Biased managers, organizational design, and incentive provision

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Abstract

We model the tradeoff between the balance and the strength of incentives implicit in the choice between hierarchical and matrix organizational structures. We show that managerial biases determine which structure is optimal: hierarchical forms are preferred when biases are low, while matrix structures are preferred when biases are high.

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1. Introduction

Some organizations are designed as hierarchies, with each subordinate reporting to only one superior. Others adopt a matrix structure, with each subordinate directly reporting to two or more superiors. If one views subordinates as agents and superiors as principals, the choice between these two organizational structures can also be seen as a choice between one or multiple principals.

The organizational design literature in economics has largely ignored the common agency element implicit in the choice between hierarchical and matrix structures. For example, Harris and Raviv (2002)
do not compare the two organizational structures in terms of their incentive characteristics. In contrast, our goal is to model the choice between hierarchical and matrix structures when these two structures provide different incentives for lower level agents.

We adopt the same setup as in Dixit (1996), which is actually a multiprincipal version of the well-known Holmstrom and Milgrom (1987, 1991) linear contracts model. Consider a situation in which an agent must exert effort into many different activities. There might be either one or many principals who may benefit from the outcome generated by the agent’s effort. Dixit (1996) is concerned with comparing a situation in which many principals compete with each other by offering incentive contracts to the agent, each one caring about only one of the activities, with a situation in which the same principals collude and offer a single contract to the agent. This modeling strategy is well suited for Dixit’s purpose, which is to explain the perceived low power of incentives provided to government agencies. However, this approach does not appear to be useful in explaining organizational design, because the outcome under collusion always dominates the case in which principals compete, implying that there is no tradeoff between the two cases.

We modify Dixit’s setup in the following manner. We assume that the organizational designer’s task is to choose the number of principals for a given agent, but principals are unable to collude and write binding agreements with each other. We then show that the choice between hierarchical (one principal) and matrix (many principals) structures depends crucially on the biases of the potential principals. If principals’ preferences are such that they are heavily biased towards some activities, the hierarchical structure performs poorly because the single principal’s bias will distort the incentive schedule offered to the agent towards her preferred activity. In such a case, the matrix structure might be preferable, because competition among principals partially offsets the effect of their individual biases on the incentive structure as perceived by the agent. However, the matrix structure leads to the traditional kind of distortions associated with common agency problems: the power of incentives might be too low under many principals, an effect emphasized by Martimort (1996) and Dixit (1996), or it might also be too high, as we will show below. In either case, common agency generically leads to incentives that are either too weak or too strong. We conclude by arguing that our approach can be used to describe the choice between hierarchical and matrix structures as a tradeoff between the strength and the balance of the incentives they provide to lower level managers or workers.

2. Model

For simplicity, we consider the case in which the final organizational outcome is the sum of the realization of two random variables (activities), $x_1$ and $x_2$. There is one agent who may exert effort into each activity (or task), $t_1$ and $t_2$. The technology can be expressed as $x = t + e$, where $x$, $t$, and $e$ are two-dimensional vectors, and $e$ is an error term which is normally distributed with zero mean and (diagonal) variance matrix $\Omega$. Moreover, we assume that the errors have the same finite variance $\sigma$.

The utility function of the agent is given by

$$u(w, t) = -\exp(-r[w - c(t)]).$$

(1)

where $w$ is the compensation paid to the agent, $c(t)$ is the cost of effort and $r$ is the agent’s absolute risk-aversion coefficient. We assume that the cost of effort takes a quadratic form $(1/2)t'\mathcal{C}t$, where the matrix
C is positive definite with positive cross-terms. For simplicity, we set the elements in the main diagonal to 1 and the off-diagonal terms to \( s \equiv (0, 1) \).

The organizational designer wants to maximize the expected organizational profit (or welfare), \( E(x_1 + x_2 - w) \). This can be seen as standard profits in case the organization is a firm, but it also admits other interpretations, such as social welfare, in the case of designing the structure of government agencies. We assume that the designer cannot run the organization herself.

