{i=1}^T$. Note that the same $E^{i,k}$ may appear more than once as it may correspond to two distinct β_i's. Let J^F be the subsequence of $(1, ..., T)$ that selects the actions in $\langle \beta_{x_k}^{i,F} \rangle_{k \in I^{i,F}}$. Consider the cylinders

$$\mathcal{E} = L_1 \times ... \times L_T \otimes_{a \in \mathcal{A} \setminus \{\beta_i\}_{i=1}^T} \sigma(a)$$

where each L_i is either E_i or $\overline{E_i}$. Note that for all states

$$\left(\omega^{\beta_1}, ..., \omega^{\beta_T}, \{\omega^a\}_{a \in \mathcal{A} \setminus \{\beta_i\}_{i=1}^T}\right)$$

within such a cylinder, the outcome

$$\sum_{i=1}^{T} \beta_i \left(\omega^{\beta_1}\right)$$

is constant. By Equations 2, 3, and 4 a cylinder $\mathcal{E}$ has positive probability only if :

(i): if β_i and β_j are in the same understanding class and $E_i = E_j$ then $L_i = L_j$;
(ii): if β_i and β_j are in the same understanding class and $E_i \neq E_j$ then either $L_i = E_i$ and $L_j = \overline{E_j}$ or $L_i = \overline{E_i}$ and $L_j = E_j$;
(iii): If $\beta_{i_1}, \beta_{i_2}, ..., \beta_{i_N}$ are in the same understanding class and

$$L_{i_k} = \overline{E_{i_k}},\ k = 1, ..., N$$

then

$$\bigcap_{k=1}^{N} \overline{E_{i_k}} \neq \emptyset.$$

Consider now a state $\vec{\omega} \in \Omega^{\mathcal{A}}$ in a cylinder $\mathcal{E}$ satisfying (i), (ii) and (iii). The difference in expected utility of lottery p' and q' at state $\vec{\omega}$ is

$$\sum_{p(F)>0} p(F) u \left(\sum_{j \in J^F}^{n} \beta_j(\omega^{\beta_j}) \right) - \sum_{q(F)>0} q(F) u \left(\sum_{j \in J^F}^{n} \beta_j(\omega^{\beta_j}) \right) \tag{5}$$

Conditions (i) and (ii) imply that if $\beta_i(\omega^{\beta_i}) \neq 0$ for some i, there exists $\omega' \in \Omega$ such that $\beta_j(\omega') = \beta_j(\omega^{\beta_j})$ for all β_j in the same understanding class as β_i. Condition (iii) implies that if $\beta_j(\omega^{\beta_j}) = 0$ for all β_j in the same understanding class, there exists $\omega' \in \Omega$ such that $\beta_j(\omega') = 0$ for all β_j in the same understanding class. Thus, if $f_p \geq f_q$, the expected utility of p' is greater than the expected utility of q' for the PCR $(\{a\}_{a \in \mathcal{A}}, \pi_0, u_0)$. Thus, $p' \succeq q'$

and $p \succeq q$. Repeating the remaining steps of Theorem 1 completes the sufficiency proof. Uniqueness of u is standard. □

A.3. **Proof of Theorem 3.** To prove Theorem 3, we first show some preliminary results.

Lemma 6. *Consider a set of* 1*-dimensional rectangles* $\{\mathcal{E}_i\}_{i=1}^N$ *corresponding to distinct understanding classes, that is, for distinct* $C_1, ..., C_n \in \mathcal{U}$ *and* $E_i \in \sigma(a_i)$*,* $a_i \in C_i$ *for* $i = 1, ..., N$*,*

$$\mathcal{E}_i = \{\vec{\omega} : \omega^{C_i} \in E_i\}.$$

Then $\pi(\bigcap_{j \leq n} \mathcal{E}_j \bigcap_{j>n} \overline{\mathcal{E}_j})$ *equals*

$$\sum_{i=0}^{N-n-1} (-1)^i \pi(\bigcap_{j \leq n+i} \mathcal{E}_j \bigcap_{j>n+1+i} \overline{\mathcal{E}_j}) + (-1)^{N-n} \pi(\bigcap_j \mathcal{E}_j).$$

