Debt Deleveraging, Debt Relief and Liquidity Traps

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Abstract

I study debt relief policies during debt-driven slumps using a model of deleveraging. Deleveraging can push the economy into a liquidity trap characterized by involuntary unemployment and low inflation. A debt relief policy, captured by a transfer of wealth from creditors to debtors, increases aggregate demand, employment and output. Debt relief may benefit creditors as well as debtors and lead to a Pareto improvement in welfare. The benefits from a policy of debt relief are greater the more the central bank is concerned with stabilizing inflation. Moreover, targeting inflation during a liquidity trap can generate multiple equilibria. In this case it is possible to design debt relief policies that eliminate undesirable equilibria.

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1 Introduction

Since the onset of the recent global crisis several countries have embarked in a process of private debt deleveraging (figure 1).[1] The path toward lower debt has been characterized by a severe global recession, taking place in a low interest rate environment limiting the scope for conventional monetary policy stimulus. Against this background, some commentators have argued that debt relief policies, that is policies that reduce the debt burden of indebted households, could play a key role in easing the recovery.[2] However, we still lack a clear understanding of the macroeconomic channels through which debt relief policies might affect the economy and of their implications for welfare.

I tackle these issues using an analytical framework suitable to study the positive and normative implications of debt relief policies during episodes of debt deleveraging. I derive two key results. First, I show that a program of debt relief leads to an expansion in employment and output if deleveraging pushes the economy in a liquidity trap, that is a case in which the nominal interest rate hits the zero lower bound. Second, I show that debt relief during a liquidity trap may benefit both debtors and creditors and generate a Pareto improvement in welfare.

I reach these results studying a tractable model of debt deleveraging. The model is simple enough so that its properties can be derived analytically, without resorting to local approximations. This is important since local approximations might perform poorly when employed to study liquidity traps.[3] Despite its simplicity the model captures salient features of debt deleveraging episodes. There are two groups of households, debtors and creditors. Deleveraging is triggered by a shock that forces debtors to reduce their debt. The process of debt reduction generates a fall in aggregate demand and in the interest rate. If the shock is large enough the economy falls in a liquidity trap characterized by low inflation, leading to involuntary unemployment due to the presence of downward nominal wage rigidities. In this context I study the impact of a policy of debt relief, which I capture with a transfer of wealth from creditors to debtors.

Debt relief leads to an increase in aggregate demand, because borrowing-constrained debtors have a higher propensity to consume out of income than creditors. If the economy is in a liquidity trap the increase in demand generates an increase in output, since in a liquidity trap there is involuntary unemployment precisely because aggregate demand is weak. Through this channel a program of debt relief has an expansionary impact on employment and output.

Debt relief can also give rise to a Pareto improvement in welfare. While it is not surprising that debtors should gain from a policy of debt relief, it is not obvious that creditors could

[1]McKinsey (2010, 2012) and Koo (2011) describe the process of international deleveraging that began with the 2007-2008 financial crisis.

[2]For example, this view is maintained by Geanakoplos and Koniak (2009) and Sufi (2012). In fact, debt relief is not just a theoretical possibility. Iceland has been implementing debt relief programs for financially distressed households since the end of 2008. Ireland is now in the process of implementing similar policies.

[3]Braun et al. (2012) show that local approximations can lead to qualitatively, as well as quantitatively, inaccurate results when employed to study economies experiencing a liquidity trap.

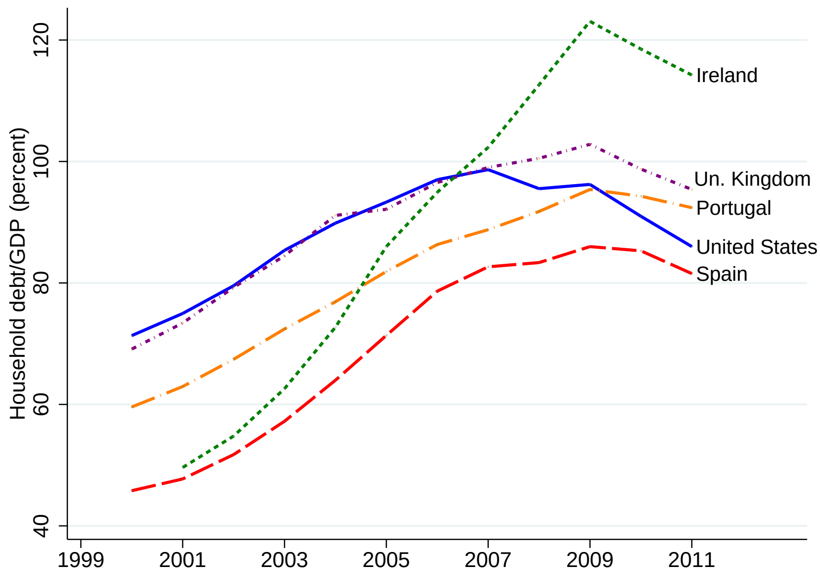

Figure 1: Household debt-to-GDP ratio, 2000 – 2011. Notes: data are from the OECD.

benefit too. In fact, a Pareto improvement in welfare is possible only if debt relief generates an expansion in output large enough to compensate creditors for the loss in wealth due to the transfer to debtors. I show that this is more likely to be the case the more the central bank is concerned with stabilizing inflation. To understand this result, consider that during the recovery from a liquidity trap real wages have to fall to a level consistent with full employment. Since nominal wages are downwardly rigid, higher inflation speeds up the process of wage adjustment and leads to a faster recovery, while low inflation during the recovery is associated with persistent unemployment.[4] Because of this effect, a policy of debt relief that limits the rise in unemployment during the liquidity trap has a larger positive impact on employment, output and welfare the more the central bank is concerned with keeping inflation low during the recovery. Moreover, I show that targeting inflation during a liquidity trap can open the door to multiple equilibria. In this case, an appropriate transfer scheme can eliminate undesirable equilibria.

The rest of the paper is structured as follows. I start with a discussion of the related literature. Section 2 introduces the model. Section 3 shows how an episode of deleveraging can generate a liquidity trap. Section 4 studies the normative and positive impact of debt relief. Section 5 discusses several extensions, including the case of a monetary union. Section 6 concludes.

Related literature. This paper is related to several strands of the literature. First, the paper is about debt deleveraging and liquidity traps. Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) study the impact of deleveraging shocks on the interest rate in closed economies, while Benigno and Romei (2012) and Fornaro (2012) consider deleveraging in open economies. I contribute to this literature by studying the impact of debt relief policies

[4]This feature of the model is consistent with the empirical findings of Calvo et al. (2012), who show that recoveries from financial crises are characterized by a trade-off between inflation and unemployment.

in economies undergoing a period of debt deleveraging.

The paper is also related to the literature on fiscal policy and liquidity traps. A non-exhaustive list of papers studying fiscal policy during liquidity traps is Eggertsson and Woodford (2006), Christiano et al. (2011), Mertens and Ravn (2010), Correia et al. (2011), Mankiw and Weinzierl (2011), Werning (2011), Bilbiie et al. (2012), Braun et al. (2012), Carlstrom et al. (2012), Farhi and Werning (2012) and Rendahl (2012). While these contributions focus on government expenditure or public debt, this paper considers the role of pure transfers from creditors to debtors.

The focus on transfers connects this paper to Werning and Farhi (2012), who study transfers among members of a monetary union. My model describes a closed economy, but most of its insights can be extended to the case of a monetary union as I discuss in section 5.3. While the rationale for transfers in Werning and Farhi (2012) arises because of the presence of idiosyncratic shocks and nominal rigidities, in this paper transfers are welfare improving because of the interaction between an aggregate deleveraging shock and the zero lower bound on the nominal interest rate. The role of transfers in stabilizing economic fluctuations is also studied in McKay and Reis (2012). While McKay and Reis (2012) consider the impact of automatic stabilizers on business cycle fluctuations, this paper analyzes the role of debt relief, a discretionary and exceptional form of policy intervention, during sharp recessions. Bianchi (2012) studies bailout policies in the form of transfers from households to firms. Bianchi (2012) focuses on a real economy, while in this paper the interaction between debt relief and monetary policy is crucial.

A key feature of the model is the presence of nominal wage rigidities. There is extensive evidence in support of the existence of downward nominal wage rigidities. Fehr and Goette (2005), Gottschalk (2005), Barattieri et al. (2010) and Fabiani et al. (2010) document the existence of downward wage rigidities using micro data. From a macro perspective Olivei and Tenreyro (2007, 2010) and Christiano et al. (2005) highlight the key role of nominal wage rigidities as a transmission channel for monetary policy. There is also evidence suggesting that nominal wages fail to adjust downward during deep recessions. Eichengreen and Sachs (1985), Bernanke and Carey (1996) and Bordo et al. (2000) discuss the role of wage rigidities during the Great Depression. Schmitt-Grohé and Uribe (2011) document the importance of downward nominal wage rigidities in the context of the 2001 Argentine crisis and of the Great Recession in countries at the Eurozone periphery. This paper also relates to models of downward nominal wage rigidities, such as Akerlof et al. (1996), Benigno and Ricci (2011), Schmitt-Grohé and Uribe (2011, 2012) and Daly and Hobijn (2013).

2 Model

Consider a closed economy inhabited by households and firms. There is also a central bank that conducts monetary policy. Time is discrete and indexed by t and there is perfect foresight.

2.1 Households

There is a continuum of households of measure one. The lifetime utility of a generic household i is

$$\sum_{t=0}^{\infty} \beta^t U\left(C_t^i\right). \tag{1}$$

In this expression, C_t^i denotes consumption, β is the subjective discount factor and the period utility function $U(\cdot)$ is specified as

$$U\left(C_t^i\right) = \frac{C_t^{i\,1-\gamma} - 1}{1-\gamma},$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In every period each household is endowed with $\bar{L}$ hours of labor. Households supply inelastically their labor endowment to the labor market, but, due to the presence of nominal wage rigidities, a household may be able to work only $L_t^i < \bar{L}$ hours.[5] Hence, when $L_t^i = \bar{L}$ for every household i the economy is operating at full employment, otherwise there is involuntary unemployment.

Households trade in one period riskless bonds. Bonds are denominated in units of consumption good and pay the real interest rate r_t.[6] The budget constraint of the household is

$$P_t C_t^i + \frac{P_t B_{t+1}^i}{1+r_t} = W_t L_t^i + P_t B_t^i + \Pi_t^i + T_t^i. \tag{2}$$

The left-hand side of this expression represents the household's expenditure. P_t is the nominal price level in period t, hence $P_t C_t^i$ is the expenditure of the household in consumption expressed in units of money. B_{t+1}^i denotes the purchase of bonds made by the household at time t at price $P_t/(1+r_t)$. If $B_{t+1}^i < 0$ the household is a borrower.