If the designer can hire a manager with preferences aligned with the organizational objectives, the solution to this problem is straightforward: the manager offers a standard linear contract to the agent as in Holmstrom and Milgrom (1991) and the second-best outcome is achieved. However, we assume that this alternative is usually not available: it is either impossible or prohibitively costly to fully align managers’ interests with organizational goals.

This assumption is perhaps most natural in the organization of government, but it is also reasonable for profit-maximizing organizations. For many reasons, such as private nonpecuniary benefits from performing some activities, the consumption of perks, or concerns about career development, managers might be naturally biased towards one of the two activities. Thus, we assume that contracts that would completely eliminate managerial biases are not available.

The absence of incentive contracts to managers is justified if only managers can observe and enforce contracts based on \((x_1, x_2)\). Another reason may be that managers’ biases are nonverifiable and the only possible contracts between the designer and managers are the ones that allocate authority over the design of workers’ compensation schemes to managers.

In order to focus only on the essentials, we assume that the task of the designer is to choose between hiring one or two managers to act as principals for the agent, taking the managers’ preferences as givens. We ignore the costs arising from managerial compensation.

We model the lack of congruence between managerial and organizational objectives in the following manner. Although the organization profit is given by \( i x - w \), where \( i \) is a vector of ones, a manager’s expected utility is given by \( b x - w \), where the vector \( b \) may not be a vector of ones.

The optimal solution to the organizational design problem obviously depends on the underlying distribution of managers’ types in the population. We will only analyze a very simple and symmetric case in which managers can be of one of only two possible types.

Let \( v \equiv [0, 1] \). We say that a manager is of “type 1” if she has a vector of benefits given by \( b_1 = [1 \ v] \) and of “type 2” if her vector of benefits is given by \( b_2 = [v \ 1] \). We choose this case in order to better illustrate the effect we are emphasizing in this paper; it is not meant to be realistic.

The advantage of the symmetric case comes from the fact that biases can be defined by only one parameter. We say that a manager is “biased towards activity \( j \)” if she is of type \( j \) and \( v < 1 \). Moreover, the level of managerial bias is given by \( \theta = 1 - v \).

We have now all the elements to solve the organizational design problem. Suppose first that a hierarchical structure is chosen. The (single) principal will therefore be biased towards one activity. Whether the manager is of type 1 or 2 is immaterial. The outside utility of the agent is normalized to \( -1 \).

We omit most of the steps in the derivation of the optimal linear contracts, because these are already fairly known in the literature spurred by Holmstrom and Milgrom (1987, 1991).

The manager will offer a compensation schedule \( w = \alpha x + \beta \) to the agent, where \( \alpha \) is a two-dimensional vector and \( \beta \) is a fixed transfer.
The optimal bonus schedule chosen by the manager (the principal) will be
\[ \alpha_h = (I + rC\Omega)^{-1}b_j, \] (2)
where \( b_j \) is the vector of benefits of the manager if she is of type \( j \) and \( \alpha_h \) denotes the optimal solution under a hierarchical structure.

The optimal fixed transfer under a hierarchical structure is given by
\[ \beta_h = \frac{1}{2} r\alpha_h' \Omega \alpha_h - \frac{1}{2} \alpha_h' C^{-1} \alpha_h. \] (3)

Finally, the expected organizational profit under a hierarchical structure is given by
\[ L_h = (i - \alpha_h)' C^{-1} \alpha_h - \beta_h. \] (4)

We note that when the bias is zero (\( \theta = 0 \)), the expressions above characterize the second-best solution from the organization’s standpoint (the first-best solution is obtained only if effort is observable). In what follows, we denote the second-best outcomes by \( \alpha_s, \beta_s \) and \( L_s \).

Our first proposition is then straightforward (all proofs are in the appendix):

**Proposition 1.** Under a hierarchical structure, expected organizational profit is strictly decreasing (and concave) in the level of managerial bias.

This proposition simply states the intuitive result that, as the manager’s interests become more aligned with the organization’s interests, organizational profit increases.

Consider now the case of a matrix structure. We assume that managers’ types are observable. In such a case, if the designer chooses to hire two managers of the same type, profits will always be lower than under a hierarchical structure. Thus, the relevant case is the one in which the designer chooses one manager of each type.