Proof. The claim follows by recursive substitutions, noting that

$$\pi(\bigcap_{j \leq n+i} \mathcal{E}_j \bigcap_{j>n+i} \overline{\mathcal{E}_j})$$

equals

$$\pi(\bigcap_{j \leq n+i} \mathcal{E}_j \bigcap_{j>n+1+i} \overline{\mathcal{E}_j}) - \pi(\bigcap_{j \leq n+1+i} \mathcal{E}_j \bigcap_{j>n+1+i} \overline{\mathcal{E}_j})$$

and $\pi(\bigcap_{i \leq N-1} \mathcal{E}_i \cap \overline{\mathcal{E}_N}) = \pi(\bigcap_{i \leq N-1} \mathcal{E}_i) - \pi(\bigcap_i \mathcal{E}_i)$. □

Let $Q_{m,N}$ be the set of all m element subsets of $\{1, ..., N\}$ and

$$S_N((x_i)_{i=1}^N) = \sum_{m=0}^{N} \sum_{q \in Q_{m,N}} (-1)^{m+1} u(\sum_{i \in q} x_i). \tag{6}$$

Lemma 7. *Condition* (N) *holds if and only if there exist* $(x_i)_{i=1}^N$ *such that*

$$S_N((x_i)_{i=1}^N) \neq 0.$$

Proof. For a sequence $\{y_i\}_{i=0}^\infty$, let $\sum_O^N y_m$ and $\sum_E^N y_m$ be, respectively, the sum of all odd and even indexed y_i with index between 0 and N, inclusive. Note that $S_1((x_1)) = u(x_1)$, so Condition (1) clearly implies there exists x_1 so that $S_1((x_1)) \neq 0$. For $N > 1$, note that $S_N((x_i)_{i=1}^N) = 0$ if and only if

$$\sum_E^N \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i) = \sum_O^N \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i)$$

Now, simple calculations show that

$$2U(p_2) = \sum_O^2 \sum_{q \in Q_{m},2} u(\sum_{i \in q} x_i)$$

and

$$2U(p_2') = \sum_E^2 \sum_{q\in Q_{m,2}} u(\sum_{i\in q} x_i).$$

Furthermore, by simple recursion if

$$2^{N-2}U\left(p_{N-1}\right) = \sum_O^{N-1} \sum_{q\in Q_{m,N-1}} u(\sum_{i\in q} x_i)$$

and

$$2^{N-2}U\left(p_{N-1}'\right) = \sum_E^{N-1} \sum_{q\in Q_{m,N-1}} u(\sum_{i\in q} x_i)$$

then one obtains that

$$\begin{aligned}2^{N-1}U\left(p_N\right) &= \sum_O^{N-1} \sum_{q\in Q_{m,N-1}} u(\sum_{i\in q} x_i) + \sum_E^{N-1} \sum_{q\in Q_{m,N-1}} u(\sum_{i\in q} x_i + x_N)\\ &= \sum_O^{N} \sum_{q\in Q_{m,N}} u(\sum_{i\in q} x_i)\end{aligned}$$

and

$$2^{N-1}U\left(p_N'\right) = \sum_E^{N} \sum_{q\in Q_{m,N}} u(\sum_{i\in q} x_i).$$

Note $S_N((x_i)_{i=1}^N) = 2^{N-1}\left(U\left(p_N\right) - U\left(p_N'\right)\right)$, completing the proof. □

Proof of Theorem 3. Suppose that $(\mathcal{U}, \pi, u)$ and $(\mathcal{U}, \mu, u)$ both represent the preference $\succeq$. Let V_π and V_μ be the respective utility indexes. Proceed by induction to establish uniqueness. The case of 1-dimensional rectangles is standard since both PCRs are rich. Suppose that $\pi(\mathcal{E}') = \mu(\mathcal{E}')$ whenever $\mathcal{E}'$ is an $N-1$-dimensional rectangle. Let $\mathcal{E}$ be an arbitrary N-dimensional rectangle, generated by $E_1, ..., E_N$ where $E_i \in \sigma(a_i')$ for some $a_i' \in C_i \in \mathcal{U}$ with distinct $C_1, ..., C_N$. By Lemma 7, let $x_1, ..., x_n \in X$ be so that $S_N((x_i)_{i=1}^N) \neq 0$. Consider the profile $F = \langle a_i \rangle_{i=1}^N$ where $a_i \in C_i$ and $a_i(\omega)$ equals x_i if $\omega \in E_i$ and equals 0 otherwise. Define