The right-hand side captures the household's income. W_t denotes the nominal wage, so $W_t L_t^i$ is the household's labor income. Labor is homogeneous across households and every household receives the same wage W_t. $P_t B_t^i$ is the gross return on investment in bonds made at time $t-1$ expressed in units of money. Π_t^i are the nominal profits that the household receives from firms. Firms are wholly owned by households and equity holdings within these firms are evenly divided among them. Finally, T_t^i is a lump sum transfer taken as given by the household.

There are frictions in the financial markets and households are subject to a borrowing limit. In particular, each period debt repayment cannot exceed the exogenous limit κ_t, so that the bond position has to satisfy

$$B_{t+1}^i \geq -\kappa_t. \tag{3}$$

This constraint captures in a simple form a case in which a household cannot credibly commit in period t to repay more than κ_t units of the consumption good to its creditors in period $t+1$.

[5] In section 5.2 I discuss the case of elastic labor supply.

[6] I focus on bonds denominated in real terms to simplify the analysis. Considering nominal bonds should not alter the key results of the paper.

Each period the household chooses B^i_{t+1} to maximize the present discounted value of utility (1), subject to the budget constraint (2) and the borrowing limit (3). The household's optimal choice of bonds satisfies

$$U'\left(C^i_t\right) = \beta\left(1 + r_t\right) U'\left(C^i_{t+1}\right) + \mu^i_t \tag{4}$$

$$\mu^i_t\left(B^i_{t+1} + \kappa_t\right) = 0, \text{ with } \mu^i_t \geq 0, \tag{5}$$

where $U'(\cdot)$ is the first derivative of the period utility function and μ^i_t is the Lagrange multiplier on the borrowing limit, normalized by the gross real interest rate $1 + r_t$. Expression (4) is the standard Euler equation for bonds, which guarantees optimal consumption smoothing over time. Expression (5) is the complementary slackness condition on constraint (3), which ensures that the borrowing limit is not violated.

2.2 Firms

There is a large number of firms that use labor as the only factor of production. Each period a firm that employs L_t units of labor produces L^α_t units of the consumption good, where $0 < \alpha < 1$.[7] The nominal profits of the representative firm are

$$\Pi_t = P_t L^\alpha_t - W_t L_t. \tag{6}$$

Each firm chooses employment L_t to maximize profits, taking the price of the consumption good and the wage as given. Profit maximization implies

$$\alpha L^{\alpha-1}_t = \frac{W_t}{P_t}. \tag{7}$$

At the optimum firms equate the marginal product of labor, the left-hand side of the expression, to the real marginal cost, the right-hand side.

2.3 Downward nominal wage rigidities

Nominal wages are downwardly rigid, and wage dynamics must satisfy

$$W_{t+1} \geq \phi\left(u_t\right) W_t,$$

where $u_t = 1 - L_t/\bar{L}$ is the unemployment rate and the function $\phi(\cdot)$ satisfies $\phi'(\cdot) \leq 0$. The term $\phi(u_t)$ introduces a feedback from the unemployment rate to wage dynamics. Specifically, when $\phi'(\cdot) < 0$ a higher unemployment rate is associated with more downward wage flexibility.

Given this constraint on wage dynamics, employment satisfies the complementary slackness

[7] To introduce constant returns-to-scale in production one could assume that a firm that employs L_t units of labor produces $L^\alpha_t K^{1-\alpha}$ units of the consumption good, where K is a fixed production factor owned by the firm, for example physical or organizational capital. The production function in the main text corresponds to the normalization $K = 1$.

condition

$$\left(\bar{L}-L_t\right)\left(W_{t+1}-\phi\left(u_t\right)W_t\right)=0,$$

which says that unemployment arises only if wages cannot fall enough for the labor market to clear.

2.4 Central bank

The central bank uses the nominal interest rate i_t as its policy instrument.[8] The nominal interest rate is related to the real interest rate by the Fisher equation

$$1+i_t=\left(1+r_t\right)\pi_{t+1}, \tag{8}$$

where $\pi_{t+1}=P_{t+1}/P_t$ is the gross inflation rate between period t and period $t+1$.

I focus on central banks that follow targeting rules.[9] First, I consider a central bank that targets an inflation rate $\bar{\pi}$.[10] Second, I consider a central bank whose main objective is to guarantee full employment and that, conditional on having reached full employment, also targets inflation $\bar{\pi}$.[11] However, it might not always be possible for the central bank to attain its desired target because of the zero bound on the nominal interest rate $i_t \geq 0$.

2.5 Market clearing and equilibrium

I consider equilibria in which every household works the same number of hours. Hence, equilibrium on the labor market is attained when

$$L_t^i=L_t. \tag{9}$$

Moreover, I focus on equilibria in which transfers are balanced every period across households so that

$$\int_0^1 T_t^i \, \mathrm{d}i=0. \tag{10}$$

[8]More formally, assume that there are government bonds paying the nominal interest rate i_t. Also assume that households cannot take a negative position in government bonds. The central bank can set the nominal interest rate through open market operations in government bonds, and in equilibrium government bonds are in zero net supply.

[9]Another possibility would be to assume a benevolent central bank that maximizes households' welfare. However, since households are heterogeneous modeling an optimizing central bank involves taking a stance on how the central bank values the utility of different individuals. I prefer not to follow this approach and I consider central banks that target aggregate variables, because in reality the mandate of most central banks is specified in terms of inflation and employment targets.

[10]In this paper I employ a notion of inflation targeting that is perhaps more restrictive than the one commonly understood in the literature on monetary policy. In fact, in general adhering to a policy of inflation targeting does not prevent the central bank from changing its inflation target in response to changes in the economy. Instead, the inflation targeting policy that I consider in this paper does not allow for changes in the target. In practice central banks in advanced economies are extremely reluctant to change their inflation target, even following major shocks such as the 2007-2008 financial crisis and the following recession.

[11]Later, in section 5.1, I study the case of a central bank that sets monetary policy according to an interest rate rule.

Market clearing for the consumption good is reached when aggregate consumption is equal to aggregate output

$$\int_0^1 C_t^i \,\mathrm{d}i = L_t^\alpha. \tag{11}$$

These conditions imply that bonds are in zero net supply in every period, $\int_0^1 B_{t+1}^i \,\mathrm{d}i = 0$. We are now ready to define an equilibrium.

Definition 1 *An equilibrium is a set of processes* $\{C_t^i, L_t^i, B_{t+1}^i, \mu_t^i, L_t, r_t, P_t, W_t\}_{t=0}^\infty$ *and a sequence of distributions for bond holdings* $\Psi_t(B)$*, such that given an exogenous process for* $\{\kappa_t\}_{t=0}^\infty$*, a sequence of interest rates and transfers* $\{i_t, T_t^i\}_{t=0}^\infty$ *and the initial distribution* $\Psi_0(B)$*, in every period t*

- *The households' decisions are optimal given prices* $\{r_t, W_t, P_t\}_{t=0}^\infty$*, that is for every household i they satisfy*

$$U'\left(C_t^i\right) = \beta\left(1+r_t\right) U'\left(C_{t+1}^i\right) + \mu_t^i$$

$$B_{t+1}^i \geq -\kappa, \ \textit{with equality if } \mu_t^i > 0.$$

- *Firms' maximize profits given prices* $\{r_t, W_t, P_t\}_{t=0}^\infty$

$$\alpha L_t^{\alpha-1} = \frac{W_t}{P_t}.$$

- *The complementary slackness for the wage setting condition holds*

$$\left(\bar{L} - L_t\right)\left(W_{t+1} - \phi\left(u_t\right) W_t\right) = 0.$$

- *Transfers are balanced every period*

$$\int_0^1 T_t^i \,\mathrm{d}i = 0.$$

- $\Psi_t(B)$ *is consistent with the decision rules*

- *Markets for bonds and labor clear*

$$L_t^i = L_t \leq \bar{L}.$$

$$\int_0^1 B_{t+1}^i \,\mathrm{d}i = 0.$$

2.6 Steady state

I focus on an economy that features a deterministic steady state in which there is no conflict between targeting inflation or employment. This requires the following assumptions.

Assumption 1 *The parameters $\bar{\pi}$ and β are such that*

$$\bar{\pi} \geq \beta.$$

The function $\phi(\cdot)$ is such that

$$\phi(0) \leq \bar{\pi}.$$

Hence, in steady state inflation is equal to its target $\bar{\pi}$ and the economy is at full employment. I also limit the analysis to steady states in which transfers are equal to zero for every household, that is $T^i = 0$ for every i.

In steady state each household features a constant consumption stream. Combining this condition with the Euler equation (4) gives the steady state real interest rate

$$r = \frac{1}{\beta} - 1,$$

where the absence of a time subscript denotes variables referring to the steady state. The steady state consumption of a generic household i is

$$C^i = \bar{L}^\alpha + \frac{rB^i}{1+r}, \tag{12}$$

where B^i is the stock of bonds owned by the household in steady state. This expression implies that the only source of heterogeneity across households in steady state consumption is due to differences in wealth. In particular, households that have a higher wealth consume more in steady state.

3 Debt deleveraging and liquidity traps

In this section I consider an episode of deleveraging and show how deleveraging can push the economy into a liquidity trap characterized by low inflation and involuntary unemployment. I start by considering economies without transfers and set $T_t^i = 0$ for every i and t.

Assume that at the start of period 0 some households are debtors and some are creditors. In particular, a fraction n of the households are debtors and each debtor starts with initial assets $-B_0 < 0$. The remaining fraction of households $1-n$ are creditors and each creditor starts with assets $n/(1-n)B_0 > 0$.[12] In what follows, I will denote debtors with the superscript d and creditors with the superscript c. This simple form of initial heterogeneity in bond holdings makes the analysis particularly tractable, while preserving the fundamental insights that could be derived from a model featuring a more realistic initial wealth distribution.

In period 0 the economy is hit by an unexpected *deleveraging shock*, that is a sudden tightening of credit conditions that forces debtors to reduce their debt positions. I capture the

[12] The existence of initial heterogeneity in bond holdings can be due to past idiosyncratic shocks, as in Guerrieri and Lorenzoni (2011) and Fornaro (2012).

deleveraging shock with an unexpected fall in the borrowing limit κ, so that $\kappa_0 = \bar{\kappa}$ where $0 < \bar{\kappa} < B_0$. The tightening of the borrowing constraint forces debtors to reduce their debt by the amount $B_0 - \bar{\kappa}$ and triggers a process of deleveraging. To simplify the analysis, I assume that the shock to κ is permanent, so that $\kappa_t = \bar{\kappa}$ in every period $t \geq 0$.