Each manager will simultaneously and noncooperatively offer a compensation schedule \( w_j = c_j x + b_j \) to the agent, \( j = 1, 2 \). The aggregate incentive schedule in a Nash equilibrium, as perceived by the agent, is denoted by \( \alpha_m = \alpha_1 + \alpha_2 \) and \( \beta_m = \beta_1 + \beta_2 \).

Solving for the equilibrium aggregate schedule gives us
\[ \alpha_m = (I + 2rC\Omega)^{-1}(b_1 + b_2). \] (5)

We note that we obtain the solution in Dixit (1996) as a particular case of our setup; when managers are totally biased (\( \theta = 1 \)), we get \( \alpha_m = (I + 2rC\Omega)^{-1}i \), which implies that incentives are too low-powered if compared with the second-best solution, which is \( \alpha_s = (I + rC\Omega)^{-1}i \).

In our setup, it is no longer true that common agency would lead to low-powered incentives, as in Dixit (1996) and Martimort (1996). If biases are sufficiently low, common agency distortions take the form of excessively high-powered incentives, as compared to the ones obtained in the second-best solution. What remains true is the fact that common agency in general distorts the power of incentives in both activities. Unlike the case of a hierarchical structure, this distortion is symmetric: incentives in both activities are either too low or too high as compared to their optimal levels. In this sense, incentives under a matrix structure are more balanced across activities than the ones under a hierarchical structure.

The following proposition formalizes this result.
Proposition 2. Under a matrix structure, the power of the incentive schedule strictly decreases with the level of bias in both activities. If managers are not biased, the power of the incentive schedule is above the optimal level, and if managers are fully biased, the power is below the optimal level.

The optimal fixed transfer under a matrix structure is given by

\[ \beta_m = \frac{1}{2} r \gamma_m \Omega \gamma_m - \frac{1}{2} \gamma_m C^{-1} \gamma_m \]  \hspace{1cm} (6)

and the expected organizational profit is given by

\[ L_m = (i - \gamma_m)' C^{-1} \gamma_m - \beta_m . \]  \hspace{1cm} (7)

The next proposition describes how profits change in response to changes in managerial biases.

Proposition 3. Under a matrix structure, there is a unique level of bias (\( \theta^* \)) for which the organizational profit (\( L_m \)) reaches its optimal level (\( L_s \)). If \( \theta < \theta^* \), organizational profit is increasing in the level of bias. If \( \theta > \theta^* \), organizational profit is decreasing in the level of bias.

The intuition for this result is as follows. When biases are high, the traditional weakening incentives effect of common agency problems is in place, thus by reducing the bias one is able to increase the power of incentives in both activities, increasing total profits. However, if biases are sufficiently low, common agency leads to a situation in which both principals care too much about both activities, resulting in a compensation schedule in which incentives are too strong. This reduces profits because excessive payments are made to the agents.

Fig. 1. Expected organizational profits (\( r=1, s=0.15 \) and \( \sigma=0.6 \)).
We finally state our main result:

**Proposition 4.** There is a unique level of bias $\tilde{\theta}$ such that:

- If $\theta > \tilde{\theta}$, the matrix structure is optimal;
- If $\theta < \tilde{\theta}$, the hierarchical structure is optimal;
- If $\theta = \tilde{\theta}$, both the matrix and the hierarchical structures are optimal.

Fig. 1 provides an illustration of this last result.

We interpret our main result as an illustration of the tradeoff between the strength and the balance of the incentives implied by the choice between hierarchical and matrix structures. When managerial biases towards certain activities are substantial, the more balanced nature of the incentives generated by a matrix structure becomes more valuable. On the other hand, when managerial biases are not a major concern, common agency distortions on the level of incentives make matrix structures less attractive.