$$\mathcal{E}_i = \{\vec{\omega} : \omega^{C_i} \in E_i\}.$$

Note that

$$\begin{aligned}V_\pi(F) &= \sum_{m=0}^{N} \sum_{q\in Q_{m,N}} \pi(\bigcap_{i\in q} \mathcal{E}_i \bigcap_{j\notin q} \overline{\mathcal{E}_j}) u(\sum_{i\in q} x_i)\\ &= \sum_{m=0}^{N} \sum_{q\in Q_{m,N}} [\mathcal{K}(q,m,N) + (-1)^{N-m}\pi(\mathcal{E})] u(\sum_{i\in q} x_i)\\ &= K + S_N((x_i)_{i=1}^N)\pi(\mathcal{E})\end{aligned}$$

where $\mathcal{K}(q,m,N)$ and K are weighted sums of probabilities of $(N-1)$-dimensional rectangles. Such a decomposition exists by Lemma 6. Since μ agrees with π on $N-1$ dimensional rectangles,

$$V_\mu(F) = K + S_N((x_i)_{i=1}^N)\mu(\mathcal{E}).$$

There exists a lottery q such $q \sim F$. Then $V_\mu(F) = \sum_{q(y)>0} q(y)u(y) = V_\pi(F)$. Hence $V_\mu(F) = V_\pi(F)$, which requires that $\mu(E) = \pi(E)$. Conversely, suppose that Condition (N) fails, and that π agrees with μ on all $N-1$ dimensional rectangles. Consider any profile $\langle a_i \rangle_{i=1}^m$, and assume WLOG that each a_i belongs to a distinct understanding class C_i; we show that

$$V_\pi(\langle a_i \rangle_{i=1}^m) = V_\mu(\langle a_i \rangle_{i=1}^m).$$

This is trivially true if $m < N$. The claim is proved if we show that, when $m \geq N$, we can replace each $V_\pi(\langle a_i \rangle_{i=1}^m)$ and $V_\mu(\langle a_i \rangle_{i=1}^m)$ with the (possibly negatively) weighted sum of the utilities of "sub-profiles" of $\langle a_i \rangle_{i=1}^m$ with at most $N-1$ elements. To do this, we rely on the following implication of Lemma 7 when Condition (N) fails: for any $x_1, ..., x_n$,

$$u(\sum_{i=1}^N x_i) = (-1)^N \left[\sum_O^{N-1} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i) - \sum_E^{N-1} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i) \right]. \tag{7}$$

Now,

$$\begin{aligned} V_\pi(\langle a_i \rangle_{i=1}^m) &= \int u\left(\sum_{i=1}^m a_i(\omega^{C_i})\right) d\pi \\ &= \int u\left(\sum_{i=1}^{N-1} a_i(\omega^{C_i}) + [\sum_{i=N}^m a_i(\omega^{C_i})]\right) d\pi \end{aligned}$$

Then, by equation 7 where $x_i = a_i(\omega^{C_i})$, $i = 1, ..., N-1$, and $x_N = \sum_{i=N}^m a_i(\omega^{C_i})$, each term

$$u\left(\sum_{i=1}^{N-1} a_i(\omega^i) + [\sum_{i=N}^m a_i(\omega^i)]\right)$$

can be written as the sum of utilities where each argument contains the sum of at most $m-1$ terms. We can repeat this procedure until the arguments of each $u(\cdot)$ contain the sum of at most $N-1$ terms. Naturally, the exact same procedure can be applied to V_μ. This establishes the result. □

A.4. **Proof of Corollary 1.**

Proof. For necessity, note that we can write

$$u(y) = \sum_{n=1}^{N^*-1} \frac{\partial^n u(x)}{\partial x^n}(y-x)^n + u(x).$$

If $N^* < \infty$, then, as in Theorem 3, $u(x_1, ..., x_N)$ is a weighted sum of the utility of subsets of $\{x_1, ..., x_N\}$ with at most N^* elements. Arguments as in Theorem 3 establish the result.