Irrespective of whether the central bank targets inflation or employment, the central bank responds to the deleveraging shock by decreasing the nominal interest rate. To see this point it is useful to start by considering a case in which the central bank is not constrained by the zero lower bound on the nominal interest rate, and hence in which inflation is always equal to the target and the economy always operates at full employment. In this case, creditors' consumption in period 0 is given by

$$C_0^c = \bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \frac{\bar{\kappa}}{1+r_0}\right).$$

From period 1 on the economy enters a steady state in which creditors' consumption is constant and equal to

$$C^c = \bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}.$$

Let us now consider the implications for the interest rate. Suppose that the real interest rate does not respond to the deleveraging shock and so $r_0 = r$. In this case $C_0^c > C^c$ and creditors experience a decrease in consumption between period 0 and period 1. But if $r_0 = r$ the Euler equation implies that creditors' consumption must be constant between periods 0 and 1, a contradiction. Hence r_0 must respond to the deleveraging shock.

In fact, it is possible to show that during period 0 the real interest rate falls below its steady state value.[13] Intuitively, the deleveraging shock forces debtors to increase their savings so as to reduce their debt positions. At full employment the interest rate must fall so that creditors, which are not borrowing constrained, become willing to absorb the forced savings coming from debtors.[14]

By the Fisher equation (8), the fall in the real interest rate exerts a depressive impact on the nominal interest rate. Hence, a deleveraging shock exposes the economy to the risk of experiencing a liquidity trap, that is a case in which the nominal interest rate hits the zero lower bound.[15] Indeed, for a sufficiently large shock, that is if $B_0 - \bar{\kappa}$ is sufficiently large, the economy enters a liquidity trap for sure.

[13] See the proof of proposition 1.

[14] See Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) for more discussion on the link between deleveraging and low interest rates.

[15] As emphasized by Krugman (1998), the central bank could avoid hitting the zero lower bound constraint by increasing expected inflation, that is by setting π_1 high enough so that $(1 + r_0)\pi_1 > 0$. However, this strategy conflicts with our assumptions about the objectives of the central bank. In fact, in the absence of a liquidity trap in period 0, once period 1 comes the central bank will want to set $\pi_1 = \bar{\pi}$. Hence, any announcement of a higher π_1 is not credible. Eggertsson and Woodford (2003) discuss these credibility issues in the context of a standard New-Keynesian model.

Condition 1 *The parameters satisfy*

$$\frac{\beta}{\bar{\pi}} \frac{U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right)} > 1.$$

Proposition 1 *If condition* 1 *holds and $T_t^i = 0$ for every i and t the economy is in a liquidity trap in period 0, $i_0 = 0$. Then there is unexpected undershooting of the inflation target, $\pi_0 < \bar{\pi}$, and involuntary unemployment, $L_0 < \bar{L}$. Moreover, the economy exits the liquidity trap in period 1, i.e. $i_t > 0$ and $\pi_t \geq \bar{\pi}$ for $t > 0$.*

Proof. See appendix. ■

In a liquidity trap the nominal interest rate hits the zero lower bound and the real interest rate is equal to the inverse of expected inflation. There is unemployment because consumption demand is too weak to sustain full employment. Intuitively, the interest rate cannot fall enough to induce creditors to absorb the forced savings of debtors at full employment. Hence, firms react to the excess supply of consumption good by cutting prices, and so inflation is lower than the target. Low inflation coupled with nominal wage stickiness leads to high real wages, which discourage firms' labor demand and employment. This adjustment process goes on until output has fallen enough so as to eliminate the excess supply on the goods' market.

Though, as stated by proposition 1, the liquidity trap lasts only one period the impact on inflation and employment can be more persistent. The persistence arises because real wages increase during the liquidity trap, and so during the recovery real wages have to fall to restore full employment. Due to the presence of downward nominal wage rigidities, inflation may affect the speed at which real wages fall during the recovery. In particular, if inflation is too low nominal wages may not fall fast enough to immediately restore full employment once the liquidity trap is over. Hence, during the recovery a trade-off between inflation and employment may arise.[16] Indeed, we can distinguish two regimes. For sufficiently mild recessions the recovery is immediate and involves no trade-off between inflation and employment. I will refer to this case as mild recessions. Instead, for large recessions the central bank faces a trade-off between inflation and employment during the recovery.

More formally, the economy is in a mild recession if the following condition holds.

Condition 2 *L_0 satisfies*

$$U'\left(L_0^{\alpha} + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right) = \frac{\beta}{\bar{\pi}} U'\left(\bar{L}^{\alpha} + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)$$

$$\left(\frac{L_0}{\bar{L}}\right)^{1-\alpha} \bar{\pi} \geq \phi(0)$$

$$L_0 < \bar{L}.$$

[16] The existence of a trade-off between inflation and employment during the recovery is consistent with the empirical evidence provided by Calvo et al. (2012).

Proposition 2 *If condition 2 holds and $T_t^i = 0$ for every i and t the economy is in a liquidity trap in period 0. Moreover, the economy is at full employment, $L_t = \bar{L}$, and inflation is equal to its target, $\pi_t = \bar{\pi}$, for all $t > 0$.*

Proof. See appendix. ■

4 Debt relief and liquidity traps

We are now ready to consider the impact of debt relief policies. I model debt relief as a lump-sum transfer from creditors to debtors occurring when the deleveraging shock hits the economy, that is in period 0. Specifically, in period 0 each debtor receives T units of the consumption good, financed with a tax $n/(1-n)T$ levied on each creditor. Formally, $T_0^d = T$, $T_0^c = -n/(1+n)T$ and $T_t^i = 0$ for every i and for $t > 0$. The period 0 budget constraints respectively of debtors and creditors now become

$$P_0 C_0^d + \frac{P_0 B_1^d}{1+r_0} = W_0 L_0 - P_0 B_0 + T + \Pi_0 \tag{13}$$

$$P_0 C_0^c + \frac{P_0 B_1^c}{1+r_0} = W_0 L_0 + \frac{n}{1-n}\left(P_0 B_0 - T\right) + \Pi_0. \tag{14}$$

This transfer scheme captures a variety of policies aiming at transferring wealth from creditors to debtors: a program of debt relief, fiscal transfers from creditors to debtors or even defaults. I am interested in inspecting the impact of a transfer from creditors to debtors on employment and output and in deriving conditions under which such a transfer is Pareto improving in welfare terms. I start to analyze the impact of transfers during mild recessions, which represent a particularly tractable case useful to build up intuition. I then move to the more complex case of large recessions.

4.1 A simple case: debt relief during mild recessions

In this section I focus on debt relief during mild recessions characterized by immediate recoveries, and I will thus assume that condition 2 holds. Let us start by considering the impact of a marginal transfer.

Proposition 3 *If conditions 2 holds, that is if in the absence of transfers the economy is in a liquidity trap characterized by a mild recession, a marginal transfer from creditors to debtors leads to an increase in employment and to a Pareto improvement in welfare. Moreover, a liquidity trap is a necessary condition to obtain a Pareto improvement in welfare from a marginal transfer from creditors to debtors.*

Proof. See appendix. ■

Proposition 3 states that a marginal transfer from creditors to debtors is Pareto improving if the economy is in a liquidity trap characterized by a mild recession. To grasp the intuition behind this result, consider that when condition 2 holds the economy reaches the steady state in period 1, right after the liquidity trap is over. Inspecting equation (12) one can see that a transfer in period 0 cannot affect steady state consumption, and so to trace the impact of a marginal transfer on welfare we just have to take into account the impact on consumption in period 0.

During the recovery from a mild recession inflation is equal to its target and so the real interest rate during the liquidity trap is equal to the inverse of the inflation target. We can then write creditors' Euler equation as

$$U'\left(C_0^c\right) = \frac{\beta}{\bar{\pi}} U'\left(C^c\right).$$

Differentiating this expression with respect to T and using the fact that $\partial C^c / \partial T = 0$ gives $\partial C_0^c / \partial T = 0$, so the transfer does not affect creditors' consumption in period 0. Hence, the transfer is Pareto improving if it leads to an increase in debtors' consumption in period 0.

To derive the impact of the transfer on C_0^d, first differentiate creditors' budget constraint (14) in period 0 with respect to T

$$\frac{\partial C_0^c}{\partial T} = -\frac{n}{1-n} + \alpha L_0^{\alpha-1} \frac{\partial L_0}{\partial T}.$$

Since by the Euler equation $\partial C_0^c / \partial T = 0$, we have

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n} \frac{L_0^{1-\alpha}}{\alpha} > 0,$$

so that the transfer leads to an increase in employment and output. In fact the expansion in output must be just enough to compensate creditors' for the loss in consumption due to the transfer, so as to leave period 0 creditors' consumption unchanged. Finally, differentiating debtors' budget constraint (13) with respect to T gives

$$\frac{\partial C_0^d}{\partial T} = 1 + \alpha L_0^{\alpha-1} \frac{\partial L_0}{\partial T} > 0.$$

From this expression it is clear that the transfer has a positive impact on debtors' consumption, both because of its direct effect and because of its positive impact on employment, and hence it is Pareto improving.

A transfer from creditors to debtors is expansionary because it stimulates aggregate demand. On the one hand, debtors' consumption demand rises one for one with income, because debtors are borrowing constrained. Hence, the transfer positively affects debtors' consumption demand. On the other hand, creditors' consumption demand is determined by the real interest rate and by expected consumption, which are not affected by the transfer if the economy is in a mild recession. Consequently, the transfer does not affect creditors' demand for consumption. The result is that the transfer generates an increase in aggregate demand which leads to an increase in inflation and production.

To understand why the transfer leads to a Pareto improvement in welfare, consider that the zero lower bound constraint on the interest rate negatively affects welfare both for creditors and debtors. The increase in aggregate demand due to the transfer relaxes the zero lower bound constraint, because it generates an increase in the interest rate that would clear the market for consumption. The relaxation of this constraint allows for a Pareto improvement in welfare.

The second part of proposition 3 states that a transfer cannot lead to a Pareto improvement in welfare if the deleveraging shock does not push the economy in a liquidity trap. To understand this result, consider that if the zero lower bound constraint never binds the economy always operates at full employment, which means that a transfer cannot induce an expansion in output. But without an increase in output creditors cannot be compensated for the loss due to the transfer. Hence, a transfer cannot be Pareto improving if the economy never enters a liquidity trap.

Having characterized the impact on welfare of a marginal transfer, I now turn to the Pareto optimal policy. I define a Pareto optimal transfer as the transfer that maximizes debtors' welfare, leaving creditors at least as well off as in the equilibrium without transfer.[17]

Definition 2 *The Pareto optimal transfer maximizes debtors' welfare, leaving creditors at least as well off as in the equilibrium without transfer.*

Proposition 4 *If condition 2 holds, the Pareto optimal transfer restores full employment. The optimal transfer T^* satisfies*

$$U'\left(\bar{L}^{\alpha}+\frac{n}{1-n}\left(B_0-\bar{\pi}\bar{\kappa}-T^*\right)\right)=\frac{\beta}{\bar{\pi}}U'\left(\bar{L}^{\alpha}+\frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

Proof. In the appendix. ■

Proposition 4 says that the Pareto optimal transfer during a mild recession restores full employment. To visually illustrate the impact of the Pareto optimal transfer during a mild recession I use a numerical example. Though the model is too simple to lend itself to a calibration exercise, I choose the parameters to target salient features of the US, so as to give a feeling of the magnitude of the effects implied by the model.