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**Appendix A**

**Proof of Proposition 1.** Take derivatives to get:

$$
\frac{\partial L_h}{\partial \theta} = \frac{(rs^2 + 1 + r\sigma)\theta}{(1 + 2r\sigma + r^2\sigma^2 - r^2s^2\sigma^2)(s^2 - 1)} < 0 \quad \text{for } \theta \in (0,1]
$$

and

$$
\frac{\partial^2 L_h}{\partial \theta^2} = \frac{(rs^2 + 1 + r\sigma)}{(1 + 2r\sigma + r^2\sigma^2 - r^2s^2\sigma^2)(s^2 - 1)} < 0.
$$

**Proof of Proposition 2.** From Eq. (5) we get $\frac{\partial \alpha_m(i)}{\partial \theta} = \frac{-1}{2r\sigma + 2r\sigma + 1} < 0$ for $i = 1, 2$. Moreover, it is easy to see that

$$
\alpha_m(i) \big|_{\theta = 0} = \frac{2}{2rs\sigma + 2r\sigma + 1} > \frac{1}{rs\sigma + r\sigma + 1} = \alpha_s(i)
$$

and that

$$
\alpha_m(i) \big|_{\theta = 0} = \frac{1}{2rs\sigma + 2r\sigma + 1} < \frac{1}{rs\sigma + r\sigma + 1} = \alpha_s(i) \quad \text{for } i = 1, 2.
$$
Proof of Proposition 3. The expected organizational profit under a matrix structure is given by

\[ L_m = \frac{-(\theta - 2)(rs\sigma + 2rs\sigma + \theta + r\sigma\theta + 2r\sigma)}{(s + 1)(2rs\sigma + 2r\sigma + 1)^2}. \]

Taking derivatives, we get

\[ \frac{\partial L_m}{\partial \theta} = -2\frac{\theta(rs\sigma + 1 + r\sigma) - 1}{(s + 1)(2rs\sigma + 2r\sigma + 1)^2}. \]

Therefore, if \( \theta(rs\sigma + 1 + r\sigma) < 0 \), then \( \sigma L_m / \sigma \theta > 0 \); if \( \theta(rs\sigma + 1 + r\sigma) > 0 \), then \( \sigma L_m / \sigma \theta < 0 \); and if \( \theta(rs\sigma + 1 + r\sigma) = 0 \), then \( \sigma L_m / \sigma \theta = 0 \) and \( \theta = 0* = (rs\sigma + 1 + r\sigma)^{-1} \). Moreover,

\[ \frac{\partial^2 L_m}{\partial \theta^2} = -2\frac{(rs\sigma + 1 + r\sigma)}{(s + 1)(2rs\sigma + 2r\sigma + 1)^2} < 0 \quad \text{for all } \theta \in [0, 1]. \]

Plugging \( \theta = \theta^* \) in \( L_m \) reveals that \( L_m(\theta^*) = L_s \).

Proof of Proposition 4. Define \( Y(\theta) = \frac{L_m(\theta) - L_m}{L_m} \). Initially, set \( \theta = 0 \). Therefore

\[ Y(0) = \frac{1}{(2rs\sigma + 2r\sigma + 1)^2(s + 1)(rs\sigma + r\sigma + 1)} > 0, \]

implying that the hierarchical structure dominates the matrix structure when the bias is zero. Now set \( \theta = 1 \). In this case,

\[ Y(1) = \frac{2r^3s^3(s^4 + 4s^3 + 6s^2 + 4s + 1) + 2r^2s^2(3s^3 + 5s^2 + 5s + 3) + r\sigma(s^2 + 4s + 5) + 1}{-2(1 + 2s\sigma + r^2s^2 - r^2s^2s^2)(2rs\sigma + 2r\sigma + 1)^2(1 - s^2)} < 0, \]

implying that the matrix structure dominates when the bias is 1. Notice that the function \( Y(\theta) \) is continuous (and differentiable). It is known that \( Y(0) > 0 \) and \( Y(1) < 0 \), thus the Intermediate Value Theorem guarantees that there is at least one \( \theta \in [0, 1] \) such that \( Y(\theta) = 0 \). Simple inspection reveals that \( Y(\theta) \) is quadratic, thus there are at most two values of \( \theta \) such that \( Y(\theta) = 0 \). Because \( Y(0) > 0 \) and \( Y(1) < 0 \), only one of these points belongs to the interval \([0, 1]\). Thus, we define \( \tilde{\theta} \) as the unique value in \([0, 1]\) such that \( Y(\tilde{\theta}) = 0 \), which completes the proof.

References