For sufficiency, recall Condition (N) is equivalent to

$$\sum_{p_N(x)>0} p_N(x)u(x) \neq \sum_{p'_N(x)>0} p'_N(x)u(x)$$

for some N order apportioned lotteries p_N and p'_N. Say *Condition (N) holds for $v \in C^\infty$* if the above holds when u is replaced by v. We show that for any v where for all $n \leq N$ there exists y_n such that $\frac{\partial^n v(x)}{\partial x^n}|_{x=y_n} \neq 0$ implies Condition (N) for v. Proceed by induction. If $N = 1$, and $\frac{\partial v}{\partial x} \neq 0$ for some x, then there clearly exists x_1 with $v(x_1) \neq v(0)$, establishing Condition (1) for v. For the induction hypothesis, suppose that if for all $n \leq N$ there exists y_n with $\frac{\partial^n v(x)}{\partial x^n}|_{x=y_n} \neq 0$, then Condition (N) holds for v. Fix any u s.t. for all $n \leq N+1$ there exists y_n with $\frac{\partial^n u(x)}{\partial x^n}|_{x=y_n} \neq 0$. Consider any $(x_i)_{i=1}^{N+1}$ and the resulting $N+1$-order apportioned lotteries p_{N+1}, p'_{N+1}. As in Lemma 7, $V(p_{N+1}) - V(p'_{N+1})$ equals

$$\sum_{O}^{N} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i) + \sum_{E}^{N} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i + x_{N+1})$$
$$- [\sum_{O}^{N} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i + x_{N+1}) + \sum_{E}^{N} \sum_{q \in Q_{m,N}} u(\sum_{i \in q} x_i)]$$

Dividing by x_{N+1} and letting x_{N+1} go to zero yields

$$\sum_{m=0}^{N} \sum_{q \in Q_{m,N}} (-1)^m u'(\sum_{i \in q} x_i) \tag{8}$$

Note (8) is the difference in utility of two N order apportioned lotteries with utility index u'. For $n \leq N$,

$$0 \neq \frac{\partial^{n+1} u(x)}{\partial x^{n+1}}|_{x=y_{n+1}} = \frac{\partial^n u'(x)}{\partial x^n}|_{x=y_{n+1}}.$$

Thus the induction hypothesis implies that there exists $x_1, ..., x_N$ such that (8) does not equal zero, and there must exist x_{N+1} so that the $N+1$-order apportioned lotteries for $x_1, ..., x_{N+1}$ are not indifferent, establishing Condition (N+1) and completing the induction. The result now follows from observing that whenever $n < N^*$, there exists y_n such that $\frac{\partial^n u(x)}{\partial x^n}|_{x=y_n} \neq 0$. □

A.5. **Proof of Theorem 7.**

Proof. (i) Since Σ_C is finite for each $C \in \mathcal{U}$, π is countably additive when restricted to any finite collection of understanding classes. The result follows by applying Kolmogorov's extension theorem (Theorem 15.26, Aliprantis and Border (2006)) as in the proof of Theorem 1.

(ii) Suppose not and let $B \in \mathcal{U}$ be such that

$$\bigcup_{C\in\mathcal{U}, C\neq B} C = \mathcal{A}.$$

Take $b \in B$ for which $\sigma(b) = \Sigma_B$. Then, by Theorem 5 (ii),

$$B = \{c \in \mathcal{A} : bKc\}.$$

By hypothesis, $b \in C$ for some $C \in \mathcal{U}$. Applying Theorem 5 (ii) we have $B \subseteq C$, a contradiction since $\mathcal{U}$ is universal.

(iii) follows from Theorem 5 (ii). □

A.6. **Proof of Lemma 3.**

Proof of Lemma 3. Necessity is trivial, as is the case where $\mathcal{A}_j$ is a singleton. Thus, suppose the family is Markov and there are at least two actions . Identify $a_j \in \mathcal{A}_j$ with degenerate lotteries in $\Delta\mathcal{A}_j$ and write $(p,q)_t$ for the strategy "player j plays p if the type is t_j and q otherwise." Fix a_j. Note that

$$\overline{(b_j,a_j)_{t_j}}(\tau_i,\tau_j)(a_j) = \overline{(c_j,a_j)_{t_j}}(\tau_i,\tau_j)(a_j)$$

for any $a_j, b_j, c_j \in \mathcal{A}_j$ with $b_j, c_j \neq a_j$, since

$$\begin{aligned}
&\frac{1}{2}\overline{(b_j,a_j)_{t_j}}(\tau_i,\tau_j)(a_j) + \frac{1}{2}\overline{(c_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j) \\
&= \overline{\frac{1}{2}(b_j,a_j)_{t_j} + \frac{1}{2}(c_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j) \\
&= \overline{\frac{1}{2}(c_j,a_j)_{t_j} + \frac{1}{2}(b_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j) \\
&= \frac{1}{2}\overline{(c_j,a_j)_{t_j}}(\tau_i,\tau_j)(a_j) + \frac{1}{2}\overline{(b_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j)
\end{aligned}$$