Every period corresponds to one year. I set the discount factor to $\beta = 0.9756$, so that in steady state the real interest rate is equal to 2.5 percent. This is close to the real interest in the US in 2007, at the onset of the financial crisis. The coefficient of relative risk aversion is set to $\gamma = 2$, a standard value in the real business cycle literature. The labor share is set to $\alpha = 0.65$, consistent with US data.

The fraction of debtors is set to $n = 0.6044$ to target a share of constrained consumption in the initial steady state of 58 percent, which is the same target used by Hall (2011). Moreover,

[17] Alternatively, one could define a Pareto optimal transfer as the transfer that maximizes creditors' welfare, leaving debtors at least as well off as in the equilibrium without transfer. In the case of a mild recession this definition would lead to an indeterminate transfer, because there is a range of transfers that leave creditors' utility unchanged, while having a positive impact on debtors' welfare.

Table 1: Parameters

	Value	Source/Target
Discount factor	$\beta = 0.9756$	$r = 0.025$
Risk aversion	$\gamma = 2$	Standard value
Labor share	$\alpha = 0.65$	Standard value
Fraction of debtors	$n = 0.6044$	Share of constrained consumption = 58%
Labor endowment	$\bar{L} = 1$	Normalization
Initial debt per debtor	$B_0 = 1.6546$	Debt/GDP in initial steady state = 100%
Inflation target	$\bar{\pi} = 1.02$	Fed inflation target
Wage rigidities	$\phi_0 = 1$	At full employment wages cannot fall
	$\phi_1 = 0.3939$	Wages fall by 2 percent per year at 5 percent unemployment

I normalize the labor endowment to one $\bar{L} = 1$ and set the initial debt per borrower to $B_0 = 1.6546$, so as to target a debt-to-GDP ratio in the initial steady state of 100 percent. This is the household debt-to-GDP ratio in the US in 2007. The inflation target is set to $\bar{\pi} = 1.02$, in line with the Fed's definition of price stability.

To model wage rigidities I adopt the same functional form proposed by Schmitt-Grohé and Uribe (2012) and assume

$$\phi(u) = \phi_0 (1 - u_t)^{\phi_1}.$$

I set $\phi_0 = 1$, so that in absence of involuntary unemployment nominal wages cannot fall. Following Schmitt-Grohé and Uribe (2012), I set ϕ_1 so that at an unemployment rate of 5 percent nominal wages can fall frictionlessly by 2 percent per year. This target implies $\phi_1 = 0.3939$.

Figure 2 displays the transitional dynamics following a deleveraging shock calibrated so that the debt-to-GDP ratio in the final steady state is equal to 94 percent. The solid lines refer to an economy without transfers. The drop in the borrowing limit forces debtors to deleverage and so the debt-to-GDP ratio falls. The central bank responds to the deleveraging shock by lowering the nominal interest rate and the economy falls into a liquidity trap that lasts one period. Inflation undershoots its target, and, due to the presence of nominal wage rigidities, real wages rise generating involuntary unemployment. Aggregate consumption falls, and the fall in consumption is particularly sharp for debtors.[18] Since we are considering a mild recession, inflation goes back to target and the economy is at full employment starting from period 1.

The dashed lines in figure 2 illustrate the impact of the Pareto optimal transfer. The transfer stimulates debtors' consumption and this has a positive impact on aggregate demand. The increase in aggregate demand brings the economy to full employment, thus closing the output gap and leaving inflation equal to the target. Creditors' consumption is not affected by the transfer because the transfer has no impact on creditors' expected consumption and on the real interest rate during the liquidity trap. Finally, though the optimal transfer restores full employment it does not lift the economy out of the liquidity trap, and the nominal interest

[18] In fact, in this example creditors' consumption rises during the liquidity trap. However, there are cases in which the fall in output during the liquidity trap is so severe that also creditors' consumption falls.

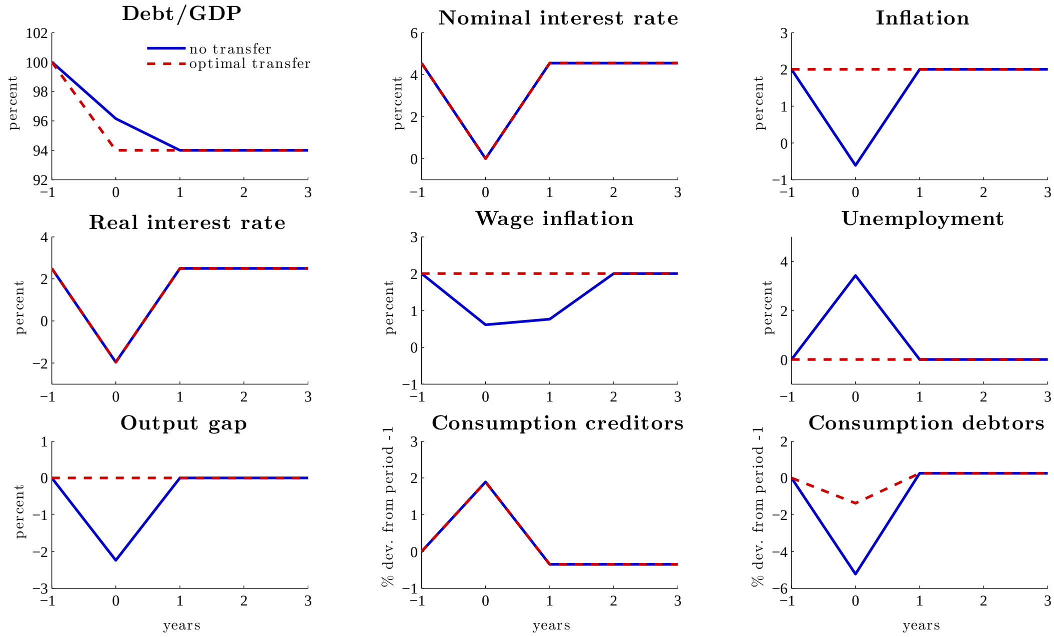

Figure 2: Deleveraging and Pareto optimal transfer during a mild recession.

rate hits the zero lower bound during period 0.

4.2 Debt relief during large recessions

I now turn to the more complex case of large recessions. Large recessions generate a trade-off between inflation and employment during the recovery and so the strategy followed by the central bank has an impact on how the economy behaves once the liquidity trap is over. Moreover, since households are forward looking monetary policy decisions that affect the recovery also have an impact on the behavior of the economy during the liquidity trap. Because of this, the impact of transfers on employment and welfare during large recessions crucially depends on whether the central bank targets inflation or employment. To illustrate this point I start by analyzing the case of a central bank that targets employment, before turning to a central bank that targets inflation.

4.2.1 Employment targeting

Suppose that the economy is in a large recession and that the central bank targets employment. Then during the recovery the central bank overshoots its inflation target, so as to make real wages fall to a level consistent with full employment. In turn, the inflation burst that the economy experiences during the recovery leads to a lower real interest rate during the liquidity trap, thus mitigating the impact of the binding zero lower bound constraint on the economy. These dynamics are illustrated by the solid lines in figure 3, which show the response of the economy to a deleveraging shock sufficiently large so as to violate condition 2.[19]

[19] The shock is calibrated so that the final steady state features a debt-to-GDP ratio of 90 percent. The other parameters are kept as in section 4.1.

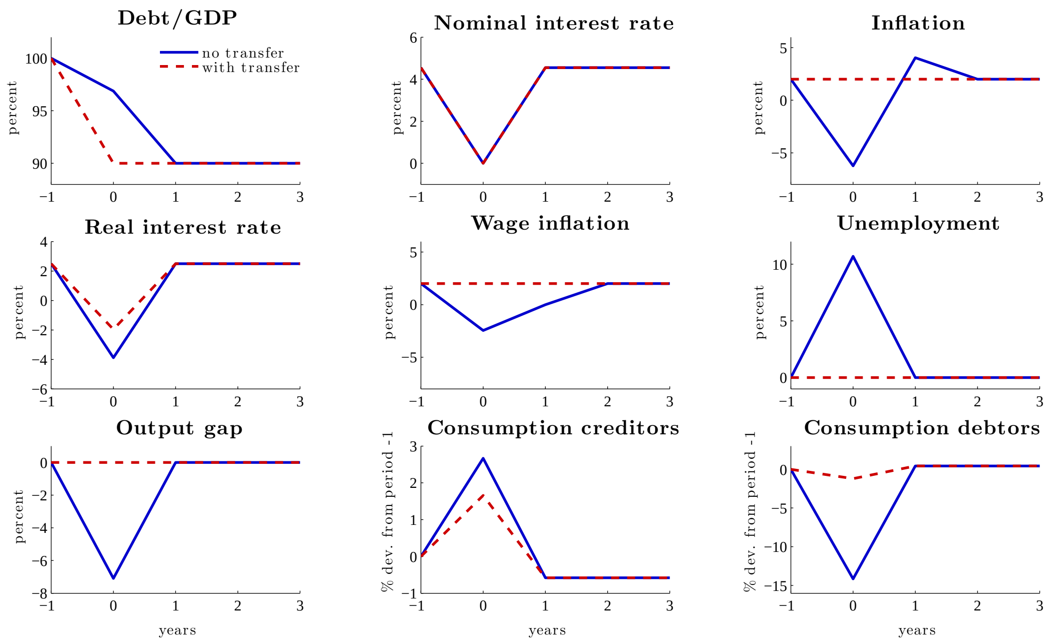

Figure 3: Deleveraging and impact of transfer during a large recession with employment targeting.

The following proposition summarizes the impact of a transfer on an economy undergoing a large recession with a central bank targeting employment.

Proposition 5 *Assume that condition* 1 *holds, that condition* 2 *is violated and that the central bank targets employment. Then a marginal transfer has an expansionary impact on employment. Moreover, a marginal transfer cannot lead to a Pareto improvement in welfare.*

Proof. See appendix. ■

As in the case of mild recessions, a transfer is expansionary because it transfers wealth to debtors, who have a higher propensity to consume out of wealth than creditors. Thus the transfer stimulates aggregate demand, relaxes the zero lower bound constraint on the nominal interest rate and raises inflation and employment.

However, a transfer unambiguously reduces creditors' consumption during the liquidity trap and so it cannot lead to a Pareto improvement in welfare. The intuition is as follows. A transfer stimulates employment during the trap, thus leading to an increase in L_0. Combining firms' optimality conditions in periods 0 and 1 with the wage setting equation in period 1 and the condition $L_1 = \bar{L}$ gives a relationship between period 1 inflation π_1 and employment during the liquidity trap L_0

$$\pi_1 = \phi(0) \left(\frac{\bar{L}}{L_0} \right)^{1-\alpha}.$$

This expression implies that an increase in L_0 leads to a reduction in expected inflation π_1. This happens because a rise in L_0 limits the fall in prices during the trap, thus limiting the rise in future inflation needed to reduce real wages to a level consistent with full employment. In turn, the fall in expected inflation leads to a rise in the real interest rate during the liquidity trap, which induces creditors to cut their consumption. Hence, if the central bank targets employment a transfer during a large recession generates a fall in creditors' consumption and it cannot be Pareto improving.

This point is illustrated by the dashed lines in figure 3, which display the impact of a transfer that restores full employment. The transfer leads to an increase in employment, but it also causes a fall in expected inflation that induces creditors to reduce their consumption. The result is that the Pareto optimal transfer in the case of a large recession with employment targeting is equal to zero.[20]

4.2.2 Inflation targeting

Under inflation targeting deleveraging tends to generate deeper recessions than under employment targeting. This happens because during a liquidity trap expected inflation is lower, and thus the real interest rate is higher, if the central bank follows a policy of inflation targeting. Through this channel, targeting inflation deepens the shortage of aggregate demand and the fall in output during a liquidity trap compared to a policy of employment targeting.

Perhaps more worryingly, targeting future inflation during a liquidity trap opens the door to multiple equilibria. To grasp the intuition behind this result it is useful to express the behavior of the economy during a liquidity trap under inflation targeting in terms of aggregate supply and demand schedules. To derive an aggregate supply (AS) schedule combine firms' optimality conditions in periods 0 and 1 with the wage setting equation in period 1 and the condition $\pi_1 = \bar{\pi}$ to obtain

$$Y_0 = Y_1 \left(\frac{\phi(u_1)}{\bar{\pi}} \right)^{\frac{\alpha}{1-\alpha}},$$

where $Y_t = L_t^{\alpha}$ denotes aggregate output. The AS curve implies a positive relationship between current and future output during a liquidity trap. Intuitively, lower production during the trap is associated with lower inflation and higher real wages. Since the inflation rate in period 1 is given by the inflation target and the adjustment in wages is constrained by the downward rigidities, also real wages in period 1 are increasing in Y_0. Hence lower output in period 0 is associated with higher real wages and lower output in period 1, creating a positive relationship between Y_0 and Y_1.

The aggregate demand (AD) schedule can be derived rearranging the Euler equation for creditors and imposing $r_0 = 1/\bar{\pi} - 1$

$$Y_0 = U'^{-1} \left(\frac{\beta}{\bar{\pi}} U' \left(Y_1 + \frac{n}{1-n} \frac{r_1 \bar{\kappa}}{1+r_1} \right) - \frac{n}{1-n} \left(B_0 - \bar{\pi}\bar{\kappa} - T \right) \right).$$

Also the AD curve describes a positive relationship between Y_0 and Y_1.[21] Intuitively, if creditors

[20] Of course, this does not necessarily mean that a transfer is not desirable on welfare terms. In fact, the transfer generates an increase in debtors' consumption and welfare. Depending on the weights that society attaches to the welfare of debtors and creditors a transfer might have a positive impact on aggregate welfare.

[21] This is because C_1^c is increasing in Y_1, despite the fact that r_1 is decreasing in Y_1. See the proof of proposition 1.

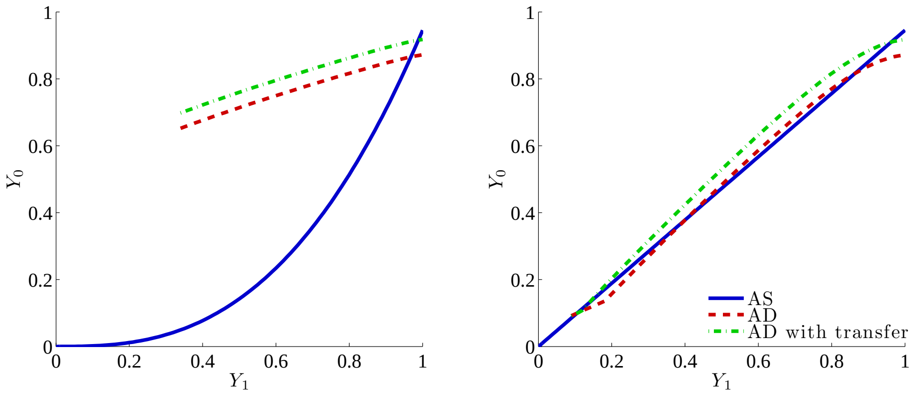

Figure 4: Multiple equilibria and transfers under inflation targeting.

expect income to be higher in period 1, i.e. a higher Y_1, they also anticipate that period 1 consumption will be higher and so their demand for consumption in period 0 increases. This in turn stimulates aggregate demand during the liquidity trap, leading to a higher production, i.e. a higher Y_0.

Combining the AS and AD curves together can generate multiple equilibria during a liquidity trap. Suppose that agents expect future output to be high. Then they will want to consume more during the trap and also output during the trap will be high. In turn a high output during the trap validates expectations of a high future output because it implies lower real wages during the liquidity trap, which lead to lower future real wages and higher future production.

Figure 4 illustrates two possible shapes of the AS and AD curves. The solid lines refer to the AS curve, while the dashed lines refer to the AD curve in the absence of transfers.[22] The left panel is obtained using the same parameters as in figure 3. In this case the curves intersect only once and so the equilibrium is unique. The right panel captures the possibility of multiple equilibria. In this example all the parameters are kept as in the example on the left panel, except that $\phi_1 = 0$ so that wages do not respond to unemployment. In this case the curves intersect three times and so there are three possible equilibria.

The impact of a marginal transfer on employment is potentially ambiguous. This is illustrated by the dash-dotted lines in figure 4. Graphically, a transfer makes the AD curve shift up. In the case depicted by the left panel the transfer unambiguously leads to an expansion in output and employment during the liquidity trap. However, in the case depicted in the right panel the impact of the transfer on output and employment is a priori ambiguous. In fact, even with the transfer there are three possible equilibria, and the impact of the transfer on employment depends on the starting equilibrium and on how the transfer affects expectations. This suggests that implementing a policy of debt relief during a liquidity trap might have a perverse impact on employment if the central bank follows a policy of inflation targeting.

One implication of this result is that implementing a transfer such as the one described in

[22]The AD curves are truncated because for some values of Y_1 the model does not have a solution.

proposition 4 might not restore full employment, because other equilibria might be consistent with that transfer in addition to the full employment one. Luckily, it is possible to design transfer schemes that eliminate multiple equilibria and lead to full employment.

Proposition 6 *Suppose that condition* 1 *holds and that the central bank targets inflation. Then a transfer scheme defined as*

$$T = \tilde{T} + \chi \left(L_0^\alpha - \bar{L}^\alpha \right)$$

where $\chi > (1-n)/n$ *and* $\tilde{T}$ *solves*

$$U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - \tilde{T}\right)\right) = \frac{\beta}{\bar{\pi}} U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

restores full employment and leads to a Pareto improvement in welfare.

Proof. In the appendix. ■

Proposition 6 describes the transfer scheme involving the smallest transfer from creditors to debtors consistent with full employment. The transfer described in proposition 6 is decreasing in output. Intuitively, multiple equilibria arise because expectations of low future output translate into weak aggregate demand by creditors leading to low output during the trap. The transfer reduces the response of creditors' demand to changes in expected future output, ruling out multiple equilibria.

The proposition also states that an appropriately designed transfer leads to a Pareto improvement in welfare.[23] The key to this result is the fact that the transfer produces an increase in output during the trap, which during a large recession generates an increase in future output and future consumption. The expectation of higher future consumption, and the fact that the interest rate is given by $r_0 = 1/\bar{\pi} - 1$ and not affected by the transfer, stimulates creditors' consumption during the trap. Hence creditors' consumption stream increases following the transfer. Debtors experience an even larger increase in their consumption stream, because this indirect effect is complemented by the direct increase in income due to the transfer. Hence, the transfer makes both creditors and debtors better off.

These effects are illustrated by figure 5.[24] Without transfers deleveraging generates a deep and persistent recession. Unemployment is persistent because with inflation equal to the target it takes a few periods for wages to fall back to a level consistent with full employment. Instead, the transfer described in proposition 6 restores full employment. Moreover, the transfer has a positive impact both on creditors' and debtors' consumption.

[23]Notice that by invoking condition 1 proposition 6 does not refer to sunspot liquidity traps, that is cases in which a liquidity trap equilibrium coexist with an equilibrium in which the zero lower bound constraint is not binding. I do not address this case because, despite an extensive search, I could not find a parameter configuration that leads to sunspot liquidity traps. In the case of sunspot liquidity traps the proposition should be qualified by acknowledging that the transfer scheme proposed implies a Pareto improvement in welfare with respect to equilibria in which the zero lower bound constraint is binding.

[24]The parameters are the same as in figure 3. Under this parameter configuration the equilibrium without transfer is unique.

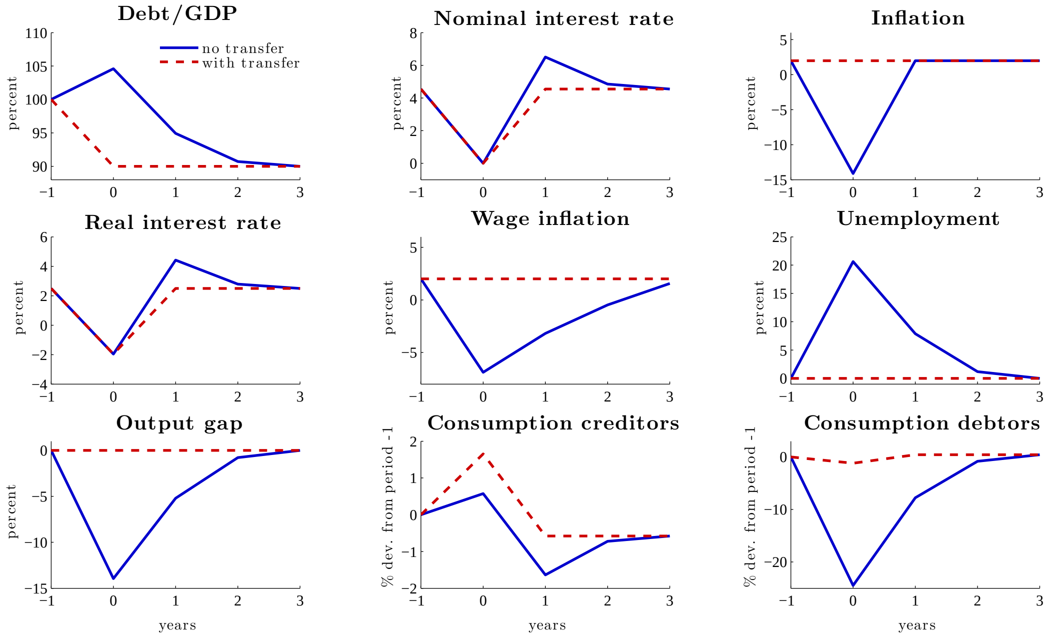

Figure 5: Deleveraging and impact of transfer during a large recession with inflation targeting.