where the first and third equalities follow from (2) and the second from the Bayesian strategies being identical. Then by (1),

$$\overline{(c_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j) = \overline{(b_j,c_j)_{t_j}}(\tau_i,\tau_j)(a_j) = 0,$$

which proves the claim. Note that an analogous reasoning yields

$$\overline{(b_j,a_j)_{t_j}}(\tau_i,\tau_j)(b_j) = \overline{(c_j,a_j)_{t_j}}(\tau_i,\tau_j)(c_j).$$

Define

$$K_j(\tau_i,\tau_j)(t_j) = \overline{(b_j,a_j)_{t_j}}(\tau_i,\tau_j)(b_j)$$

for every τ_i, τ_j, and t_j. By (1),

$$\sum_{(\tau_i,\tau_j)\in T_i\times T_j} q(\tau_i,\tau_j)K_j(\tau_i,\tau_j)(t_j) = q(t_j).$$

Now note that, by (1)

$$(a_j, a_j)_{t_j} = \overline{(a_j, a_j)_{t_j}}$$

Thus, (2) implies that for any strategy σ_j such that $\sigma_j(t_j) = \sigma_j(t'_j)$ for any $t_j, t'_j \in T_j$, we have $\bar{\sigma}_j = \sigma_j$. Since

$$\sum_{t_j \in T_j} \frac{1}{\#T_j}(b_j, a_j)_{t_j}$$

is a strategy in which every type plays b_j with probability $\frac{1}{\#T_j}$ and a_j with probability $\frac{\#T_j - 1}{\#T_j}$, we have

$$\#T_j \sum_{t_j \in T_j} \frac{1}{\#T_j} \overline{(b_j, a_j)_{t_j}}(\tau_i, \tau_j)(b_j) = 1.$$

Hence,

$$\sum_{t_j \in T_j} K_j(\tau_i, \tau_j)(t_j) = 1$$

for any τ_i, τ_j. To complete the proof it suffices to show that for any pure strategy σ_j and $b_j \in \mathcal{A}_j$,

$$\bar{\sigma}_j(\tau_i, \tau_j)(b_j) = \sum_{t_j : \sigma(t_j) = b_j} K_j(\tau_i, \tau_j)(t_j).$$

Assuming $\#T_j = n$, taking $a_j \neq b_j$

$$\frac{1}{n}\sigma_j + \frac{n-1}{n}(a_j, a_j)_{t_j} = \sum_{t_j \in T_j} \frac{1}{n}(\sigma(t_j), a_j)_{t_j}.$$

Thus,

$$\frac{1}{n}\bar{\sigma}_j + \frac{n-1}{n}\overline{(a_j, a_j)_{t_j}} = \sum_{t_j \in T_j} \frac{1}{n}\overline{(\sigma(t_j), a_j)_{t_j}}.$$

By the above arguments, $\overline{(\sigma_j(t_j), a_j)_{t_j}}(\tau_i, \tau_j)(b_j)$ equals $K_j(\tau_i, \tau_j)(t_j)$ if $\sigma_j(t_j) = b_j$ and equals zero otherwise. Thus,

$$\frac{1}{n}\bar{\sigma}_j(\tau_i, \tau_j)(b_j) = \sum_{t_j : \sigma(t_j) = b_j} \frac{1}{n} K_j(\tau_i, \tau_j)(t_j).$$

since $(a_j, a_j)_{t_j}(b_j) = 0$. □

Appendix B. Supplementary Results (Online)

B.1. **Full Correlation Neglect Characterization.** We now turn to the behavioral characterization of the full correlation neglect model, that is Equation (1) holds whenever $C \neq C' \in \mathcal{U}$. In this model, either the DM perfectly understands the connection between a and b or believes that a and b are independent. To compactly state the axiom, for any $a, b \in \mathcal{A}$, let $a + b$ be an arbitrary action with outcome $a(\omega) + b(\omega)$ in state ω, and for any

$p,q \in \Delta(X)$, let $p \oplus q = (p(x)q(y), x+y)_{p(x)+q(y)>0}$, the independent sum of p and q. Consider the following axiom.