Deriving the Pareto optimal transfer, defined as the transfer that maximizes debtors' welfare leaving creditors at least as well off as in the initial equilibrium, in the case of large recessions with inflation targeting can be cumbersome and I will leave it to future research.[25] Here I just notice that the Pareto optimal transfer is larger than $\tilde{T}$, the smallest transfer that restores full employment. As shown in the proof to proposition 6, under that transfer both creditors and debtors are better off compared to an equilibrium with large recession and no transfer. Hence, a marginal increase in the transfer generates a rise in debtors' utility, while leaving creditors still better off compared to the equilibrium without transfer. This also implies that the Pareto optimal transfer lifts the economy out of the liquidity trap, because for creditors' Euler equation to hold the interest rate must satisfy $r_0 > 1/\bar{\pi} - 1$ if $T > \tilde{T}$.[26]

Summarizing, the case for debt relief policies during a liquidity trap is particularly strong if the central bank follows a policy of inflation targeting. Not only a transfer can lead to an increase in welfare both for creditors and debtors, but an appropriately designed transfer scheme can also eliminate the possibility of multiple equilibria.

5 Extensions

I now consider a few extensions to the baseline model. I start by analyzing the case of a central bank that conducts monetary policy according to an interest rate rule. I then consider the role of disutility from working. I conclude this section with a discussion of the similarities and

[25]The main difficulty comes from the fact that under the Pareto optimal transfer the borrowing limit may not be binding for debtors, which complicates significantly the analysis.

[26]Instead, characterizing the transfer that maximizes creditors' welfare leaving debtors at least as well off as in the initial equilibrium is easier. Indeed, this happens when the transfer described in proposition 6 is implemented.

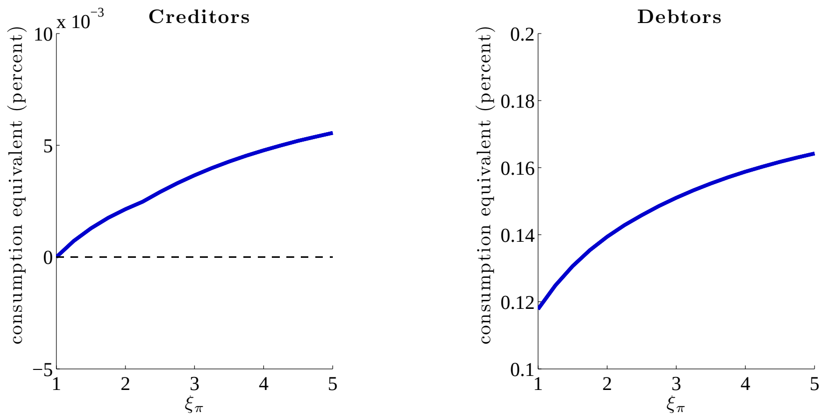

Figure 6: Welfare gains from transfer with interest rate rule.

differences between debt relief in a closed economy and in a monetary union.

5.1 Interest rate rule

One popular way of modeling monetary policy is through interest rate rules. In this section I consider a central bank that sets the policy rate according to the simple rule

$$1 + i_t = \max\left(1, (1+i)\left(\frac{\pi_t}{\bar{\pi}}\right)^{\xi_\pi}\right), \tag{15}$$

where $i = (1+r)\bar{\pi} - 1$ is the steady state nominal interest rate and $\xi_\pi \geq 1$ is a parameter determining how aggressively the central bank responds to deviations of inflation from the target. A higher value of ξ_π is associated with a stronger aversion to inflation variability, and when $\xi_\pi \to \infty$ the central bank is effectively implementing a policy of inflation targeting. Notice that the rule takes into account the fact that monetary policy is constrained by the zero lower bound on the nominal interest rate.[27]

Based on the analysis of section 4.2, one could conjecture that a transfer is more likely to lead to a Pareto improvement in welfare the more aggressively the central bank responds to deviations of inflation from the target, i.e. the higher ξ_π. In this section I show that this conjecture is correct.

Considering a central bank that conducts monetary policy according to rule (15) makes it difficult to derive analytical results, hence I will resort to numerical simulations. To investigate whether a transfer is more likely to be Pareto improving the more aggressively the central bank responds to inflation, I compute the welfare gains for creditors and debtors from a transfer equal to 1 percent of full employment GDP, that is $T = 0.01\bar{L}^\alpha/n$, for a range of values of ξ_π given a shock that pushes the economy into a large recession.[28] I compute the welfare gains

[27] Benhabib et al. (2001) show that an interest rate rule such as the one described in expression (15) can give rise to expectation-driven liquidity traps. I consider a central bank that is able to avoid expectation-driven liquidity traps, for instance by implementing the exit strategy proposed by Schmitt-Grohé and Uribe (2012).

[28] The other parameters are the same as in figure 3.

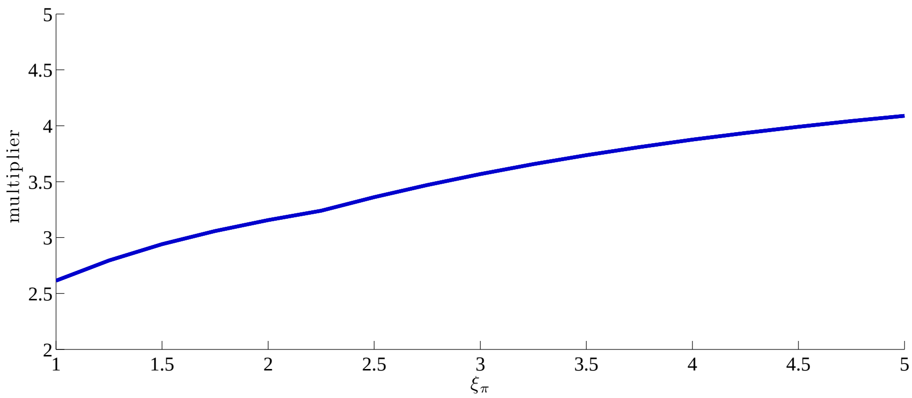

Figure 7: Impact on output of transfer with interest rate rule.

from implementing a transfer as the proportional increase in the consumption stream that a household living in the economy with no transfer must receive in order to be indifferent between remaining in the no-transfer economy and switching to an economy with the transfer.[29]

Figure 6 shows the results. The transfer leads to a Pareto improvement in welfare for any value of $\xi_\pi > 1$.[30] Moreover the welfare gains of both debtors and creditors are increasing in ξ_π, confirming the conjecture that a transfer is more likely to lead to a Pareto improvement in welfare the more aggressively the central bank responds to deviations of inflation from the target.

This result is due to the fact that the impact of a transfer on output is larger the more the central bank is concerned with stabilizing inflation. This happens because the speed of the recovery from a large recession is increasing in inflation. To illustrate this point I computed the transfer multiplier, defined as

$$\frac{\mathbf{Y}^T - \mathbf{Y}^{NT}}{nT},$$

where

$$\mathbf{Y} = \sum_{t=0}^{\infty} \frac{Y_t}{(1+r)^t},$$

is the present value of output and the superscripts T and NT denote respectively allocations with and without transfer.[31] Figure 7 shows that the multiplier increases with ξ_π, so that the impact of the transfer on output is larger the more aggressively the central bank responds to deviations of inflation from the target.

[29]Formally, the welfare gain η^i for $i = c, d$ is defined as

$$\sum_{t=0}^{\infty} \beta^t U\left(\left(1+\eta^i\right) C_t^{i,NT}\right) = \sum_{t=0}^{\infty} \beta^t U\left(C_t^{i,T}\right),$$

where the superscripts NT and T denote allocations respectively in the economy without and with transfer.

[30]If $\xi_\pi = 1$ the transfer makes debtors better off, but it leads to a small welfare loss for creditors.

[31]I discount output with the steady state real interest rate because I want to abstract from the impact of the transfer on the interest rate.

5.2 Disutility from working

In the baseline model households do not experience disutility from working, a typical assumption in the literature on involuntary unemployment.[32] However, the literature on monetary policy commonly assumes that households experience disutility from working and that the labor supply is elastic.[33]

The presence of disutility from working makes it less likely for a debt relief policy to produce a Pareto improvement in welfare. This happens because creditors need to be compensated not only for the loss in wealth due to the transfer, but also for the disutility due to the increase in labor effort. However, the presence of disutility from working does not eliminate the possibility of Pareto improving transfers.

To make this point I use a numerical example. Suppose that households experience disutility from working during period 0. Specifically assume that the lifetime utility of a household is given by

$$\frac{C_0^{i\,1-\gamma}-1}{1-\gamma}-\psi\frac{L_0^{i\,1+\theta}}{1+\theta}+\sum_{t=1}^{\infty}\beta^t U\left(C_t^i\right),$$

where $\psi > 0$ is a parameter determining the disutility from working and $\theta \geq 0$ determines the elasticity of labor supply. Notice that to simplify the analysis I assume that labor disutility arises only during period 0, while from period 1 on the model is exactly identical to the baseline.

The solid lines in figure 8 illustrate the impact on welfare of a transfer equal to 1 percent of full employment GDP as a function of the elasticity of labor supply θ. The economy is hit by a shock large enough to generate a large recession and the central bank targets inflation. For each value of θ I calibrated ψ so that given the pattern of consumption in the initial steady state aggregate labor is exactly equal to $\bar{L}$.[34] For comparison, the dashed lines show the welfare gains from the transfer in the baseline model without disutility from working.

Figure 8 shows that introducing disutility from labor does not eliminate the possibility of Pareto improving transfers. The figure also shows that, perhaps unsurprisingly, a Pareto improvement in welfare is more likely to materialize the more inelastic the labor supply, i.e. the higher θ. Indeed, for this particular numerical example the transfer is Pareto improving for values of θ greater than 5.

It is important to stress that the model with elastic labor supply is likely to bias downward the gains from a policy of debt relief. The reason is that the preferences considered in this section threat equally voluntary and involuntary leisure. Instead, the empirical evidence suggests that involuntary leisure has a negative impact on welfare.[35]

[32] See Pissarides (2000).

[33] An example of a monetary model with involuntary unemployment and disutility from labor effort is Erceg et al. (2000).

[34] The other parameters are the same as in figure 2.

[35] See Winkelmann and Winkelmann (1998). Moreover, there is empirical evidence suggesting that, everything else held constant, people living in countries with a lower unemployment rate are happier, as documented by Di Tella et al. (2001).

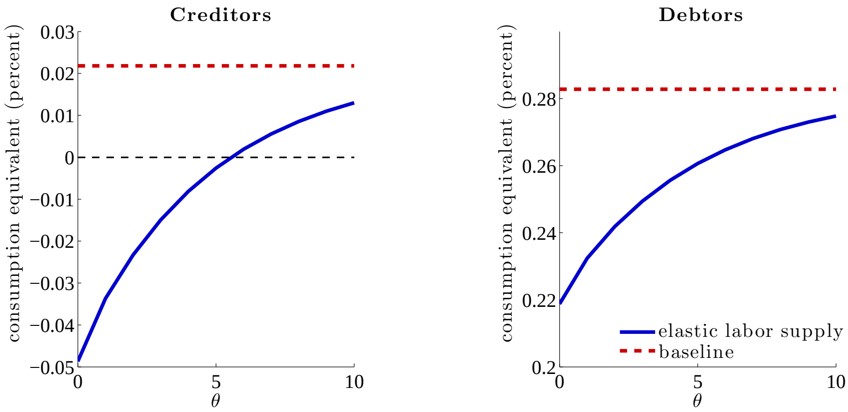

Figure 8: Welfare gains from transfer with disutility from working.

5.3 Debt relief policies in monetary unions

Though the model describes a closed economy, its fundamental insights apply to the case of a monetary union undergoing an episode of deleveraging, as long as countries are heterogeneous in their debt positions.[36] In particular, a transfer from creditor to debtor countries should lead to an economic expansion and possibly to a Pareto improvement in welfare, especially if the central bank of the union is mainly concerned with targeting inflation.[37]

However, there is an important difference between the case of a closed economy and a monetary union. In fact, in a closed economy a benevolent government will implement a policy of debt relief if this leads to a Pareto improvement in welfare. This might not be the case in a monetary union. To see this point, imagine a monetary union composed of a continuum of countries, each one of them being infinitesimally small. In this world, a creditor country does not have an incentive to unilaterally forgive its debtors. In fact, being infinitesimally small a single country does not take into account the impact of its actions on aggregate demand and output. Hence, in a monetary union the implementation of a Pareto improving debt relief policy requires coordination across member countries. I am exploring these coordination issues in ongoing research.

6 Conclusion

Debt deleveraging can push the economy into a liquidity trap characterized by involuntary unemployment and low inflation. During these episodes, debt relief policies lead to an expansion

[36]Indeed, from a modeling perspective the only difference would be that in a monetary union in which labor is immobile across countries differences in wages could arise. See Benigno and Romei (2012) and Fornaro (2012) for models of deleveraging in monetary unions.

[37]This seems to fit the case of the Eurozone well. In the Eurozone a group of countries, the periphery, is characterized by high foreign debt and is undergoing a period of private debt deleveraging, while the rest of the union, the core, has low foreign debt, or even a positive stock of foreign assets, and is not experiencing a contraction in credit. Moreover, the mandate of the European Central Bank is to maintain price stability. The analysis above suggests that in this case a transfer from the core to the periphery should lead to an expansion in output and potentially to a Pareto improvement in welfare.

in employment and output and can benefit both creditors and debtors.

One natural direction in which the analysis could be extended is to consider the impact of debt relief on moral hazard. In fact, the anticipation of a future debt relief might give an incentive to borrowers to increase their debt during times in which access to finance is plentiful. Moral hazard could thus partly counteract the positive impact of debt relief on welfare, and the interactions between the two represents a fruitful area for future research.[38]

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Appendix

A Proofs

A.1 Proof of proposition 1

I first show that if condition 1 holds then the economy is in a liquidity trap in period 0 and $i_0 = 0$. Suppose that the zero lower bound constraint on the nominal interest rate does not bind. Then the central bank hits the inflation target, $\pi_0 = \bar{\pi}$ and the economy operates at full employment, $L_0 = \bar{L}$. Moreover, the real interest rate satisfies $r_0 \geq \bar{\pi}^{-1} - 1$. The Euler equation for creditors, equation (4), then implies

$$\beta \left(1 + r_0\right) \frac{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \frac{\bar{\kappa}}{1+r_0}\right)\right)} = 1.$$

The left-hand side of this expression is increasing in r_0, hence we have that

$$\frac{\beta}{\bar{\pi}} \frac{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)}{U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right)} \leq 1,$$

which contradicts condition 1. Hence, we must have $i_0 = 0$.

I now characterize the equilibrium in a liquidity trap. Since $i_0 = 0$ then $r_0 = \pi_1^{-1} - 1$. The Euler equation for creditors is then

$$U'\left(L_0^\alpha + \frac{n}{1-n}\left(B_0 - \pi_1\bar{\kappa}\right)\right) = \frac{\beta}{\pi_1} U'\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right). \tag{A.1}$$

Suppose that the economy is at full employment in period 0 ($L_0 = \bar{L}$). Then the central bank can hit both its inflation and employment targets in period 1 and so $L_1 = \bar{L}$, $\pi_1 = \bar{\pi}$ and $r_1 = r$. So the Euler equation for creditors writes

$$U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa}\right)\right) = \frac{\beta}{\bar{\pi}} U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right),$$

which contradicts condition 1. Hence, equilibrium labor is given by the $L_0 < \bar{L}$ that solves equation $(A.1)$ and there is involuntary unemployment.

Moreover, the fact that there is involuntary unemployment implies that the constraint on wage setting, equation (2.3), binds and so

$$\frac{W_0}{W_{-1}} = \phi\left(L_0\right) < \bar{\pi},$$

where the last inequality follows from assumption 1. Combining the optimality conditions for firms, equation (6) in periods -1 and 0, the fact that the economy starts in steady state and so

$L_{-1} = \bar{L}$ and the wage setting equation gives

$$\left(\frac{\bar{L}}{L_0}\right)^{1-\alpha} \pi_0 = \frac{W_0}{W_{-1}} = \phi\left(L_0\right) < \bar{\pi}.$$

This condition implies that $\pi_0 < \bar{\pi}$.

I now show that the economy exits the liquidity trap in period 1 and so $i_t > 0$ and $\pi_t \geq \bar{\pi}$ for $t > 0$. There are two cases to consider. First, consider a central bank that targets employment. Assume that in periods $t > 0$ the zero lower bound constraint does not bind and so the employment target is hit, that is $L_t = \bar{L}$ for $t > 0$. Then the economy enters a steady state with $r_t = r = 1/\beta - 1$, which, by assumption 1, implies $i_t > 0$ for $t > 0$. This means that the economy exits the liquidity trap in period 1, validating our conjecture that the central bank hits the employment target in periods $t > 0$. Moreover, we can write the equation that determines the evolution of employment as

$$\pi_t = \left(\frac{L_t}{L_{t-1}}\right)^{1-\alpha} \frac{W_t}{W_{t-1}} \geq \left(\frac{L_t}{L_{t-1}}\right)^{1-\alpha} \phi\left(L_t\right). \tag{A.2}$$

Recall that $\phi(0) < \bar{\pi}$ and that $\phi'(\cdot) \leq 0$. For $t > 1$ we have $L_{t-1} = \bar{L}$ and so the central bank can satisfy $(A.2)$ by setting $\pi_t = \bar{\pi}$. In $t = 1$ $L_{t-1} = L_0 < \bar{L}$ and so $\pi_1 \geq \bar{\pi}$.

Let us now consider a central bank that targets inflation. Suppose that the inflation target is hit in periods $t > 0$, so $\pi_t = \bar{\pi}$ for $t > 0$. Then L_t evolves according to

$$L_t = \min\left(\bar{L}, \left(\frac{\bar{\pi}}{\phi\left(L_t\right)}\right)^{\frac{1}{1-\alpha}} L_{t-1}\right) \geq L_{t-1},$$

where the inequality follows from $\phi(0) < \bar{\pi}$ and $\phi'(\cdot) \leq 0$. Hence the growth rate of labor is nonnegative.

This implies that also creditors' consumption C_t^c grows over time and increases with L_t. Suppose this is not the case in $t = 1$. Then $r_1 < r$. Moreover

$$C_1^c > C_2^c \leftrightarrow L_1^\alpha + \frac{r_1}{1+r_1}\bar{\kappa} > L_2^\alpha + \frac{r_2}{1+r_2}\bar{\kappa}.$$

Since $L_1 \leq L_2$ this condition implies that $r_2 < r_1 < r$. Repeating this logic forward we see that the economy converges to the steady state only if creditors' consumption grows over time. This means that $r_t \geq r_{t+1} \geq r$ for all $t > 0$. By assumption 1 this also implies that $i_t > 0$ for all $t > 0$, so that the economy exits the liquidity trap in period 1 and inflation is never lower than the target in periods $t > 0$.

Throughout the proof I have assumed that debtors are against their borrowing limit in $t = 0$ and during the transition to the new steady state. To conclude the proof I show that this is indeed the case. Let us start with period $t = 0$. Suppose that debtors are not against their borrowing limit, then the Euler equation for debtors implies

$$U'\left(L_0^\alpha - B_0 + \pi_1\bar{\kappa}\right) \leq \frac{\beta}{\pi_1}U'\left(L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}\right) \leq U'\left(L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}\right),$$

where the last inequality follows from $\pi_1 \geq \bar{\pi} \geq \beta$. Then it must be that

$$L_0^\alpha - B_0 + \pi_1\bar{\kappa} \geq L_1^\alpha - \frac{r_1}{1+r_1}\bar{\kappa}. \tag{A.3}$$

The Euler equation for creditors implies

$$U'\left(L_0^\alpha + \frac{n}{1-n}\ B_0 - \pi_1\bar{\kappa})\right) = \frac{\beta}{\pi_1}U'\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right).$$

Since $L_0 \leq L_1$ this implies that

$$B_0 > \bar{\kappa}\left(\pi_1 + \frac{r_1}{1+r_1}\right),$$

which contradicts condition $A.3$, proving that debtors are against their borrowing limit during the liquidity trap.

Also in $t > 0$ debtors are against their borrowing limit, i.e. $B_t^d = -\bar{\kappa}$ for all t. In the case of a central bank that targets employment this follows from the fact that the economy enters in steady state in period 1, and in steady state the bond positions are continually rolled over.[39] In the case of a central bank that targets inflation, it is possible to show that in absence of the borrowing constraint B^d grows at rate $B_{t+1}^d/B_t^d = (1+g_{t+1})^\gamma/\beta$, where $g_{t+1} = L_{t+1}/L_t - 1$. Since $g_t > 0$ during the transition, this means that debtors would like to increase their debt during the transition. This would violate the borrowing constraint and so $B_t^d = -\bar{\kappa}$ for all t.■

A.2 Proof of proposition 2

Since condition 2 implies condition 1, the proof of the first part of the proposition follows directly from the proof of proposition 1.