Axiom 7 (Independent Bets)**.** For all $\omega_1, \omega_2 \in \Omega$, $x \in X$, lotteries p_1, p_2 over $\{0, x\}$, and $a_1, a_2 \in \mathcal{A}$ so that $a_i(\omega) = \begin{cases} x & \omega = \omega_i \\ 0 & otherwise \end{cases}$ for $i = 1, 2$:
If $\langle a_1, a_2 \rangle \not\sim a_1 + a_2$, $a_1 \sim p_1$ and $a_2 \sim p_2$, then $\langle a_1, a_2 \rangle \sim p_1 \oplus p_2$.

To interpret the axiom, consider a_1, a_2, p_1, p_2 satisfying its hypothesis. If $\langle a_1, a_2 \rangle \not\sim a_1 + a_2$, then the DM misunderstands the connection between a_1 and a_2. If the DM treats a_1 and a_2 as independent, then she is indifferent between the independent sum of p_1 and p_2, $p_1 \oplus p_2$, and $\langle a_1, a_2 \rangle$. This follows from observing that $p_i \sim a_i$ only if the objective distribution of p_i equals the subjective distribution of a_i.

Proposition 5. *Suppose $\succeq$ has a rich PCR with consistent marginal beliefs and strictly concave utility. Then, $\succeq$ satisfies Independent Bets if and only if $\succeq$ has a full correlation neglect PCR.*

Proof. Suppose Independent Bets, and a rich PCR $(\mathcal{U}, \pi, u)$ with consistent marginal beliefs and maximal $\mathcal{U}$. WLOG, $\pi(\omega) > 0$ for all ω Fix arbitrary $C, C' \in \mathcal{U}$. Label $\Omega = \{\omega_1, ..., \omega_n\}$ and pick $a_i \in C$ and $b_j \in C'$ so that $a_i(\omega)$ $(b_j(\omega))$ equals x if $\omega = \omega_i$ $(= \omega_j)$ and 0 otherwise. If $\langle a_i, b_j \rangle \sim a_i + b_j$ for all i, j, then $C = C'$ because $\mathcal{U}$ is universal.

Otherwise, there exists i, j such that $\langle a_i, b_j \rangle \not\sim a_i + b_j$. Pick lotteries p_i and p_j over $\{x, 0\}$ such that $p_i \sim a_i$ and $p_j \sim b_j$. Note this requires $p_i(x) = \pi(\omega_i)$ and $p_j(x) = \pi(\omega_j)$. Thus, $p_i \oplus p_j \sim \langle a_i, b_j \rangle$ if and only if $\pi(\{\omega_i\}_C \cap \{\omega_j\}_{C'}) = \pi(\omega_i)\pi(\omega_j)$. Define $E \in \Sigma$ to be one of the largest sets (according to set inclusion) such that $\pi(\{\omega_i\}_C \cap E_{C'}) = 0$. We will show that $E = \emptyset$. By above, $\omega_j \notin E$, implying $\bar{E} \neq \emptyset$. Consider any $\omega_k \in \bar{E}$. Since $\pi(\{\omega_i\}_C) \cap \{\omega_k\}_{C'}) \neq 0$, $\langle a_i, b_k \rangle \not\sim a_i + b_k$. As above, $\pi(\{\omega_i\}_C) \cap \{\omega_k\}_{C'}) = \pi(\omega_i)\pi(\omega_k)$. Using additivity, $\pi(\{\omega_i\}_C \cap \bar{E}_{C'}) = \pi(\omega_i)\pi(\bar{E})$. If $\pi(\bar{E}) \neq 1$, then there must be another state that we can add to E, contradicting that E is one of the largest sets. Hence $E = \emptyset$ and for any ω, we can find $\omega' \neq \omega$ such that $\pi(\{\omega\}_C \cap \{\omega'\}_{C'}) \neq 0$, and apply the above steps to conclude that $\pi(\{\omega\}_C \cap \{\omega'\}_{C'}) = \pi(\omega)\pi(\omega')$, completing the proof. □

B.2. **Comparative Correlation Coefficients.** In this subsection, we consider when DM2 believes that any pair of actions is more strongly correlated than DM1.[9] We focus on one common measure, the correlation coefficient, which is given for a and b by

$$\rho(a,b) = \frac{E[ab] - E[a]E[b]}{\sqrt{var[a]var[b]}},$$

[9]We thank David Ahn for suggesting this comparison

where $E[\cdot]$ is the expectation operator and $var[\cdot]$ is the variance operator. Thus we assume throughout the subsection that $X = \mathbb{R}$.

Formally, we compare the behavior of two risk-averse DMs, one of whom thinks that the magnitude of the correlation coefficient between a and b is larger than the other. As correlation has a "second order" effect on preferences, we focus in the main text only on the case of mean variance preferences, where $u(x) = x - \beta x^2$ for all $x \in X$. The general case for concave u can be studied in a similar way, but must rely on approximations, specifically sequences converging to zero.

Before stating our definition formally, we need some preliminary defintions. First, say that $b \in \mathcal{A}$ *is* $\succeq_i$ *mean-zero* if for any $x > 0$ and sequence of actions $\{b_n\}_{n\in\mathbb{N}}$ with $b_n(\omega) = \frac{1}{n}b(\omega)$ for all ω and n, there exists N such that $n > N$ implies that $x \succ_i b_n$ and $b_n \succ_i -x$. This will be shown to be equivalent to $E_i[b] = 0$, as shown below. Second, for any $b \in \mathcal{A}$, define $-b \in \mathcal{A}$ to be any action for which $[-b](\omega) + b(\omega) = 0$ for all ω and there exists a rich, $\succeq_1$-understood B with $b, -b \in B$. If $\succeq$ has a rich PCR, then by Theorem 2, there exists a $-b$ for every b.

Definition 14. Say that $\succeq_1$ *perceives more correlation* than $\succeq_2$ if any $a, b \in \mathcal{A}$ where b is $\succeq_i$ mean-zero and any $x \in X$,

$$\langle a, b\rangle \succ_1 \langle a, -b\rangle \textit{ and } x \succeq_1 \langle a, b\rangle \implies x \succeq_2 \langle a, b\rangle$$

and

$$\langle a, -b\rangle \succ_1 \langle a, b\rangle \textit{ and } x \succeq_2 \langle a, b\rangle \implies x \succeq_1 \langle a, b\rangle.$$

Pick two actions a and b, where $E_i[b] = 0$. Suppose that DM1 likes $\langle a, -b\rangle$ better than $\langle a, b\rangle$. This implies that DM1 thinks b is not a good hedge for a: a and b must be positively correlated. If DM2 thinks a and b are more strongly correlated than DM1 does, she should think that b is an even worse hedge for a. Hence, DM2's certainty equivalent of $\langle a, b\rangle$ should be higher than DM1's. In contrast, she must also thinks that $-b$ a better hedge than DM1 does. Consequently, her certainty equivalent of $\langle a, -b\rangle$ should be at least as high as DM1's. Relative to DM1, she *overvalues* a positively correlated b and *undervalues* a negatively correlated b.

The next result establishes that the behavioral comparison above corresponds to higher perceived correlation. Assuming each $\succeq_i$ has a PCR, let $\rho_i(\cdot)$ and $E_i[\cdot]$ be the correlation and expectation operators according to π_i.

Theorem 8. *Let $\succeq_i$ have a rich PCRs $(\mathcal{U}, \pi_i, u)$ for $i = 1, 2$, where π_1 and π_2 have the same marginals and $u(x) = x - \beta x^2$ for some $\beta > 0$.*

If $\succeq_1$ perceives more correlation than $\succeq_2$, then for any $a, b \in \mathcal{A}$, $\rho_1(a, b) > 0$ implies $\rho_1(a, b) \geq \rho_2(a, b)$ and $\rho_1(a, b) < 0$ implies $\rho_1(a, b) \leq \rho_2(a, b)$.

Conversely, if for every a, b for which there does not exist $C \in \mathcal{U}$ with $a, b \in C$, $\rho_1(a,b) > 0$ implies $\rho_1(a,b) > \rho_2(a,b)$ and $\rho_1(a,b) < 0$ implies $\rho_1(a,b) < \rho_2(a,b)$, then $\succeq_1$ perceives more correlation than $\succeq_2$.