We have to prove that the economy reaches a steady state with full employment and inflation equal to target in period 1. Combining firms' optimality conditions in period 0 and 1 with the constraint on wage setting in period 1 gives

$$\left(\frac{L_0}{L_1}\right)^{1-\alpha}\pi_1 \geq \phi\left(1-u_1\right).$$

This equation implies that if condition 2 holds, it is feasible for the central bank to hit both the inflation and the employment target in period 1 and to set $\pi_1 = \bar{\pi}$, $L_1 = \bar{L}$ and $u_1 = 0$. ■

[39]Strictly speaking, debtors are not constrained in steady state since their desired bond position is exactly $B^d = -\bar{\kappa}$.

A.3 Proof of proposition 3

I first show that a marginal transfer from creditors to debtors can lead to a Pareto improvement in welfare only if the economy is in a liquidity trap. Suppose that the economy is not in a liquidity trap in period 0. Then a marginal transfer from creditors to borrowers in period 0 has the following impact on creditors' consumption

$$\frac{\partial C_0^c}{\partial T} = \frac{n}{1-n}\left(\frac{B_1}{(1+r_0)^2}\frac{\partial r_0}{\partial T} - \frac{1}{1+r_0}\frac{\partial B_1}{\partial T} - 1\right) \tag{A.4}$$

$$\frac{\partial C_t^c}{\partial T} = \frac{n}{1-n}\left(\frac{\partial B_1}{\partial T}(1-\beta)\right)$$

for all t> 0,where I have used the fact that out of a liquidity trap $L_t = \bar{L}$ and $\partial L_t/\partial T = 0$. Because of the borrowing constraint $\partial B_1/\partial T \leq 0$ so $\partial C_t^c/\partial T \leq 0$ for all $t > 0$. Moreover, differentiating the period 0 creditors' Euler equation with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = \frac{\beta}{U''(C_0^c)}\left(\frac{\partial r_0}{\partial T}U'(C_1^c) + (1+r_0)\frac{\partial C_t^c}{\partial T}U''(C_1^c)\right). \tag{A.5}$$

Equation $(A.4)$ implies that $\partial C_0^c/\partial T > 0$ only if $\partial r_0/\partial T > 0$. But equation $(A.5)$ implies that if $\partial r_0/\partial T > 0$ then $\partial C_0^c/\partial T < 0$, and hence $\partial C_0^c/\partial T < 0$. This implies that out of a liquidity trap a transfer from creditors to debtors unambiguously hurts creditors and hence cannot lead to a Pareto improvement in welfare.

I now show that a marginal transfer from creditors to debtors leads to a Pareto improvement in welfare if the deleveraging shock pushes the economy into a mild recession, that is if condition 2 is satisfied. Proposition 2 implies that the economy reaches a steady state with full employment in period 1 and so a marginal transfer does not affect C_t^c or C_t^d for $t > 0$. Consumption in period 0 are

$$C_0^d = L_0^\alpha - B_0 + \bar{\pi}\bar{\kappa} + T$$

$$C_0^c = L_0^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T\right).$$

Differentiating these expressions with respect to T gives

$$\frac{\partial C_0^d}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} + 1 \tag{A.6}$$

$$\frac{\partial C_0^c}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} - \frac{n}{1-n}. \tag{A.7}$$

These expressions imply that a transfer can be Pareto improving only if $\partial L_0/\partial T > 0$, otherwise the transfer will unambiguously hurt creditors. The Euler equation for creditors, equation $(A.1)$ can be written as

$$C_0^c = U'^{-1}\left(\frac{\beta}{\pi}U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right)\right).$$

Differentiating this expression with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = 0.$$

Combining this expression with equations $(A.7)$ and $(A.6)$ yields

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n}\frac{1}{\alpha}L_0^{1-\alpha} > 0 \tag{A.8}$$

$$\frac{\partial C_0^d}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} + 1 > 0, \tag{A.9}$$

which completes the proof. ■

A.4 Proof of proposition 4

I start by showing that if 1 holds an appropriate transfer restores full employment. Suppose that the optimal choice of bonds by debtors is $B_1^d = -\bar{\kappa}$. Then, full employment is restored by a transfer T^*, implicitly defined by

$$U'\left(\bar{L}^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T^*\right)\right) = \frac{\beta}{\bar{\pi}}U'\left(\bar{L}^\alpha + \frac{n}{1-n}\frac{r}{1+r}\bar{\kappa}\right).$$

Notice that since $\bar{\pi} \geq \beta$, T^* satisfies

$$T^* \leq B_0 - \bar{\kappa}\left(\bar{\pi} + \frac{r}{1+r}\right). \tag{A.10}$$

This condition implies that the optimal transfer does not lift the economy out of the liquidity trap and so $i_0 = 0$. Finally, we must show that $T = T^*$ is consistent with $B_1^d = -\bar{\kappa}$. Suppose this is not the case then the Euler equation for debtors implies

$$U'\left(L_0^\alpha - B_0 + \bar{\pi}\bar{\kappa} + T^*\right) < \frac{\beta}{\bar{\pi}}U'\left(\bar{L}^\alpha - \frac{r}{1+r}\bar{\kappa}\right).$$

This expression implies

$$T^* > B_0 - \bar{\kappa}\left(\frac{r}{1+r} + \bar{\pi}\right),$$

which contradicts $(A.10)$, thus proving that it is optimal for borrowers to set $B_1^d = -\bar{\kappa}$.

We have proved that a transfer T^* restores full employment, I now show that T^* is the Pareto optimal policy. Expressions $(A.7)$, $(A.8)$ and $(A.9)$ imply that an increase in the transfer T leads to Pareto improvement in welfare if $\frac{\partial L_0}{\partial T} > 0$, which is the case if $L_0 \leq \bar{L}$. Hence the Pareto optimal policy sets $L_0 = \bar{L}$. ■

A.5 Proof of proposition 5

I start by showing that if the central bank targets employment and the economy is in a large recession a marginal transfer from creditors to debtors leads to an expansion in employment. Differentiating the euler equation for creditors and using the fact that if the central bank targets employment the transfer has no impact on C_t^c for $t > 0$ gives

$$\frac{\partial C_0^c}{\partial T} = -\frac{\beta}{\pi_1^2}\frac{U'\left(C_1^c\right)}{U''\left(C_0^c\right)}\frac{\partial \pi_1}{\partial T}. \tag{A.11}$$

Differentiating creditors' budget constraint in period 0 with respect to T gives

$$\frac{\partial C_0^c}{\partial T} = \alpha L_0^{\alpha-1}\frac{\partial L_0}{\partial T} - \frac{n}{1-n}\left(\bar{\kappa}\frac{\partial \pi_1}{\partial T} + 1\right). \tag{A.12}$$

Moreover, using the fact that the wage setting condition binds in period 1 gives

$$\pi_1 = \phi(0)\left(\frac{\bar{L}}{L_0}\right)^{1-\alpha}.$$

Differentiating this expression with respect to T gives

$$\frac{\partial \pi_1}{\partial T} = -(1-\alpha)\phi\left(0\right)\bar{L}^{1-\alpha}L_0^{\alpha-2}\frac{\partial L_0}{\partial T}. \tag{A.13}$$

Combining $(A.11)$, $(A.12)$ and $(A.13)$ yields

$$\frac{\partial L_0}{\partial T} = \frac{n}{1-n}\left(\alpha L_0^{\alpha-1} + (1-\alpha)\phi(0)\bar{L}^{1-\alpha}L_0^{\alpha-2}\left(\frac{n}{1-n}\bar{\kappa} - \frac{\beta}{\pi_1^2}\frac{U'\left(C_1^c\right)}{U''\left(C_0^c\right)}\right)\right).$$

This expression implies $\partial L_0/\partial T > 0$ so that a marginal transfer has an expansionary impact on employment.

To prove that a marginal transfer cannot be Pareto improving it is sufficient to notice that by equation $(A.13)$ if $\partial L_0/\partial T > 0$ then $\partial \pi_1/\partial T < 0$ and by equation $(A.11)$ $\partial \pi_1/\partial T < 0$ implies $\partial C_0^c/\partial T < 0$. ■

A.6 Proof of proposition 6

I start by showing that the transfer scheme proposed in proposition 6 leads to a unique equilibrium characterized by full employment. Assuming that the borrowing constraint always binds for debtors, the Euler equation for creditors can be written as

$$L_0^\alpha + \frac{n}{1-n}\left(B_0 - \bar{\pi}\bar{\kappa} - T\right) = \left(\frac{\beta}{\bar{\pi}}\right)^{-\frac{1}{\gamma}}\left(L_1^\alpha + \frac{n}{1-n}\frac{r_1}{1+r_1}\bar{\kappa}\right).$$

Plugging $T = \tilde{T} + \chi \left(L_0^\alpha - \bar{L}^\alpha \right)$ in this expression gives

$$\left(L_0^\alpha - \bar{L}^\alpha \right) \left(1 - \chi \frac{n}{1-n} \right) = \left(\frac{\beta}{\bar{\pi}} \right)^{-\frac{1}{\gamma}} \left(L_1^\alpha - \bar{L}^\alpha + \frac{n}{1-n} \left(\frac{r_1}{1+r_1} - \frac{r}{1+r} \right) \bar{\kappa} \right).$$

The right-hand side of this expression is not greater than zero.[40] Since $\chi > (1-n)/n$ the only possible solution to this equation is $L_0 = \bar{L}$, which proves that the transfer scheme proposed gives rise to a unique equilibrium characterized by full employment.[41] The proof that $B_t^d = -\bar{\kappa}$ for all t follows along the lines of the proof of proposition 4.

To prove that the transfer leads to a Pareto improvement in welfare, first consider that if the central bank targets inflation C_t^d and C_t^c for $t > 0$ are weakly increasing in L_0. Moreover, the tax leads to an increase in period 0 debtors' consumption, both because of its direct effect and because of the increase in employment. Hence, to show that the transfer leads to a Pareto improvement in welfare we need to show that its impact on period 0 creditors' consumption is nonnegative. To see that this is the case, consider that the transfer leads to an increase in C_1^c, while leaving $r_0 = 1/\bar{\pi} - 1$ unchanged. By creditors' Euler equation this implies that the transfer also leads to an increase in C_0^c and thus leads to a Pareto improvement in welfare. ■

[40]Recall that C_1^c is increasing in L_1, as shown in the proof of proposition 1.
[41]Notice that when $L_0 = \bar{L}$ the economy enters the steady state in period 1 and so $L_1 = \bar{L}$ and $r_1 = r$.