For an example, let each $\succeq_i$ have a η_i-misperception representations, as in the introduction, for $i = 1, 2$. Then $\succeq_1$ perceives more correlation than $\succeq_2$ if and only if $\eta_1 \geq \eta_2$. This follows from linearity of the correlation with respect to η in the η-misperception model. In particular, if $\rho_\pi(a,b)$ is the true correlation between a and b according to π, then $\rho_i(a,b) = \eta_i \rho_\pi(a,b)$. Hence, when $\eta_1 > \eta_2$, $\rho_\pi(a,b) \gtrless 0$ implies $\rho_1(a,b) \gtrless \rho_2(a,b)$.

Proof. First, b is $\succeq_1$ mean-zero $\iff$ $E_1[b] = 0$. To see this, let b_n be as above and note that by a second order Taylor expansion $V(b_n) = \frac{1}{n}E_1[b] + O(\frac{1}{n^2}) = O(\frac{1}{n^2})$ and $V(\frac{x}{n}) = \frac{1}{n}x + O(\frac{1}{n^2})$. Similarly, $\langle a, b\rangle \succ_i \langle a, -b\rangle$ if and only if $\rho_i(a,b) < 0$ because

$$\begin{aligned} V_1\langle a, b\rangle &= E_1[a+b] - \beta E_1[a^2 + b^2 + ab] \\ &\gtrless E_1[a+b] - \beta E_1[a^2 + b^2 - ab] = V_1\langle a, -b\rangle \end{aligned}$$

if and only if $E_1[ab] \lesseqgtr 0 = E_1[a]E_1[b]$ if and only if $\rho_1(a,b) \lesseqgtr 0$.

Now, suppose $\succeq_1$ perceives more correlation than $\succeq_2$. Normalize so that $u(0) = 0$. Pick any a, b with $\rho_1(a,b) > 0$. If $a, b \in C \in \mathcal{U}$, then $\rho_2(a,b) = \rho_1(a,b)$, so suppose $a \in C \implies b \notin C$. Pick $C_a, C_b \in \mathcal{U}$ with $a \in C_a$ and $b \in C_b$. There exists $c \in C_b$ with $c(\omega) = b(\omega) - E_1[b]$; note $\rho_i(a,b) = \rho_i(a,c)$ for $i = 1, 2$. By above c is mean-zero and $\langle a, -c\rangle \succ_1 \langle a, c\rangle$. Letting $x \in X$ be such that $x \sim_1 \langle a, c\rangle$, it follows that $x \succeq_2 \langle a, c\rangle$. This implies that $V_2(\langle a, c\rangle) \geq V_1(\langle a, c\rangle)$. Writing out the utility function as above, conclude that $E_1[ac] \geq E_2[ac]$ since $E_1[a] = E_2[a]$, $E_1[c] = E_2[c]$, $E_1[a^2] = E_2[a^2]$, and $E_1[c^2] = E_2[c^2]$. As above, this implies $\rho_1[a,b] = \rho_1[a,c] \geq \rho_2[a,c] = \rho_2[a,b]$. Repeating the exercise with $-b$ replacing b gives that $-\rho_2(a,b) = \rho_2(a,-b) \leq \rho_1(a,-b) < -\rho_1(a,b)$. Hence, $|\rho_1(a,b)| > |\rho_2(a,b)|$. The negative correlation case is identical.

Now, suppose $\rho_1(a,b) \gtrless 0$ implies $\rho_1(a,b) \gtrless \rho_2(a,b)$ for every a, b with satisfying the implication $a \in C \implies b \notin C$. Pick arbitrary $a, b \in \mathcal{A}$ where b is $\succeq_1$ mean-zero and $\langle a, b\rangle \succ \langle a, -b\rangle$. By above, $E_1[b] = 0$ and $E_1[ab] < 0$, implying $\rho_1(a,b) < 0$. If there exists $C \in \mathcal{U}$ with $a, b \in C$, then $x \succeq_1 \langle a, b\rangle$ if and only if $x \succeq_2 \langle a, b\rangle$. If no such C exists, then $\rho_2(a,b) > \rho_1(a,b)$ implying $E_2[ab] > E_1[ab]$. Therefore, $V_2(\langle a, b\rangle) < V_1(\langle a, b\rangle)$ and since $V_1(x) = V_2(x)$, $x \succeq_1 \langle a, b\rangle$ implies $x \succeq_2 \langle a, b\rangle$. The case where $\langle a, -b\rangle \succ_1 \langle a, b\rangle$ is similar. Conclude $\succeq_1$ perceives more correlation than $\succeq_2$. □

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